Late time behavior of codimension-0 observables in JT gravity

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Abstract

Recently, complexity has been studied intensively as a holographic dual of bulk quantities which grows for very long time, far beyond the thermalization of entanglement entropy. Originally, the volume of wormhole was proposed as a complexity dual in bulk, but it has been pointed out that wide classes of bulk observables are candidate for gravitational dual of complexity. In previous study, the length of wormhole was evaluated using gravitational path integral including non-trivial geometry, and the transition from linear growth to saturation was observed. In this study, we focus on the codimension-0 observables in JT gravity, and evaluate the late time behavior. We compare these behaviors to the complexity and discuss the difference between the volume case.

I. Introduction

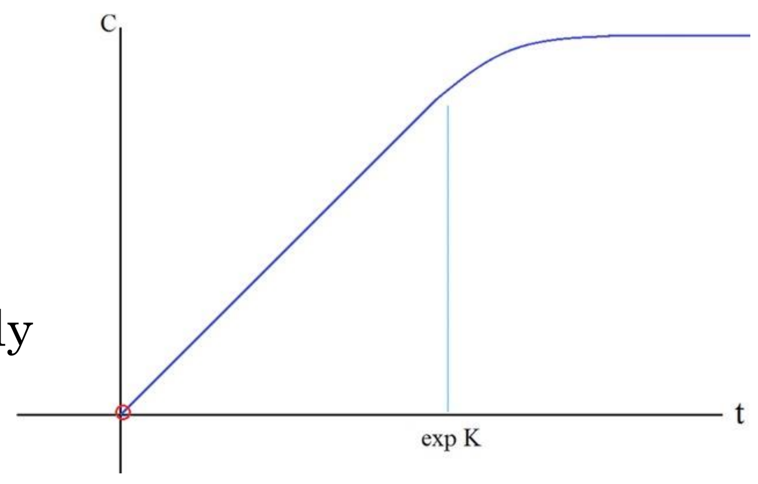
◆Complexity

- Complexity is the minimum number of simple operators needed to transform a state to other states :

$$|\psi_f\rangle = g_n g_{n-1} \cdots g_1 |\psi_i\rangle$$

g_i : unitary operators

- Complexity is known to grow linearly at first, and then saturate.



L. Susskind, *Fortsch. Phys.* 64 (2016) 49

◆Question : What's the dual bulk observables ?

- Volume of Einstein-Rosen bridge (CV conjecture)
- Action of Wheeler-de Witt patch (CA conjecture)
- Spacetime volume of WdW patch (CV 2.0 conjecture)

II. JT Gravity

- JT gravity is a 2 dimensional gravity theory, which is obtained from higher dimensional charged black hole.

$$S_{\mathrm{JT}} = -\frac{S_0}{4\pi}\underbrace{\left(\int_{\mathcal{M}} \sqrt{g}R + 2\int_{\partial\mathcal{M}} \sqrt{h}K\right)}_{\text{Topological term : }\chi(\mathcal{M})} - \frac{1}{2}\underbrace{\left(\int_{\mathcal{M}} \sqrt{g}\phi(R+2) + \int_{\partial\mathcal{M}} \sqrt{h}\phi K\right)}_{\text{Fixing } R=-2}$$

- JT gravity is equivalent to the random matrix theory
- CV conjecture was tested in JT gravity, and observed linear growth in early time and saturation in late time

  But several discrepancy from complexity in chaotic system

  e.g. Growing fluctuation in late time $\propto \sqrt{t}$

- We discuss the action in codimension-0 region using gravitational path integral including non trivial topologies.

III. Codimension-0 Observables

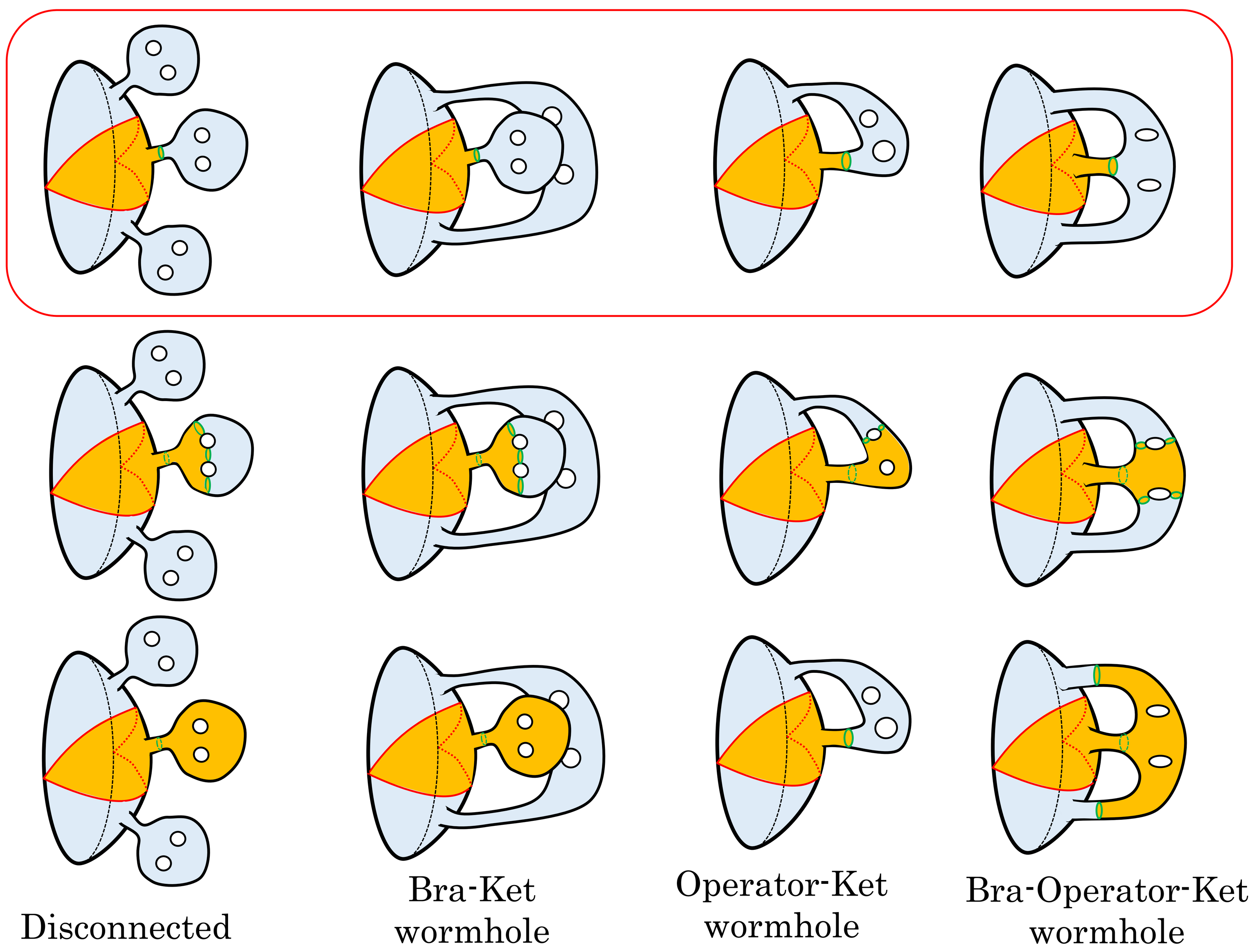
- We consider the codimension-0 observable in Euclidean spacetime.
- There are several choices of codimension-0 region
- Action in codimension-0 region $\mathcal{U}$ (orange region) is given by

$$\left\langle e^{-\Delta A[\mathcal{U}]}\right\rangle\Big|_{(x_1,x_2)} = \int_0^\infty dE dE_1 dE_2 \mathcal{M}[E_1,E,\Delta]\mathcal{M}[E,E_2,\Delta]e^{-x_1E_1-x_2E_2}D(E_1,E_2,E,\Delta)$$

$$\begin{aligned}&D(E_1,E_2,E_3,\Delta)\\ &\approx D(E_1)D(E_2)D(E_3) + \langle D(E_1)D(E_2)\rangle_c D(E_3) + \langle D(E_1)D(E_3)\rangle_c D(E_2)e^{-\Delta S_0}\\ &+ \langle D(E_2)D(E_3)\rangle_c D(E_1)e^{-\Delta S_0} + \langle D(E_1)D(E_2)D(E_3)\rangle_c e^{-\Delta S_0}\end{aligned}$$

$$\mathcal{M}[E_1,E_2,\Delta] = \frac{\left|\Gamma\big(\Delta+i(s_1+s_2)\big)\Gamma\big(\Delta+i(s_1-s_2)\big)\right|^2}{2^{2\Delta-1}\Gamma(2\Delta)} \qquad (s_i = \sqrt{2E_i})$$

$$\Longrightarrow \quad \langle A[\mathcal{U}]\rangle = -\lim_{\Delta\to 0}\frac{\partial}{\partial\Delta}\left\langle e^{-\Delta A[\mathcal{U}]}\right\rangle$$



IV. Late Time Behavior

- We expect the behavior of $\langle A[\mathcal{U}]\rangle$:

$$\langle A[\mathcal{U}]\rangle = 2e^{S_0}\rho(\bar{s})\tilde{t}\ ? \qquad (\tilde{t} \ll 1) \quad \text{Linear growth in early time}$$

$$\langle A[\mathcal{U}]\rangle = const \qquad (\tilde{t} \gg 1) \quad \text{Saturation in late time}$$

Behavior of $\langle A[\mathcal{U}]\rangle$ is consistent with complexity $(\tilde{t} \gg 1)$!

- How about fluctuations ? Is it consistent with complexity ?

V. Summary and Discussion

- We analyzed the time dependence of codimension-0 observable in JT gravity ;

$$\langle A[\tilde{t}]\rangle \propto \tilde{t} \quad (\tilde{t} \ll 1), \qquad \langle A[\tilde{t}]\rangle \sim \text{const} \quad (\tilde{t} \gg 1)$$

- For future work, we will address these issues ;
  - Fluctuations of action : the volume case shows the growing fluctuation as $\sqrt{\tilde{t}}$, doesn't match the chaotic system
  - Non zero extrinsic curvature case : Behavior of WdW action
  - Propose new complexity measure in quantum system like spectral complexity