Interior-structure Dependence of Logarithmic Corrections to Bekenstein-Hawking Formula

RIKEN iTHEMS

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- **Work in progress**

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What is the quantum definition of black hole?

- Classical definition of black holes = existence of horizons

However,…

- No observational data showing the existence of horizons yet.
- Black-hole entropy $S = \frac{A}{4\hbar G} = \log \Omega$

  ⇒ black hole = essentially quantum object consisting of (still unknown) d.o.f.
- In quantum gravity, spacetime should fluctuates.

  ⇒ No a priori reason to follow the classical geometric definition.

⇒A possible approach is to consider

What is the quantum definition/characterization of black holes?

⇒Black hole? Information problem? D.o.f for quantum gravity?

Maximization of entropy

[Yokokura 2023]

- A *candidate* quantum definition motivated by thermodynamics and (local) holography is

<u>Black hole maximizes thermodynamic entropy for a given surface area.</u>

BH $=$ S_{max}

Similar ideas:
[Dvali-Gomez 2013,
Oriti-Pranzetti-Sindoni 2016]

- <u>Motivations</u>

(1) Entropy is quantum mechanical:

surface area A

$$S = \log\, \Omega\,, \qquad \Omega = \#\text{ of } \{|\psi\rangle \text{ consistent with } (\cancel{E,V})\}$$

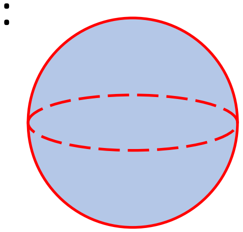
(2) Any configuration with mass $\frac{R}{2G}$ in a strong-gravity limit becomes a BH with size R.

⇒BH = a macroscopic state (phase) with maximum entropy according to 2nd law?

(3) Bekenstein-Hawking formula saturates the holographic entropy bound:

(still conjecture) $\quad S[L] \leq \dfrac{A(B)}{4l_p^2}$

[Bousso 1999]



Area of B: $A(B)$

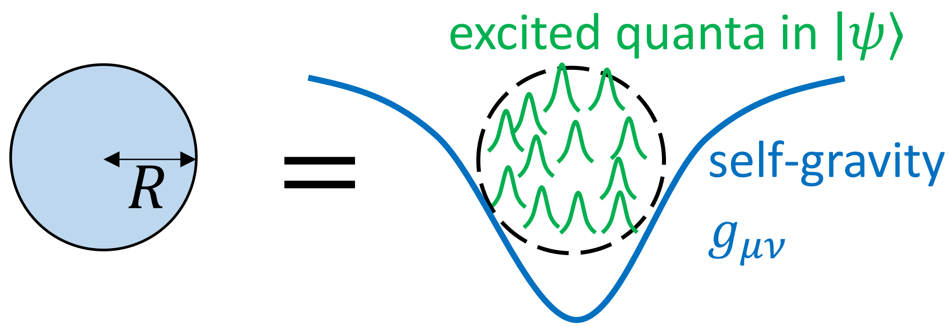
- <u>In the future</u>, this should lead to a quantum many-body description of black holes.

Semi-classical Einstein eq.

- *As a first trial*, we consider the 4D semi-classical Einstein eq with many matter fields in a self-consistent manner:

$$\boxed{G_{\mu\nu} = 8\pi G \langle \psi | T_{\mu\nu} | \psi \rangle}$$

$g_{\mu\nu}$:classical metric
ϕ_i:quantum operators



spherical static region with a surface area $4\pi R^2$ $=$ a collection of many excited quanta in $(|\psi\rangle, g_{\mu\nu})$

↓

thermodynamic entropy

$$S[g_{\mu\nu}, R) \underset{\text{Assume } \nabla_\mu \tilde{s}^\mu = 0}{=} \int_\Sigma d\Sigma_\mu \, \tilde{s}^\mu \underset{\text{spherical and static}}{=} \int_0^R dr \sqrt{g_{rr}(r)} \, s(r)$$

$s(r)$: entropy density per proper radial length

Semi-classical gravity condensate

- We have reached uniquely the entropy-maximizing configuration: [Yokokura 2023]

$$ds^2 = -\frac{2\sigma}{r^2} e^{-\frac{R^2 - r^2}{2\sigma\eta}} dt^2 + \frac{r^2}{2\sigma} dr^2 + r^2 d\Omega^2. \qquad (\leftarrow \text{Locally } AdS_2 \times S^2)$$

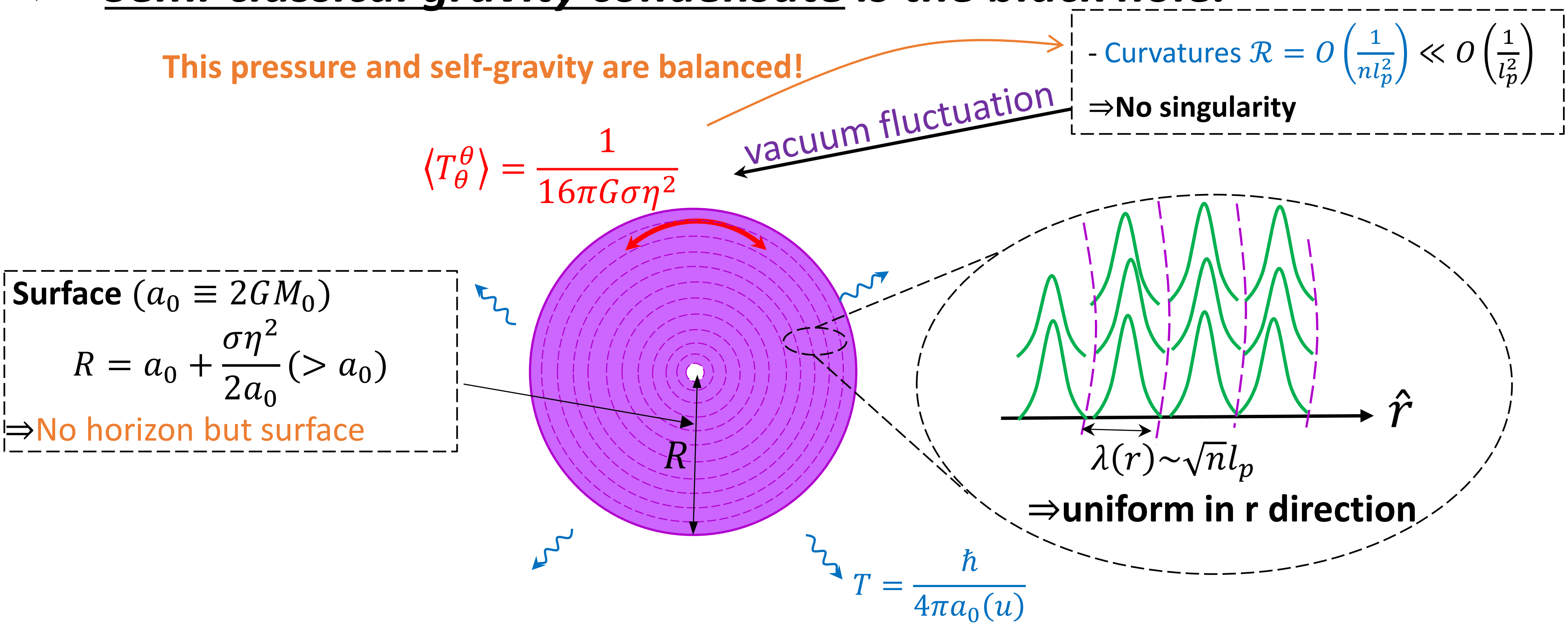
- Satisfies $G_{\mu\nu} = 8\pi G \langle \psi | T_{\mu\nu} | \psi \rangle$ with $n(\gg 1)$ scalar fields non-perturbatively in $\hbar$ for

$$\sigma = \frac{n l_p^2}{120\pi\eta^2}, \qquad 1 \le \eta < 2.$$

[Kawai-Yokokura 2020]

- Represents that self-gravitating quanta condensates into a dense configuration.

⇒ ***Semi-classical gravity condensate is the black hole.***



Maximum entropy S_{max}

[Yokokura 2022, 2023]

- The excited quanta behave like a local thermal state at

$$T_{loc}(r) = \frac{\hbar\alpha(r)}{2\pi} = \frac{\hbar}{2\pi\sqrt{2\sigma\eta^2}}. \quad \sim\frac{m_p}{\sqrt{n}}$$

Unruh temperature

- In this local equilibrium, we have

$$T_{loc}s = \rho_{1d} + p_{1d}, \qquad p_{1d} = \frac{2-\eta}{\eta}\rho_{1d}.$$

1D Gibbs relation — equation of states

$$\rho_{1d} = 4\pi r^2(-\langle T^t_t\rangle), \quad p_{1d} = 4\pi r^2\langle T^r_r\rangle$$

$$-\langle T^t_t\rangle = \frac{1}{8\pi G r^2}, \quad \langle T^r_r\rangle = \frac{2-\eta}{\eta}(-\langle T^t_t\rangle)$$

- Then, we can evaluate the entropy density: $s(r) = \frac{2\pi\sqrt{2\sigma}}{l_p^2}$

⇒Integrating it over the volume produces the Bekenstein-Hawking formula:

$$S_{max} = \int_0^R dr\sqrt{g_{rr}(r)}s(r) = \int_0^R dr\sqrt{\frac{r^2}{2\sigma}\frac{2\pi\sqrt{2\sigma}}{l_p^2}} = \frac{A}{4l_p^2}$$

$$A \equiv 4\pi R^2 \approx 4\pi a_0^2$$

σ cancels out!

<u>Note: The self-gravity changes the volume law to the area law.</u>

- This derives the Bousso bound for thermodynamic entropy: $S \leq S_{max} = \frac{A}{4l_p^2}$.

Question

[Yokokura, work in progress]

- To study the gravity condensate in more detail, let's ask

Can we consider shorter fluctuations of the gravity condensate and see the interior-structure dependence in thermodynamic quantities (e.g. entropy)?

- We determine the sub-leading terms $(a_1(r), b_1(r))$ for the $1/r$ expansion:

$$ds^2 = -\left(1 - \frac{a(r)}{r}\right) e^{b(r)} dt^2 + \left(1 - \frac{a(r)}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$a(r) = r - \frac{2\sigma}{r} + a_1(r), \qquad b(r) = \frac{r^2}{2\eta\sigma} + b_1(r)$$

Leading part of
semi-classical gravity condensate
for $r \gg l_p$

⇒Let's examine macroscopic and microscopic approaches.

Approach 1 (1/2): Conformal matter and Thermodynamics

[Yokokura, work in progress]

- Consider conformal matter fields because $\mathcal{R} = O\left(\frac{1}{n l_p^2}\right)$.

⇒The key eq:

$$G^{\mu}_{\mu} = 8\pi G \langle\psi|T^{\mu}_{\mu}|\psi\rangle = 8\pi G\hbar(c_W\mathcal{F} - a_W\mathcal{G} + b_W\Box R)$$

4D Weyl anomaly

$$\mathcal{F} \equiv C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}, \qquad \mathcal{G} \equiv R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

$c_W, a_W, b_W \sim$ d. o. f. of matter fields.

⇒Self-consistent value:

$$\sigma = \frac{8\pi l_p^2 c_W}{3\eta^2}$$

- The other key eq:

$$\langle T^r_r\rangle = \frac{2-\eta}{\eta}(-\langle T^t_t\rangle).$$

equation of state

- We can solve the two eqs order by order.

Approach 1 (2/2): Conformal matter and Thermodynamics

[Yokokura, work in progress]

- Apply thermodynamics locally.

$$A = 4\pi R^2 \approx 4\pi a_0^2$$
$$a_0 \equiv 2GM$$
$$L \sim \sqrt{n} l_p$$

- The entropy is given by

$$S = \int_L^R dr \sqrt{g_{rr}(r)} s(r) = \frac{A}{4l_p^2} + \frac{4\pi\sigma\eta}{l_p^2} \log \frac{R}{L}$$

⇒Using $\sigma = \frac{8\pi l_p^2 c_W}{3\eta^2}$, we reach

$$S = \frac{\pi a_0^2}{l_p^2} + \frac{32\pi^2 c_w}{\underbrace{3\eta}_{\frac{n_s}{180\eta}}} \log \frac{a_0}{l_p}$$

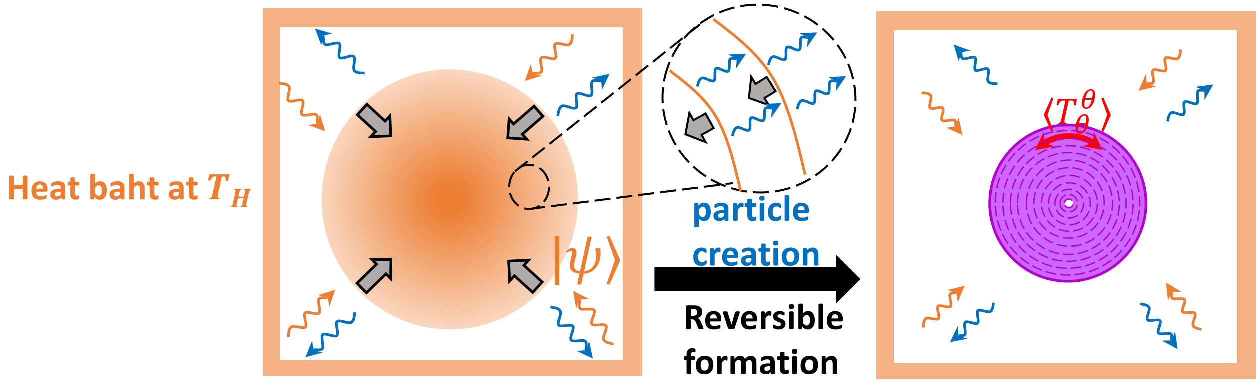
for n_s scalar fields: $c_w = \frac{n_s}{1920\pi^2}$

$$\langle T^r_r \rangle = \frac{2-\eta}{\eta}(-\langle T^t_t \rangle)$$
$$1 \le \eta < 2$$

The eq-of-state dependence appears!

Approach 2 (1/2): Microscopic S-wave model

- The entropy-maximizing configuration should be formed by a reversible process in a heat bath with Hawking temperature.

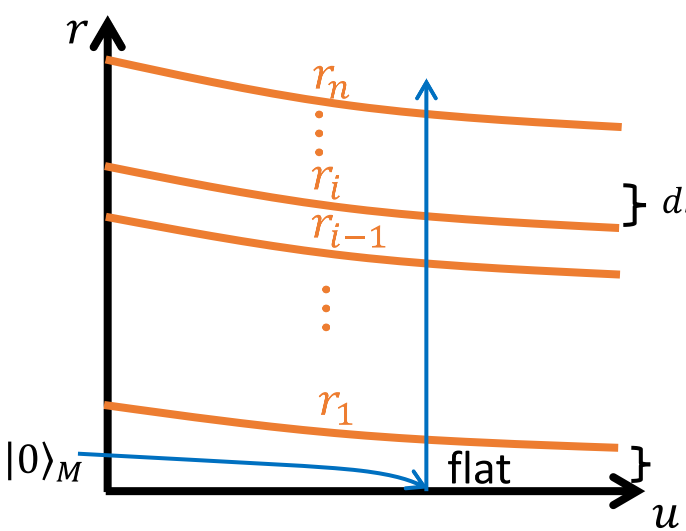


[Kawai-Yokokura 2014, 2021]

Semi-classical gravity condensate can be obtained as a result of time evolution, not the static assumption.

- We model this process by s-waves in WKB approximation. [Kawai-Matsuo-Yokokura 2013, Kawai-Yokokura 2020]

Radiation and created particle = excited s-wave of scalar field in WKB approximation



$$ds_i^2 = -\left(1 - \frac{a_i}{r}\right) du_i^2 - 2du_i dr + r^2 d\Omega^2$$

Energy flux formula of particle creation

$$J_i \equiv 4\pi r^2 \langle 0_M | T_{u_i u_i} | 0_M \rangle = \frac{\hbar n_s}{16\pi} \{u_i, U\}$$

EoM is

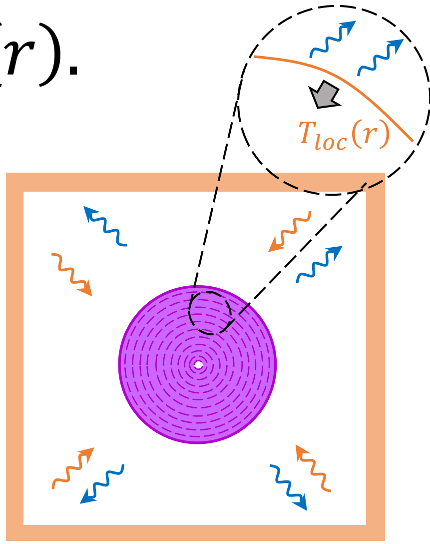
$$\frac{da_i(u_i)}{du_i} = -2GJ_i$$

Approach 2 (2/2): Microscopic S-wave model

[Yokokura, work in progress]

- Solving the time evolution self-consistently, we can obtain the consistent sub-leading terms with Approach 1!

- Particle creation $J_i = \frac{\hbar n_s}{16\pi}\{u_i, U\}$ determines $T_{loc}(r)$.



- Microscopic evaluation of the entropy is given by

$$\hat{s}(r) = \frac{1}{2}\int_0^\infty d\tilde{\omega}\, D(\tilde{\omega}; r)\, g_s(\hbar\beta_{loc}(r)\tilde{\omega})$$

entropy of harmonic oscillator with $\tilde{\omega}$ and $T_{loc}(r)$

$$S = \int_L^R dr\, \hat{s}(r) = \frac{A}{4l_p^2} + \underbrace{\frac{4\pi\sigma_s}{l_p^2}}_{= \frac{n_s}{24}} \log\frac{R}{L}$$

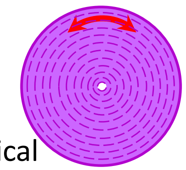
$$\sigma_s \equiv \frac{n_s l_p^2}{96\pi}$$

⇒consistent with Approach 1!

Conclusion

- A quantum characterization of BH: [Yokokura 2023]

  "Black hole maximizes thermodynamic entropy for a given surface area."

BH $\overset{\text{def}}{=}$ S_{max} $\overset{\text{result}}{=}$ (for spherical static cases)  semi-classical gravity condensate

- Two approaches (macro and micro) led to the same sub-leading term:

$$S_{max} = \frac{A}{4l_p^2} + \frac{4\pi\sigma\eta}{l_p^2}\log\frac{R}{L}$$

conformal matter case:

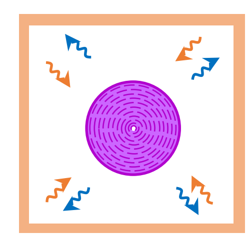
$$\sigma = \frac{8\pi l_p^2 c_W}{3\eta^2}$$

⇒Consistency of the gravity condensate

[Yokokura, work in progress]

- The eq-of-state dependence could be observed by a precise measurement of the equilibrium temperature T in the bath:

$$dM_0 = TdS_{\mathrm{max}} \Rightarrow \qquad T = \frac{\hbar}{4\pi a_0}\left(1 - \frac{4\sigma\eta}{a_0^2}\right)$$



T ?

- This could provide some clue about the effective theory for interior of BHs:

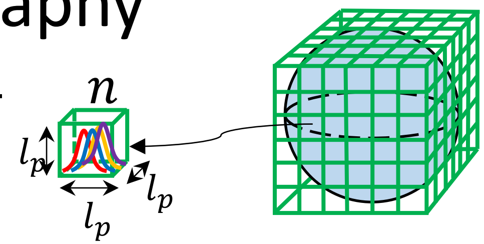
$$\eta = \eta(\alpha, \beta, \lambda) \qquad S_{\mathrm{finite}} = \int (\alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + \lambda\phi^4)$$

Future prospects（1/2）

- 1. Role of self-gravity in holography

[Yokokura 2023]

  - $s_{\text{self-gravity}}(r) \sim \frac{\sqrt{n}}{l_p r^2} \ll s_{\text{naive}}(x) \sim \frac{n}{l_p^3}$



  - The s-wave model can explain $S = \frac{A}{4 l_p^2}$.
  - The two approaches provide the same Log corrections.

⇒***Self-gravity suppresses excitations of local d.o.f. in the bulk.***
⇒Origin of holography?

- 2. Gravity-condensate phase

[Yokokura 2023]

  - The radial direction is uniform:

$$s_{1d}(r) = const., \qquad \rho_{1d}(r) = const., \qquad T_{loc}(r) = const.$$

⇒The gravity condensate is a thermodynamic phase?

In a (non-rel) material, $\lambda_T = \frac{\hbar}{\sqrt{mT}} \sim \rho_N^{-1/3}$ ⇒quantum effects governs the system.

In the gravity condensate, $\lambda(r) \sim \frac{\hbar}{T_{loc}(r)} \sim \mathcal{R}(r)^{-\frac{1}{2}}$. ⇒quantum gravitational?

⇒ Can we make an effective theory/a quantum many-body model?

[Livine and Yokokura, work in progress]

Future prospects（2/2）

[Yokokura 2023]

- 3. Relations to other models for gravity condensate

  - Model in Group Field Theory [Oriti-Pranzetti-Sindoni 2016]
    ⇒ radially uniform
  - Model in many gravitons [Dvali-Gomez 2013]
    ⇒ Their picture appears in each spherical subsystem of ours.

  ⇒My gravity condensate has

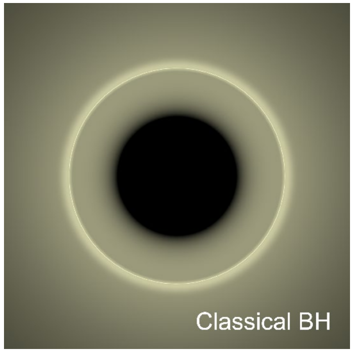
$$N_{matter}(r) \sim N_{graviton}(r) \sim n \ (\gg 1)$$

  ⇒It represents semi-classically a mixture of matter quanta and gravity quanta?

[Chen-Yokokura 2024]

- 4. Phenomenology

  -Imaging the semi-classical gravity condensate.
  ⇒consistent with current observations!
  - How about GW?



Classical Schwarzschild BH

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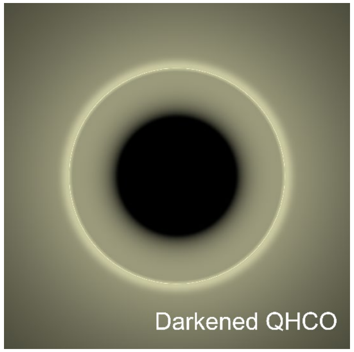

Our gravity condensate

Thank you very much!!