2次元 $\mathbb{C}P^{N-1}$ 模型における準安定な真空エネルギーの θ 角依存性

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菅野翼氏(東北大)、米倉和也氏(東北大)との現在進行中の研究に基づく

Main message today

In large N limit, the (metastable) vacua of 2d $\mathbb{C}P^{N-1}$ model are not regarded as metastable at large θ

1. Introduction

2d $\mathbb{C}P^{N-1}$ model with theta angle θ

Lagrangian

$$\mathscr{L} = -(D_\mu z)^\dagger D^\mu z - \sigma\left(z^\dagger z - \frac{N}{g_0^2}\right) + \frac{i\theta}{4\pi}\epsilon^{\mu\nu}F_{\mu\nu}$$

z : N complex scalar field, A_μ : aux. $\mathrm{U}(1)$ - gauge field, σ : aux. scalar field

g_0 : 't Hooft coupling, $D_\mu = \partial_\mu + iA_\mu$

Motivations of 2d $\mathbb{C}P^{N-1}$ model

- 2d $\mathbb{C}P^{N-1}$ model shares some properties with 4d pure $\mathrm{SU}(N)$ theory
- Exactly solvable in large N limit

$\Longrightarrow$ 2d $\mathbb{C}P^{N-1}$ model is a good toy model of 4d pure $\mathrm{SU}(N)$ theory

Instanton effect and θ - angle

Instanton in 2d spacetime

Configuration (Classical solution) which has a nontrivial winding number

Winding number $\displaystyle\int_{M_2} \frac{F}{2\pi} \in \mathbb{Z}$

θ **- angle :** Free parameter which gives weights of instanton configurations to the partition function

- $\theta \sim \theta + 2\pi$ because of winding numbers
- Instantons effect on perturbative calculations via the theta angle

Vacuum energy in DIGA

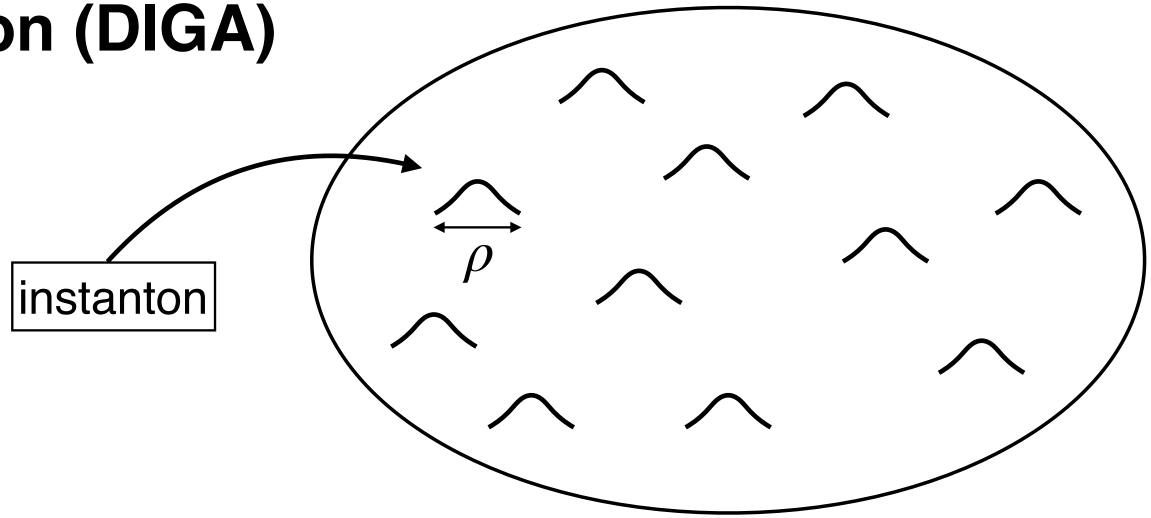
Parameter of the instanton solution

x_0 : instanton position, ρ : instanton size

Dilute instanton gas approximation (DIGA)

Instanton size ρ : sufficiently small

(Each instantons do not overlap)



Vacuum energy : $E(\theta) \propto 1 - \cos\theta$

Dilute instanton gas configuration

However this approx. is valid at the weak coupling…

The vacuum energy is nontrivial in the strong coupling regime

Vacuum energy in non-DIGA case

Expectation : If θ is changed, true vacuum → metastable vacuum, one of metastable vacua → true vacuum

$\Rightarrow$ Energies of all vacua are analytic, but the true vacuum energy is not analytic

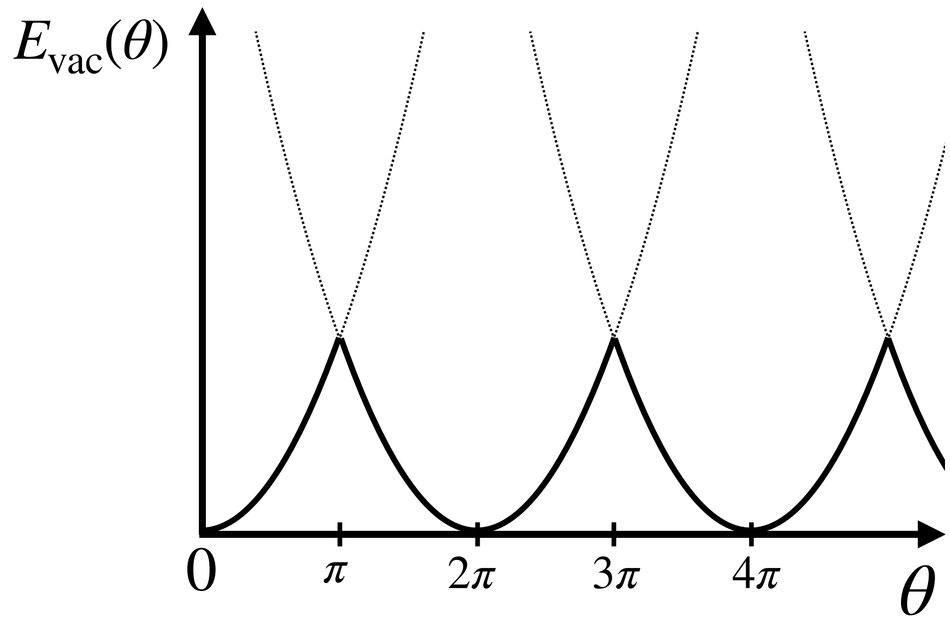
Evidence $\cdots$ Large N limit [D'adda, Lüscher, '78], 't Hooft anomaly [Nguyen, Tanizaki, Ünsal, '22]

Natural candidate (e.g. $N \geq 3$)

- Trivially gapped at $\theta \notin (2\mathbb{Z}+1)\pi$
- Charge conjugation symmetry breaking at $\theta \in (2\mathbb{Z}+1)\pi$

—— $\cdots$ true vacuum

········ $\cdots$ metastable vacuum



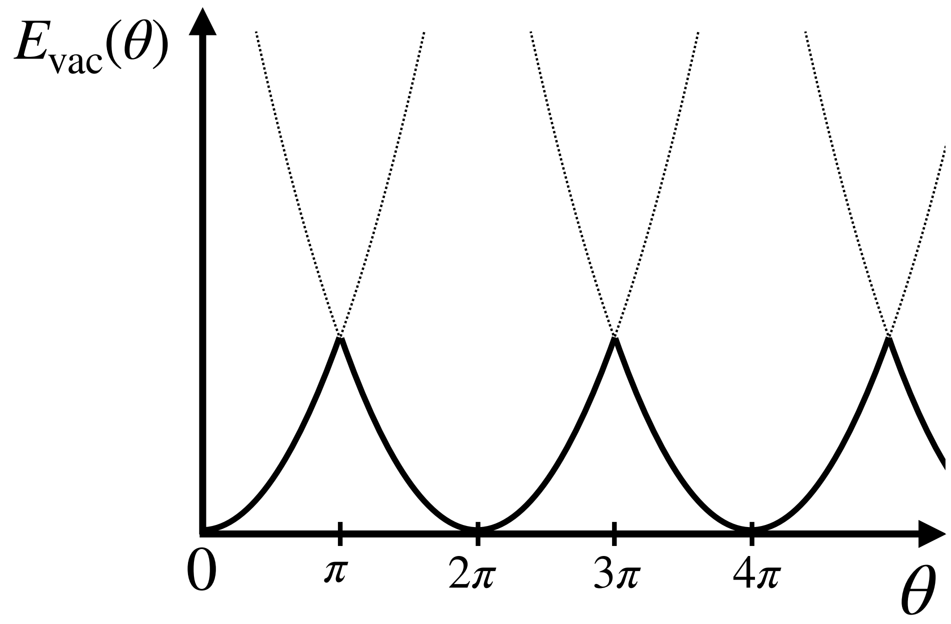
Vacuum energy in large N limit

The energy of the true vacuum

$$E_{\text{vac}}(\theta) = \frac{3\sigma_0}{\pi N}\theta^2, \; -\pi \le \theta \le \pi$$

Let us concentrate on one branch

- The energy of the true vacuum (that is the bevavior between $|\theta| \lesssim O(1)$) is well understood.
- The metastable vacuum energy (that is the behavior at $|\theta| \gtrsim O(1)$) is not understood completely.



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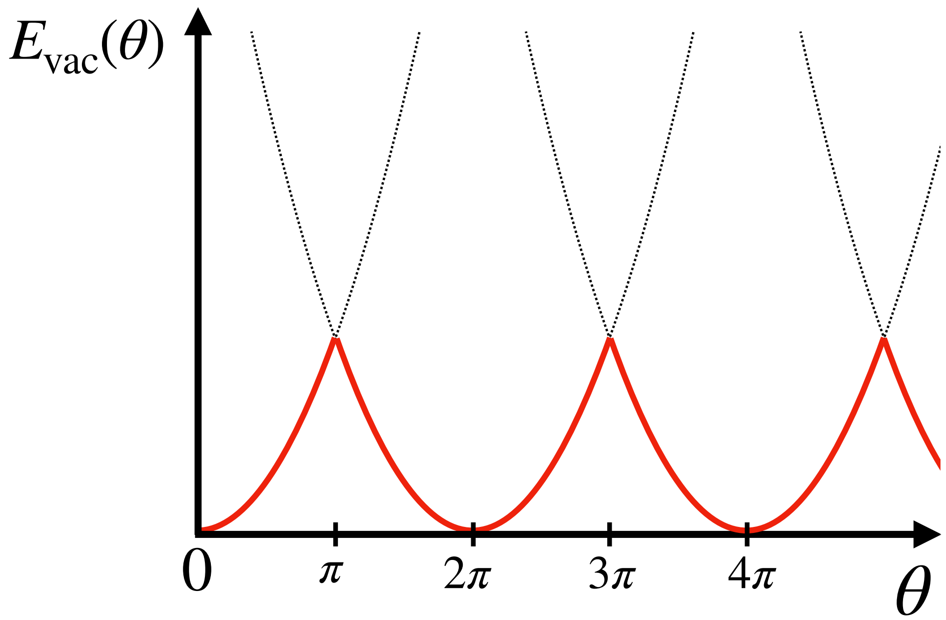

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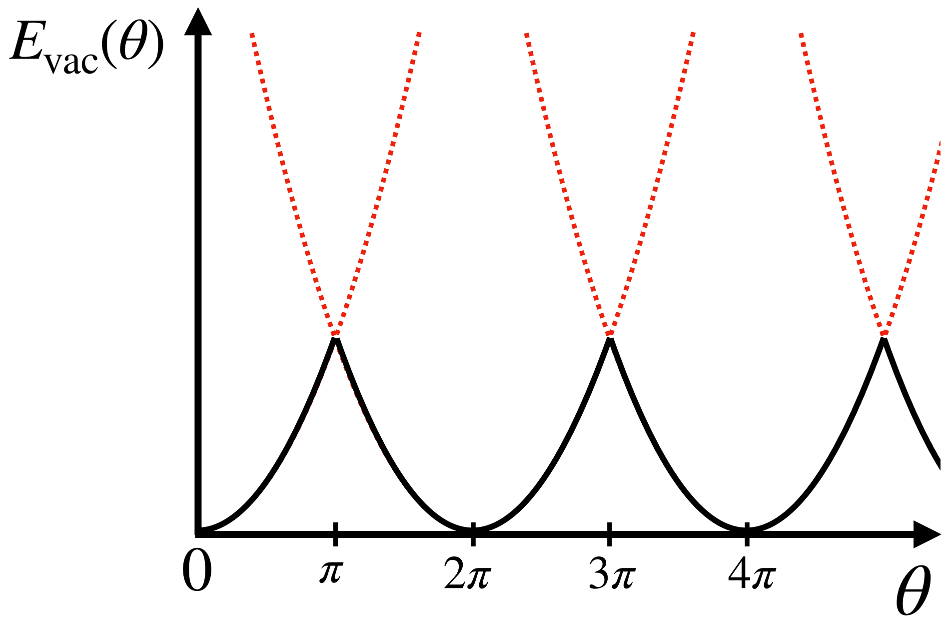
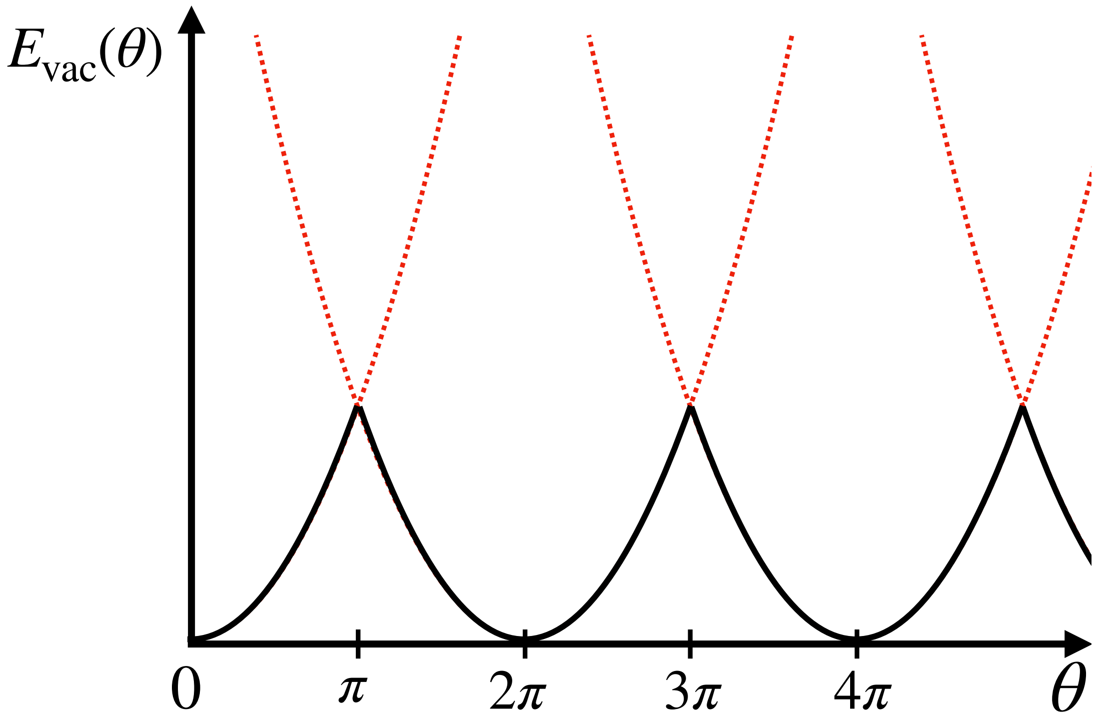
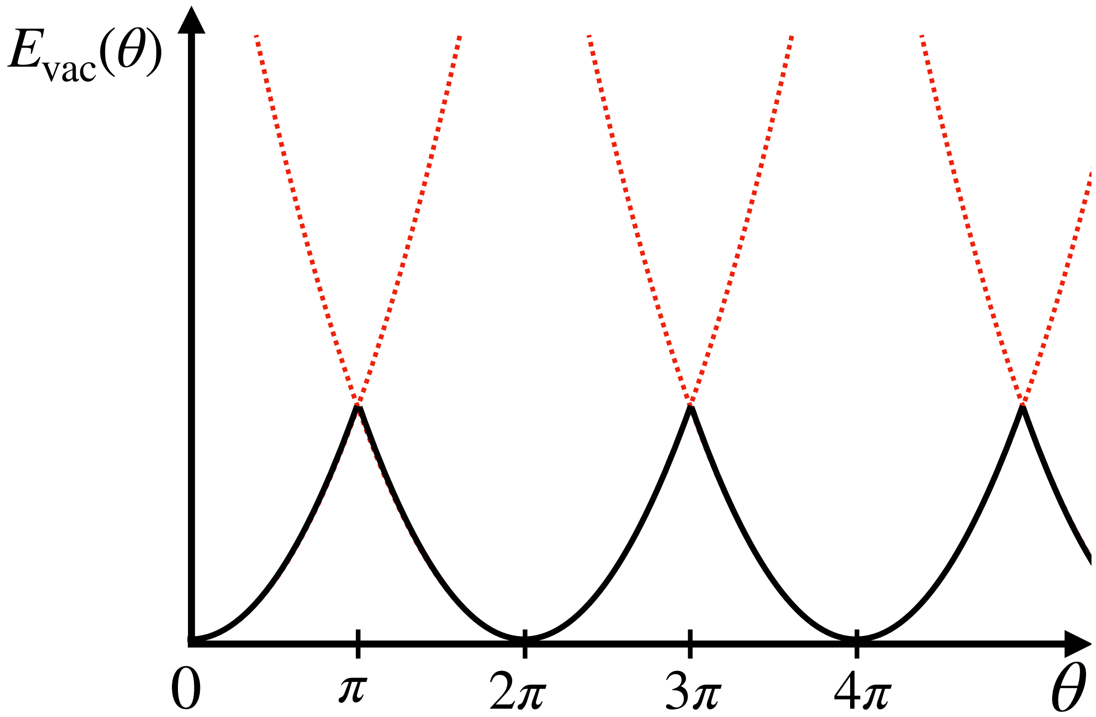


Target in this talk



Target in this talk

We would like to study the behavior at $\theta \sim O(N)$



Symmetry & ’t Hooft anomaly in 2d $\mathbb{C}P^{N-1}$ model

Symmetry in 2d $\mathbb{C}P^{N-1}$ model

$\mathrm{U}(1)$ gauge symmetry, $\mathrm{PSU}(N)$ global symmetry, Charge conjugation symmetry ($\theta = 0, \pi$)

’t Hooft anomaly in 2d $\mathbb{C}P^{N-1}$ model

$$\text{2d } \mathbb{C}P^{N-1} \text{ model} \xrightarrow[\mathrm{PSU}(N)\text{ - gauge field } B]{\text{Background}} \text{2d } \mathbb{C}P^{N-1} \text{ model w/ } B$$

$$\boxed{Z[\theta + 2\pi] = Z[\theta]} \qquad \boxed{Z[\theta + 2\pi, B] = e^{i\mathscr{A}[B]} Z[\theta, B]}$$

Constraints on vacuum energy

Existence of nontrivial phase factor = nontrivial vacuum

- SSB (Discrete symmetry breaking is only possible in 2d)
- Existence of gapless d.o.f.

between $0 \leq \theta < 2\pi$

in 2d $\mathbb{C}P^{N-1}$ model.

Does a nontrivial phase factor exist under $\theta \to \theta + 2\pi n$?

- If $n = 1, \cdots, N-1$, $n\mathscr{A}[B] \notin 2\pi\mathbb{Z} \Rightarrow$ vac$[\theta = 0] \neq$ vac$[\theta = 2\pi n]$
  The energy of the true vacuum is not analytic as a function of θ
- If $n = N$, $n\mathscr{A}[B] \in 2\pi\mathbb{Z} \Rightarrow$ vac$[\theta = 0] =$ vac$[\theta = 2\pi N]$: possible
  The vacuum energy can be $2\pi N$ - periodicity

Questions

How do the vacua energies behave as a function of θ ?
($2\pi N$ - periodicity ? # of vacua = N or not ?)

2. Method

Method

We use large N limit

- Step 1 : Evaluation of the path integral in Euclidean space (integrating out $z, z^{\dagger}$)

- Step 2 : Determination of an analytic form of effective potential

- Step 3 : Wick rotation & Find a saddle point

$\Longrightarrow$ **We will obtain the behavior of the vacua energies beyond $\theta \sim O(1)$**

Effective potential and saddle point

Effective potential

$$V_{\text{eff}} = -\frac{N}{g^2}\sigma + \frac{\theta}{2\pi}\mathscr{E} - \frac{N}{4\pi}\left[2i\mathscr{E}\log\Gamma\left(\frac{1}{2} - i\frac{\sigma}{2\mathscr{E}}\right) + \sigma\log\frac{2i\mathscr{E}}{M^2} - i\mathscr{E}\log 2\pi\right]$$

$\mathscr{E}$: Const. electric field, σ : Const. aux. scalar field

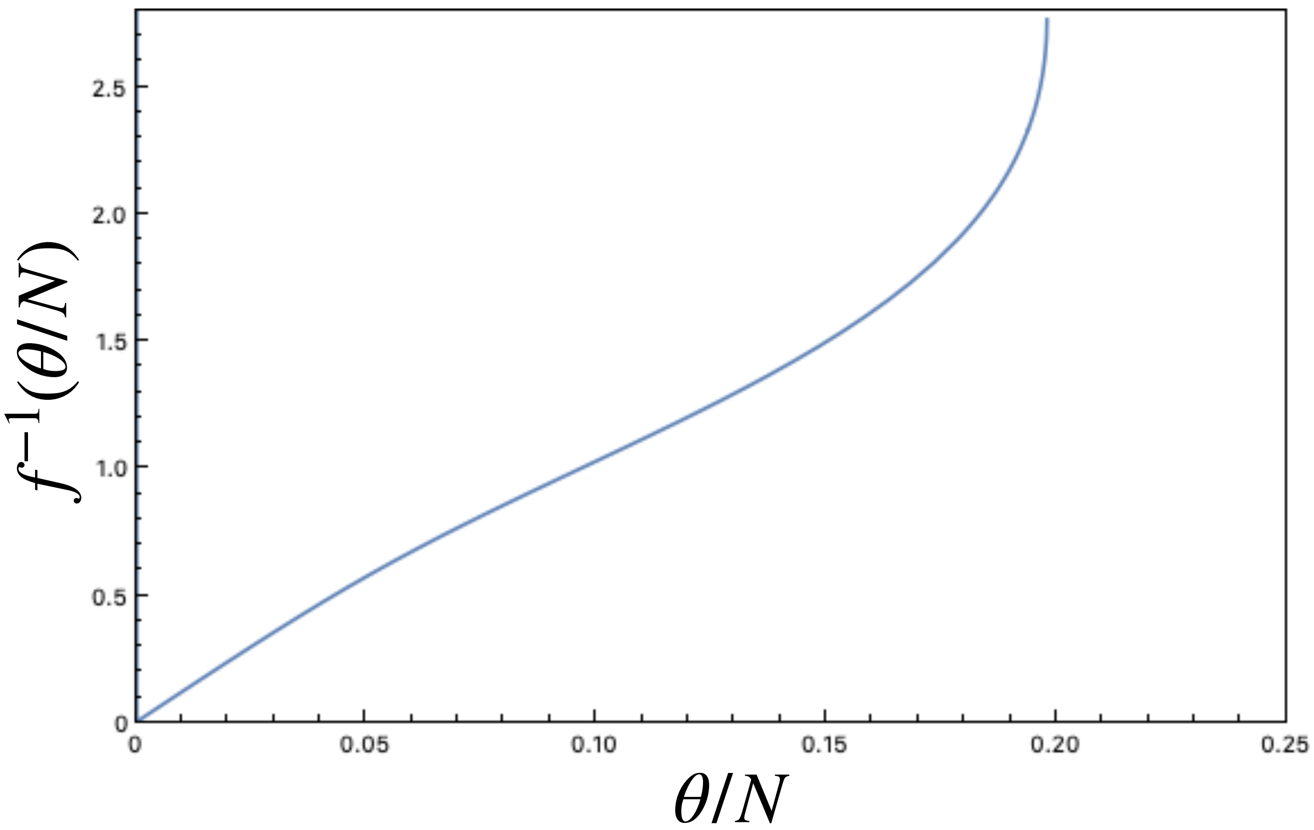
A saddle point of the real part of the effective potential

$$\boxed{\sigma/2\mathscr{E} = f^{-1}(\theta/N), \quad 2\mathscr{E} = \sigma_0 \exp\left(-\mathrm{Re}\psi\left(1/2 - \mathrm{i}f^{-1}\left(\theta/N\right)\right)\right)}$$

$\sigma_0 = M^2 \exp\left(-4\pi/g^2\right)$, ψ : Digamma function

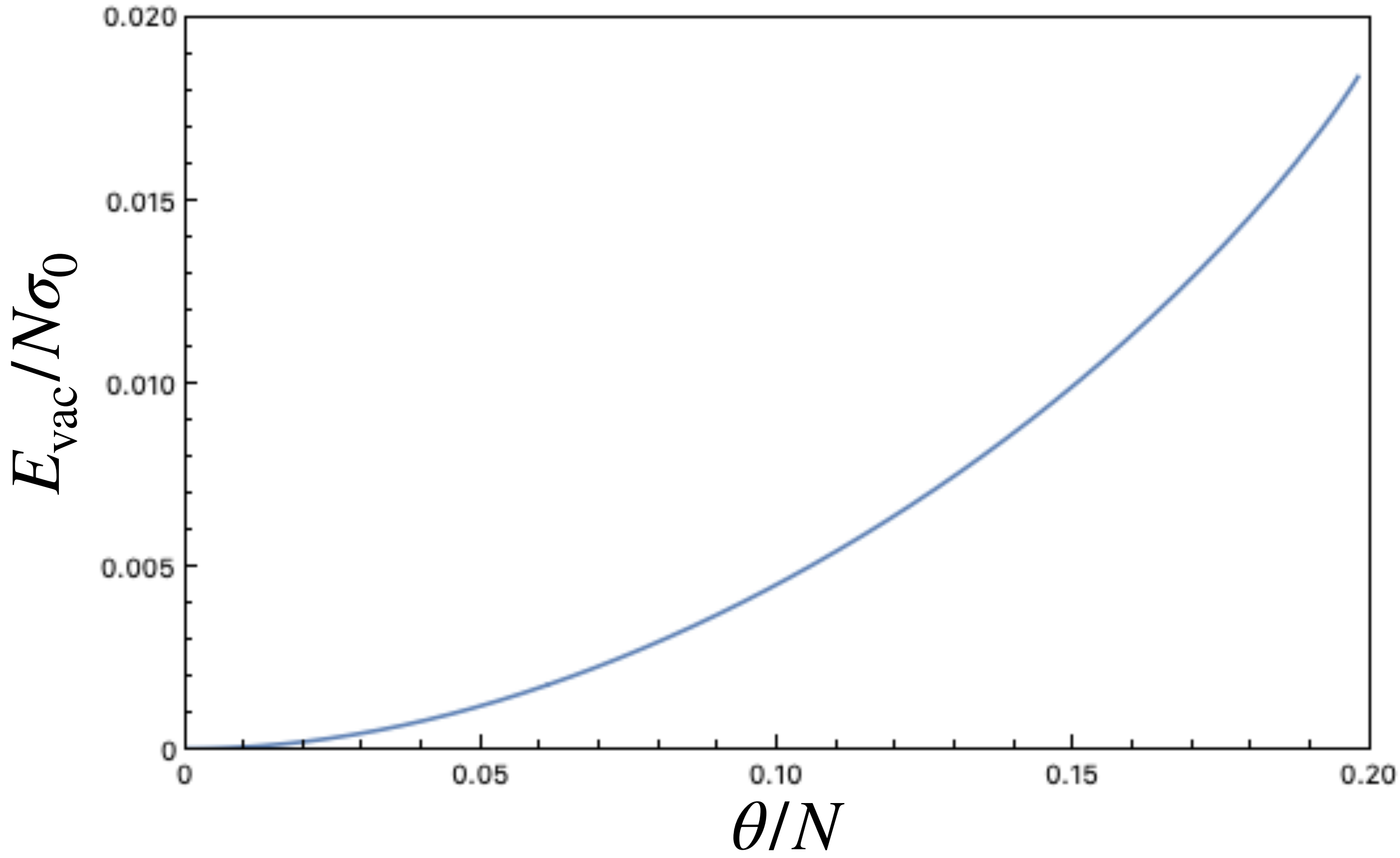
$$\Longrightarrow E_{\text{vac}} := \mathrm{Re}V_{\text{eff}}\big|_{\text{saddle point}}, \quad \Gamma := \mathrm{Im}V_{\text{eff}}\big|_{\text{saddle point}}$$

A plot of $\sigma/2\mathscr{E} = f^{-1}(\theta/N)$



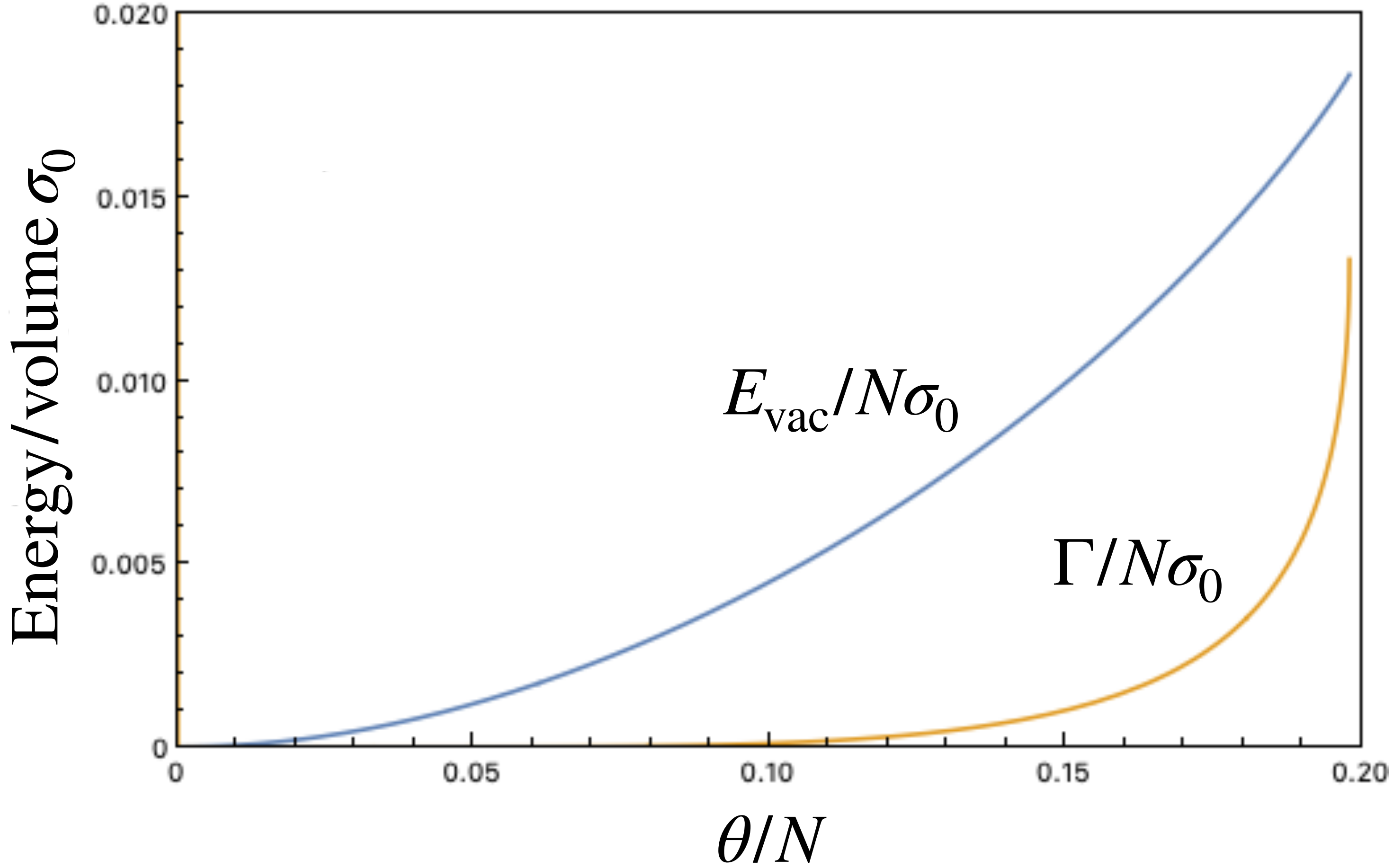
3. Result

Metastable vacuum energy



Note : It is defined between $0 \leq \theta \lesssim 0.2N$

Instability of metastable vacua



$E_{\mathrm{vac}} \sim \Gamma$ at $\theta \sim 0.2N \Rightarrow$ It is no longer regarded as metastable.

Summary

- We studied the behavior of the vacua energies beyond $\theta \sim O(1)$ of 2d $\mathbb{C}P^{N-1}$ model in large N limit.
- As a result, metastable vacua are not regarded as metastable at large θ because of high decay rates.