Shock waves and delay of hyperfast growth in dS complexity

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Motivation-1

What is holographic description of de Sitter?

- One suggestion is dS/CFT. (Analytic continuation from AdS/CFT)

Recently, it is conjectured that

- Two-dim dS $\Leftrightarrow$ DSSYK_{∞} [Susskind]
- Holographic screen is stretched horizon.
- De Sitter shows the exponential expansion $\Rightarrow$ Complexity shows "hyperfast"
- Let us consider more about dS complexity

Motivation-2

- In particular, we would like to study the responses from shockwave.

- OTOC calculation [Stanford,Shenker]
- Created S-AdS case [Chapman, Marrochio, Myers]
- De sitter without shockwave [Jørstad ,Myers, Ruanc]
- This holographic complexity calculation shows hyperfast

- Let us discuss about how small perturbations can change this property

de Sitter Complexity and Hyperfast

Complexity ⋯ minimal # of gates required to achieve a particular state from a reference state

- In holographic setting, this reflects the geometrical structure of the bulk spacetime

- There are various proposals. The complexity corresponds to;

CA ⋯ Full gravitational action of the WdW patch

CV2.0 ⋯ The spacetime volume of WdW patch

CV ⋯ Volume of codimension-1 extremal surfaces

de Sitter Complexity and Hyperfast

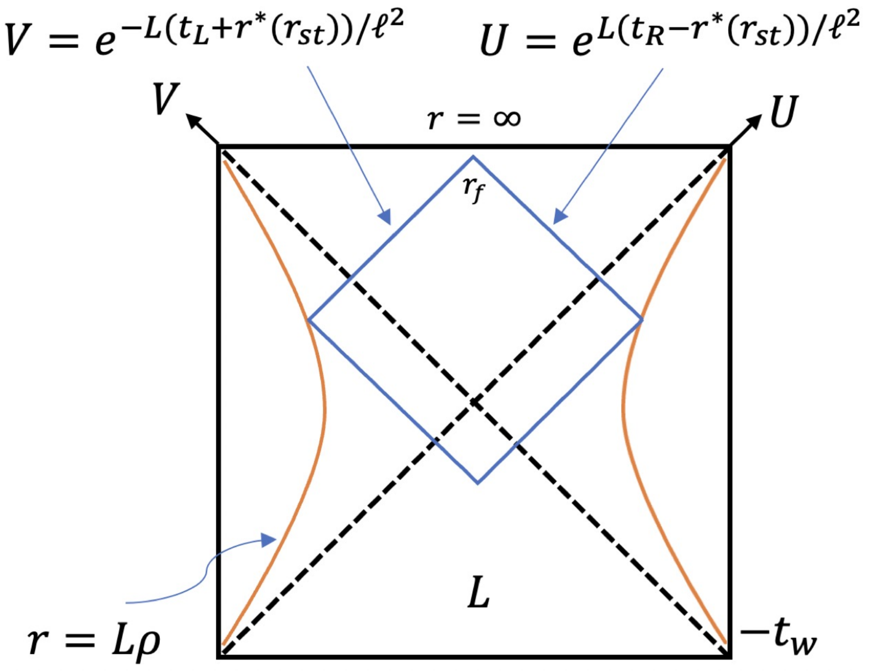
- Let us consider the Complexity of $\mathbf{dS}_{d+1\geq 3}$ spacetime[Jørstad ,Myers, Ruanc]
- The implicit assumption is that the Holographic screen is a stretched horizon, and boundary time is flowing over it.
- They showed the Complexity diverges at the critical point τ_∞

$$\lim_{\tau\to\tau_\infty} \mathcal{C}_V \to \infty, \quad \lim_{\tau\to\tau_\infty} \frac{d\mathcal{C}_V}{d\tau} \to \infty.$$

- CA and CV2.0 behave similar ways(Strictly speaking, the critical time is determined by the WdW patch).
- CV also diverges at critical time τ_∞

de Sitter Complexity and Hyperfast

- By using CV2.0, we can show complexity diverges at particular time τ_∞ and estimate it.



- Intuitively, The main contribution to complexity comes from the point r_f in WdW where r is the largest
- We can calculate τ_∞ as the time for the WdW patch to reach $r_f = \infty$

$$\tau_\infty = \mathrm{Arctanh}\rho$$

Shockwave geometry and de Sitter Complexity

- Let us look at how small perturbation affects the properties of hyper-fast, especially critical time.

- First, we will review shockwave geometry for both AdS and dS. Then we will move on to the specific calculations and the result.
- As understood from the case of pure dS, if you only want to find the critical time, you only need to check the WdW patch.

Shockwave geometry-1

- BTZ+single shockwave

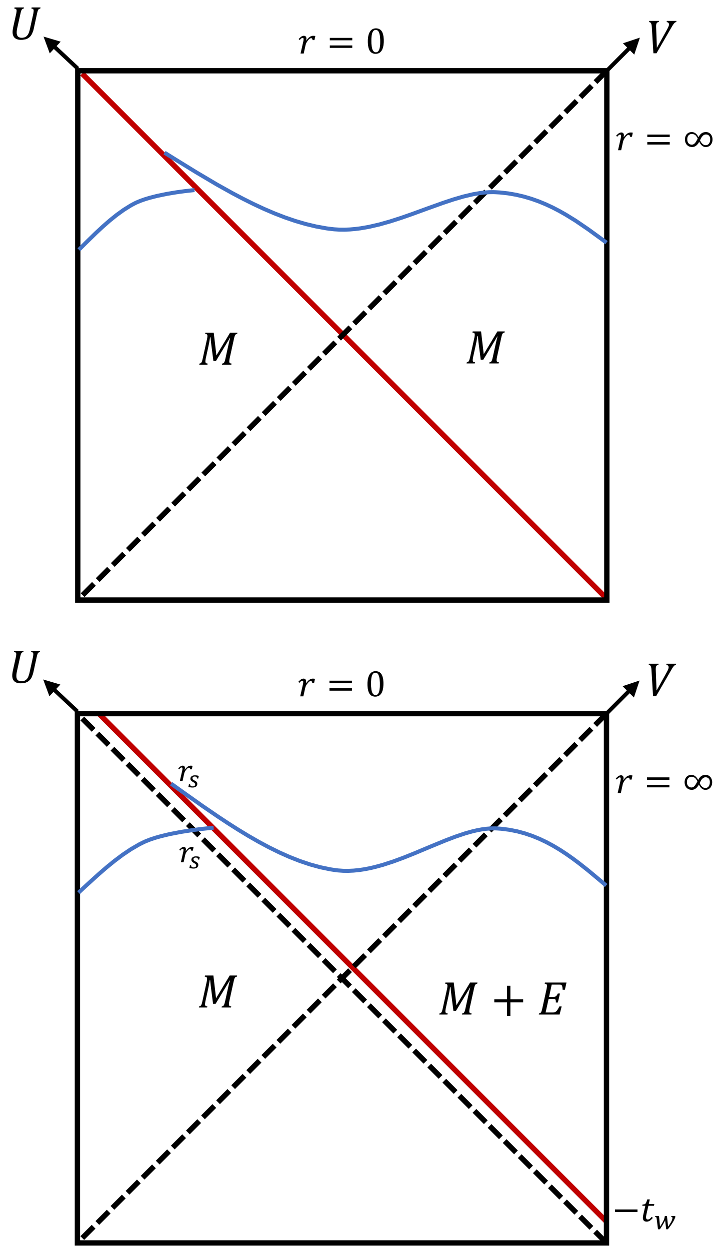
$$T_{VV} = \frac{\alpha}{4\pi G_N}\delta(V)$$

- From ANEC, $\alpha > 0$.
- This spacetime is devided by shockwave
- when shockwave is localized on horizon, the solution is well known.
- This is bh spacetime of the same mass pasted together with a constant shift $\alpha > 0$

$$\alpha = \frac{E}{4M} e^{\frac{R}{\ell^2} t_w}$$

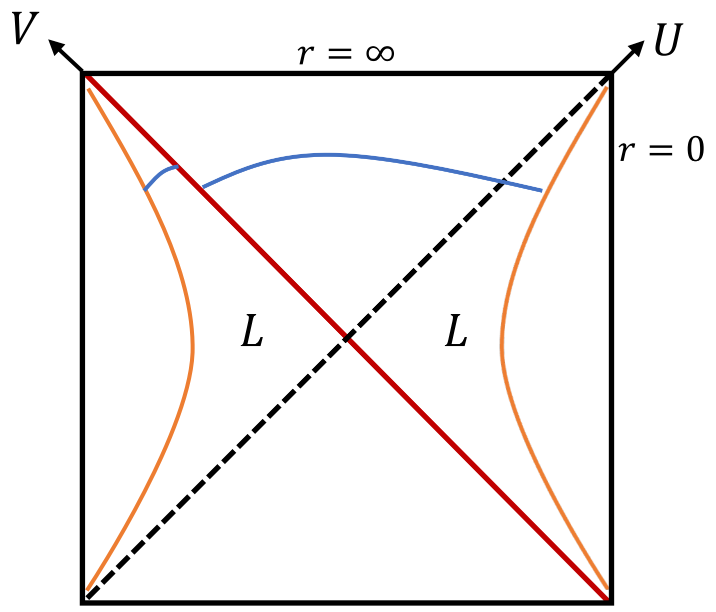
under the limit $E \to 0$.

- If the shockwave is not localized on horizon, the mass of the black hole in the two patches is different.
- On the shockwave, the continuity of r is imposed. And there is a shift, although it is not simple.

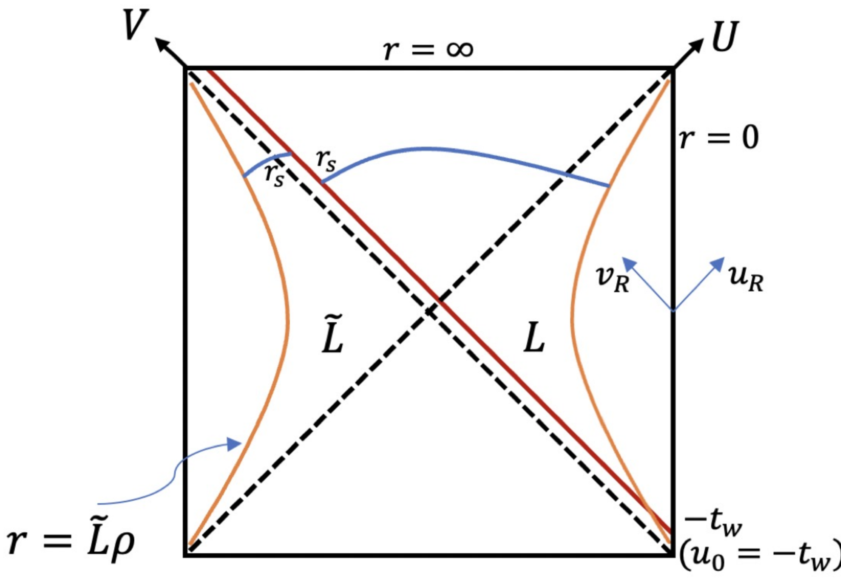


Shockwave geometry-2

- (S)dS+single shockwave $T_{UU} = \frac{\beta}{4\pi G_N}\delta(U)$
- From ANEC, $\beta > 0$.
- when shockwave is localized on horizon, This is SdS pasted together with a negative constant shift $-\beta$ under the certain limit



- If the shockwave is not localized on horizon, the pure dS and S-dS patches are pasted together.
- To be consistent with the positive energy insertion, the lower patch must be S-dS. Otherwise, we can derive that the energy insertion is negative.
- On the shockwave, the continuity of r is imposed. And there is a shift, although it is not simple.



CV2.0 argument

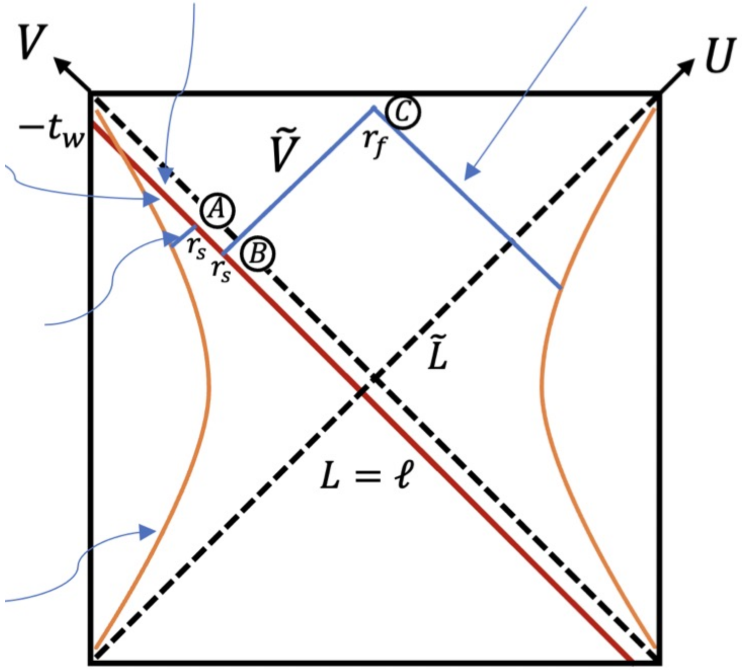
- When there is a shockwave, the future boundary of the WdW patch is shifted.
- Variation of τ_∞ can be calculated, and the general formula is

$$\Delta\tau_\infty = \tau_\infty = r^*(r_s) - \tilde{r}^*(r_s) \qquad r^*(r) = \int \frac{dr}{f(r)} = \frac{\ell^2}{2L}\log\left|\frac{L+r}{L-r}\right|$$

- If we consider shockwave→horizon limit

$$\Delta\tau_\infty \propto \beta\ (>0)$$

- Therefore, the critical time was found to be delayed
- Based on the method of finding geodesic in the Vaidya, CV reproduces the same result.



Summary

- With shockwave, dS behaves exactly opposite to AdS. In particular, the constant shift of the patches is exactly opposite.
- According to CV(2.0), critical time can be easily **delayed** by a shockwave with positive Energy. Thus, it is indeed affected by small perturbations.
- This can be understood that shockwave makes universe "radiation/matter dominance" instead of cosmological constant dominance. Then universe no more exponentially expands. This is why the hyper-fast property can be destroyed.
- We can expect to observe similar behavior in the calculations from CA. It would also be interesting to examine the general time dependence, not limited to late time.

Thank you for your attention.

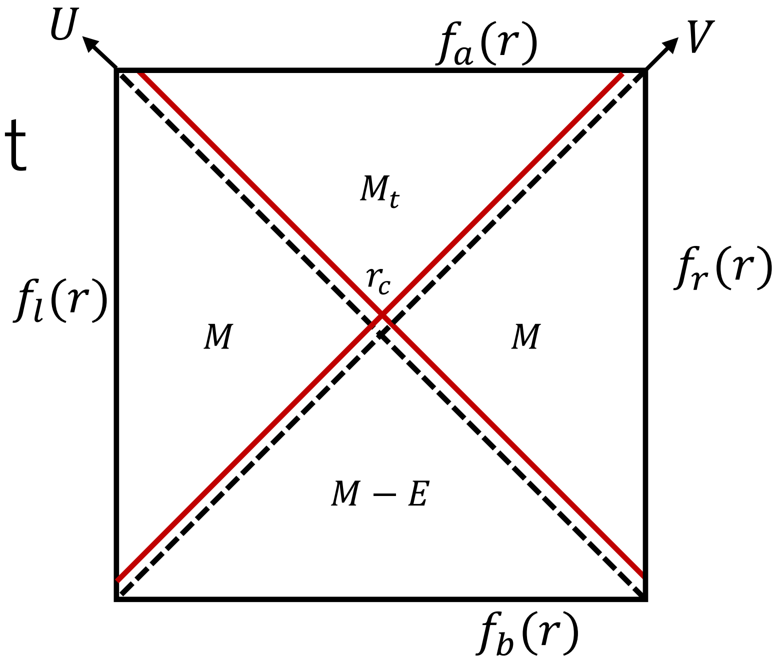
Thank you for listening.

Shockwave geometry-3

- double shockwaves
- If shockwave is localized on horizon, it's just pasting with constant shift ($\alpha \ll 1$).

$$ds^2 = \frac{-4\ell^2 dU dV}{(1+UV)^2} + 4\ell^2\alpha\delta(U)dU^2 + 4\ell^2\alpha\delta(V)dV^2 + R^2\left(\frac{1-UV}{1+UV}\right)^2 d\phi^2.$$

- On each shockwave, continuity of r is imposed as before.
- In r_c, additional independent equation is also imposed $f_a(r_c)f_b(r_c) = f_l(r_c)f_r(r_c)$
- This determines the nontrivial mass $M_t = M + 2E + \frac{8G_N\ell^2E^2}{8G_N\ell^2M - r_c^2}$



Shockwave geometry-4

- SdS

$$ds^2 = \frac{-4\ell^2 L^2 dU dV}{(L^2 - UV)^2} + L^2 \left(\frac{L^2 + UV}{L^2 - UV}\right)^2 d\phi^2$$

$$L^2 = (1 - 8G_N M)\ell^2$$

- Method1, If the shockwave is localized on horizon

$$T_{UU} = \frac{\alpha}{4\pi G_N \ell^2}\delta(U)$$

$$ds^2 = \frac{-4\ell^4 dU dV}{(\ell^2 - UV)^2} - 4\alpha\delta(U)dU^2 + \ell^2 \left(\frac{\ell^2 + UV}{\ell^2 - UV}\right)^2 d\phi^2$$

$$ds^2 = \frac{-4\ell^4 dU dV}{(\ell^2 - U(V - \alpha\theta(U)))^2} + \ell^2 \left(\frac{\ell^2 + U(V - \alpha\theta(U))}{\ell^2 + U(V - \alpha\theta(U))}\right)^2 d\phi^2$$

- It is the exact opposite shift from AdS case. This allows travel from a pole to the other pole.