

Bulk Reconstruction and Gauge Invariance

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5 August 2024

YITP workshop “Strings and fields”

mainly based on the following papers:

with Sotaro Sugishita JHEP11(2022)041, 2207.06455 [hep-th], 2309.04231 [hep-th]

and a paper to appear

Introduction

One way to study quantum gravity is AdS/CFT duality

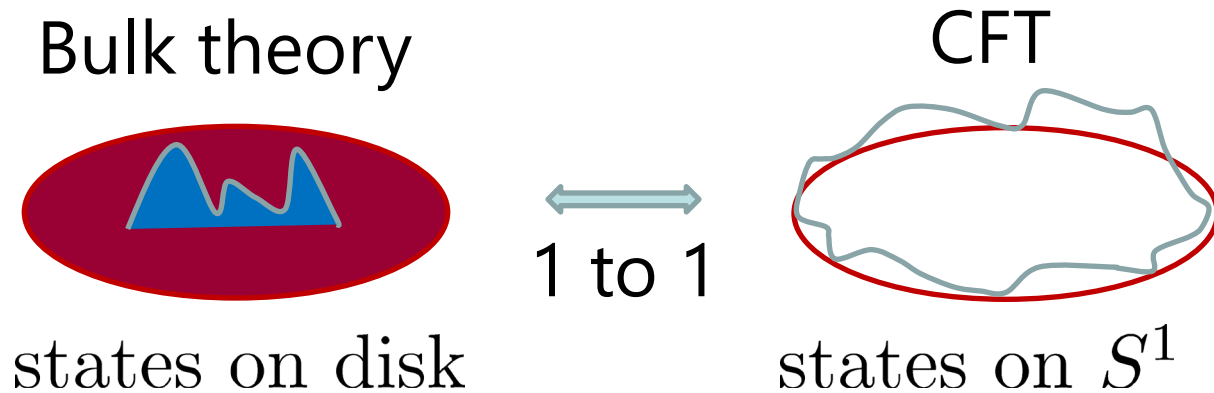
Maldacena

Quantum gravity on AdS

= conformal field theory (CFT)

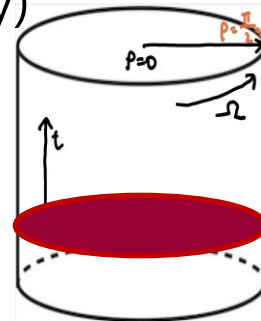
In operator formalism, AdS/CFT is
equivalence or duality between
Hilbert spaces and Hamiltonians
of gravity on AdS and CFT

Low energy states of bulk theory and CFT

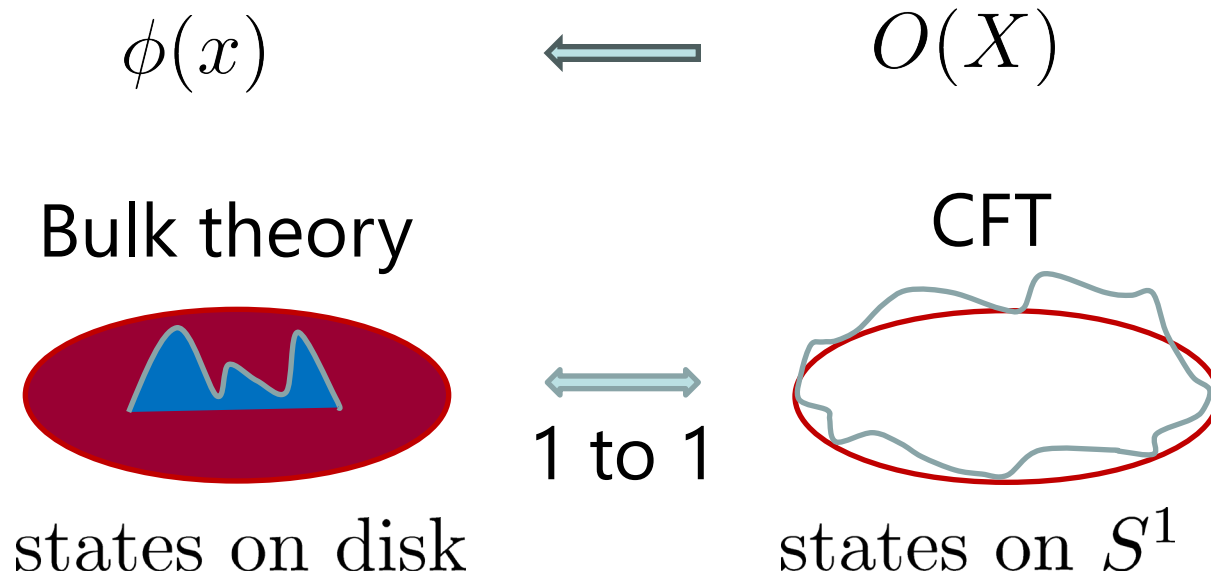


(CFT is NOT living on boundary,
CFT corresponds to whole bulk theory)

They are on a fixed time slice of AdS or cylinder



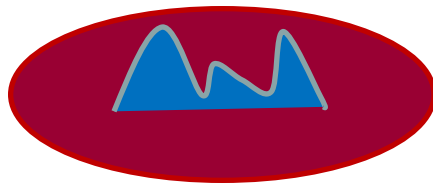
Bulk reconstruction is
a map from CFT operator to bulk operator



To understand bulk reconstruction is
to understand AdS/CFT itself

For $N=\infty$,

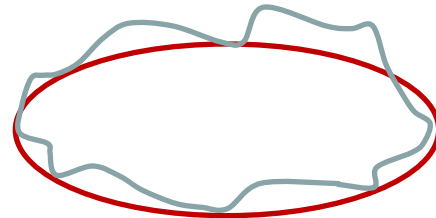
Bulk FREE theory



$$G_N = 0$$

$\longleftrightarrow$
1 to 1

GFF



" $N = \infty$ " CFT

Generalized free field (GFF) is
"dimensional reduction" of bulk free theory.

But, GFF violates fundamental properties of QFT!
just a approximation of large, but finite N

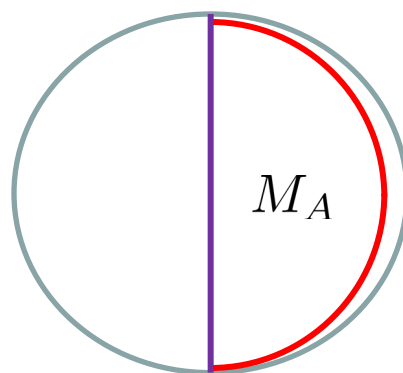
Another important subject in AdS/CFT:

Subregions in bulk and boundary

Subregions are important for quantum information
theoretical aspects of AdS/CFT

Subregion duality

Simplest example:
 $t = 0$ slice of AdS_3



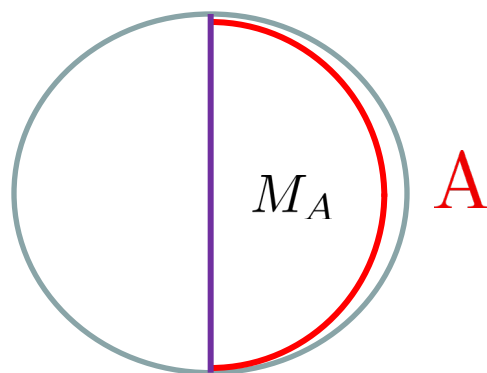
A

A is a half-circle

M_A is a half-disk (Rindler patch of AdS)
surrounded by Ryu-Takayanagi surface

Correspondence: $A \rightarrow M_A$

Entanglement wedge reconstruction (EWR) is a bulk reconstruction for subregion



$t = 0$ slice of AdS_3

Essentially, EWR claim that bulk operators on M_A can be reconstructed from CFT operators on A .

In this talk,

by considering the gauge symmetry (diffeomorphism)
in gravitational theory,

we demonstrate that the widely believed properties of bulk
reconstruction are either incorrect or significantly modified,

These includes holographic error correction code and
entanglement wedge reconstruction

Subregion is related to horizon and black hole

half subregion of a time slice of Minkowski space



Rindler space

Near horizon of (outside) Black hole = Rindler space

In particular,
Rindler patch of AdS = “Black hole” of hyperbolic space

Plan

- 1. Introduction**
- 2. (HKLL) bulk reconstruction**
- 3. Bulk reconstruction for subregion**
- 4. Gauge invariant operator and gravitational dressing**
- 5. Conclusion**

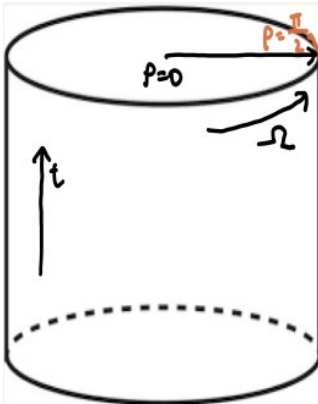
(HKLL) Bulk reconstruction

(Global) AdS_{d+1}

The metric of global AdS_{d+1} ($l_{AdS} = 1$) is

$$ds^2_{AdS} = \frac{1}{\cos^2(\rho)} (-dt^2 + d\rho^2 + \sin^2(\rho) d\Omega_{d-1}^2)$$

where $0 \leq \rho < \pi/2$



For $N=\infty$,

large N limit of holographic CFT_d

 **equivalent!**

Free bulk theory on AdS_{d+1}

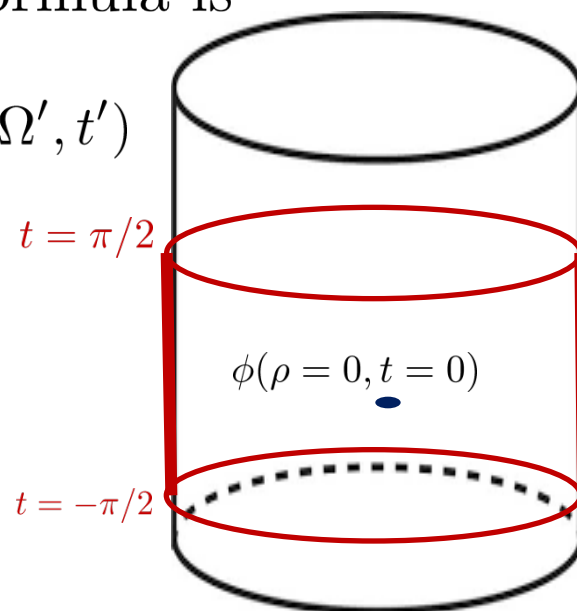
HKLL reconstruction formula

Then, the HKLL bulk reconstruction formula is

$$\phi(\rho = 0, t = 0) = \int_{-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}} dt' d\Omega' K(\Omega', t') \mathcal{O}(\Omega', t')$$

$$\text{where } K(\Omega, t) \sim \frac{1}{(\cos t)^{d-\Delta}}$$

(K is given for any ρ .)



(Global) HKLL bulk reconstruction

Now $\mathcal{O}(\theta, t)$ denotes the CFT operator and we define

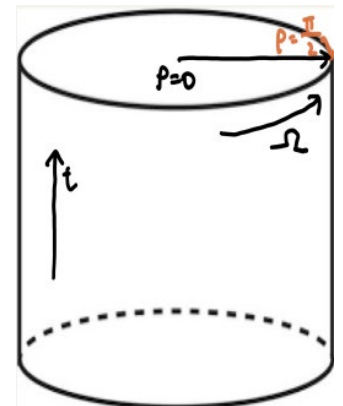
$$\phi^G(t, \rho, \theta) \equiv \int dt' d\theta' K(\theta', t'; t, \rho, \theta) \mathcal{O}(\theta', t')$$

Then, we can show

$$\langle 0 | \phi(t, \rho, \theta) \phi(t', \rho', \theta') | 0 \rangle = \langle 0 | \phi^G(t, \rho, \theta) \phi^G(t', \rho', \theta') | 0 \rangle$$

i.e. bulk 2-point function

is reproduced by CFT operator
(of any CFT)



Comment 1:

Large N factorization of holographic CFT with $N = \infty$:

vanishing of connected n -point func. for $n > 2$

i.e. $\langle \mathcal{O}_1 \mathcal{O}_2 \cdots \mathcal{O}_n \rangle = \sum \langle \mathcal{O}_1 \mathcal{O}_2 \rangle \langle \mathcal{O}_3 \mathcal{O}_4 \rangle \cdots \langle \mathcal{O}_{l-1} \mathcal{O}_l \rangle$

Those of GFF is also factorized because it is free theory



Bulk n -point function is reproduced by the CFT operator for holographic CFT

Comment 2:

The smearing function $K(\theta, t)$ in

$$\phi^G(t, \rho, \theta) \equiv \int dt' d\theta' K(\theta', t') \mathcal{O}(\theta', t')$$

has ambiguity.

$$K(\theta', t') \rightarrow K(\theta', t') + \delta K(\theta', t')$$

is OK as a bulk operator if

$$\int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dt e^{i\omega t + im\theta} \delta K(\theta, t) = 0$$

for $\omega = 2n + |m| + \Delta$ where n is non-negative integer

Comment 3:

HKLL bulk reconstruction can be extended to include $1/N$ corrections (=interaction) as

$$\begin{aligned}\phi^G(t, \rho, \theta) = & \int dt' d\theta' K(\theta', t') \mathcal{O}(\theta', t') \\ & + \frac{1}{N} \int dt' dt'' d\theta' d\theta'' K^1(\theta', t', \theta'', t'') \mathcal{O}(\theta', t') \mathcal{O}(\theta'', t'') + \dots\end{aligned}$$

by requiring e.o.m. or a kind of micro causality.

Bulk reconstruction for subregion

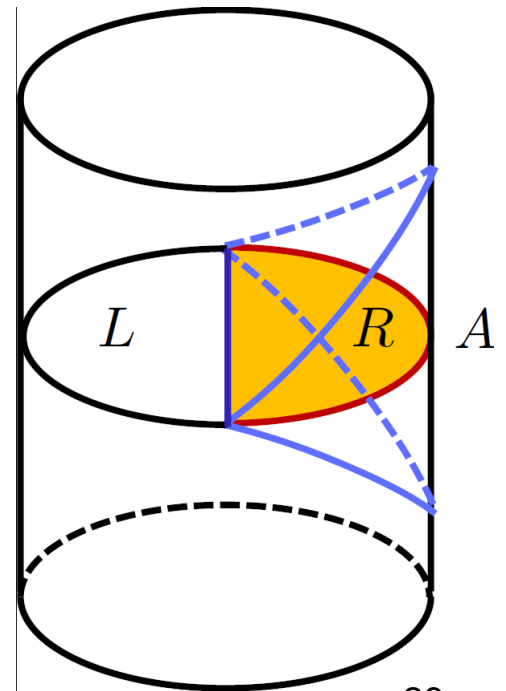
Rindler patch of AdS_3

The metric of Rindler patch of AdS_3 ($l_{AdS} = 1$) is

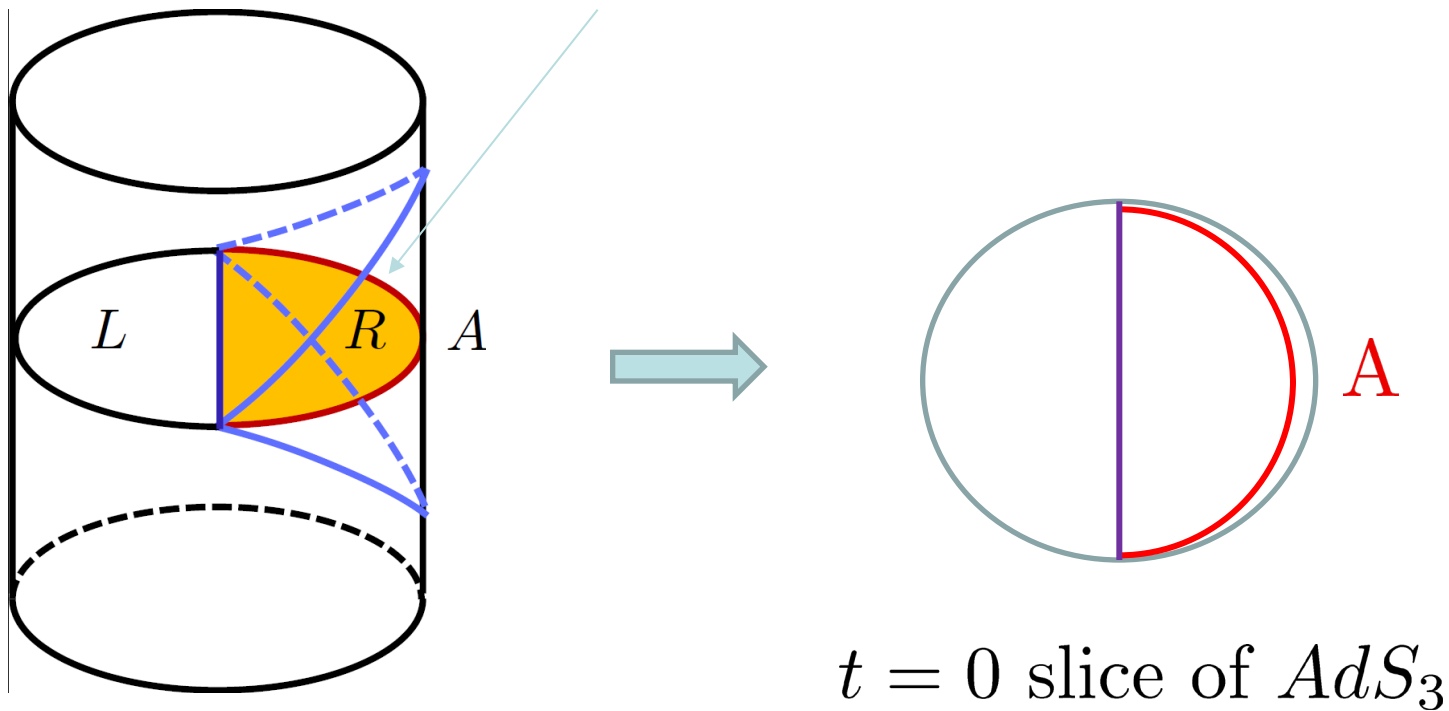
$$ds^2 = -\xi^2 dt_R^2 + \frac{d\xi^2}{1 + \xi^2} + (1 + \xi^2) d\chi^2$$

where $-\infty < t_R < \infty$, $-\infty \leq \chi < \infty$

$$0 \leq \xi < \infty$$



Entanglement wedge of A (denoted by M_A) = AdS-Rindler patch



AdS-Rindler HKLL bulk reconstruction

Finally, we regard $O(t_R, \chi)$ as CFT operator and define

$$\phi^R(t_R, \xi, \chi) \equiv \int dt'_R d\chi' K^R(\chi', t'_R) \mathcal{O}(\chi', t'_R)$$

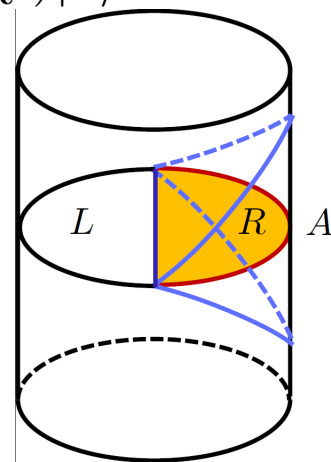
Then, if $\{t_R, \xi, \chi\}$ are in AdS-Rindler patch,

$$\langle 0 | \phi(t_R, \xi, \chi) \phi(t'_R, \xi', \chi') | 0 \rangle = \langle 0 | \phi^R(t_R, \xi, \chi) \phi^R(t'_R, \xi', \chi') | 0 \rangle$$

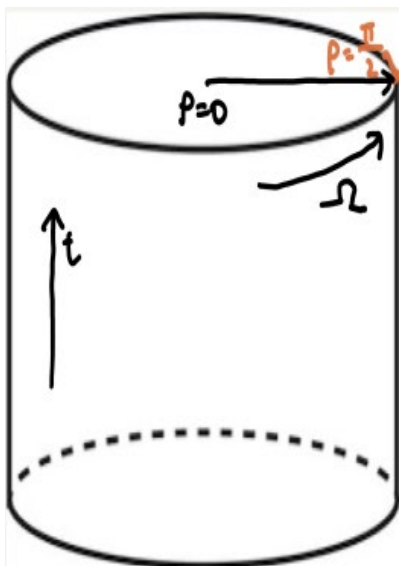
i.e. bulk correlation function in M_A

is reproduced by CFT operator in A

(It may possible that $K^R = K + \delta K$.)

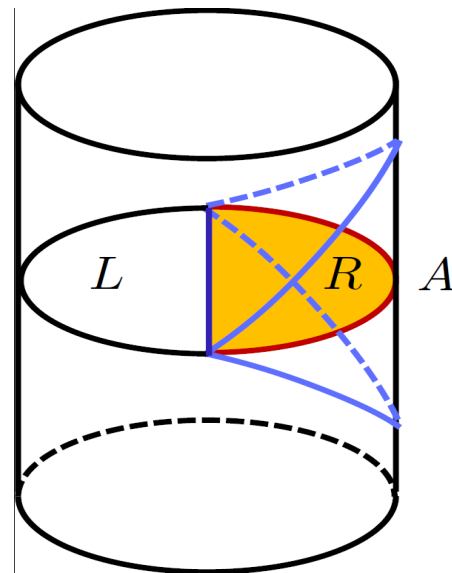


This suggests
both quantum gravity on Global and Rindler patches
are consistent



Global

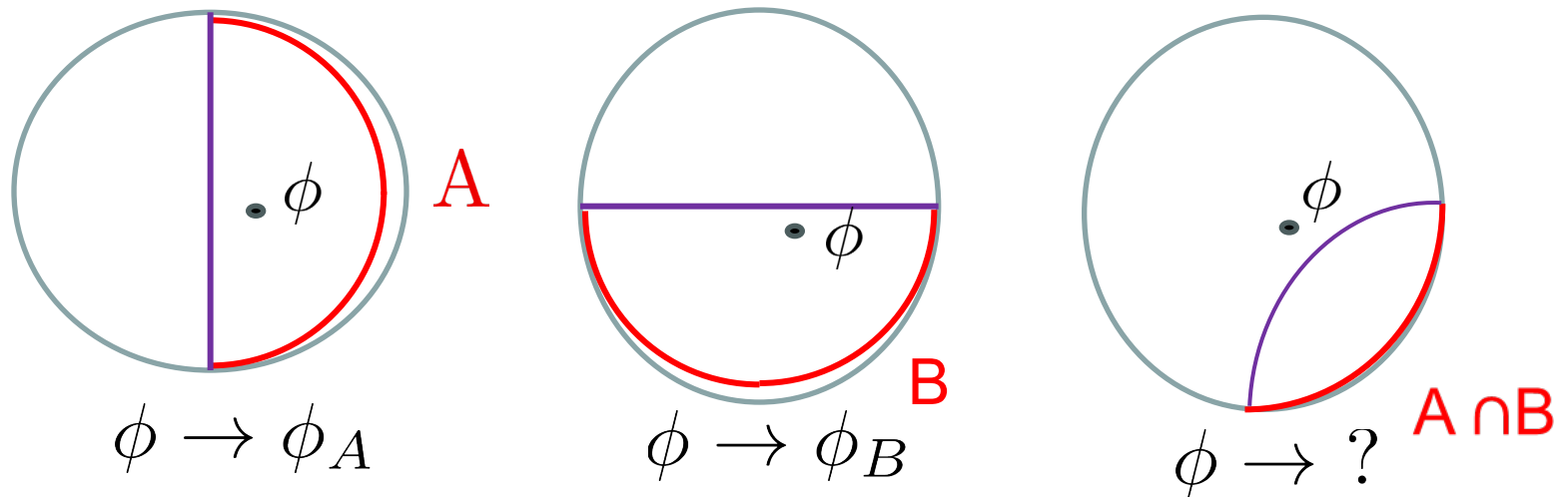
("Kruskal coord.")



Rindler

("Schwarzschild coord.")

”Paradox” for CFT with $N < \infty$



Bulk local operator ϕ can be reconstructed by CFT operator on A , and on B , but not on $A \cap B$.

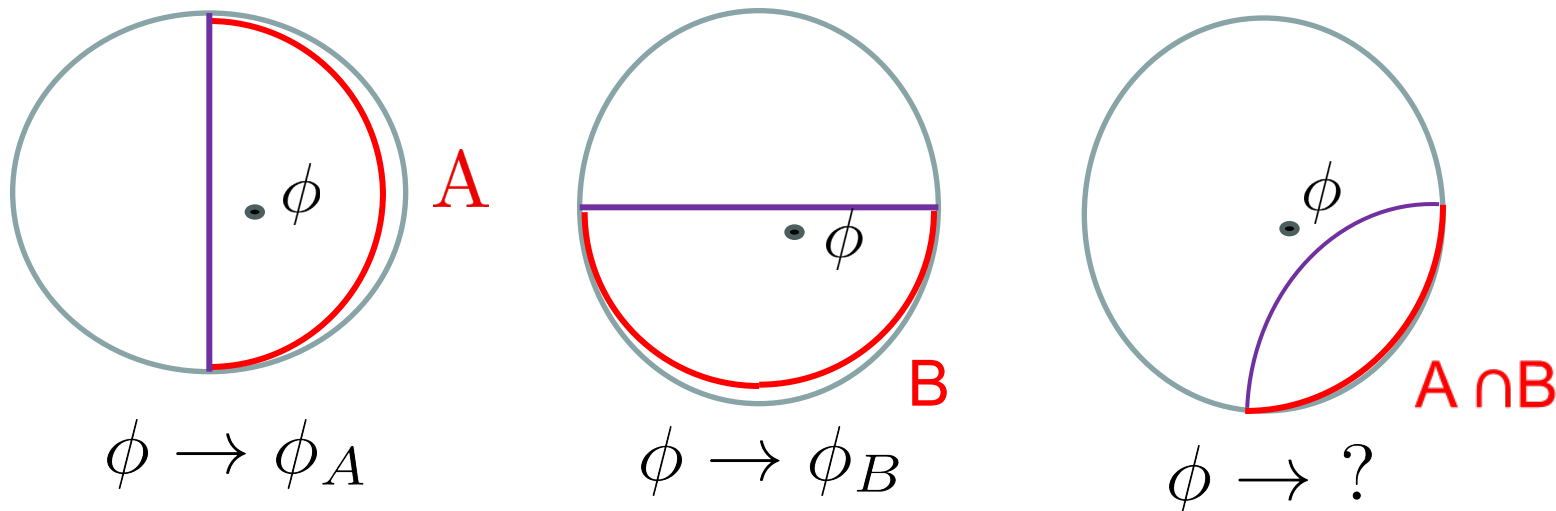


Holographic quantum error correction code proposal

Almheiri-Dong-Harlow, HaPPY model

$\phi_A \neq \phi_B$, but $\phi_A = \phi_B$ in the code (low energy) subspace
(without concrete evidences)

Our claim:



$\phi_A \neq \phi_B$ even in the code (low energy) subspace
 (bulk local operator ϕ depends on the subregion:
 "subregion complementarity")

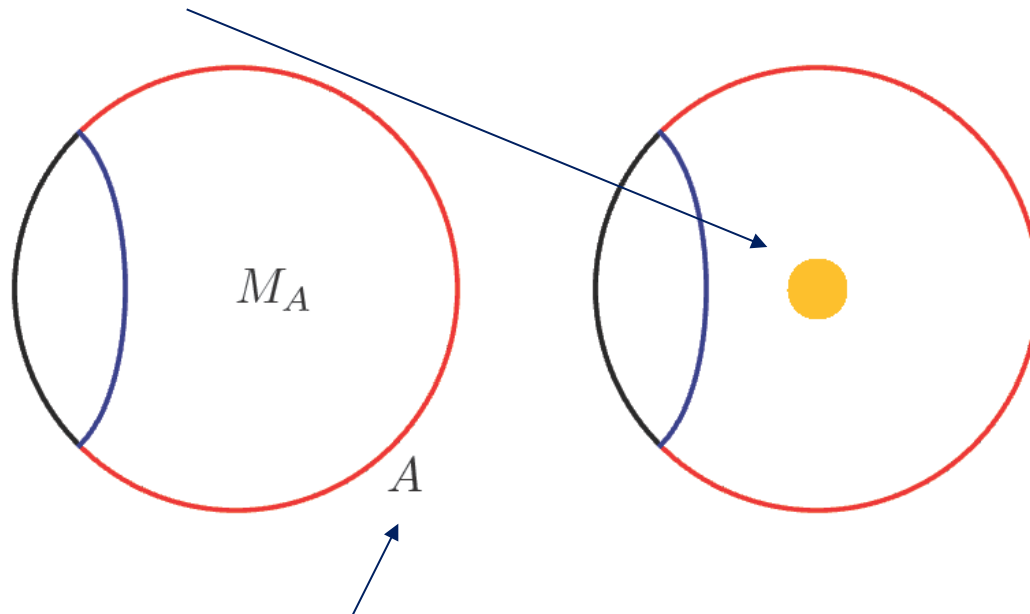
No paradox, No holographic quantum error correction code,
 No entanglement wedge reconstruction (in Dong-Harlow-Wall)

As an example, we will show that

$\phi^G \neq \phi^R$ even in the low energy subspace
even if we include $1/N$ corrections

Counterexample of entanglement wedge reconstruction

bulk operator ϕ supported on small region around the center such that ϕ is spherically symmetric and Hermite



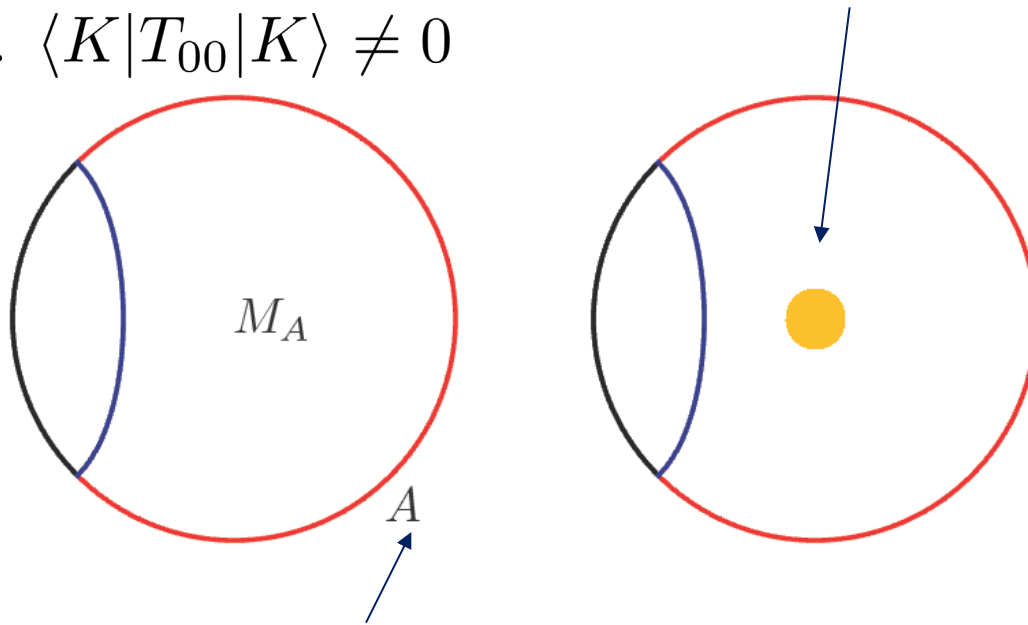
a CFT subregion A such that its causal wedge M_A includes the bulk region around the center

Consider the state $|K\rangle = e^{i\phi}|0\rangle$ (or $(1 + i\phi - \frac{1}{2}\phi^2)|0\rangle$)₃₀

Violation of entanglement wedge reconstruction

Clearly, expectation value of the CFT stress tensor for $|K\rangle$ is spherical symmetric and nonzero everywhere.

i.e. $\langle K|T_{00}|K\rangle \neq 0$



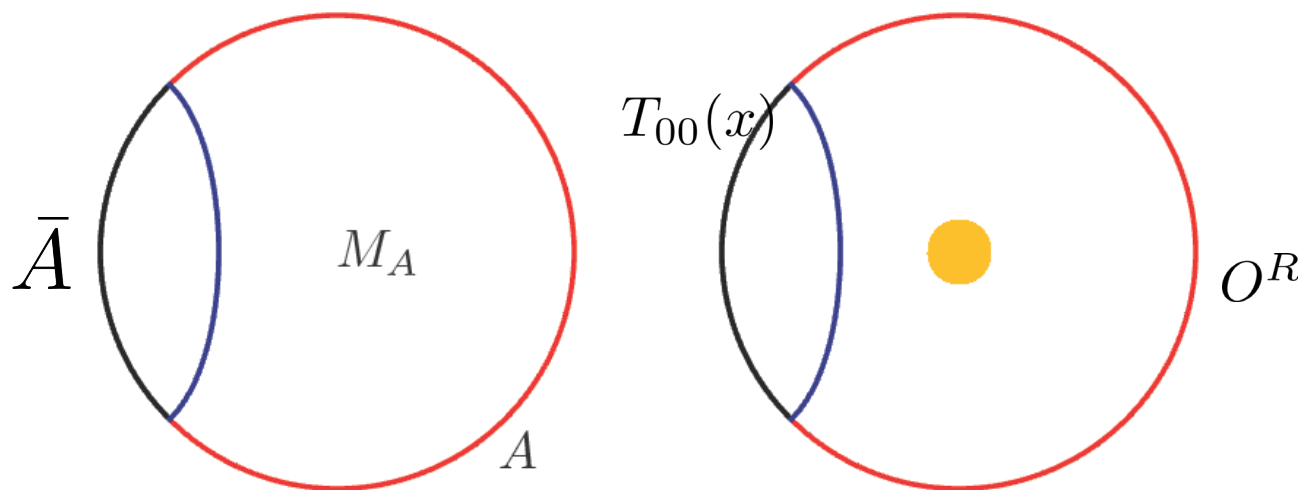
Can we find CFT operator ϕ^R , which is supported only on A , such that $e^{i\phi^R}|0\rangle = |K\rangle$?

The answer is NO!

Violation of entanglement wedge reconstruction

The answer is no because

$\langle 0|e^{-iO^R}T_{00}(x)e^{iO^R}|0\rangle = 0$ where $x \in \bar{A}$
by the causality $[T_{00}(x), O^R] = 0$.



There is no O^R such that $e^{iO^R}|0\rangle = e^{i\phi}|0\rangle$.

Thus, the entanglement wedge reconstruction is not valid.

Remarks:

(1) By smearing of the operators in spacetime, everything here is considered low energy.

(2) Reeh–Schlieder theorem gives any state, but not operator. Thus, it is not relevant here.

(3) Energy density here is $\mathcal{O}(N^0)$.

The difference in energy density means difference in 3-point functions $\langle 0 | [\phi, [T_{00}(x), \phi]] | 0 \rangle$.

The leading order of 3-point function is $\mathcal{O}(1/N)$,
i.e. $\langle OOO \rangle = \mathcal{O}(1/N)$, in $1/N$ expansion.

Here, $\langle O T_{\mu\nu} O \rangle = \mathcal{O}(1)$ because $T_{\mu\nu} \sim N h_{\mu\nu}$
where $h_{\mu\nu}$ satisfies $\langle h_{\mu\nu} h_{\mu'\nu'} \rangle = \mathcal{O}(1)$.

Thus, the difference between ϕ^G and ϕ^R is 3-point function,
which is exactly zero for GFF.

The interaction (=3-point function) fundamentally change
the bulk reconstruction!

Gauge invariant operator and Gravitational dressing

Bulk local operator is NOT gauge invariant

Under infinitesimal diffeo $x \rightarrow x + \xi$,

$$\phi(x) \rightarrow \phi(x) + \frac{\partial \xi^\mu}{\partial x^\nu} \partial_\mu \phi(x)$$

**No local gauge invariant operator in gravity theory
except free “gravity” theory.**

**Gauge invariant operator can be constructed by
gravitational dressing.**

**First, let us consider gauge field case,
instead of gravity.**

Charged operator with Wilson line is gauge invariant,

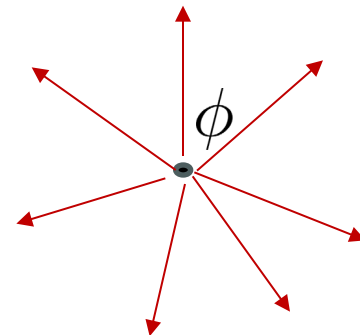
$$\phi(x) e^{ie \int_C A}$$

where C is a curve from x to boundary



We can average the direction of Wilson line.

**This gives Coulomb potential,
related to charge conservation**



Gravity case

Operator with gravitational Wilson line is gauge invariant,

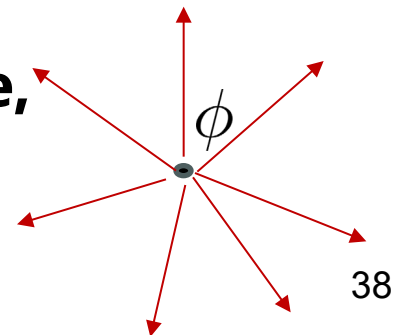
$$\phi(x) e^{i \int_C f(h)}$$

where C is a curve from x to boundary



We can average the direction of Wilson line.

**This gives Schwarzschild metric, in a sense,
related to energy conservation**



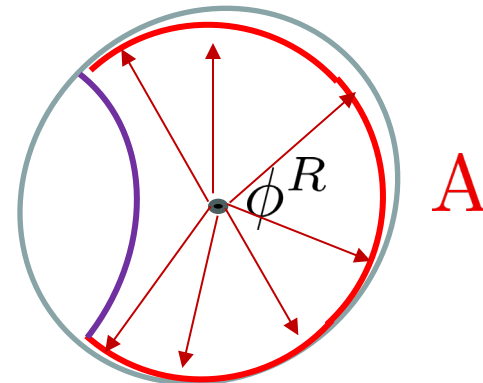
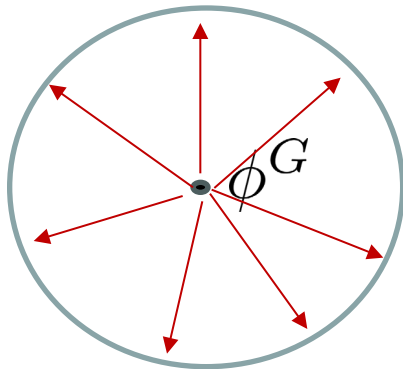
Gauge invariant operator in gravity need boundary

**(which implies that “Island” is inconsistent
except for massive gravity)**

Geng-Karch-Perez-Pardavila-Raju-Randall-Riojas-Shashi

**To consider gauge invariant operator,
subregion including boundary is needed**

Why $\phi^G \neq \phi^R$ in bulk theory?



Their gravitational dressings are different!

corresponding to the difference in 3-point functions $\langle 0 | \phi \phi T_{00}(x) | 0 \rangle$.

Gauge invariance is crucial and essential, thus
Naive picture or models from $N = \infty$ is not good!

Entanglement entropy for gravity theory

**Gauge invariant operators are non-local and
Hilbert space is not factorized.**

**It is natural to consider
subalgebra on entanglement wedge because
entanglement entropy for subalgebra is known.**

Entanglement wedge reconstruction based on subalgebra

**If we assume that relative entropy
in Jafferis-Lewkowycz-Maldacena-Suh may be also
for this subalgebra ver. although they used replica trick,
then, the entanglement wedge reconstruction of the
algebraic version will be derived.**

Harlow

But, this version is very different from the usual one.

**Moreover, this version is consistent with
the weak entanglement wedge reconstruction
which follows from the “simple bulk reconstruction”**

Conclusion

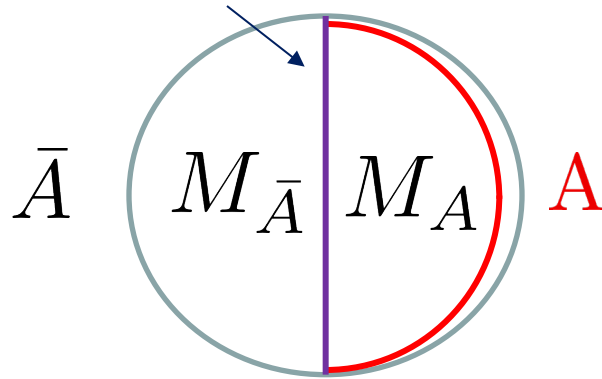
- **By considering the gauge symmetry (diffeomorphism) in gravitational theory, we demonstrate that the widely believed properties of bulk reconstruction are either incorrect or significantly modified.**
- **Many interesting future directions**

Fin

backups

Decompositions for Bulk and CFT

Ryu-Takayanagi surface



CFT space $(= S^{d-1}) = A + \bar{A}$

Bulk space $= M_A + M_{\bar{A}}$

M_A = Entanglement wedge of A

Subregion duality:

For density matrices ρ, σ ,

$$\rho_A = \sigma_A \Leftrightarrow \rho_{M_A} = \sigma_{M_A}$$

where $\rho_A = \text{tr}_{\bar{A}}(\rho)$, $\rho_{M_A} = \text{tr}_{M_{\bar{A}}}(\rho)$

Entanglement wedge reconstruction:

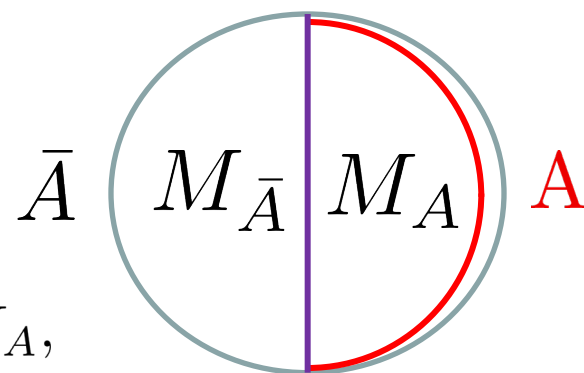
For low energy state $|\phi\rangle$,

$$\forall \mathcal{O}_{M_A} |\phi\rangle = \exists \mathcal{O}_A |\phi\rangle,$$

$\mathcal{O}_{M_A}$ is bulk operator supported in M_A ,

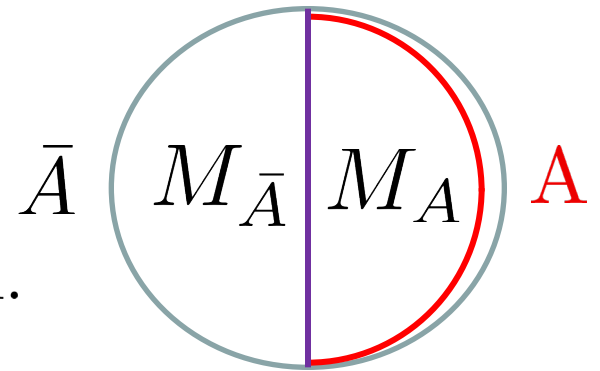
$\mathcal{O}_A$ is CFT operator supported in A

Dong-Harlow-Wall



For GFF (=free bulk theory), with $M_A = \text{AdS-Rindler case}$,
 entanglement wedge reconstruction
 and subregion duality are (seemingly) satisfied

because bulk operators on M_A
 and CFT operators on A are identical.



(More precisely, bulk operators on M_A
 and CFT operators on $D(A)$ are identical.)

Note that GFF cannot have energy momentum tensor
 and violate some axioms of QFT.

Operators on different time slices can be independent in GFF.

”Derivation”

Jafferis-Lewkowycz-Maldacena-Suh (JLMS) showed
CFT relative entropy = bulk relative entropy



Subregion duality



Entanglement wedge reconstruction

AdS/CFT for subregion

Subregion complementarity

Both of ϕ^G, ϕ^R , reconstructed by CFT operator, give bulk 2-point function for $X, X' \in M_A$:

$$\langle 0 | \phi(X) \phi(X') | 0 \rangle = \langle 0 | \phi^G(X) \phi^G(X') | 0 \rangle = \langle 0 | \phi^R(X) \phi^R(X') | 0 \rangle$$

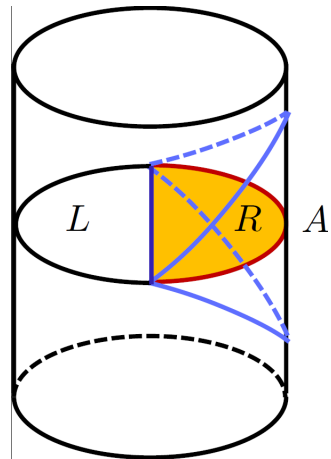
Thus, both describe bulk physics in M_A .

Nevertheless, these are different: $\phi^R(X) | 0 \rangle \neq \phi^G(X) | 0 \rangle$

Same bulk operator in different coordinate patches are realized in CFT differently!



Subregion complementarity
(similar(?) to Black hole complementarity)



What is black hole complementarity

Consider a black hole, and two observers:

Infalling observer:

According to an infalling observer, nothing special happens at the event horizon itself

Outside observer:

infalling information heats up the stretched horizon, which then reradiates it as Hawking radiation

Black hole complementarity:

These two observers see different physics, but no problems because they can not communicate.

Subregion complementarity may be similar to this.