Infinitely many new renormalization group flows between Virasoro minimal models from non-invertible symmetries

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Abstract

Based on the study of non-invertible symmetries, we propose there exist infinitely many new renormalization group flows between Virasoro minimal models $\mathcal{M}(kq+I,q) \to \mathcal{M}(kq-I,q)$ induced by $\phi_{(1,2k+1)}$.

Previous work

- $k=1, I=1$ cases by Zamolodchikov.
- $k=1, I>1$ cases by Ahn and Lässig.
- $k=2$ cases by Dorey et al.

Virasoro minimal models

Consider A-series Virasoro minimal models $\mathcal{M}(p,q)$

Basic information

$p,q \in \mathbb{Z}$: coprime.

Central charge : $c = 1 - 6\frac{(p-q)^2}{pq}$.

$\frac{(p-1)(q-1)}{2}$ primary fields : $\phi_{(r,s)} = \phi_{(q-r,p-s)}$ $(1 \le r \le q-1, 1 \le s \le p-1)$.

The conformal weight : $h_{r,s} = h_{q-r,p-s} = \frac{(pr-qs)^2-(p-q)^2}{4pq}$.

The fusion rule :

$$\phi_{(r,s)} \times \phi_{(m,n)} = \sum_{\substack{k=1+|r-m| \\ k+r+m=1 \mod 2}}^{\min(r+m-1,2q-1-r-m)} \sum_{\substack{l=1+|s-n| \\ l+s+n=1 \mod 2}}^{\min(s+n-1,2p-1-s-n)} \phi_{(k,l)}.$$

Topological defect lines

$\exists$ topological defect lines(Verlinde lines) $L_{(r,s)} (\overset{\text{one to one}}{\longleftrightarrow} \phi_{(r,s)})$

The fusion rule and the Verlinde formula

$$L_{(r,s)} \times L_{(m,n)} = \sum_{(k,l)} N_{(r,s)(m,n)}^{(k,l)} L_{(k,l)}$$

$$= \sum_{\substack{k=1+|r-m| \\ k+r+m=1 \mod 2}}^{\min(r+m-1,2q-1-r-m)} \sum_{\substack{l=1+|s-n| \\ l+s+n=1 \mod 2}}^{\min(s+n-1,2p-1-s-n)} L_{(k,l)}.$$

$$N_{ab}^{c} = \sum_{d} \frac{S_{ad}S_{bd}S_{dc}}{S_{0d}} \quad a=(r,s) \text{ etc}$$

where modular S-matrix is given by

$$S_{(r,s),(\rho,\sigma)} = 2\sqrt{\frac{2}{pq}}(-1)^{1+s\rho+r\sigma} \sin\left(\pi\frac{p}{q}r\rho\right) \sin\left(\pi\frac{q}{p}s\sigma\right).$$

Action :

$$L_{(r,s)} |\phi_{(\rho,\sigma)}\rangle = \frac{S_{(r,s),(\rho,\sigma)}}{S_{(1,1),(\rho,\sigma)}} |\phi_{(\rho,\sigma)}\rangle, \tag{1}$$

$$L_{(r,s)} |0\rangle = \underbrace{d_{(r,s)}}_{\text{quantum dimension}} |0\rangle = \frac{S_{(r,s),(1,1)}}{S_{(1,1),(1,1)}} |0\rangle. \tag{2}$$

Symmetry

Invertible $\mathbb{Z}_2$ symmetry

generated by $L_{(q-1,1)} = \eta$.

anomalous when p and q are both odd.

Non-invertible symmetry

$L_{(r,1)} (r=1,\cdots,q-1)$ form $SU(2)_{q-2}$ fusion ring.

New renormalization group flow

Main Claim

$\mathcal{M}(kq+I,q)$ flows to $\mathcal{M}(kq-I,q)$ induced by $\phi_{(1,2k+1)}$ deformation.

Step 1 : Surviving topological defect lines

$\phi_{(1,2k+1)}$ deformation preserve $SU(2)_{q-2}$ subcategory.

($\because$ we can easily check that $L_{(r,1)} (r=1,\cdots,q-1)$ commute with $\phi_{(1,2k+1)}$ from (1).)

Step 2 : Renormalization group invariants

There are two important quantities.

- quantum dimensions are RG invariant.
- spin contens of $\mathcal{H}_{L_{(r,1)}}$ in IR $\subset$ spin contents of $\mathcal{H}_{L_{(r,1)}}$ in UV

Quantum dimensions of $L_{(r,1)}$

Direct computation from (2)

$\to d_{(r,1)}$ in $\mathcal{M}(kq+I,q) = d_{(r,1)}$ in $\mathcal{M}(kq-I,q)$.

Spin contents of $\mathcal{H}_{L_{(r,1)}}$

Insert $L_{(r,1)}$ in the time direction.

From S-modular transformation and Verlinde formula,

$$Z_{L_{(r,1)}} = \sum_{(s,m),(t,n)} N_{(s,m)(t,n)}^{(r,1)} \chi_{(s,m)} \bar{\chi}_{(t,n)}.$$

Spin contents of $\mathcal{H}_{L_{(r,1)}}$:

$$\{\, h_{s,m} - h_{t,n} \mid N_{(s,m)(t,n)}^{(r,1)} \neq 0 \,\} \mod \mathbb{Z}.$$

$L_{(r,1)} (r=1,\cdots,q-1)$ form $SU(2)_{q-2}$.

$\to$ we know which $N_{(s,m)(t,n)}^{(r,1)} \neq 0$

$\to$ we can compute the spin contents of $\mathcal{H}_{L_{(r,1)}}$.

* Remark 1

From the c_{eff} theorem, $\mathcal{M}(kq+I,q)$ is UV and $\mathcal{M}(kq-I,q)$ is IR.

* Remark 2

$L_{(2,1)}$ distinguish our RG flow.

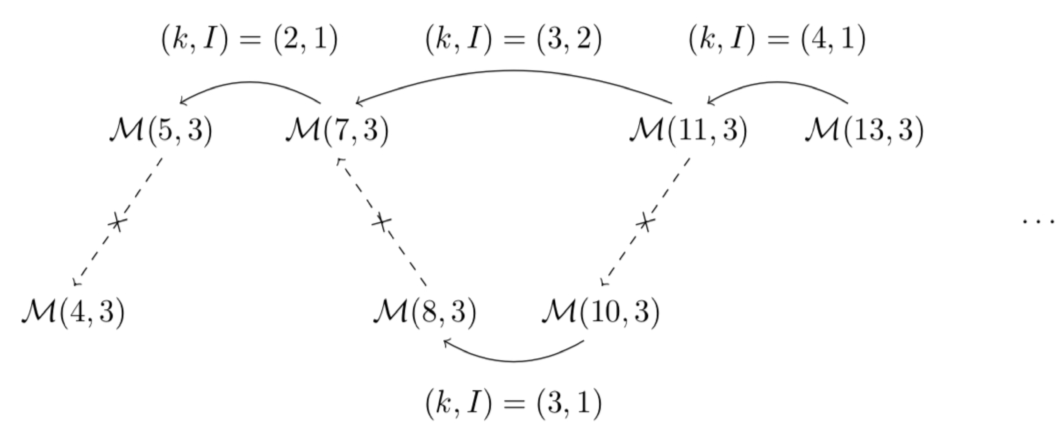
Example

Ex 1 : $\mathcal{M}(3k+I,3) \to \mathcal{M}(3k-I,3)$ and anomaly matching

When we consider $\mathcal{M}(p,3)$,

$$\text{Quantum dimensions of } L_{(2,1)} = \eta : \begin{cases} +1 & \text{for even } p \\ -1 & \text{for odd } p, \end{cases}$$

$$\text{Spin contents of } \mathcal{H}_{L_{(2,1)}} = \mathcal{H}_\eta : \begin{cases} \{0, \pm\frac{1}{2}\} \mod \mathbb{Z} & \text{for even } p \\ \{\pm\frac{1}{4}\} \mod \mathbb{Z} & \text{for odd } p. \end{cases}$$



Physical interpretation

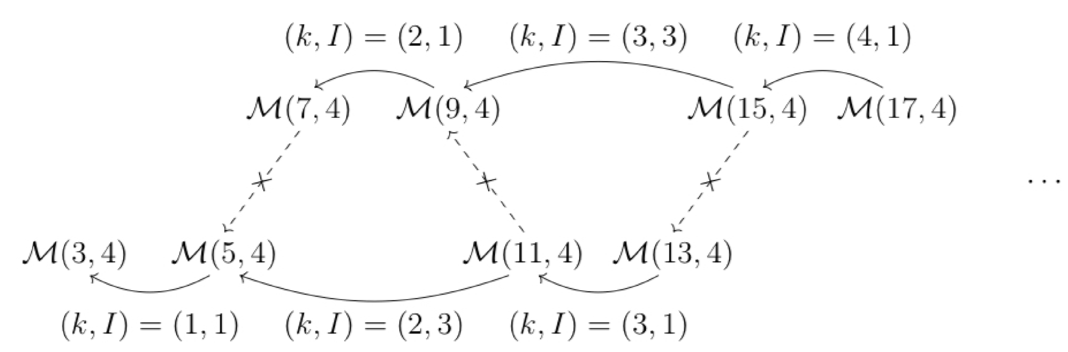
It is consistent with anomaly matching for invertible $\mathbb{Z}_2$ symmetry.

Ex 2 : $\mathcal{M}(4k+I,4) \to \mathcal{M}(4k-I,4)$ and duality defect

When we consider $\mathcal{M}(p,4)$, we have the duality defect $L_{(2,1)} = \mathcal{N}$.

$$\text{Quantum dimensions of } L_{(2,1)} = \mathcal{N} : \begin{cases} \sqrt{2} & \text{for } p=\pm 3 \mod 8 \\ -\sqrt{2} & \text{for } p=\pm 1 \mod 8, \end{cases}$$

$$\text{Spin contents of } \mathcal{H}_{L_{(2,1)}} = \mathcal{H}_\mathcal{N} : \begin{cases} \{\pm\frac{1}{16}, \pm\frac{7}{16}\} \mod \mathbb{Z} & \text{for } p=\pm 3 \mod 8 \\ \{\pm\frac{3}{16}, \pm\frac{5}{16}\} \mod \mathbb{Z} & \text{for } p=\pm 1 \mod 8. \end{cases}$$



Physical interpretation

The existence of the duality defect

$\to$ UV and IR are both invariant under $\mathbb{Z}_2$ gauging.

$\to$ They must be in $\mathcal{M}(p,4)$ series.

Future direction

Lagrangian descriptions of the Virasoro (non-unitary) minimal model

$\to$ generalized in higher dimensions

References

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