

# Decay of Kaluza-Klein Vacuum via Singular Instanton

Based on the study with Yutaka Ookouchi and Ryota Sato (arXiv: 2404.13917[hep-th]).

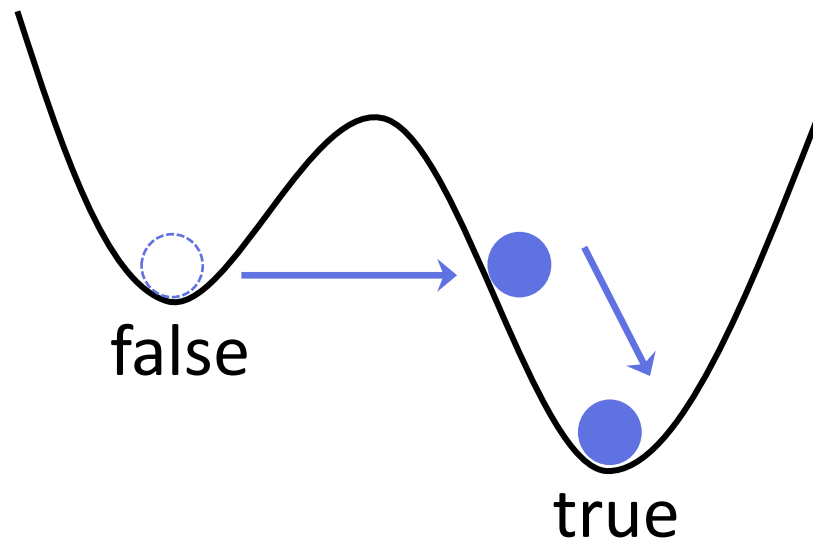
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# Vacuum decay and string theory

- Absence of guiding principle for compactification implies a huge number of (meta)stable states.

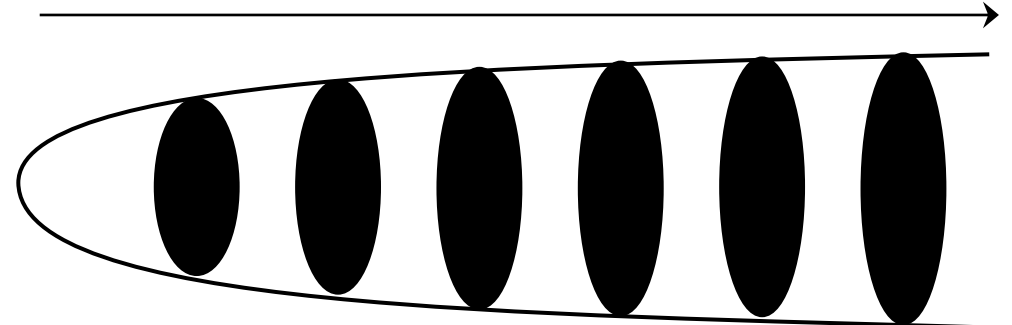
e.g. 1 : metastable (anti) de Sitter



- ✓ Potential barrier.
- ✓ Stringy vacua are often metastable (can transit to a true vacuum with a lower vacuum energy).

e.g. 2 : bubble of nothing

Expanding of "bubble of nothing"

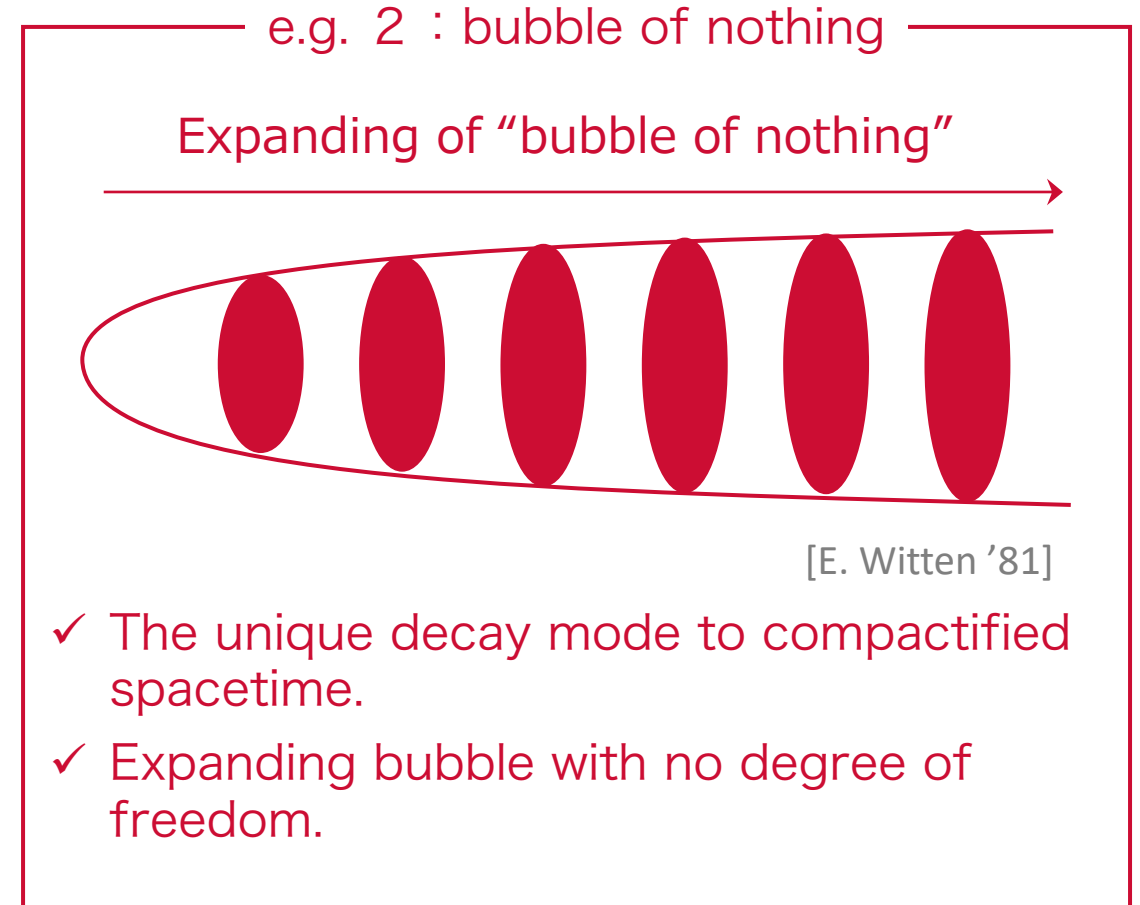
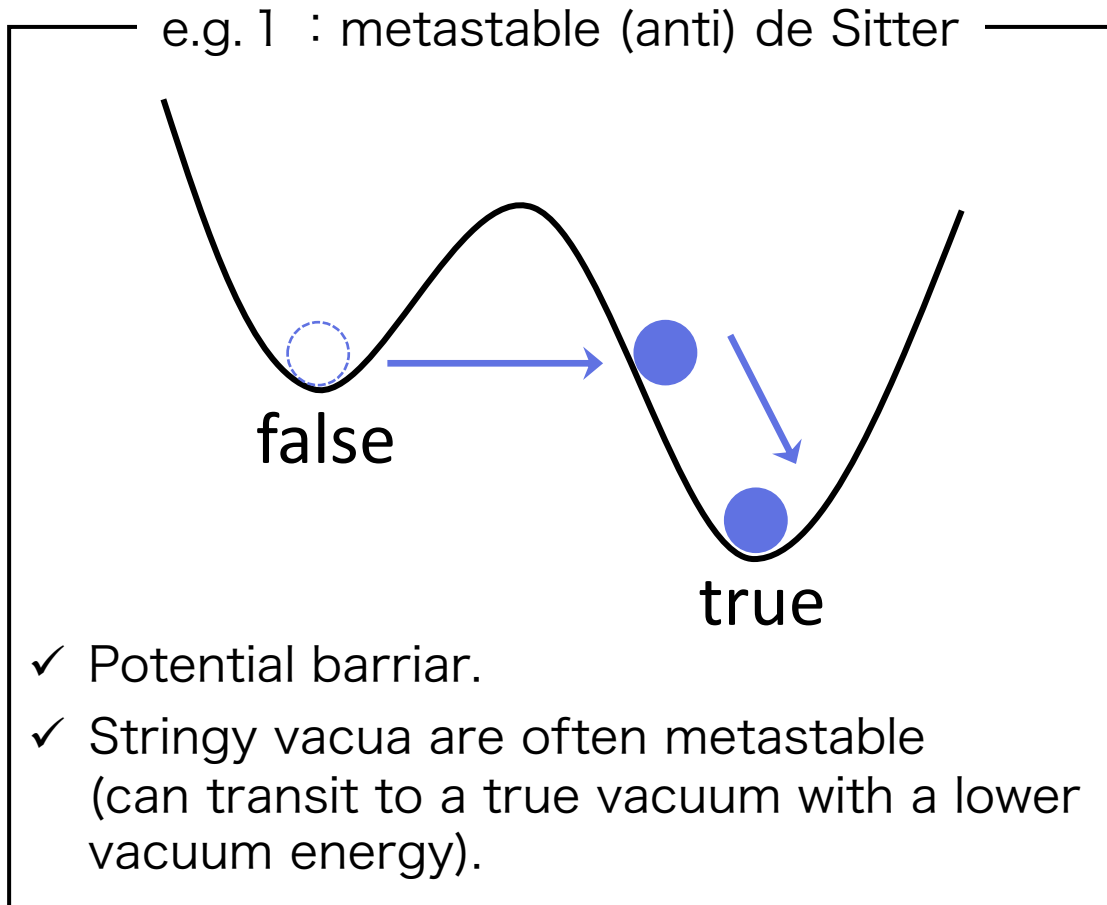


[E. Witten '81]

- ✓ The unique decay mode to compactified spacetime.
- ✓ Expanding bubble with no degree of freedom.

# Vacuum decay and string theory

- Absence of guiding principle for compactification implies a huge number of (meta)stable states.



# Current state of “Bubble of Nothing”

- Bubble of nothing (BoN) is attracting more and more attention because it would be the best candidate for a universal decay channel of non-supersymmetric string vacua.

[I. Garcia Etxebarria, M. Montero, K. Sousa and I. Valenzuela, JHEP 12 (2020) 032 [arXiv: 2005.06494[hep-th]]]

[G. Dibietto, N. Petri, and M. Schillo, JHEP 08 (2020) 040 [arXiv: 2002.01764[hep-th]]]

- While in conventional discussion, BoN is forbidden even for SUSY broken vacua if fermions with SUSY preserving boundary conditions exist, some counterexamples have been proposed in recent years.

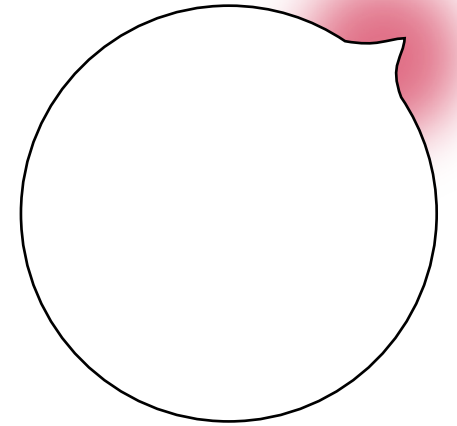
[J.J. Blanco-Pillado, B. Shlaer, K. Sousa and J. Urrestilla, JCAP 10 (2016) 002 [arXiv: 1606.03095[hep-th]]]

[P. Draper, B. Lillard and C. Skye, JHEP 10 (2023) 049 [arXiv: 2305.17838[hep-th]]]

“Bubble of nothing” is more universal than you think.



- We considered a decay of Kaluza-Klein vacuum mediated by a **singular instanton**.
- We have evaluated the on-shell contribution of the singularity and find that it reduces a total bounce action.

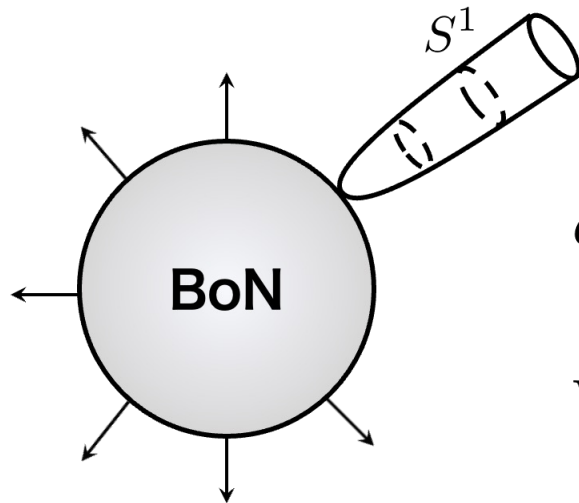


### Take-home message

Decay via singular instanton may be more dominant channel in the context of BoN.

1. Introduction
2. Review of bubble of nothing
3. Decay via singular instanton
4. Thermodynamical interpretation
5. Summary and future work

- “Bubble of Nothing” (often abbreviated as BoN) is a catastrophic decay phenomenon particular to compactified spacetime.
- Kaluza-Klein vacuum ( $M_4 \times S^1$ ) can decay as the BoN expands at the speed of light.



$$ds_5^2 = \left(1 - \left(\frac{R}{r}\right)^2\right) d\phi^2 + \left(1 - \left(\frac{R}{r}\right)^2\right)^{-1} dr^2 + r^2 ds_{dS_3}^2,$$

$$\text{where } ds_{dS_3}^2 = -d\tau^2 + \cosh^2 \tau ds_{S^2}^2$$

- BoN instantons take the form of Euclidean black hole solutions.

$$ds_E^2 = \left(1 - \left(\frac{\sqrt{\alpha}}{r}\right)^2\right) d\phi^2 + \left(1 - \left(\frac{\sqrt{\alpha}}{r}\right)^2\right)^{-1} dr^2 + r^2 ds_{S^3}^2$$

- Euclidean black hole solutions have **conical singularity** at the position of event horizon.

► Fix the periodicity of the imaginary time to appropriate value.

$$\alpha = R_{KK}^2 \quad (\text{Smoothness condition})$$

- if the contribution to the on-shell action from the singularity is finite, the condition may be relaxed.



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# Singular instanton and regularization

- The study of singular instantons was initiated by Hawking and Turok and then explored mainly in the context of open universe.

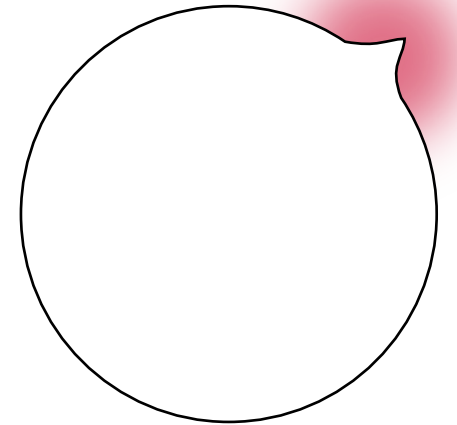
[S.W. Hawking and N. Turok, Phys. Lett. B 425 (1998) 25 [hep-th/9802030]]

[N. Turok and S.W. Hawking, Phys. Lett. B 432 (1998) 271 [hep-th/9803156]]

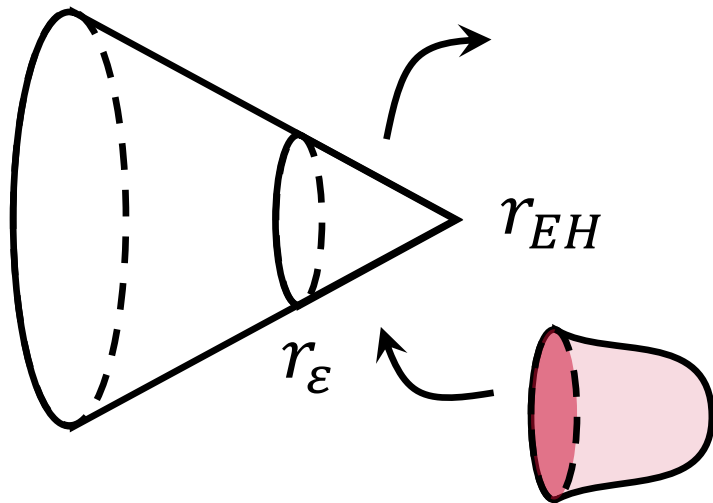
- Gregory, Moss and Withers have refined the geometrical technology for more precise treatment of singularities.

[D. V. Fursaev, A. N. Solodukhin, Phys.Rev.D 52 (1995) 2133-2143 [arXiv:9501127[hep-th]]]

[R. Gregory, I. G. Moss and B. Withers, JHEP 03 (2014) 081 [arXiv:1401.0017[hep-th]]]



dS instanton + singularity



$$I_{\mathcal{B}} = -\frac{1}{16\pi G_n} \int_{\mathcal{B}} d^n x \sqrt{g} R - \frac{1}{8\pi G_n} \int_{\partial \mathcal{B}} d^{n-1} y \sqrt{h} (K - K_0)$$

$$= -\frac{\mathcal{A}_{EH}}{4G_n}$$

Entropy-like formula

Smooth cap

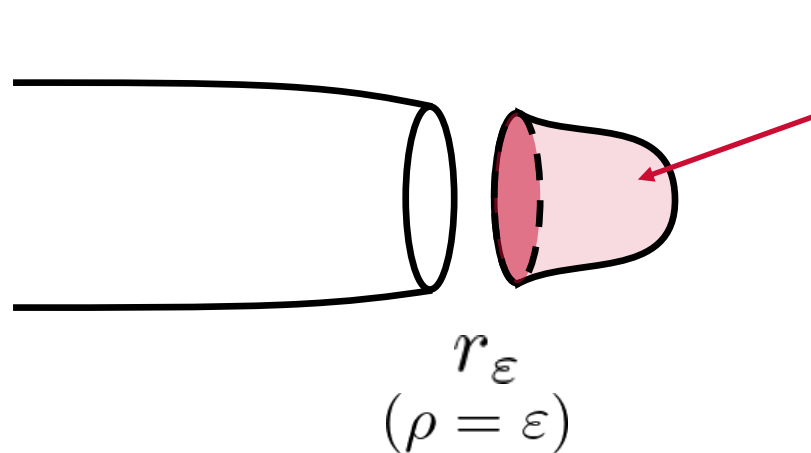
Introducing  $\rho \equiv r\sqrt{f(r)}$ , we can rewrite the instanton solution as

$$ds^2 = F(\rho)^2 \underbrace{d\chi^2}_{2\pi \text{ periodic}} + d\rho^2 + r(\rho)^2 d\Omega_3^2, \quad F(\rho)^2 \equiv \left(1 - \left(\frac{\sqrt{\alpha}}{r(\rho)}\right)^2\right) R^2$$

►  $ds^2 \simeq d\rho^2 + \rho^2 d(F'(0)\chi)^2 + r(0)^2 d\Omega_3^2$  (near the singularity)

Since  $F'(0) \neq 1$  in general, there would be a deficit angle defined as

$$2\pi\delta = 2\pi(1 - F'(0)) = 2\pi\left(1 - \frac{R}{\sqrt{\alpha}}\right)$$



$$ds^2 = \tilde{F}(\rho)^2 d\chi^2 + d\rho^2 + r(\rho)^2 d\Omega_3^2$$

$$\boxed{\tilde{F}'(0) = 1}, \text{ but } \delta \simeq 1 - \tilde{F}'(\epsilon)$$

smooth

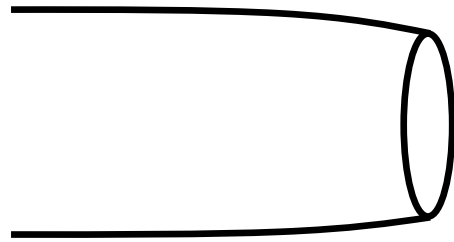
$$-\frac{1}{16\pi G_n} \int_{\mathcal{B}} \mathcal{R} = -\frac{\mathcal{A}}{4G_n} \delta, \quad -\frac{1}{8\pi G_n} \oint_{\partial\mathcal{B}} (\mathcal{K} - \mathcal{K}_0) = -\frac{\mathcal{A}}{4G_n} (1 - \delta)$$

► 
$$I_{\mathcal{B}} = -\frac{\mathcal{A}}{4G_n} \delta - \frac{\mathcal{A}}{4G_n} (1 - \delta) = -\frac{\mathcal{A}}{4G_n}$$

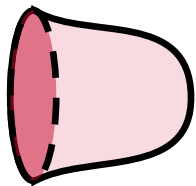
cancel each other out

## Bounce action

Split the manifold into two and calculate the Euclidean action for each.



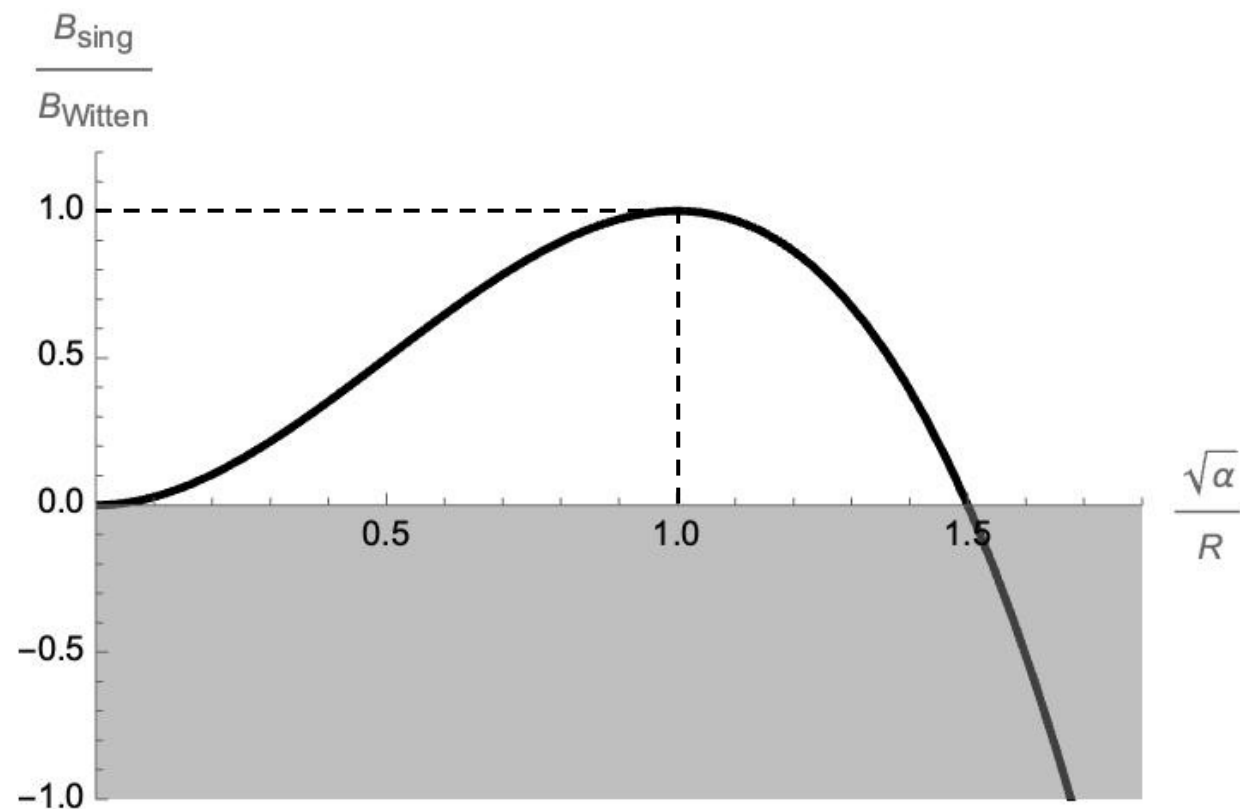
$$I_{\mathcal{M}-\mathcal{B}} = - \frac{1}{8\pi G_n} \int d^{n-1}y \sqrt{h} (K - K_0) \Big|_{r_\varepsilon, r_\infty} = \frac{3\pi\alpha}{8G_4}$$



$$I_{\mathcal{B}} = - \frac{2\pi^2 \alpha^{3/2}}{4G_5} = - \frac{\pi \alpha^{3/2}}{4G_4 R}$$

$$B = I_{\mathcal{M}-\mathcal{B}} + I_{\mathcal{B}} = \frac{\pi R^2}{8G_4} \left( \frac{3\alpha}{R^2} - \frac{2\alpha^{3/2}}{R^3} \right) \quad (\text{Bounce action})$$

- ✓ Conical singularities in Euclidean solutions play an important role as a catalyst which reduce bounce actions.
- ✓ Our semiclassical analysis is unreliable in the shaded region.



Perhaps we should pay more attention to “singular” BoN.

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We can reproduce the bounce action with thermodynamic functions.

ADM energy  $E = -\frac{1}{8\pi G_5} \oint_{S_{\phi r}} (k - k_0) \sqrt{\sigma} d^3\theta = \frac{3\pi\alpha}{8G_5}$

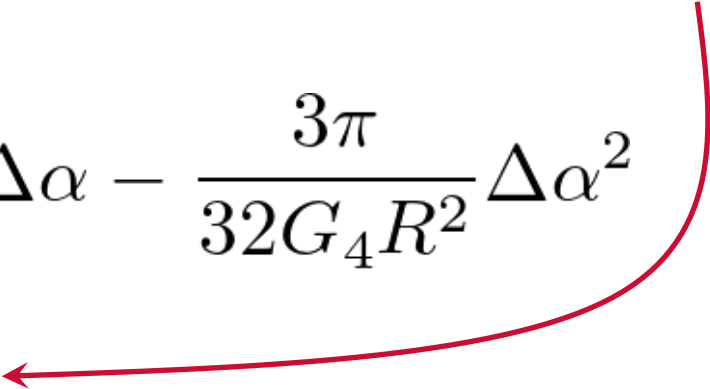
Entropy  $S = \frac{2\pi^2\alpha^{3/2}}{4G_5}$

►  $B = \frac{W}{T} = \frac{E - TS}{T} = \frac{3\pi\alpha}{8G_4} - \frac{\pi\alpha^{3/2}}{4RG_4}$



Let us consider shifting  $\alpha$  slightly from  $R^2$  up to the second order.

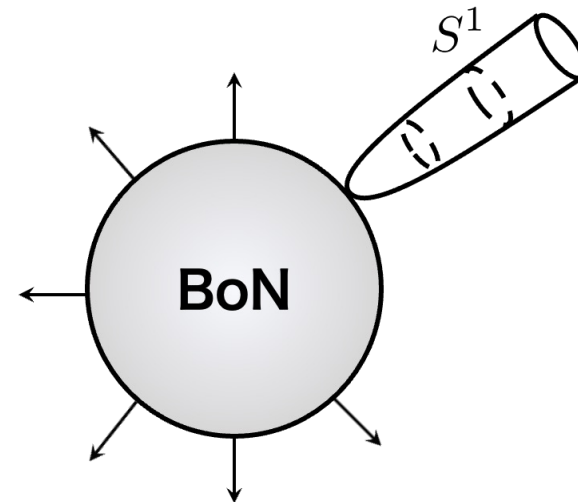
$$\Delta E = \frac{3}{16G_4 R} \Delta\alpha, \quad \Delta S = \frac{3\pi}{8G_4} \Delta\alpha + \frac{3\pi}{32G_4 R^2} \Delta\alpha^2$$


$$B \simeq B_{\text{witten}} + \frac{3\pi}{8G_4} \Delta\alpha - \frac{3\pi}{8G_4} \Delta\alpha - \frac{3\pi}{32G_4 R^2} \Delta\alpha^2$$
$$= B_{\text{witten}} - \frac{3\pi}{32G_4 R^2} \Delta\alpha^2$$


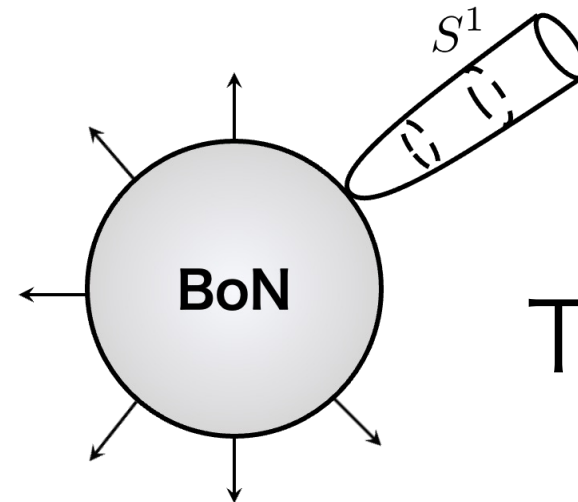
Negative contribution

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  - Singular instanton may play an important role in decays of higher-dimensional spacetime.
    - Conical singularity works as reducing the value of bounce action.
    - Our calculation is consistent with Witten’s original argument.
    - We can reproduce the bounce action with thermodynamic functions and give an interpretation.
- ✓ Validity of the regularization method.
  - ✓ Uniform flux?
  - ✓ Embedding into stringy model?
  - ✓ End of the world brane?



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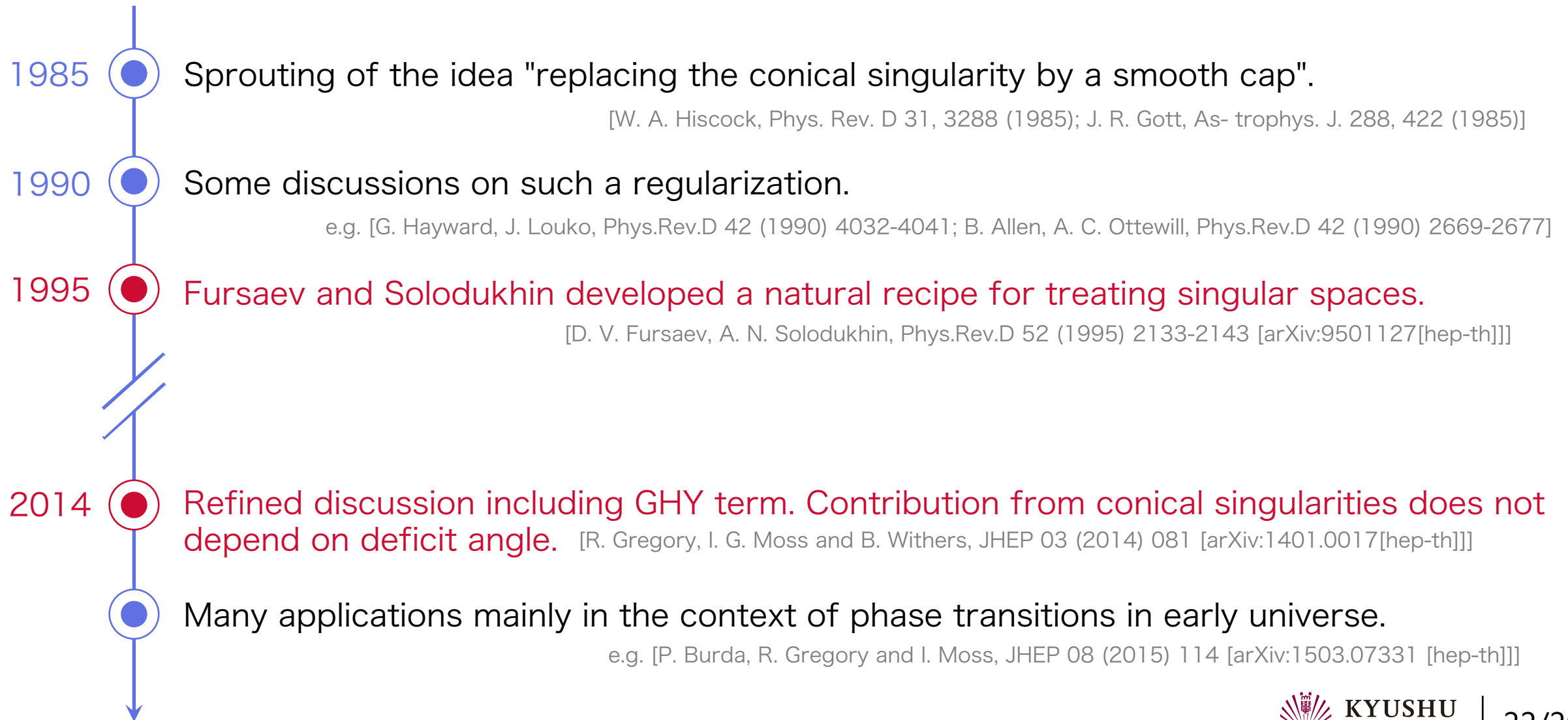


Thank you!

BACKUP

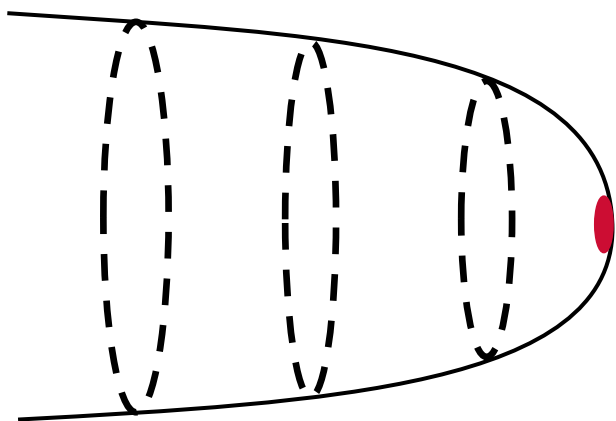
# Background of conical deficit regularization

Conical deficit regularization has been gradually established in studies since the 1980s.



Cobordism conjecture predicts that all string compactifications have defects (end-of-the-world branes) at the tip of cigar geometry.

[4D effective theory description]



$$S_{4D} = \int_{\mathcal{M}} \sqrt{g} \left( -\frac{1}{2} \mathcal{R}_4 + \frac{1}{2} (\partial\phi)^2 + V(\phi) \right) - \int_{\partial\mathcal{M}} \sqrt{h} (\mathcal{K}_4 - T_4)$$

[A bordism with defect, from Friedrich,Hebecker&Walcher'23]

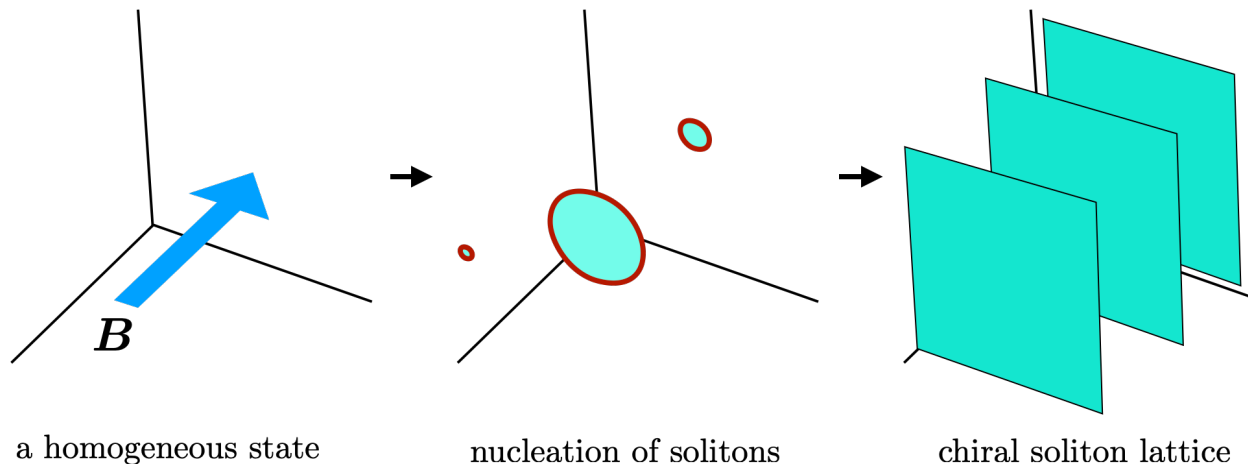
## Example: Chiral soliton lattice

They considered the nucleation of domain walls with a constant background magnetic field. Their defects have **negative energy due to the topological interaction**.

$$\mathcal{L}_{UV} = |\partial_\mu \phi|^2 - \frac{\lambda}{4} (|\phi|^2 - v^2)^2 + vm^2 (\phi + \phi^*) + \boxed{cj \cdot B}$$

$$j^\mu = -\frac{i}{2} (\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*) = |\phi|^2 \partial^\mu \theta, \quad \phi = |\phi| e^{i\theta}.$$

$$\blacktriangleright \mathcal{H}_{UV} = |\dot{\phi}|^2 + |\nabla \phi|^2 + \frac{\lambda}{4} (|\phi|^2 - v^2)^2 - vm^2 (\phi + \phi^*) - cj \cdot B$$



[Picture from Eto&Nitta'22]

