Kyoto, Sep. 26th 2023

Quantum Electrodynamics in 2+1 Dimensions as the Organizing Principle of Triangular Lattice Antiferromagnets

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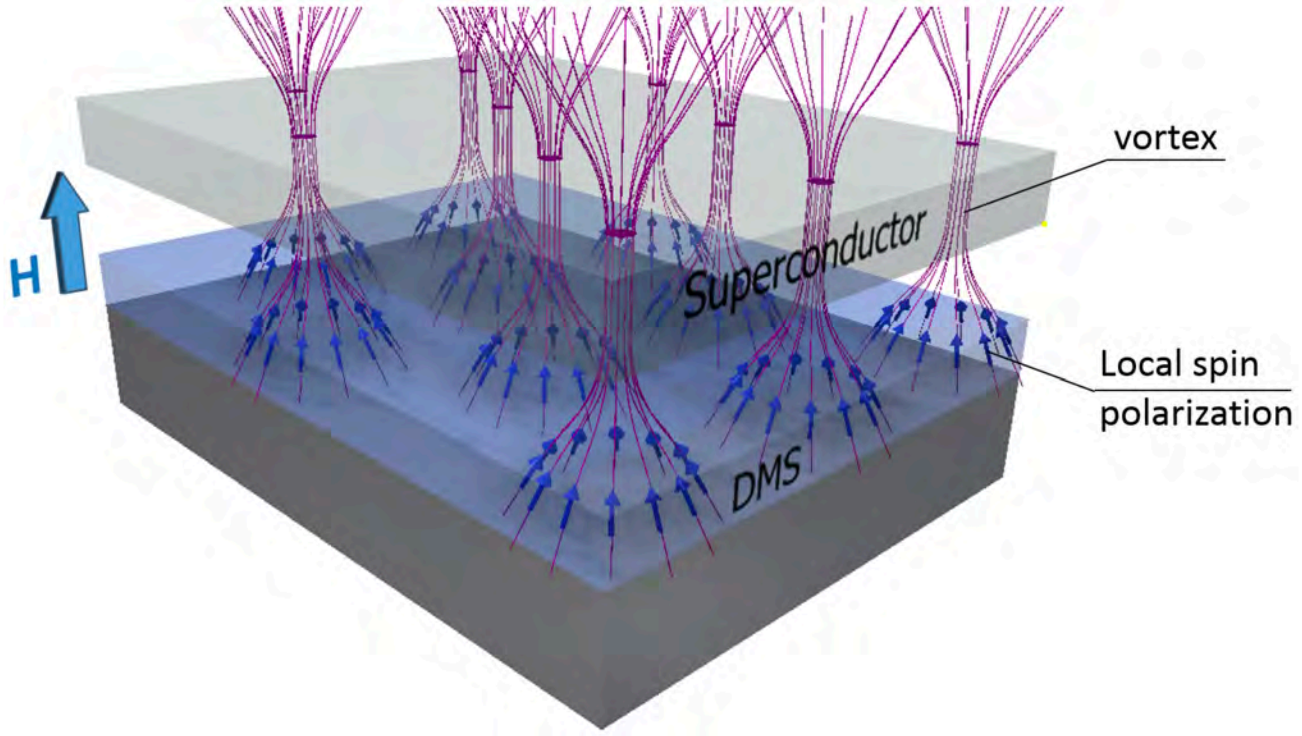
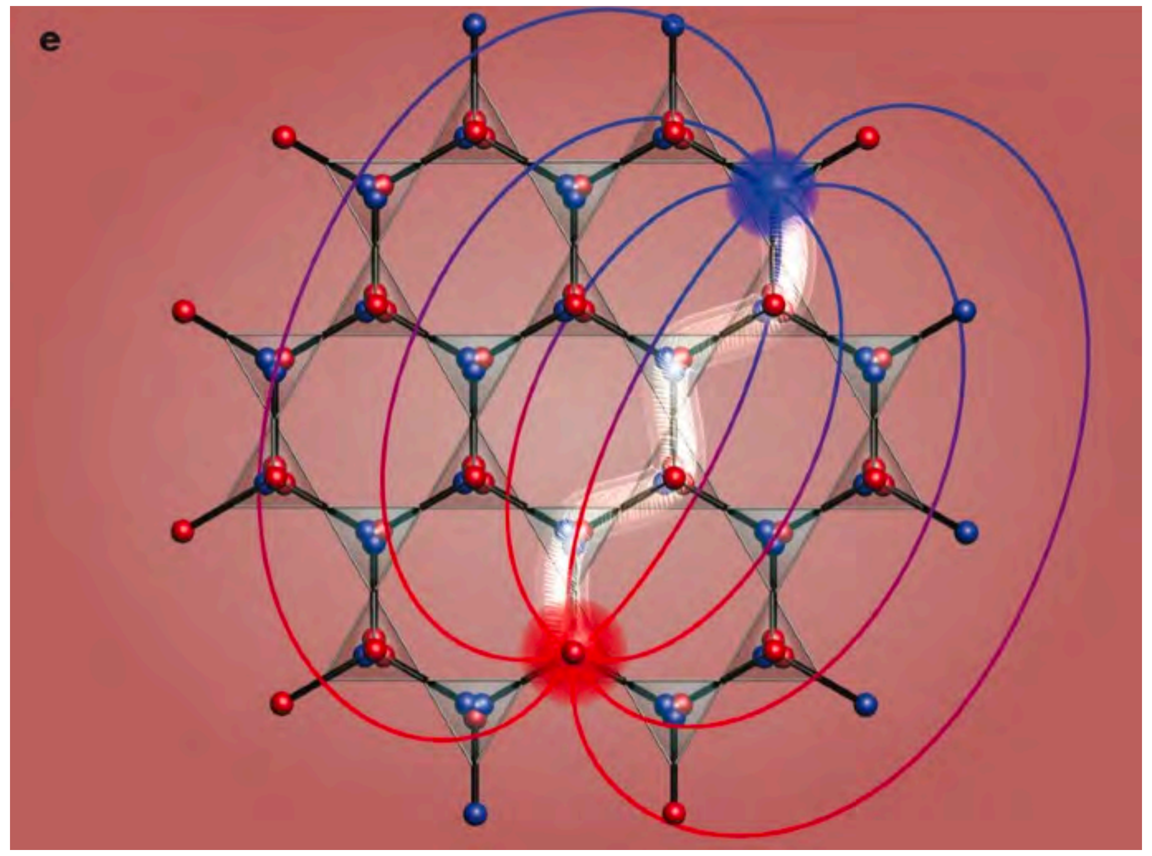
Field theory in condensed matter

Fluid dynamics, elasticity

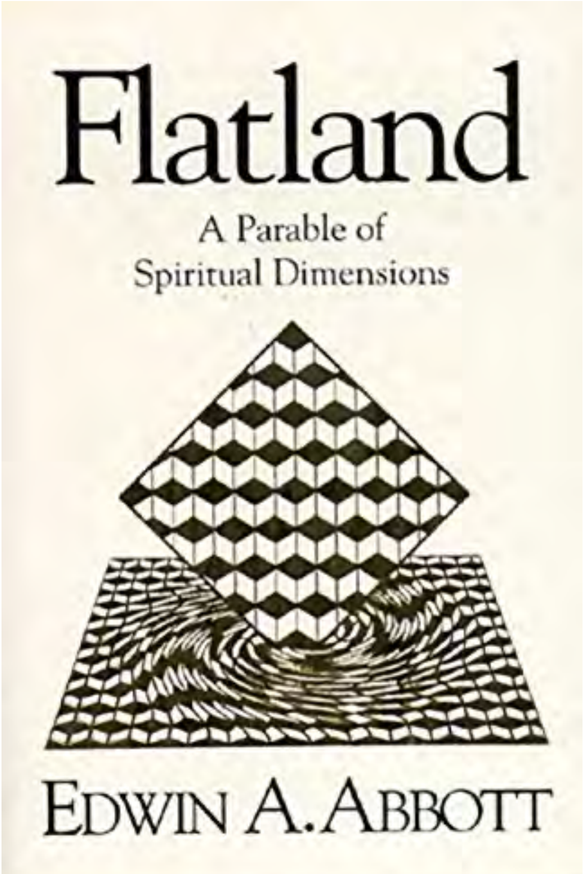
Ginzburg-Landau theory of superconductivity



Emergent electrodynamics in spin ice



- Today: Quantum Electrodynamics in 2+1 dimensions (QED_3) in triangular lattice antiferromagnets



Simulating Physics with Computers

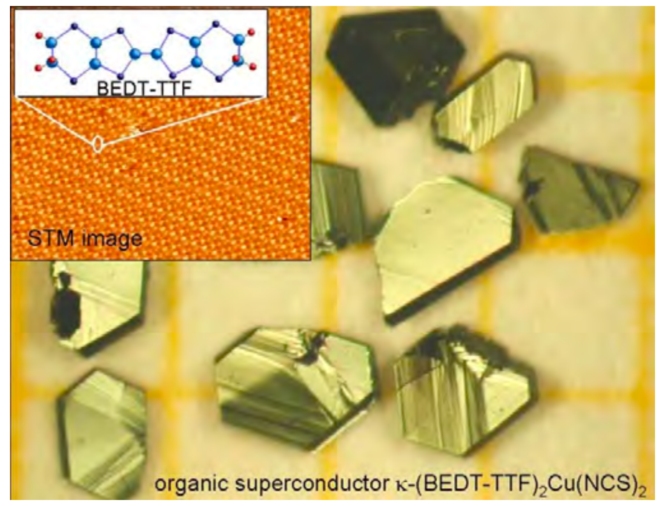
Richard P. Feynman

Richard Feynman, Int. J. Theor. Phys. 21 (1982)

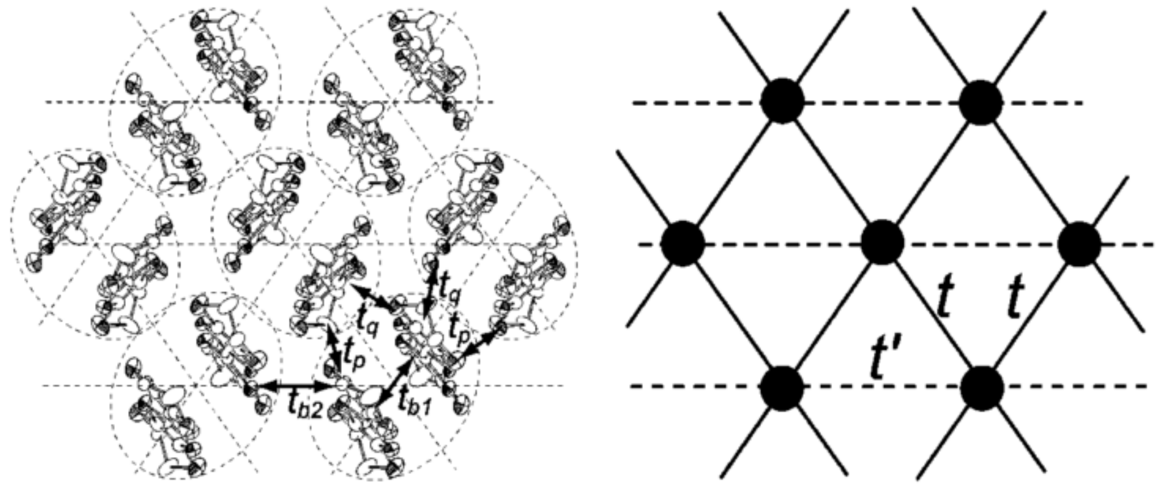
"It does seem to be true that all the various field theories have the same kind of behavior, and can be simulated in every way, apparently, with little latticeworks of spins and other things."

Antiferromagnetism in triangular magnets

- **Long-range** spin ordering: $|\langle \mathbf{S}_0 \cdot \mathbf{S}_r \rangle| \to \text{const}$ (as $r \to \infty$)
- Breaking continuous SU(2) spin rotation symmetry (at $T = 0$)
- Gapless **Goldstone** modes are excitations
- Observation of **Bragg peaks** in scattering experiments

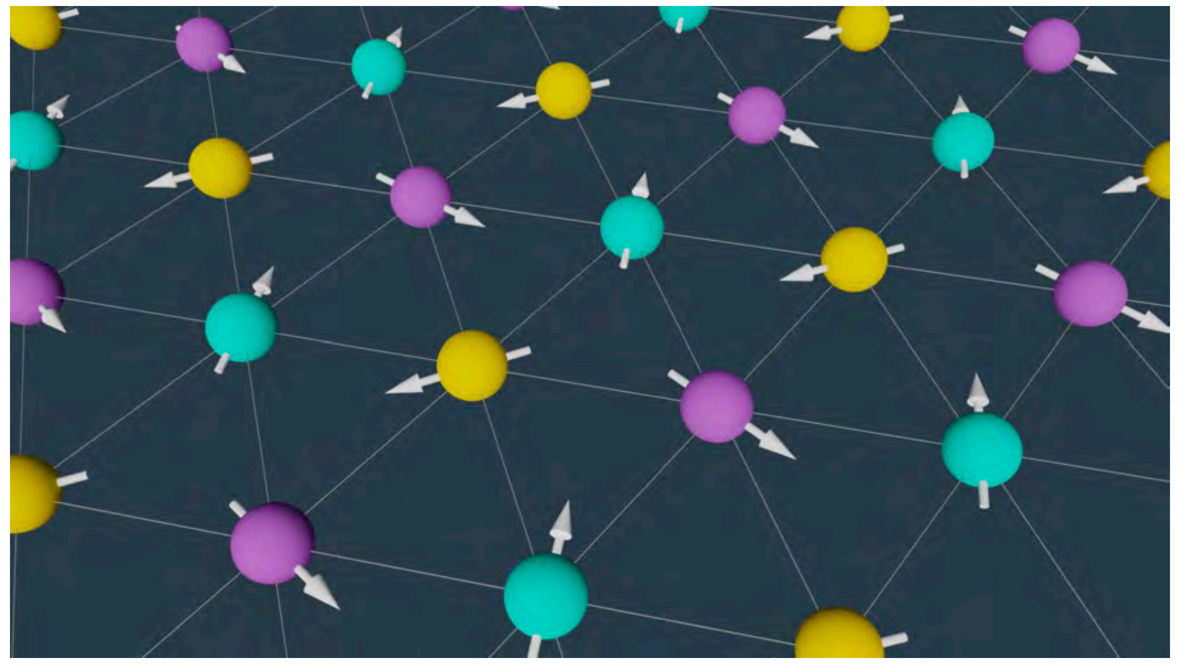


$\kappa - (\mathrm{BEDT} - \mathrm{TTF})_2\mathrm{Cu}_2(\mathrm{CN})_3$

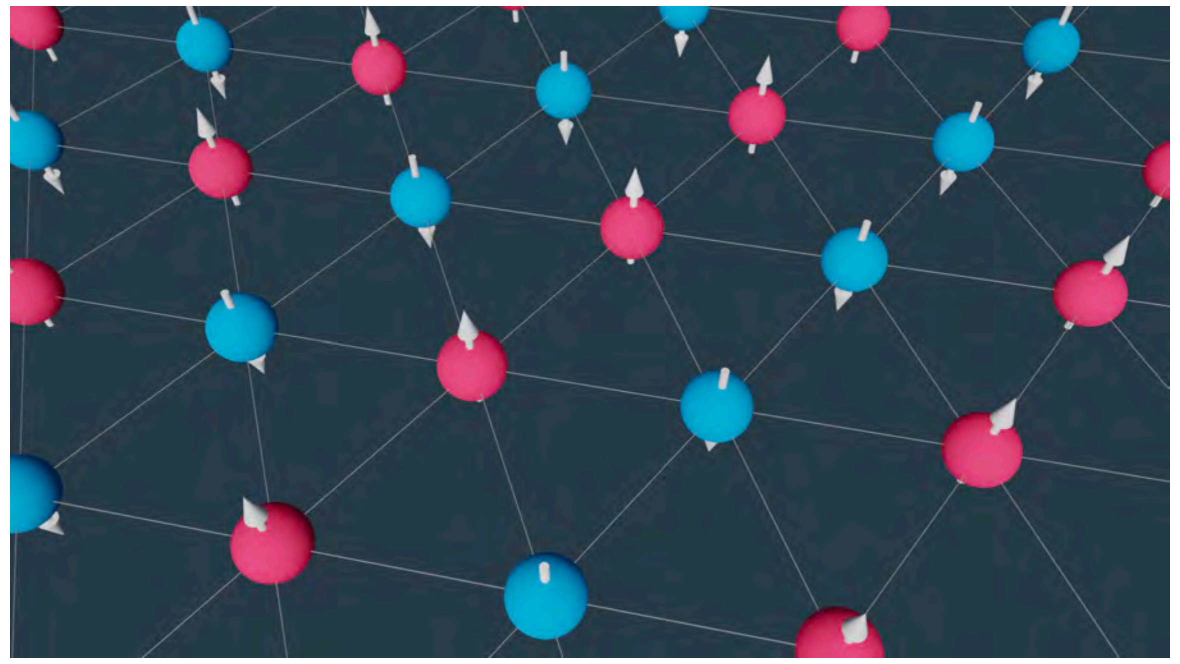


[Y. Shimizu et al., Phys. Rev. Lett. 91, 107001 (2003)]

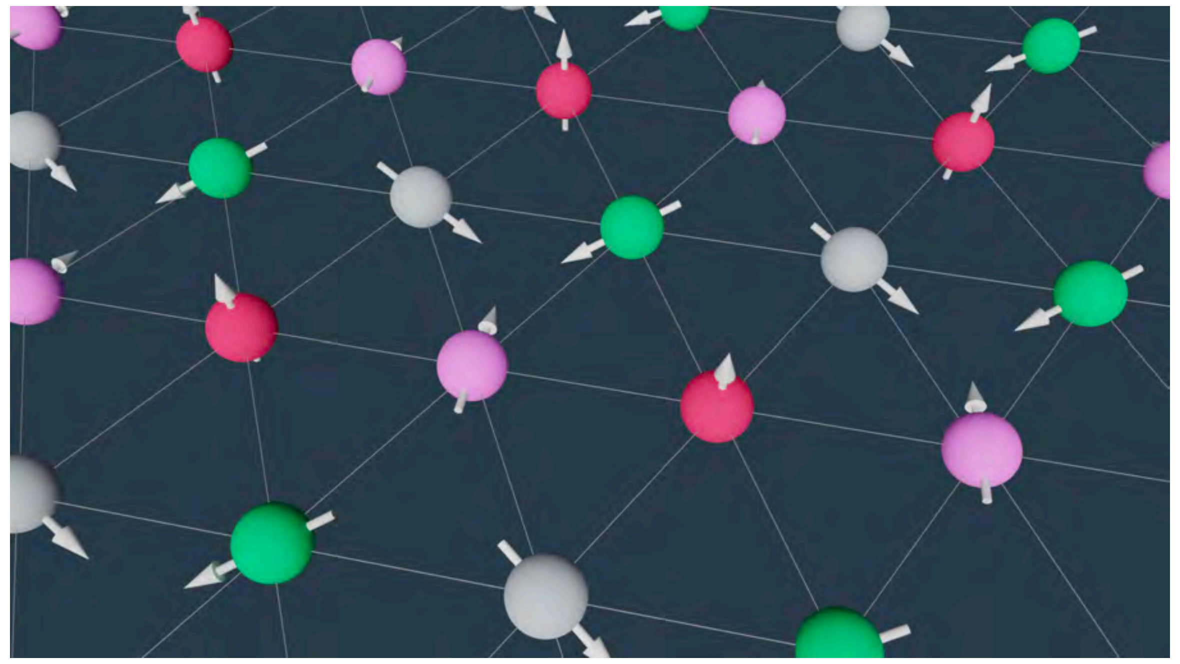
120° Néel AFM, $\mathbf{q} = K$



Stripy AFM, $\mathbf{q} = M$



Tetrahedral AFM, $\mathbf{q} = M$

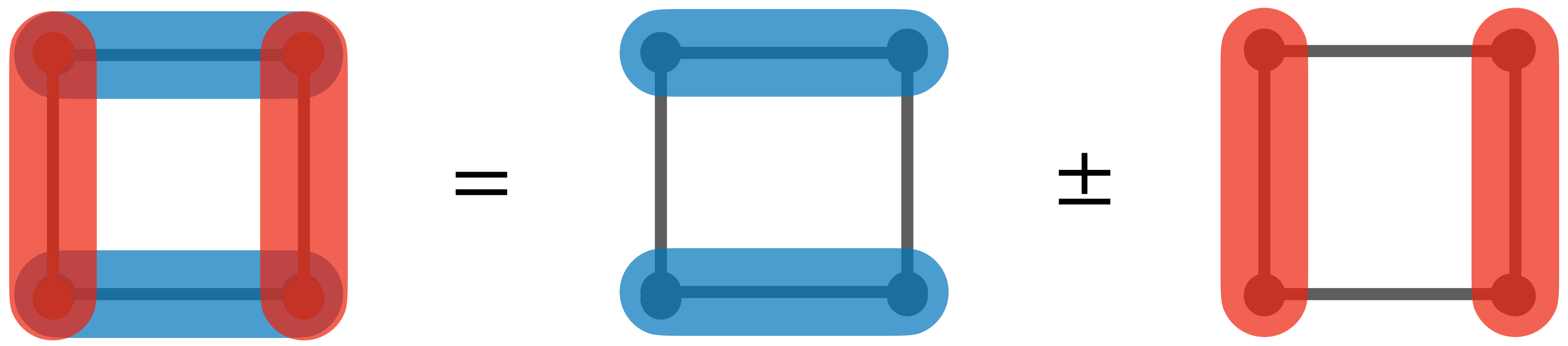


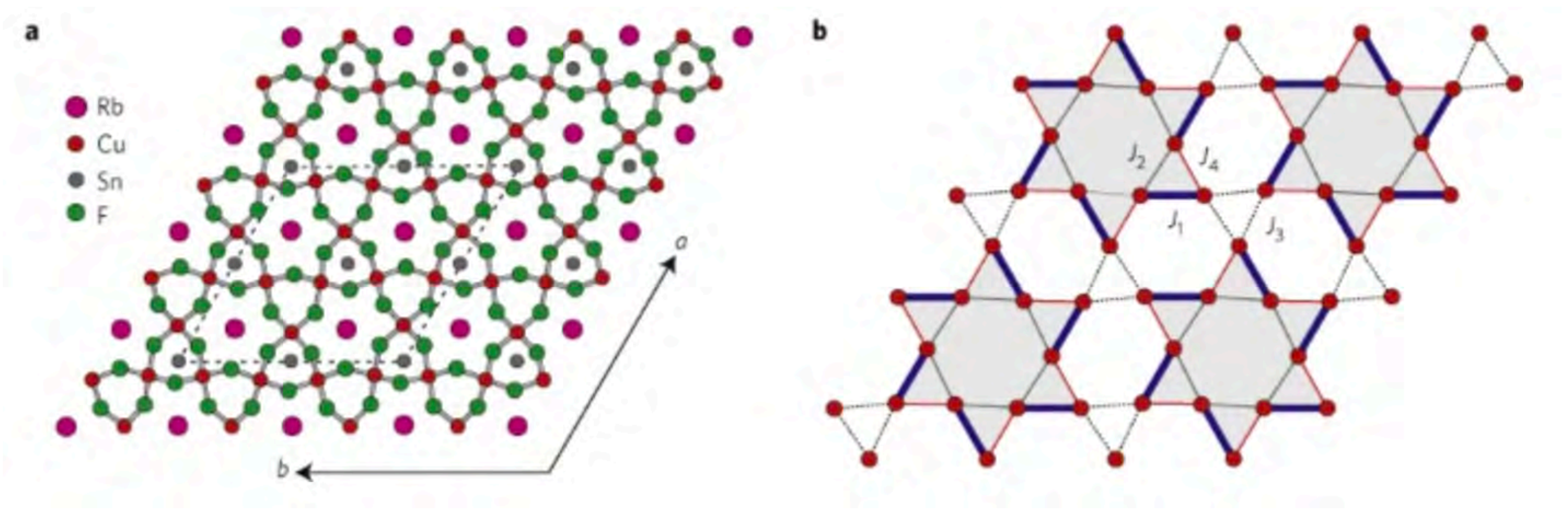
[A. Wietek, R. Rossi, F. Šimkovic IV, M. Klett, P. Hansmann, M. Ferrero, E. M. Stoudenmire, T. Schäfer, A. Georges, Phys. Rev. X 11, 041013 (2021)]

Valence bond crystals

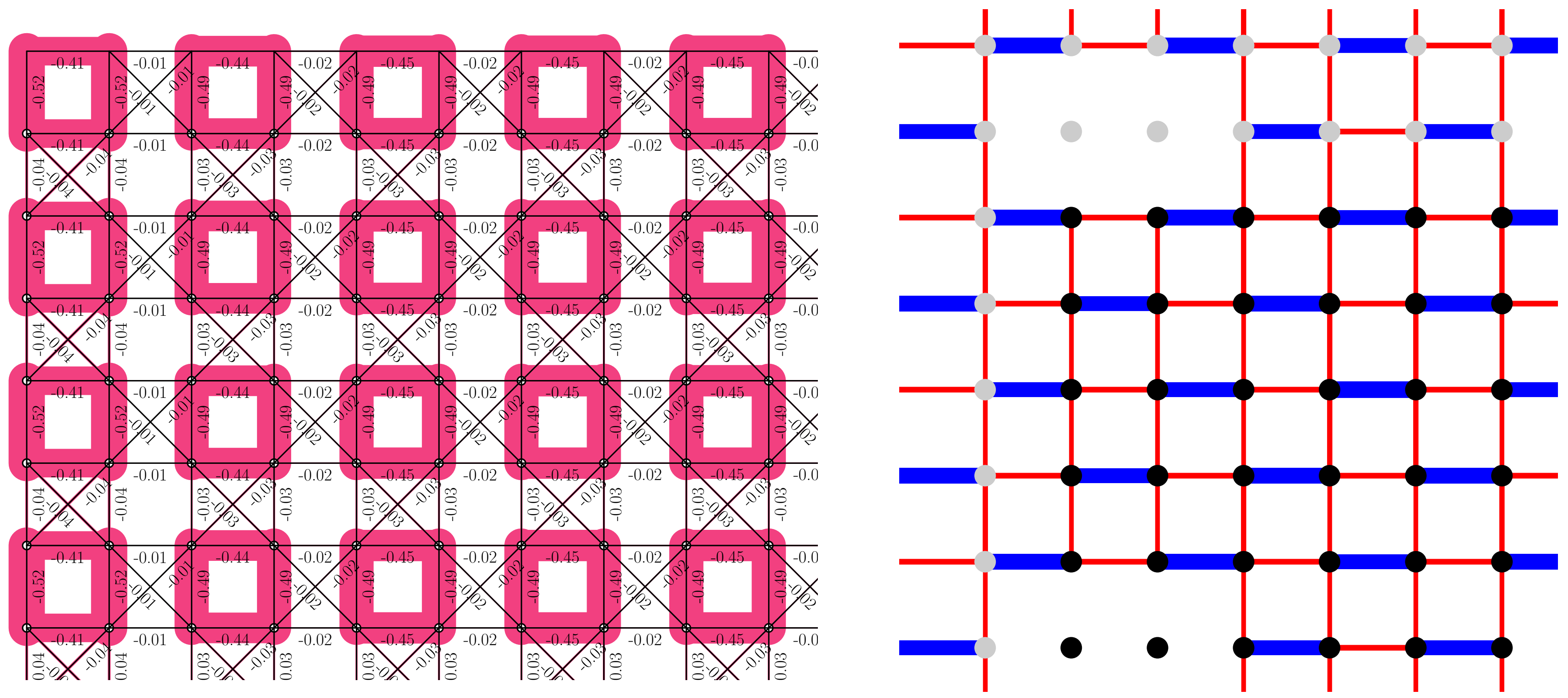
- Regular patterns of (singlet) dimers / plaquettes / … covering a lattice
- Gapped state, breaks discrete symmetry
- Long-range dimer order $|\langle(\mathbf{S}_0 \cdot \mathbf{S}_1)(\mathbf{S}_r \cdot \mathbf{S}_{r+\alpha})\rangle| \to \text{const}$
- Short-range spin correlations, $|\langle \mathbf{S}_0 \cdot \mathbf{S}_r \rangle| \sim e^{-r/\xi}$

$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$





[Matan et al., Nat. Phys. 6, 865–869 (2010)]



Chiral spin liquid (topological)

- Fractional quantum Hall effect for spins instead of electrons

  [V. Kalmeyer, R.B. Laughlin, Phys. Rev. Lett. 59 (1987)]
  [X. G. Wen, F. Wilczek, and A. Zee, Phys. Rev. B 39, 11413, (1989)]

- Fractionally quantized spin-Hall and thermal Hall effects

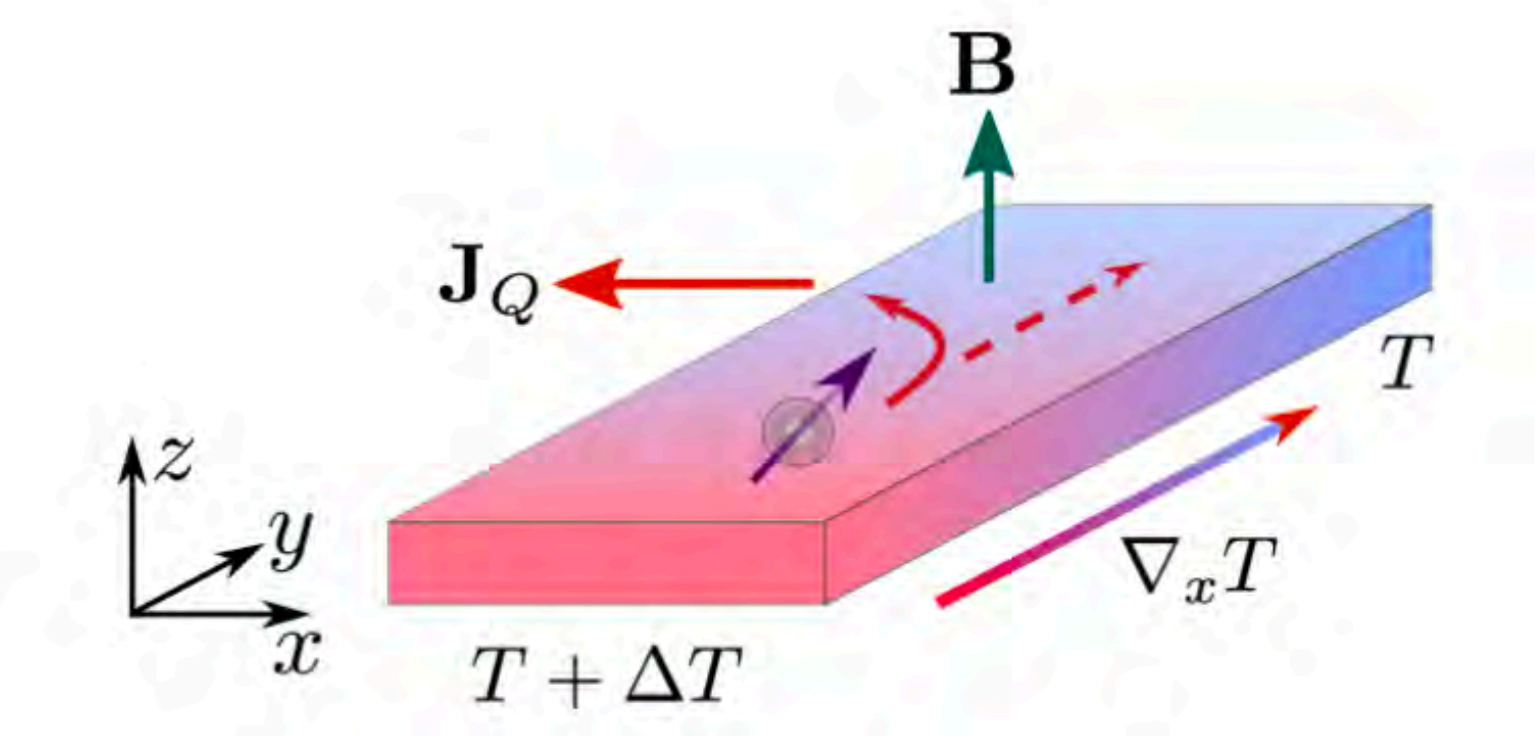
- (Spontaneous) Breaking time-reversal symmetry, $\mathcal{O} = \mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_\mathbf{k})$

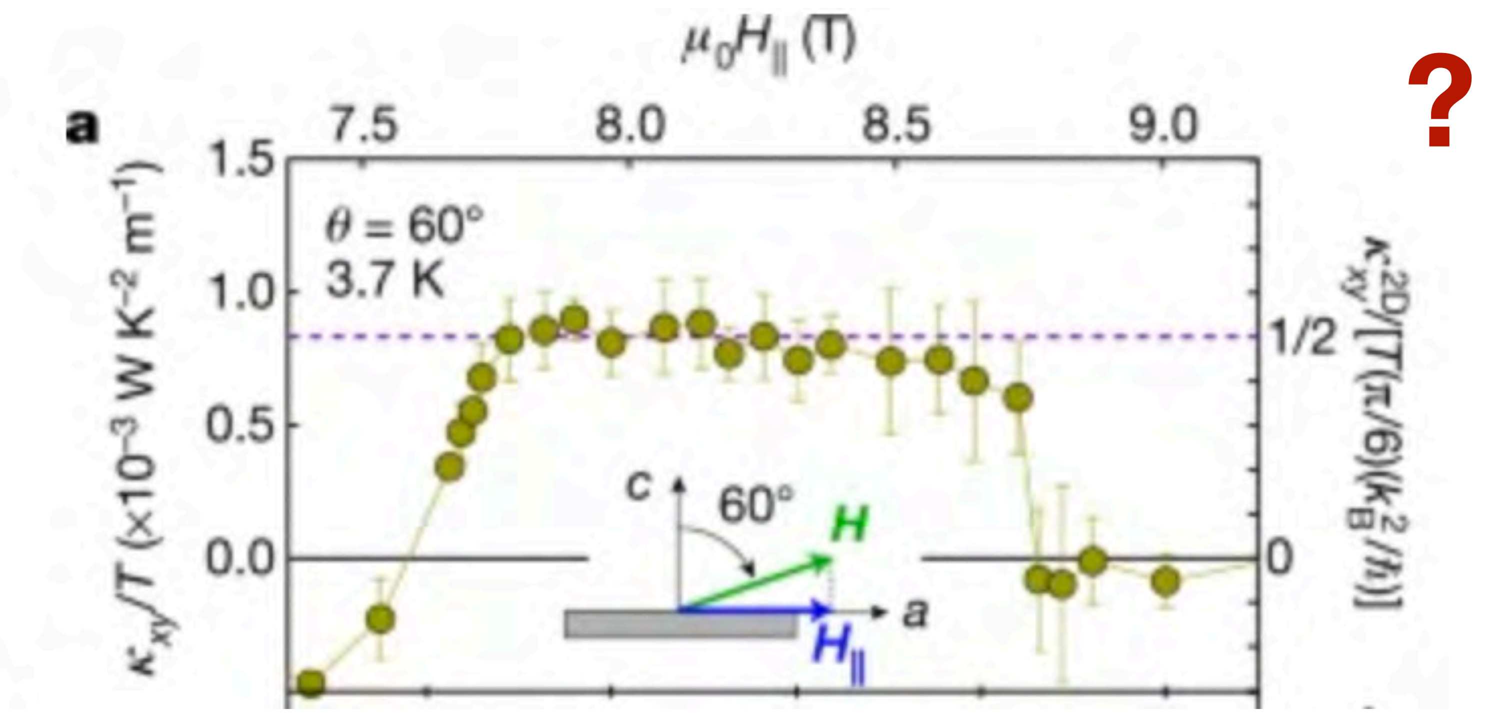
- Gapped phase, short-range spin correlations, $|\langle \mathbf{S}_0 \cdot \mathbf{S}_r \rangle| \sim e^{-r/\xi}$

- Topological order, effective Chern-Simons theory

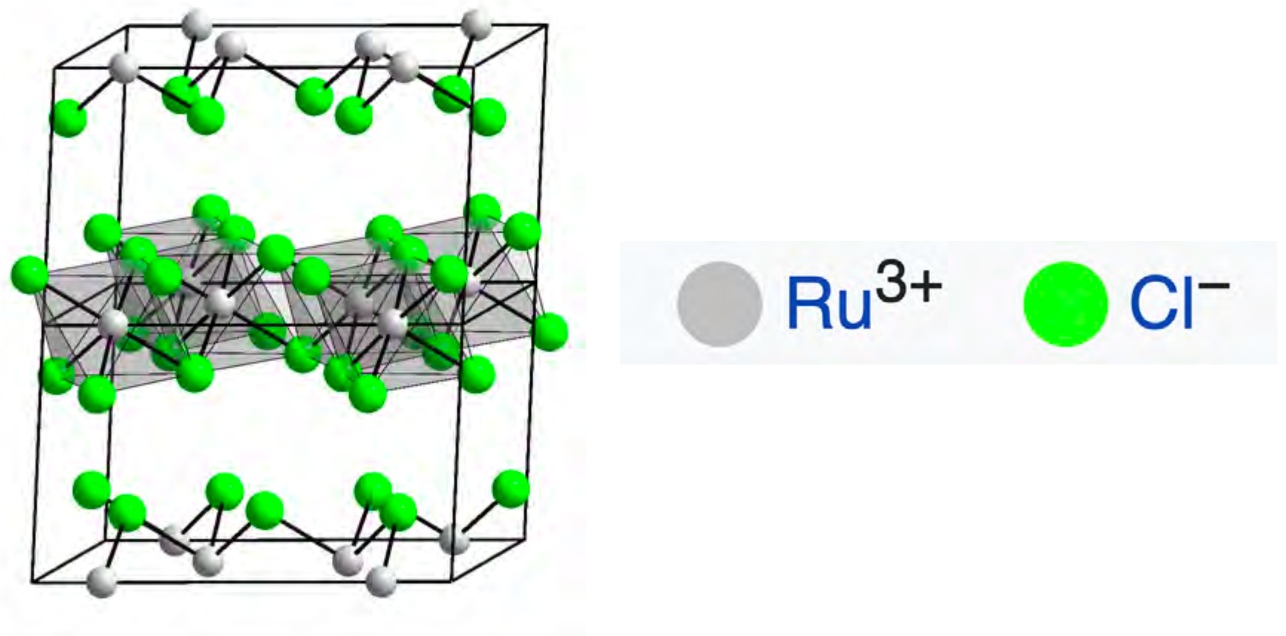
- Several simple lattice models are known stabilising this phase for SU(2) as well as SU(N)

[A. Wietek, A. M. Läuchli, Phys. Rev. B 95, 035141 (2017)]
[A. Wietek, A. Sterdyniak, A. M. Läuchli, Phys. Rev. B 92, 125122 (2015)]
[Y.-C. He, D. N. Sheng, and Yan Chen, Phys. Rev. Lett. 112, (2014)]
[B. Bauer et al., Nat. Comm. 5, 5137 (2014)]
[S. Gong, W. Zhu, D. N. Sheng, Sci. Rep. 4, 6317 (2014)]
[J.Y. Chen et al., Phys. Rev. B 104, 235104 (2021)] ...





[Kasahara et al., Nature 571, 376–380 (2019)]

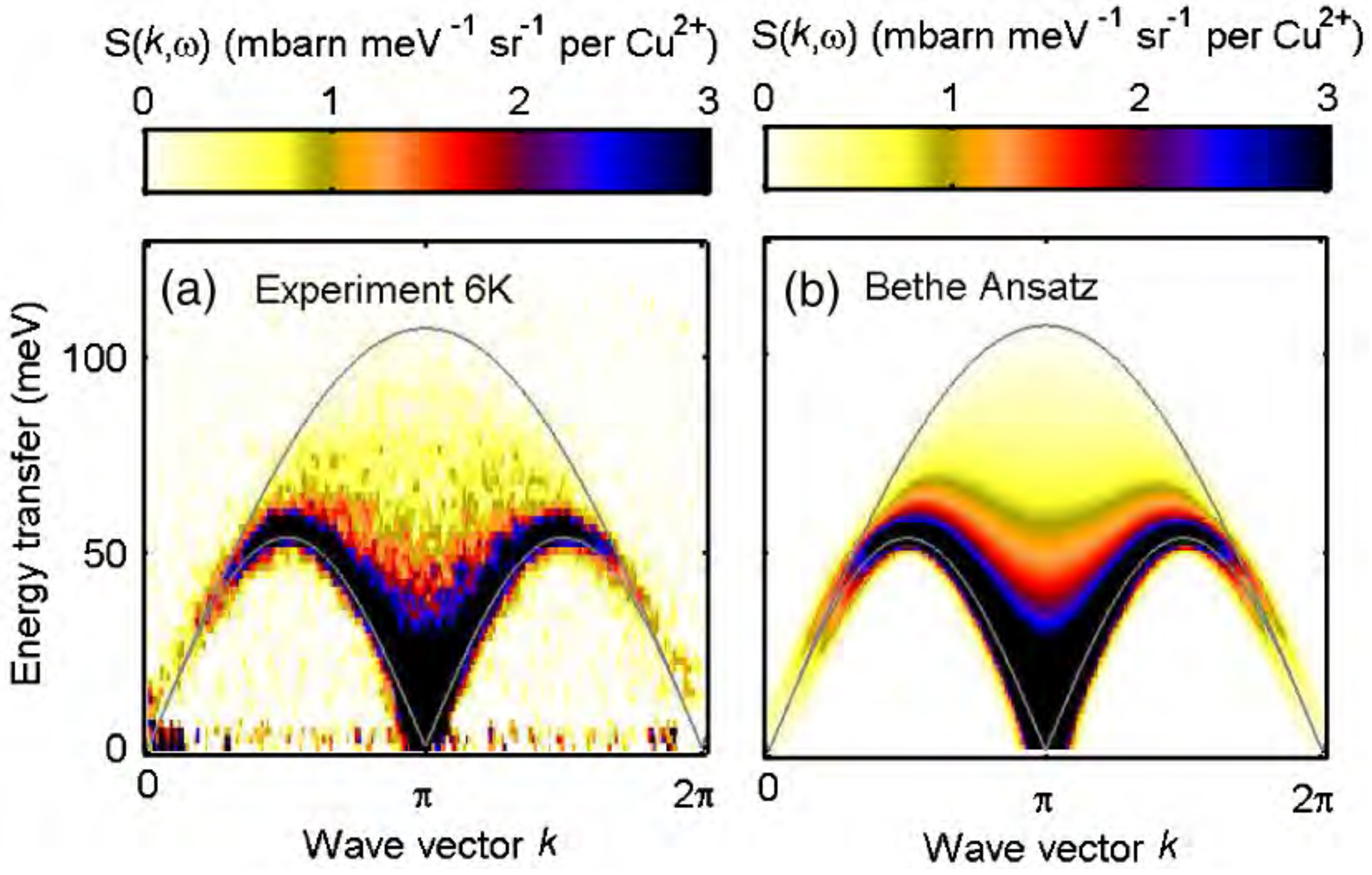


Algebraic spin liquids

- Quasi-long-range spin order, $|\langle \mathbf{S}_0 \cdot \mathbf{S}_r \rangle| \sim 1/r^{1+\eta}$
- Best known example: spin-1/2 chain in one spatial dimension
- “Critical phase”, exactly solvable with Bethe ansatz

[H. Bethe, Zeitschrift für Physik 71, 205–226 (1931)]

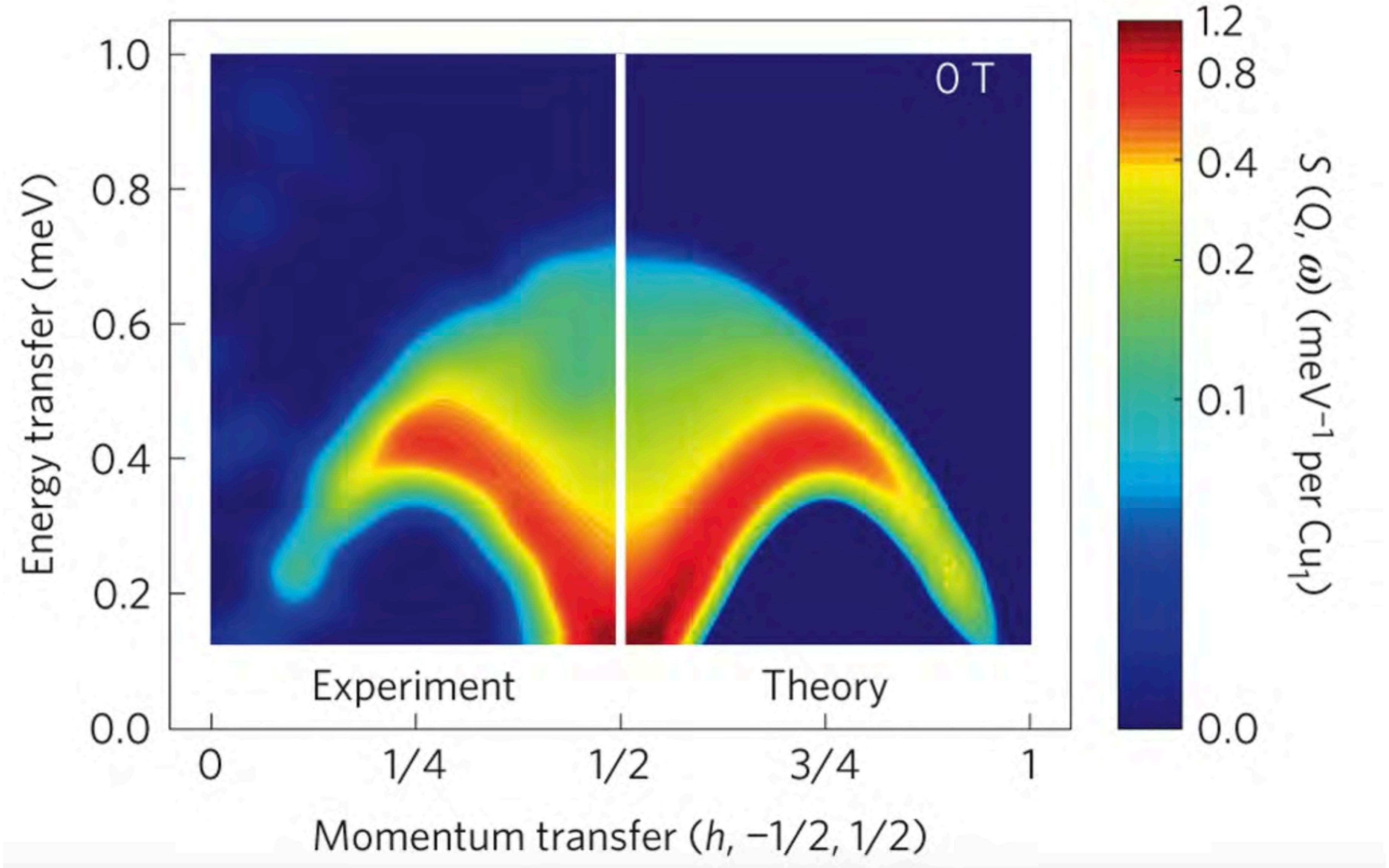
- Conformal field theory, Luttinger liquid



$KCuF_3$

[Lake et al., Phys. Rev. Lett. 111, 137205 (2013)]

$CuSO4\ (5H_2O)$



[Mourigal et al., Nat. Phys., 9, 435–441 (2013)]

Outline

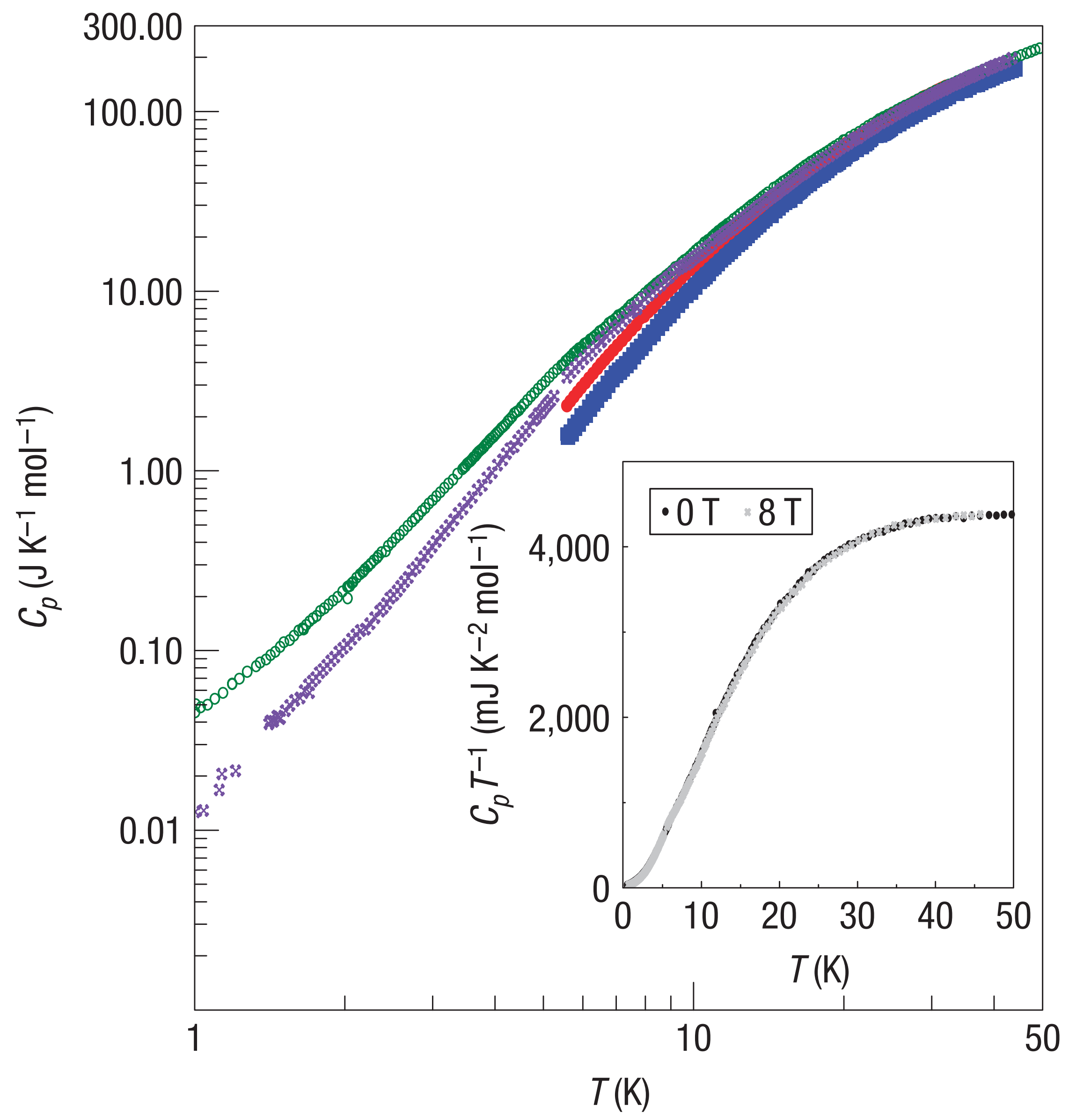
- Spin liquid on the triangular lattice
- Dirac spin liquid in a nutshell
- Comparison with numerics (Exact Diagonalization)

Alex Wietek
(Dresden)

Andreas Läuchli
(PSI/EPFL)

Experimental motivation

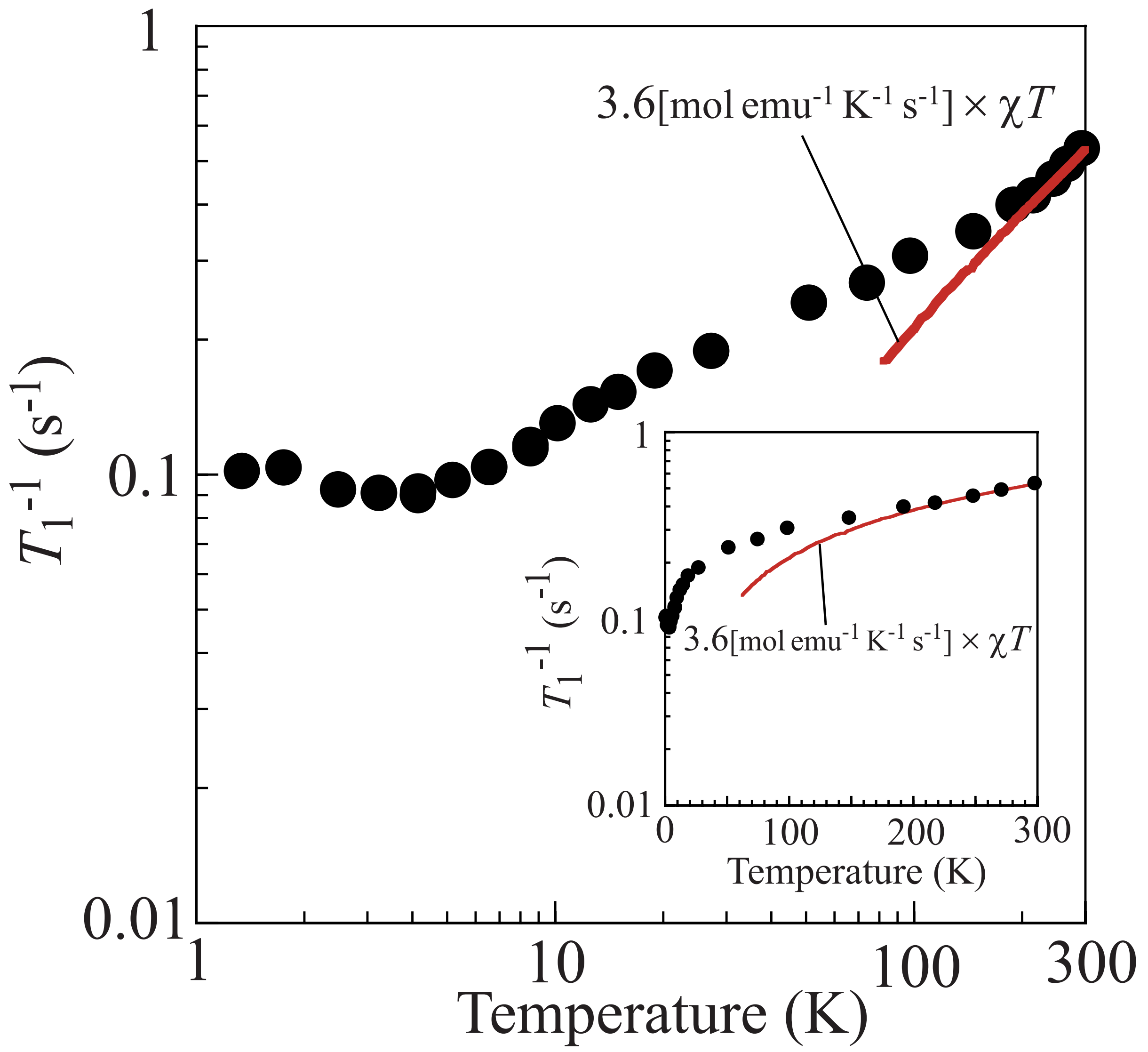
Thermodynamics



Thermodynamic properties of a spin-1/2 spin-liquid state in a κ-type organic salt

SATOSHI YAMASHITA[1], YASUHIRO NAKAZAWA[1,2]*, MASAHARU OGUNI[3], YUGO OSHIMA[2,4], HIROYUKI NOJIRI[2,4], YASUHIRO SHIMIZU[5], KAZUYA MIYAGAWA[2,6] AND KAZUSHI KANODA[2,6]

NMR

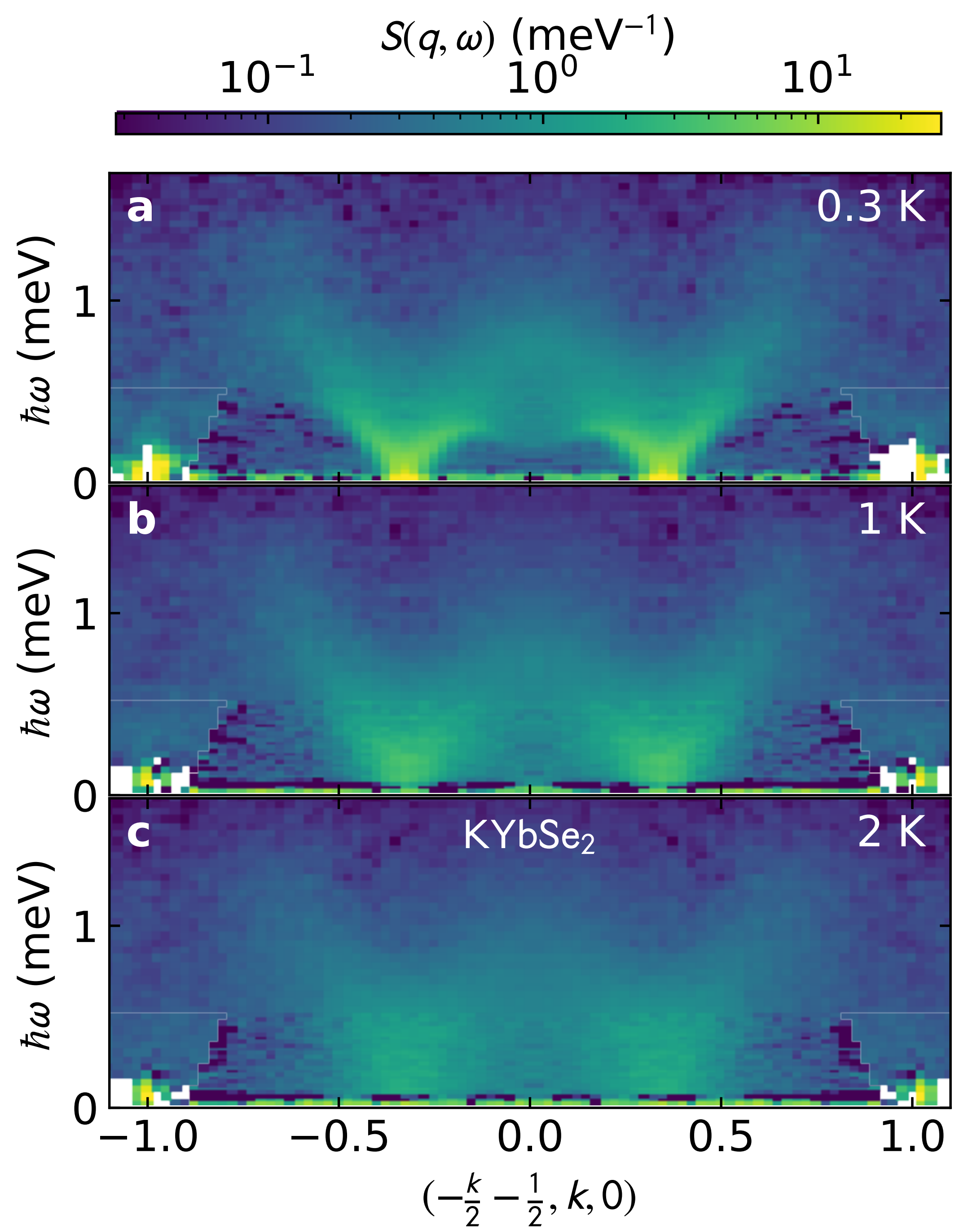


PHYSICAL REVIEW B **77**, 104413 (2008)

Quantum spin liquid in the spin-1/2 triangular antiferromagnet $EtMe_3Sb[Pd(dmit)_2]_2$

T. Itou,[1] A. Oyamada,[1] S. Maegawa,[1] M. Tamura,[2] and R. Kato[2]

Inelastic neutron Scattering



Witnessing quantum criticality and entanglement in the triangular antiferromagnet $KYbSe_2$

A. O. Scheie,[1,*] E. A. Ghioldi,[2,3] J. Xing,[4] J. A. M. Paddison,[4] N. E. Sherman,[5,6] M. Dupont,[5,6] L. D. Sanjeewa,[7,8] Sangyun Lee,[9,10] A.J. Woods,[9,10] D. Abernathy,[1] D. M. Pajerowski,[1] T. J. Williams,[1] Shang-Shun Zhang,[11] L. O. Manuel,[3] A. E. Trumper,[3] C. D. Pemmaraju,[12] A. S. Sefat,[4] D. S. Parker,[4] T. P. Devereaux,[12,13] R. Movshovich,[9,10] J. E. Moore,[5,6,10] C. D. Batista,[2,14,†] and D. A. Tennant[1,10,14]

Organic salts

delafossite $KYbSe_2$

J1-J2 triangular lattice

Spin-1/2 triangular lattice has a long history in frustrated magnetism:

AF Ising model has a residual entropy at T=0

PHYSICAL REVIEW VOLUME 79, NUMBER 2 JULY 15, 1950

Antiferromagnetism. The Triangular Ising Net

G. H. WANNIER
Bell Telephone Laboratories, Murray Hill, New Jersey

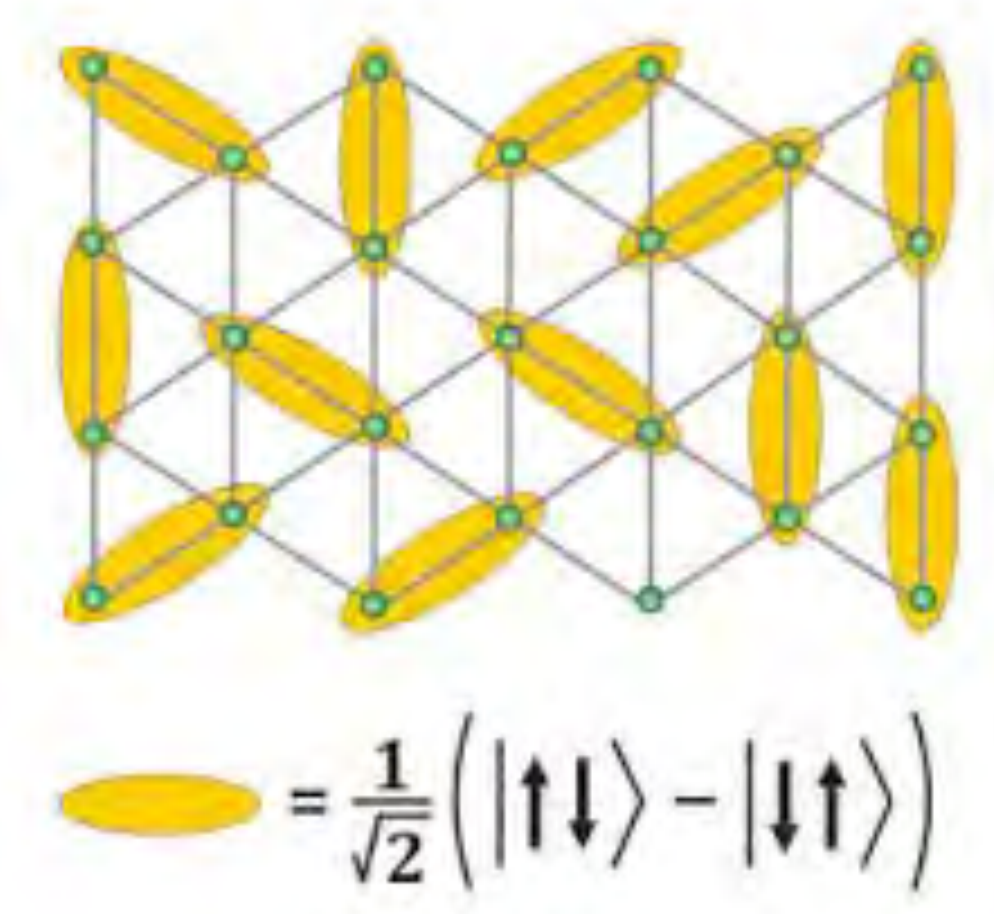
(Received February 11, 1950)

In this paper the statistical mechanics of a two-dimensionally infinite set of Ising spins is worked out for the case in which they form either a triangular or a honeycomb arrangement. Results for the honeycomb and the ferromagnetic triangular net differ little from the published ones for the square net (Curie point with logarithmically infinite specific heat). The triangular net with antiferromagnetic interaction is a sample case of antiferromagnetism in a non-fitting lattice. The binding energy comes out to be only one-third of what it is in the ferromagnetic case. The entropy at absolute zero is finite; it equals

$$S(0)=R\frac{2}{\pi}\int_0^{\pi/3}\ln(2\cos\omega)d\omega=0.3383R.$$

The system is disordered at all temperatures and possesses no Curie point.

Ground-state of AF Heisenberg model could be a spin liquid



RVB theory

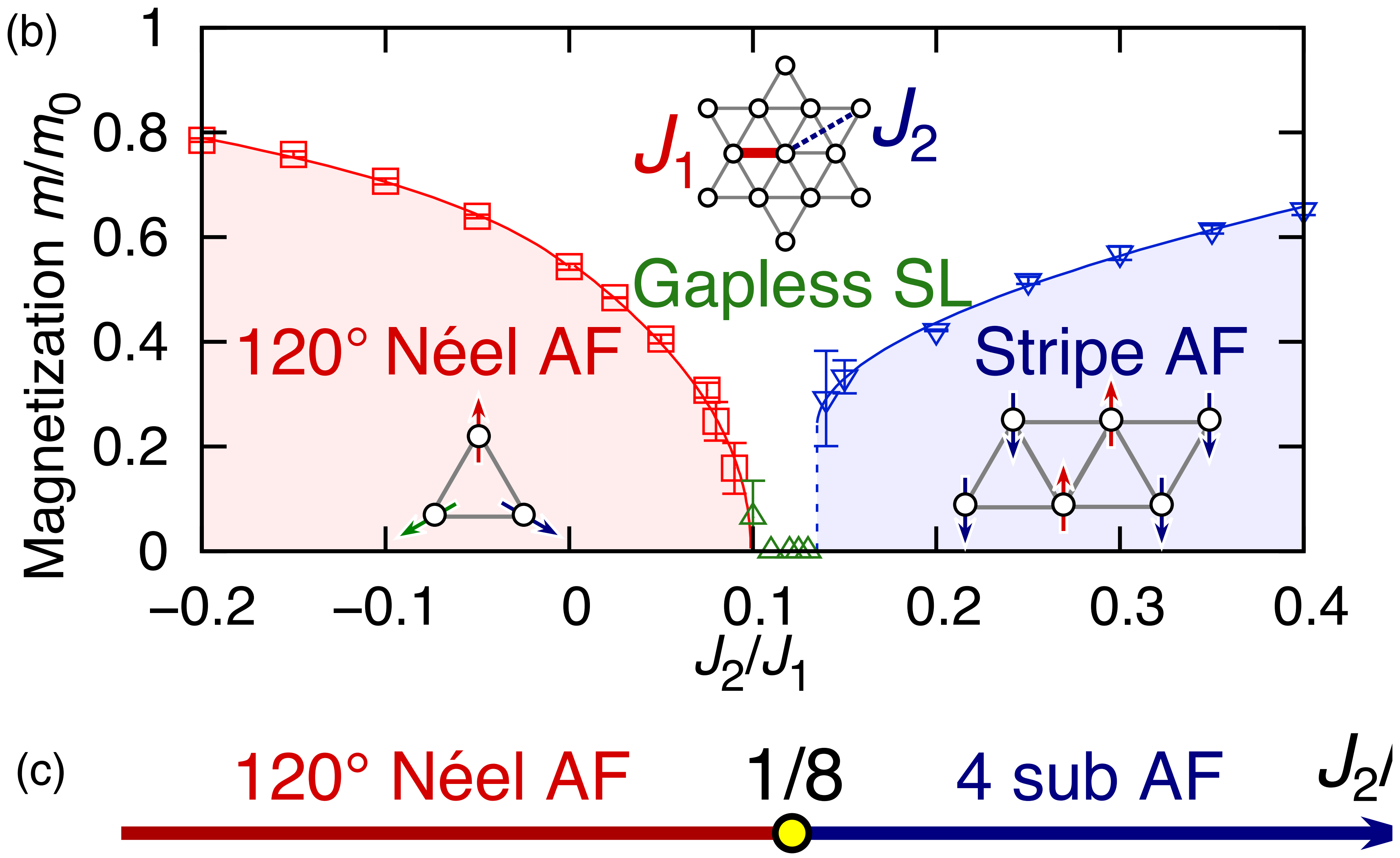
P. W. Anderson

J1-J2 triangular lattice

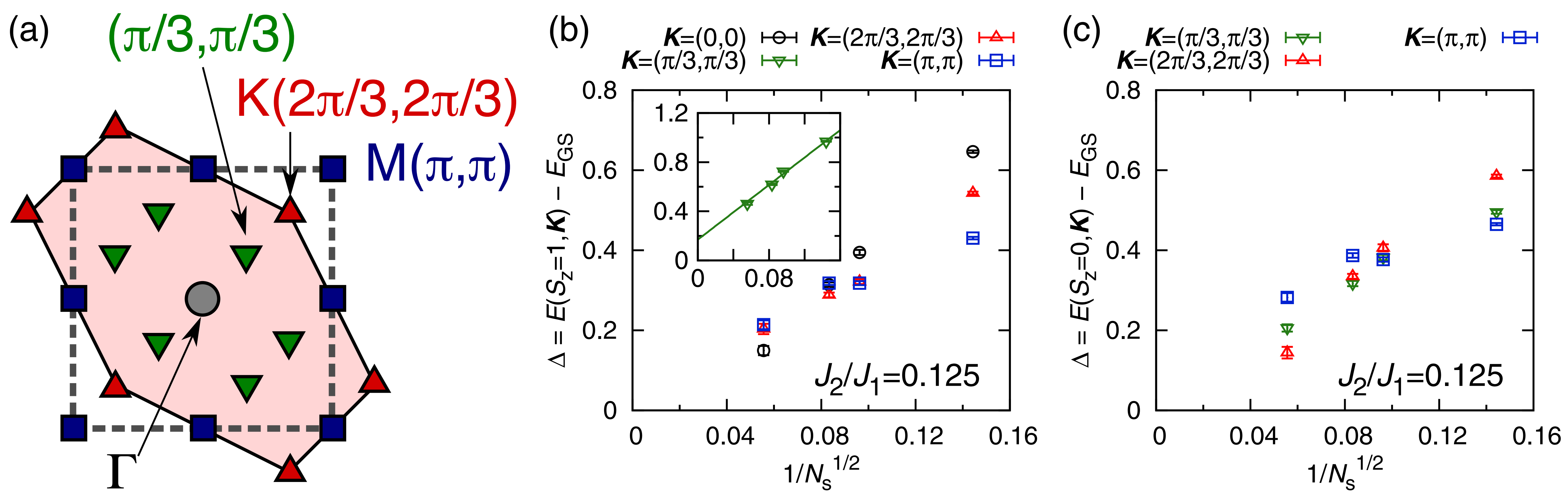
Spin-1/2 triangular lattice has a long history in frustrated magnetism:
Wannier, Anderson's RVB theory…

Variational Monte-Carlo

Ground-state



Excitations



Algebraic gap closing at several k points !?

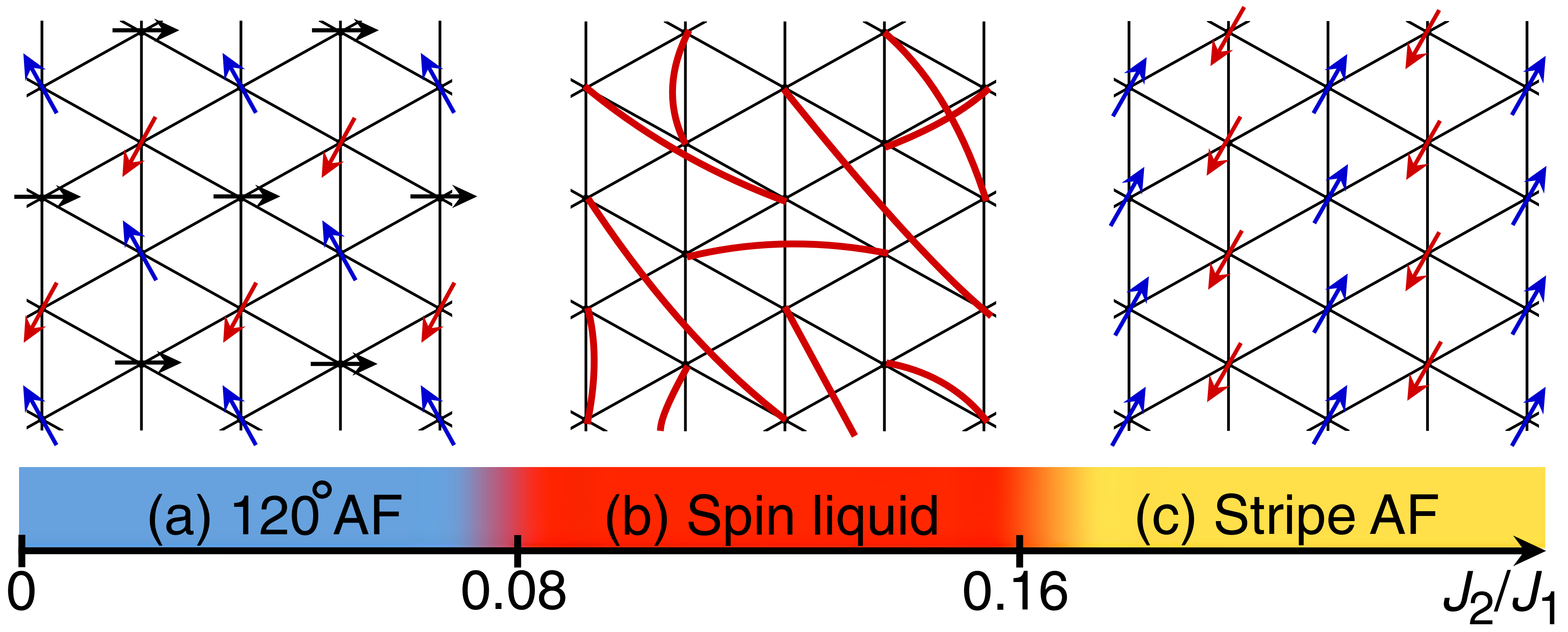
Journal of the Physical Society of Japan **83**, 093707 (2014) **Letters**
http://dx.doi.org/10.7566/JPSJ.83.093707

Gapless Spin-Liquid Phase in an Extended Spin 1/2 Triangular Heisenberg Model

Ryui Kaneko[1]*, Satoshi Morita[2], and Masatoshi Imada[3]

J1-J2 triangular lattice

Variational Monte-Carlo



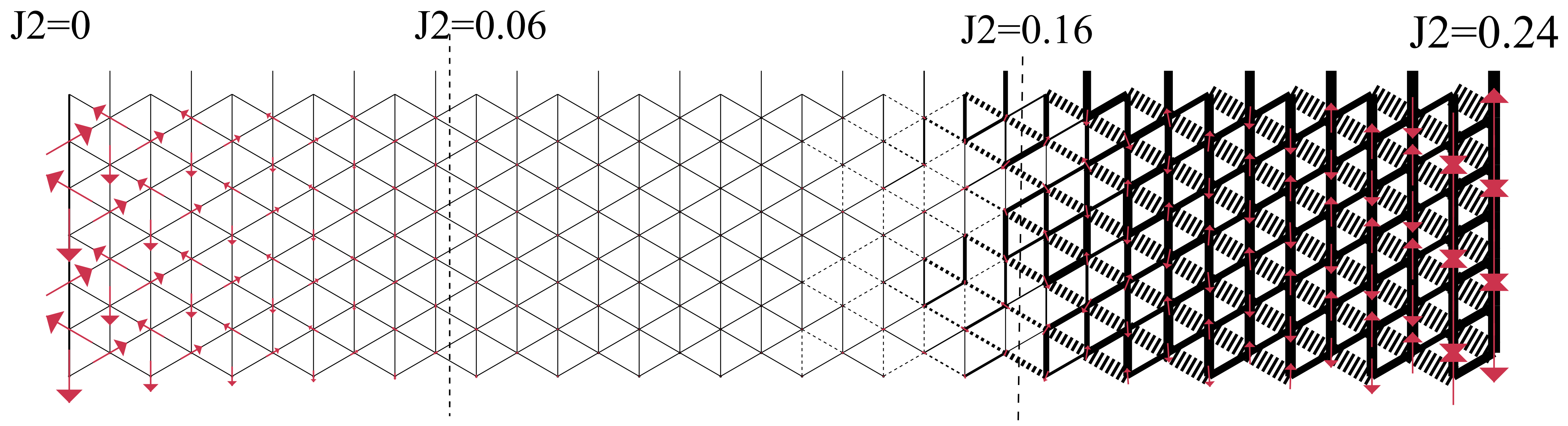
Gapless spin liquid phase

PHYSICAL REVIEW B **93**, 144411 (2016)

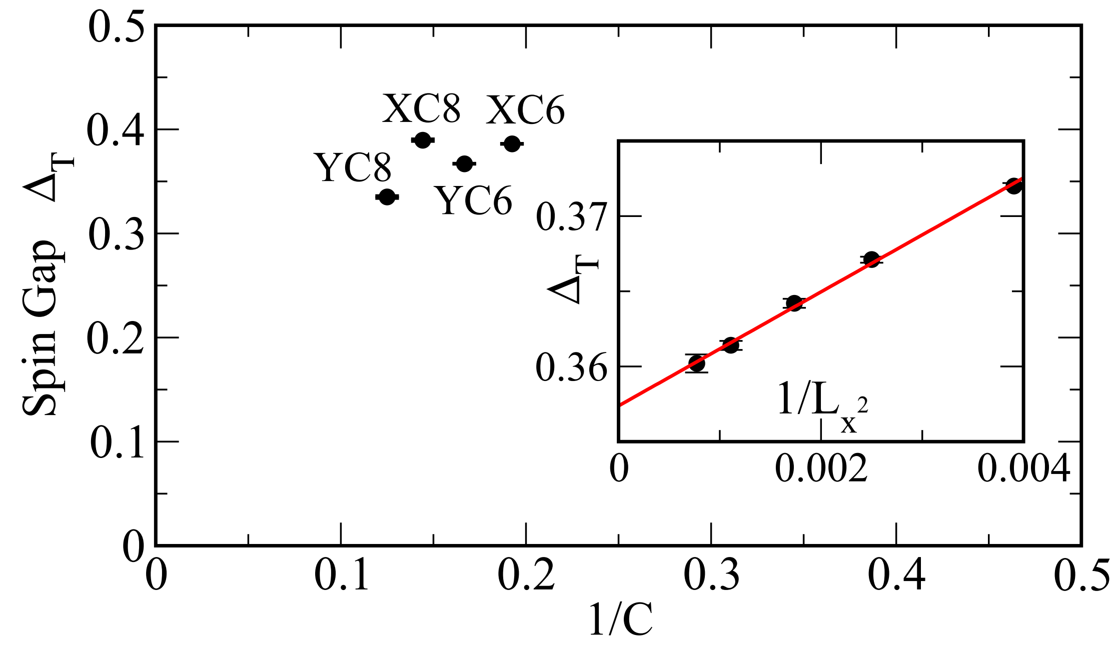
Spin liquid nature in the Heisenberg J_1-J_2 triangular antiferromagnet

Yasir Iqbal,[1,*] Wen-Jun Hu,[2,†] Ronny Thomale,[1,‡] Didier Poilblanc,[3,§] and Federico Becca[4,‖]

DMRG



Gapped spin liquid phase



PHYSICAL REVIEW B **92**, 041105(R) (2015)

Spin liquid phase of the $S = \frac{1}{2}$ $J_1 - J_2$ Heisenberg model on the triangular lattice

Zhenyue Zhu and Steven R. White

PHYSICAL REVIEW B **92**, 140403(R) (2015)

Competing spin-liquid states in the spin-$\frac{1}{2}$ Heisenberg model on the triangular lattice

Wen-Jun Hu, Shou-Shu Gong,[*] Wei Zhu, and D. N. Sheng

J1-J2 triangular lattice: DMRG

Apparent finite spin-gap due to cylinder geometry

→ **Gapless** spin liquid

Proximity to other quantum spin liquids, such as chiral spin liquids (CSL)

PHYSICAL REVIEW B **96**, 075116 (2017)

Global phase diagram and quantum spin liquids in a spin-$\frac{1}{2}$ triangular antiferromagnet

Shou-Shu Gong,[1] W. Zhu,[2] J.-X. Zhu,[2,3] D. N. Sheng,[4] and Kun Yang[5]

PHYSICAL REVIEW B **95**, 035141 (2017)

Chiral spin liquid and quantum criticality in extended $S = \frac{1}{2}$ Heisenberg models on the triangular lattice

Alexander Wietek* and Andreas M. Läuchli

PHYSICAL REVIEW LETTERS **127**, 087201 (2021)

Four-Spin Terms and the Origin of the Chiral Spin Liquid in Mott Insulators on the Triangular Lattice

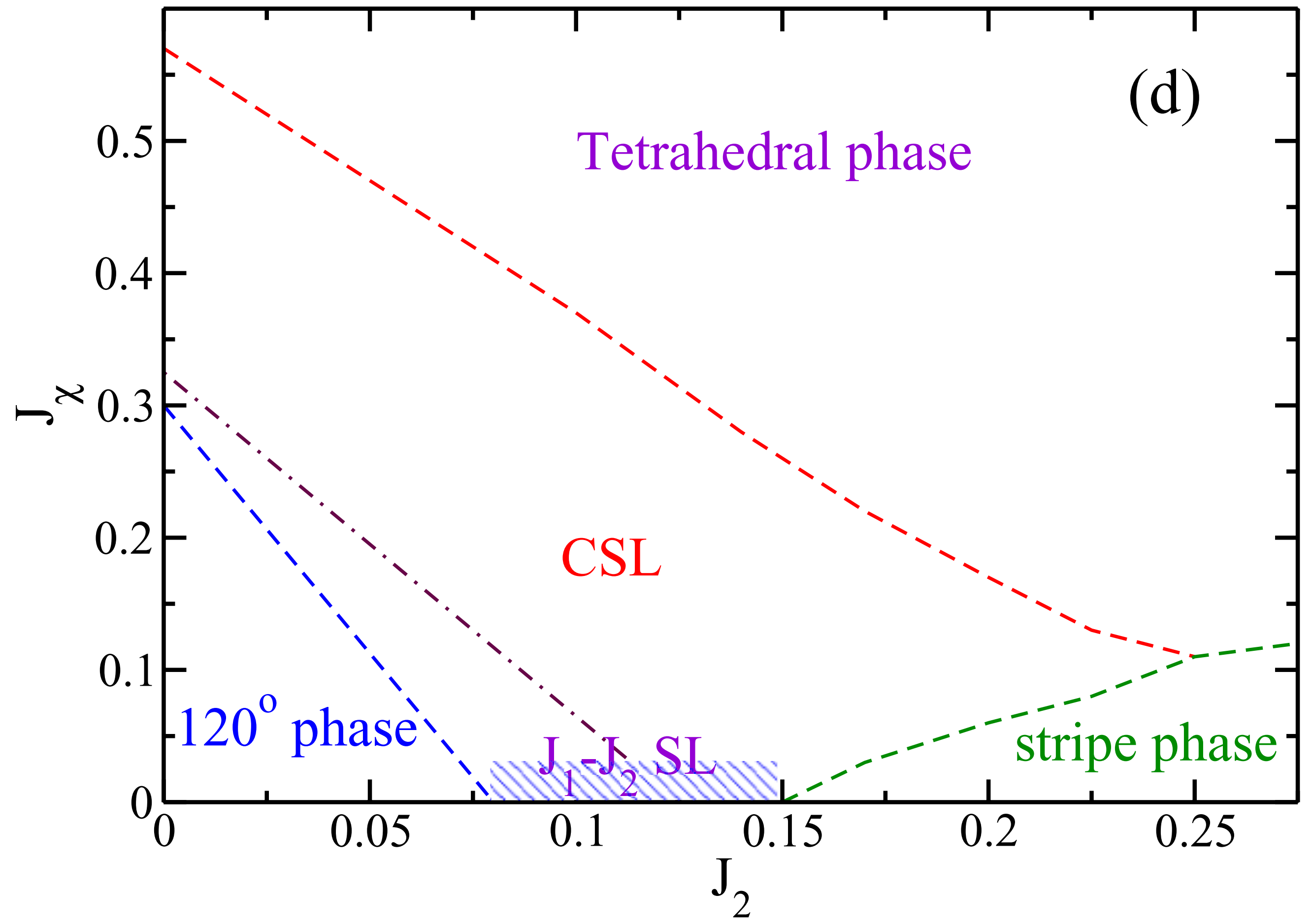
Tessa Cookmeyer,[1,2*] Johannes Motruk,[1,2,3] and Joel E. Moore[1,2]

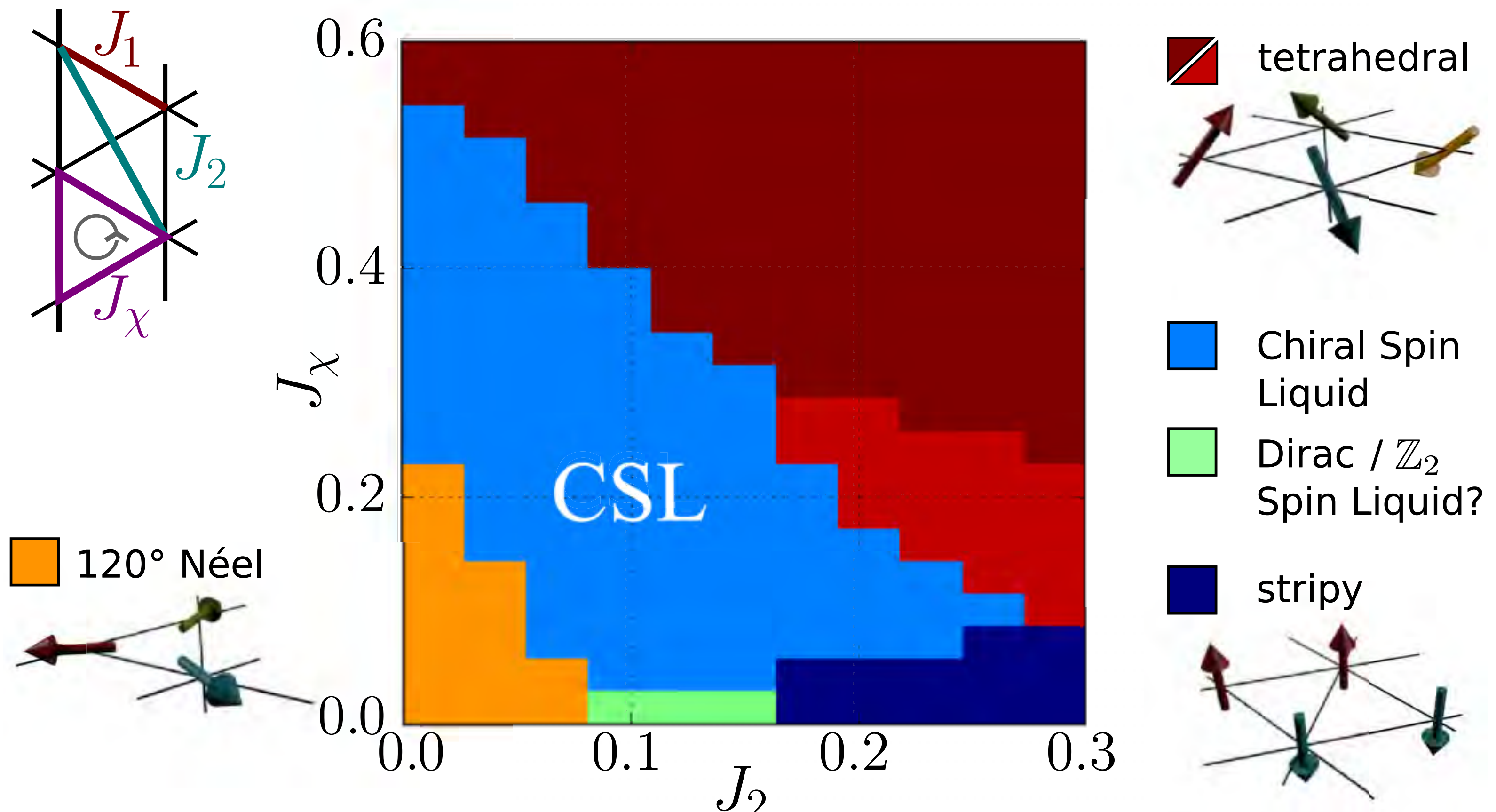
PHYSICAL REVIEW LETTERS **123**, 207203 (2019)

Editors' Suggestion

Dirac Spin Liquid on the Spin-1/2 Triangular Heisenberg Antiferromagnet

Shijie Hu,[1,*] W. Zhu,[2,†] Sebastian Eggert,[1] and Yin-Chen He[3,‡]





Construction of quantum spin liquids

[Xiao-Gang Wen, Phys. Rev. B 65, 165113 (2002)]

- Translation of a spin Hamiltonian to fermions coupled to a gauge field

$$H = \sum_{i,j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

- Breaking up a spin into Abrikosov fermionic "partons" …

$$\mathbf{S}_i = \frac{1}{2} \sum_{\alpha\beta} c^\dagger_{i\alpha} \sigma_{\alpha\beta} c_{i\beta}, \quad \alpha, \beta = \uparrow \downarrow$$

- … and rewriting the original Hamiltonian in terms of these new operators

$$H = \sum_{i,j,\alpha,\beta} -\frac{J_{ij}}{2} c^\dagger_{i\alpha} c_{j\alpha} c^\dagger_{j\beta} c_{i\beta} + \sum_{i,j} \frac{J_{ij}}{2} \left(n_i - \frac{1}{2} n_i n_j \right) .$$

- Now doubly occupied sites are explicitly allowed $|\downarrow, \downarrow\uparrow, \emptyset, \uparrow\rangle$ $|\downarrow, \uparrow, \downarrow, \uparrow\rangle$

- Interesting exact local gauge symmetry: $c^\dagger_{i\alpha} \to e^{i\theta_i} c^\dagger_{i\alpha}$

Construction of quantum spin liquids

[Xiao-Gang Wen, Phys. Rev. B 65, 165113 (2002)]

- Mean-field decoupling using an ansatz for $\chi_{ij} \equiv \langle c^\dagger_{i\alpha} c_{j\alpha} \rangle$

$$H_{\text{mean}} = \sum_{i,j,\alpha} (\chi_{ij} c^\dagger_{i\alpha} c_{j\alpha} + \text{H.c.})$$

- To reintroduce the single particle constraint, either couple to the gauge field …

$$\mathcal{Z} = \int \mathcal{D}c_i \mathcal{D}a_i \mathcal{D}\chi_{ij} \exp\left\{ i \int \mathrm{d}t\ \mathcal{L} - \sum_i a_i(t)(n_i - 1) \right\}, \quad \int \mathcal{D}a_i \exp\left\{ i \int \mathrm{d}t\ a_i(n_i - 1) \right\} = \delta\left(n_i - 1\right).$$

- … or perform a Gutzwiller projection

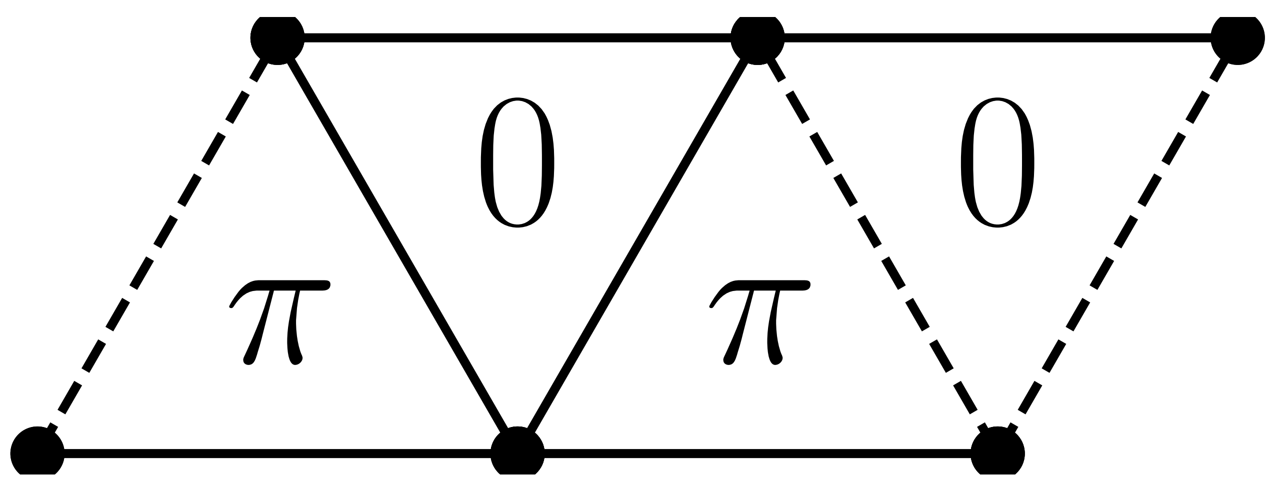
$$|\downarrow, \downarrow\uparrow, \emptyset, \uparrow\rangle \overset{\mathcal{P}}{\Rightarrow} 0$$

$$|\downarrow, \uparrow, \downarrow, \uparrow\rangle \overset{\mathcal{P}}{\Rightarrow} |\downarrow, \uparrow, \downarrow, \uparrow\rangle$$

Dirac spin liquid

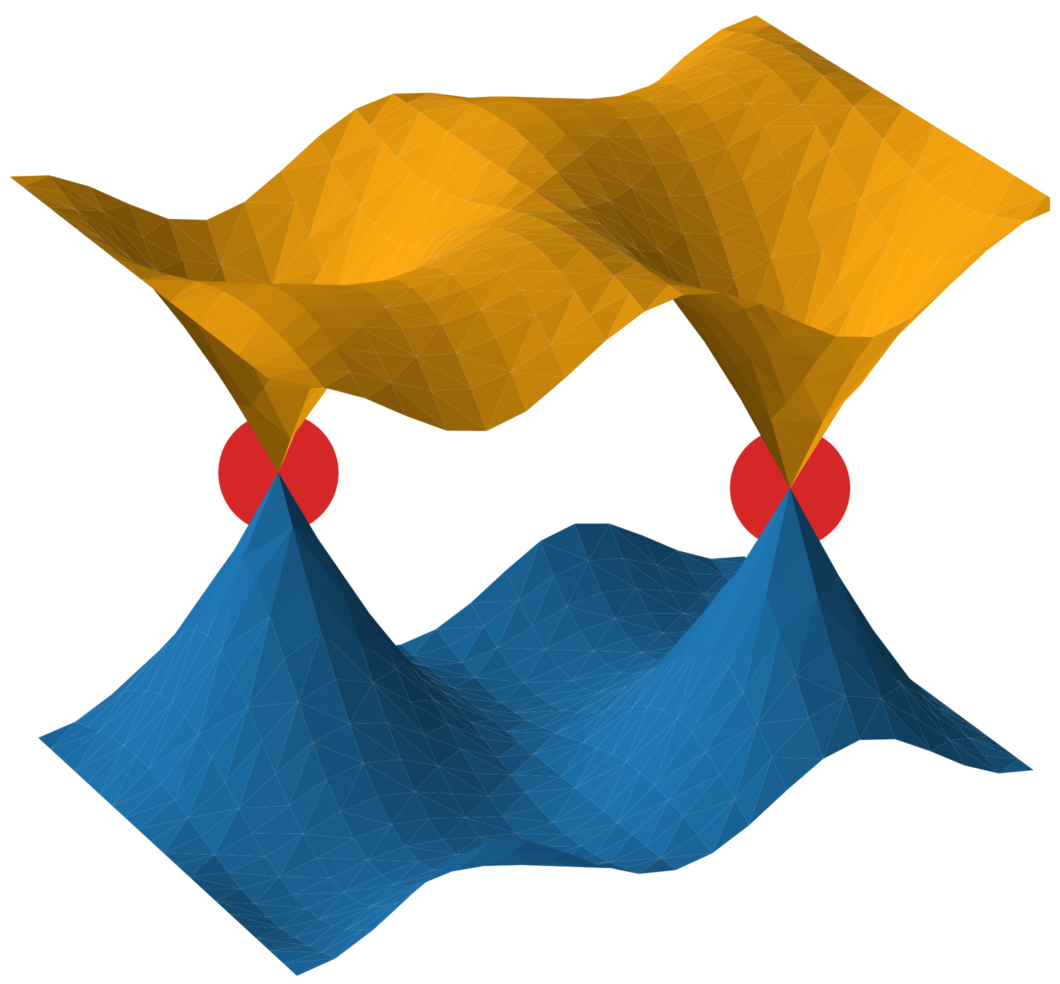
[I. Affleck and J. B. Marston, Phys. Rev. B 37, 3774(R) (1988)]

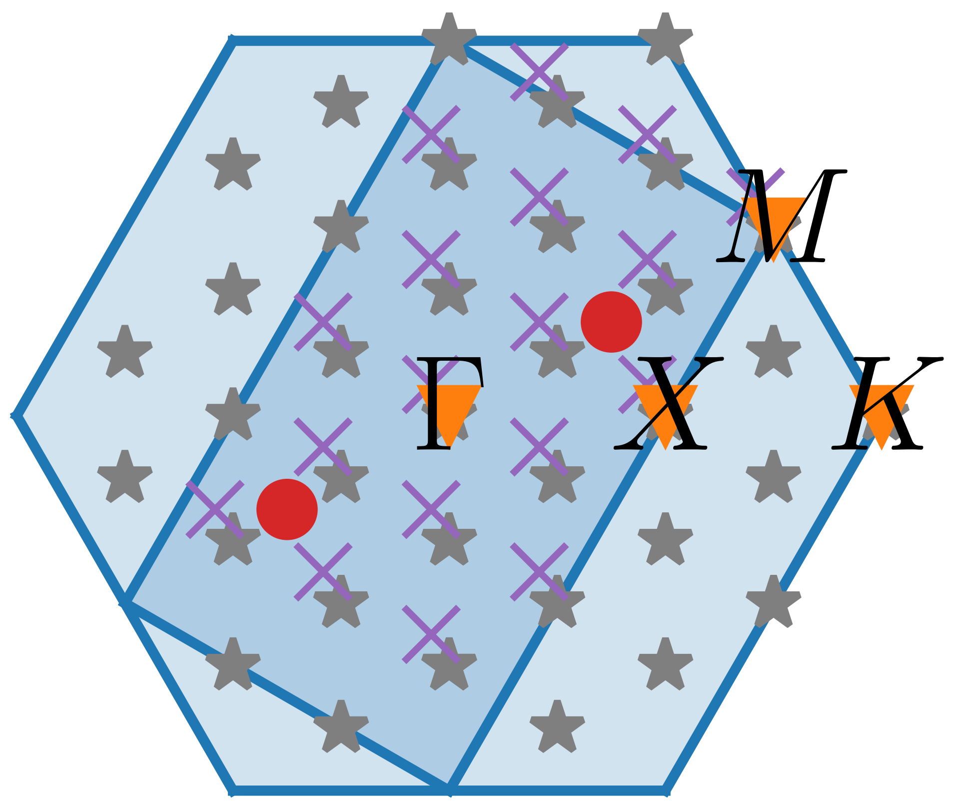
- Choosing a two-site unit cell with π- flux …



$\chi_{ij} = \pm 1$

- … yields a band structure with two Dirac cones

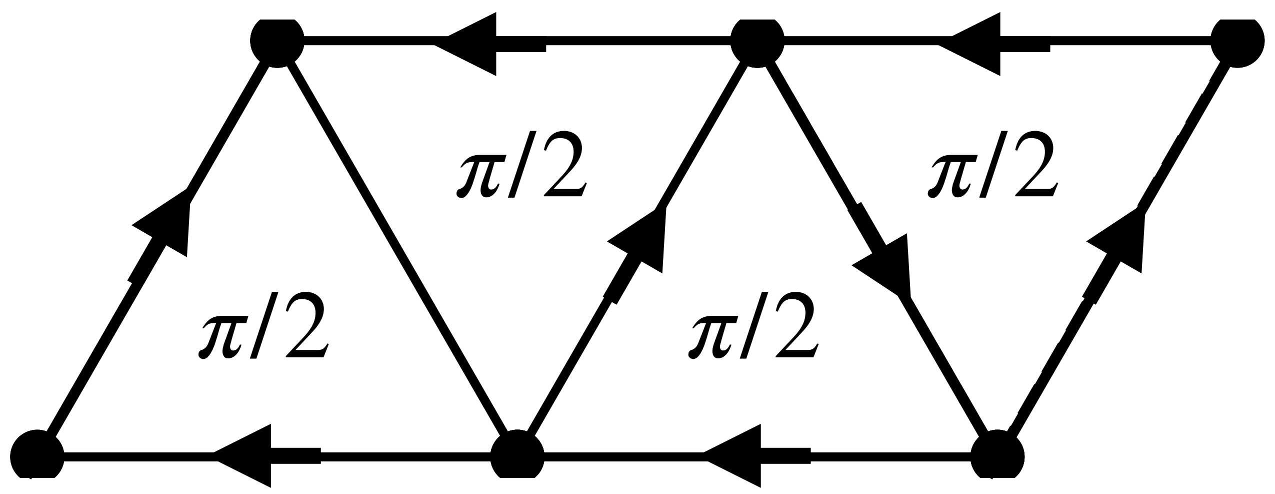




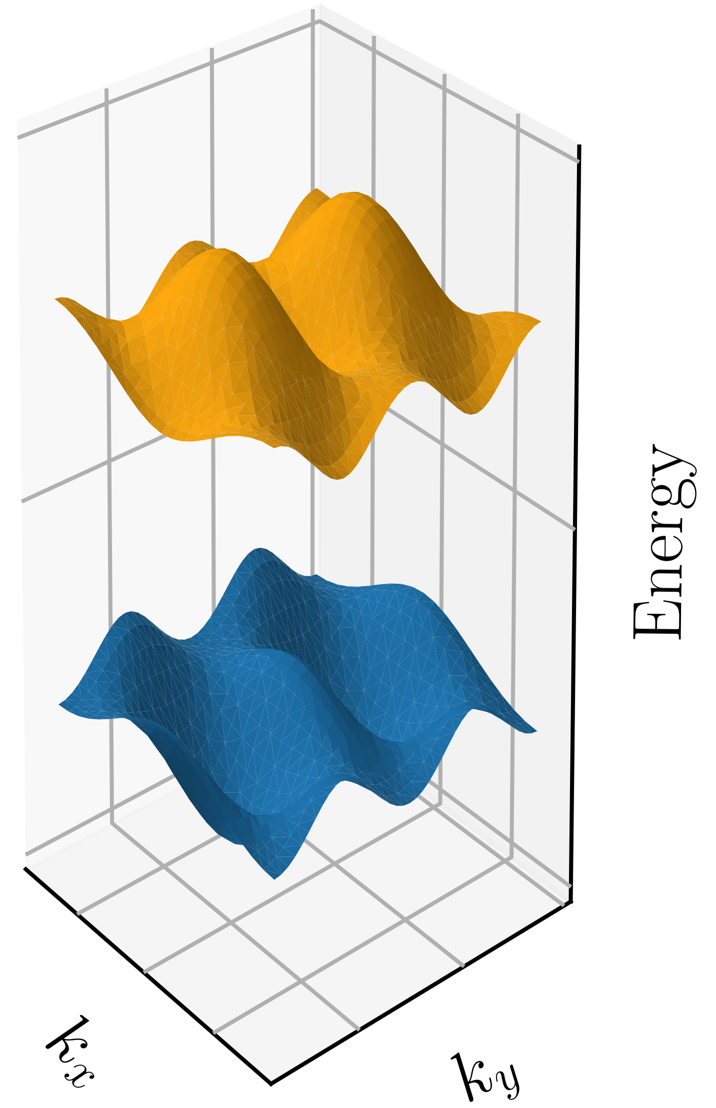
Chiral spin liquid

[X. G. Wen, F. Wilczek, and A. Zee, Phys. Rev. B 39, 11413, (1989)]

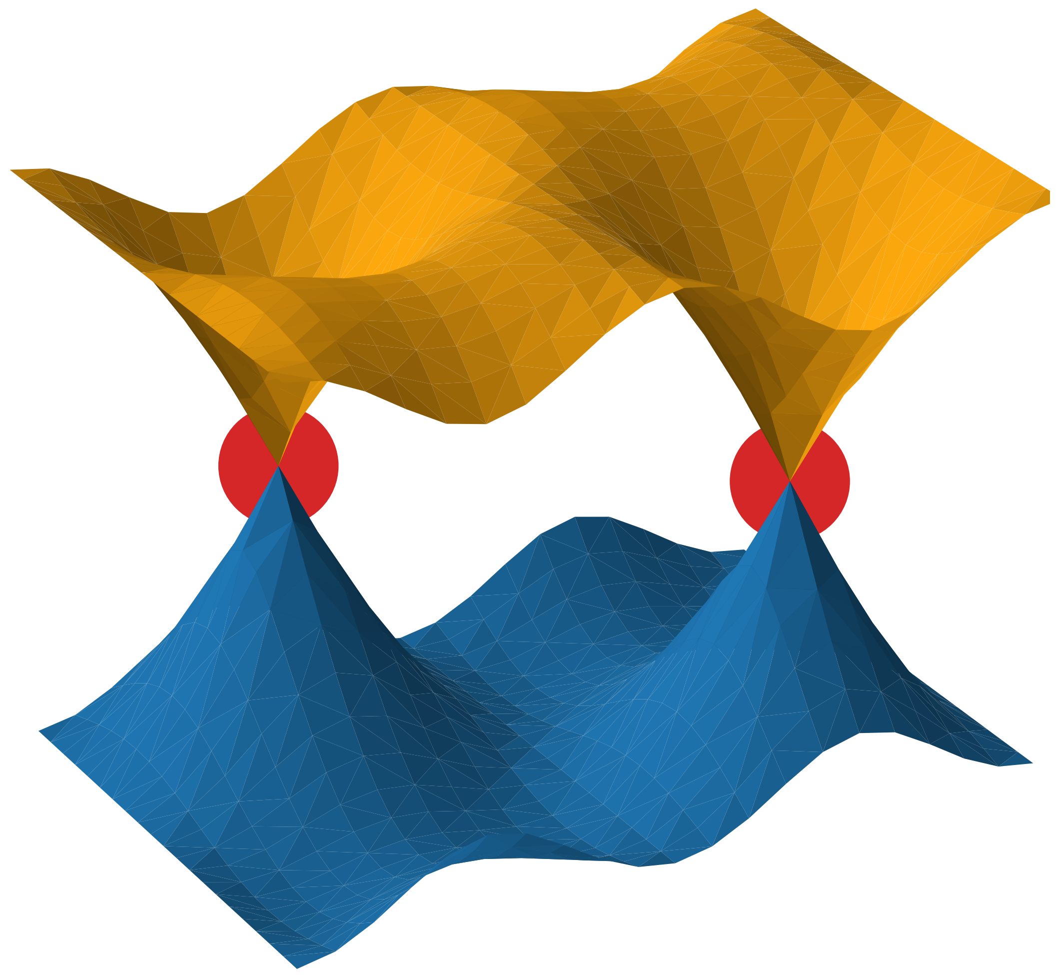
- Choosing a two-site unit cell with $\pi/2$- flux …



- … yields a band structure with two Chern bands



Dirac spin liquid summary



If the fermion (parton) band structure has (2) Dirac cones

Then, they are coupled to a compact U(1) gauge field

As a spin wavefunction, it could be gapless (several gapless excitations, see later),

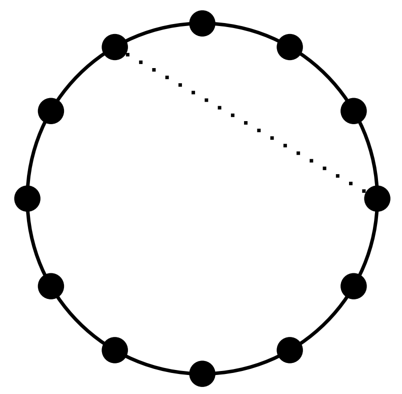
→ **algebraic spin liquid**

(Think about the S=1/2 Heisenberg chain in 1d)

It has been proposed as the groundstate of other frustrated magnets (kagome) but there is no exactly solvable model so far…

Example: Gutzwiller projection of a Fermi sea

In 1d, it is known that the ground-state of a S=1/2 sytem can be obtained as the Gutzwiller projection of a simple Fermi sea



$$\eta_\alpha = e^{\mathrm{i}\frac{2\pi}{N}\alpha}$$

$$H^{\mathrm{HS}} = \left(\frac{2\pi}{N}\right)^2 \sum_{\alpha<\beta}^{N} \frac{\boldsymbol{S}_\alpha \boldsymbol{S}_\beta}{|\eta_\alpha - \eta_\beta|^2}$$

VOLUME 60, NUMBER 7 PHYSICAL REVIEW LETTERS 15 FEBRUARY 1988

Exact Solution of an $S = \frac{1}{2}$ Heisenberg Antiferromagnetic Chain with Long-Ranged Interactions

B. Sriram Shastry[(a)]

VOLUME 60, NUMBER 7 PHYSICAL REVIEW LETTERS 15 FEBRUARY 1988

Exact Jastrow-Gutzwiller Resonating-Valence-Bond Ground State of the Spin-$\frac{1}{2}$ Antiferromagnetic Heisenberg Chain with $1/r^2$ Exchange

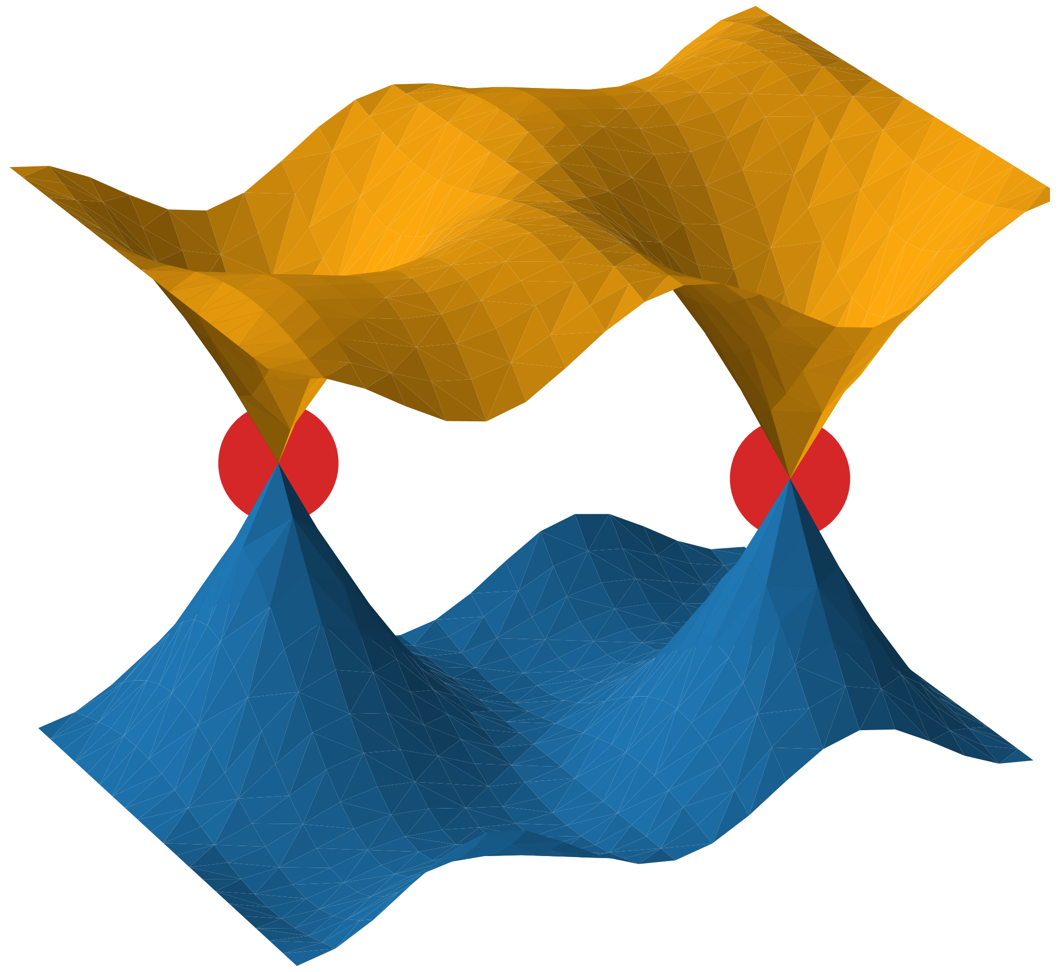
F. D. M. Haldane

Field theory in the continuum

[M. Hermele, T. Senthil, M. P. A. Fisher, P. A. Lee, N. Nagaosa, and X.-G. Wen, Phys. Rev. B 70, 214437 (2004)]

- Continuum limit, expanding close to the Dirac nodes at long wavelengths yields effective action

$$\mathscr{L} = \sum_{i=1}^{4} \bar{\Psi}_i[-i\gamma^\mu(\partial_\mu + ia_\mu)]\Psi_i + \frac{1}{2e^2}\sum_\mu (\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2$$

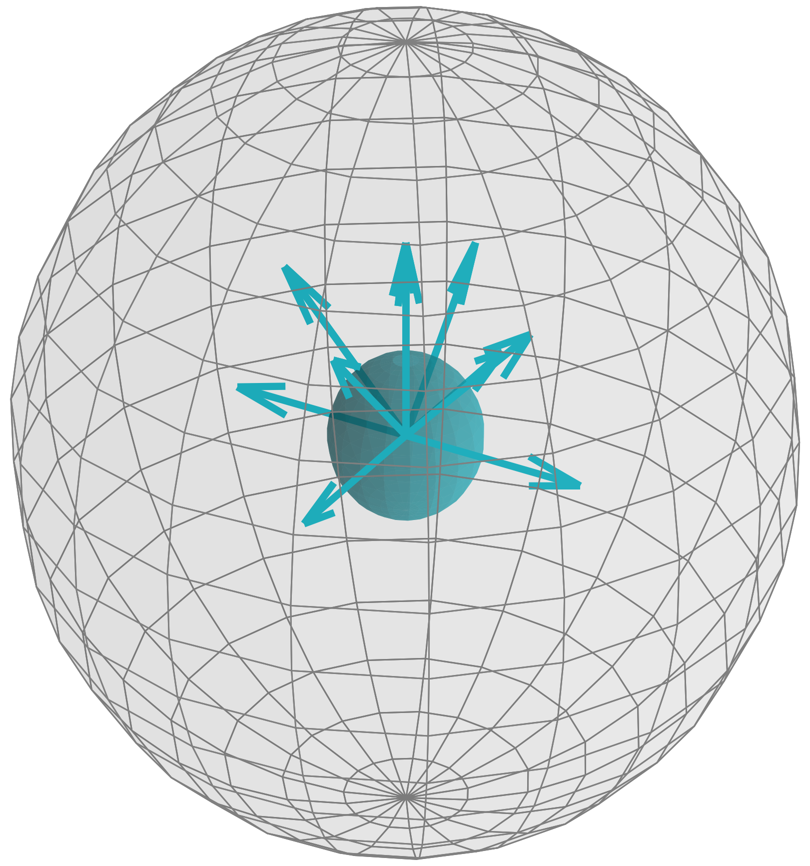


- Quantum electrodynamics in 2+1 dimensions,
- $N_f = 4$ fermions (factor 2 from $\alpha = \uparrow\downarrow$, factor 2 from 2 Dirac nodes)
- Gamma matrices: $(\gamma_0, \gamma_1, \gamma_2) = (i\sigma_y, \sigma_z, \sigma_x)$
- Gauge field a_μ stems from introducing single occupancy constraint
- Enhanced symmetry: SU(4) flavor

Properties of QED3

- Properties of QED3 strongly depend on the number of fermion flavours N_f (Dirac spin liquid $N_f = 4$)

$$\mathscr{L} = \sum_{i=1}^{4} \bar{\Psi}_i[-i\gamma^\mu(\partial_\mu + ia_\mu)]\Psi_i + \frac{1}{2e^2}\sum_\mu (\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2$$



- $N_f \to \infty$ limit supresses gauge fluctuations
- conformal field theory with gapless fermion and photon modes
- Monopole excitations at large energy scale $\propto N_f$

- $N_f = 0$ limit: pure $U(1)$ gauge field theory is confining in 2+1 dimensions

  [A. Polyakov, Nucl. Phys. B 120, 429 (1977)]

- $N_f = 4$: still subject of ongoing research (presumably strongly coupled CFT) [many refs...]

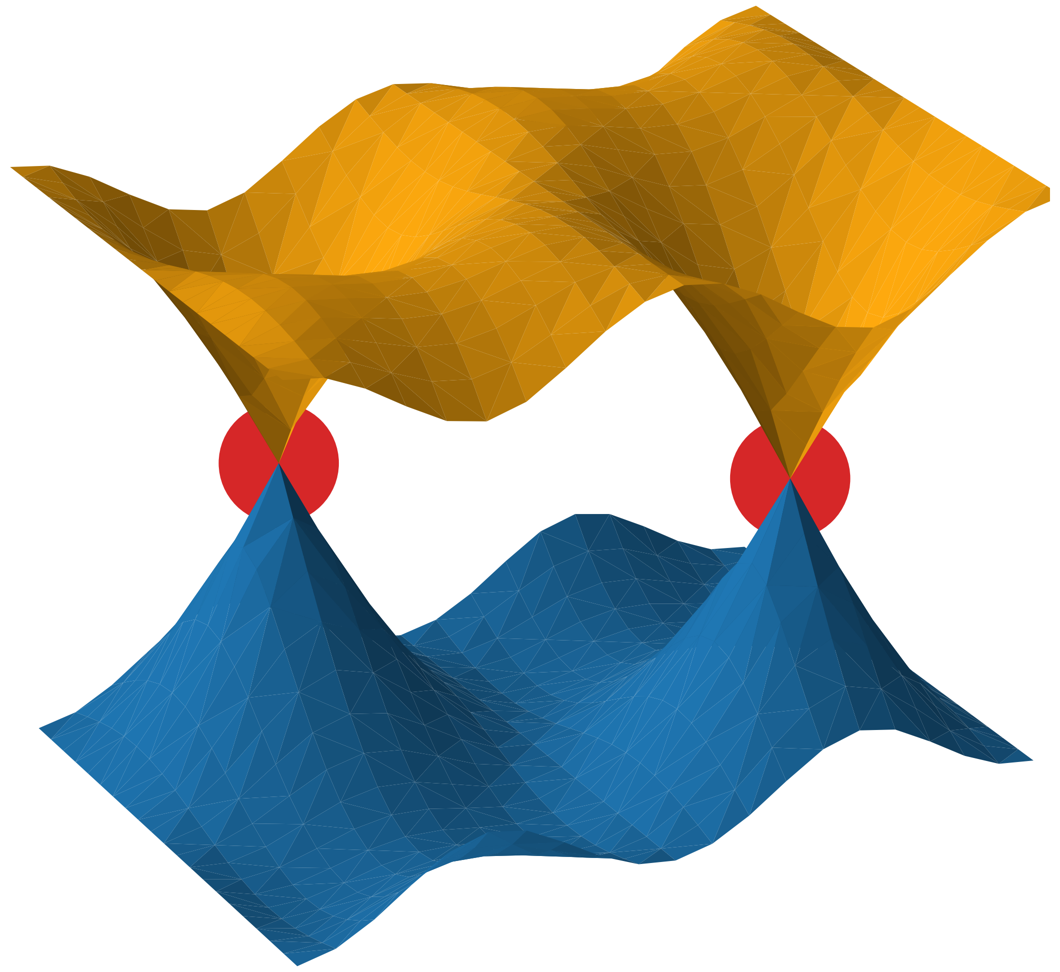
The mother of many competing orders

[M. Hermele, T. Senthil, and M.P.A. Fisher, Phys. Rev. B **72**, 104404 (2005)]

[X.-Y. Song, C. Wang, A. Vishwanath, Y.-C. He, Nat. Commun. **10**, 4254 (2019)]

- Whether QED3 with $N_f = 4$ fermions is stable is not fully settled → there can be spontaneous mass generation

$$\mathscr{L} = \sum_{i=1}^{4} \bar{\Psi}_i[-i\gamma^\mu(\partial_\mu + ia_\mu)]\Psi_i \quad + \quad g\phi \cdot \bar{\Psi}\mathbf{M}\Psi + (\partial_\mu\phi)^2 - u\phi^2 - \lambda\phi^4$$



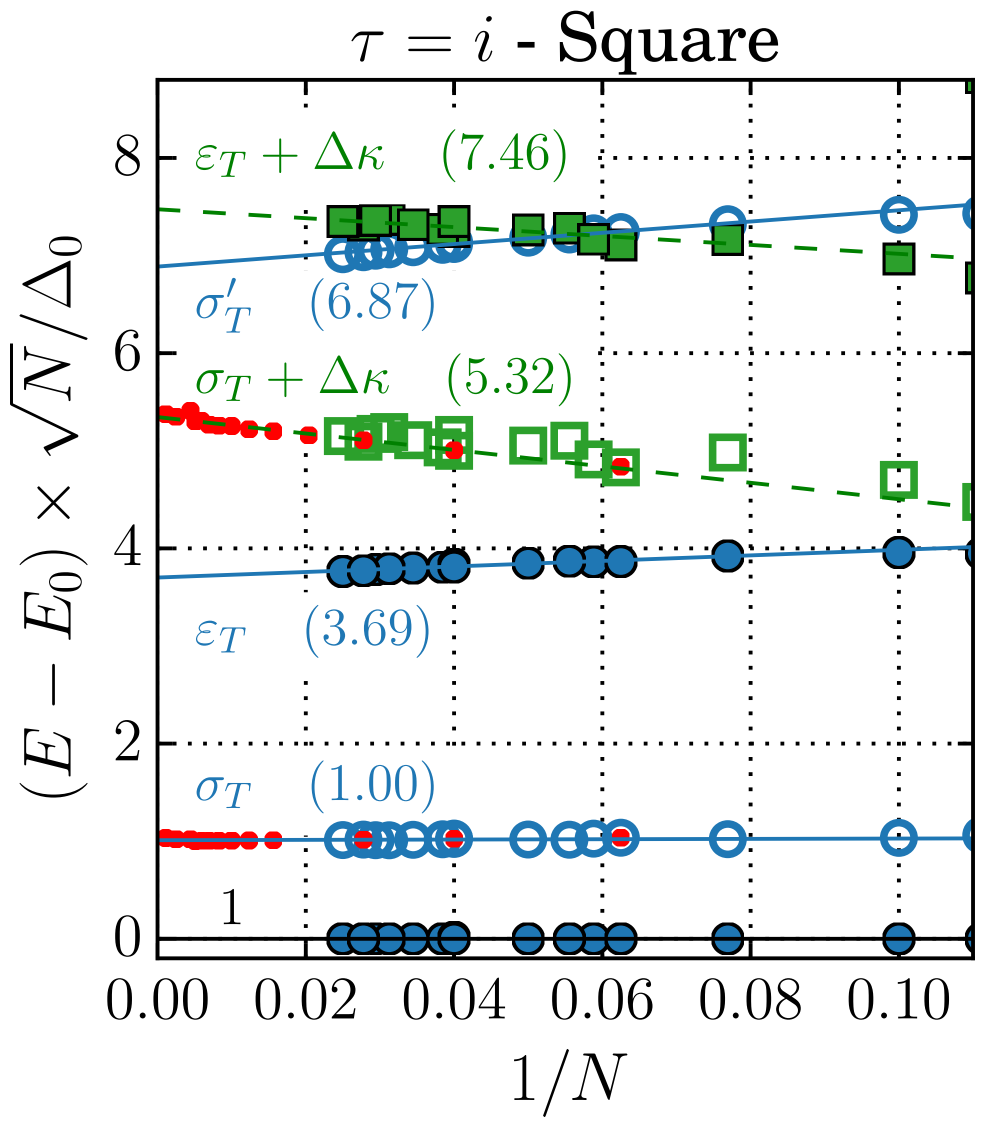
- Analogous to Higgs mechanism of gapping EM field in superconductor
- Different mass terms stabilise different phases of matter
- Mass term $\bar{\Psi}\Psi$ breaks time reversal symmetry → chiral spin liquid
- Mass terms $M_{i0} = \bar{\Psi}\sigma_i \otimes 1\Psi$ leads to non-collinear antiferromagnet → $120°$ Néel state
- Mass terms $M_{0j} = \bar{\Psi}1 \otimes \sigma_j\Psi$ leads to valence bond solid states
- the latter two cases yield a proliferation of monopole excitations

Low-energy spectrum on a torus

Operator-state correspondence in CFT valid on a sphere geometry

But numerics is more practical on a torus…

Nontrivial geometry but universal spectrum too !



2+1d Ising CFT

PRL **117,** 210401 (2016) PHYSICAL REVIEW LETTERS week ending 18 NOVEMBER 2016

Universal Signatures of Quantum Critical Points from Finite-Size Torus Spectra: A Window into the Operator Content of Higher-Dimensional Conformal Field Theories

Michael Schuler,[1] Seth Whitsitt,[2] Louis-Paul Henry,[1] Subir Sachdev,[2,3] and Andreas M. Läuchli[1]

Bilinear fermions in QED3

$\bar{\mathbf{q}} = (0,0)$		$\bar{\mathbf{q}} = (1,0)$		$\bar{\mathbf{q}} = (1,1)$	
$\bar{E}_f$	d_f	$\bar{E}_f$	d_f	$\bar{E}_f$	d_f
1.414214	$4N_f^2 - 2$	1.414214	$2N_f^2 - 1$	1.414214	$N_f^2 - 1$
		2.288246	$4N_f^2 - 2$	2.288246	$4N_f^2 - 2$
				2.828427	$2N_f^2 - 1$
3.162278	$8N_f^2 - 4$	3.162278	$2N_f^2 - 1$	3.162278	$2N_f^2 - 1$
		3.702459	$4N_f^2 - 2$		
		4.130649	$4N_f^2 - 2$	4.130649	$4N_f^2 - 2$
4.242640	$4N_f^2 - 2$				
				4.496615	$4N_f^2 - 2$
				4.670830	$4N_f^2 - 2$

PHYSICAL REVIEW B **95**, 205128 (2017)

Spectrum of conformal gauge theories on a torus

Alex Thomson[1] and Subir Sachdev[1,2]

Low-energy spectrum ?

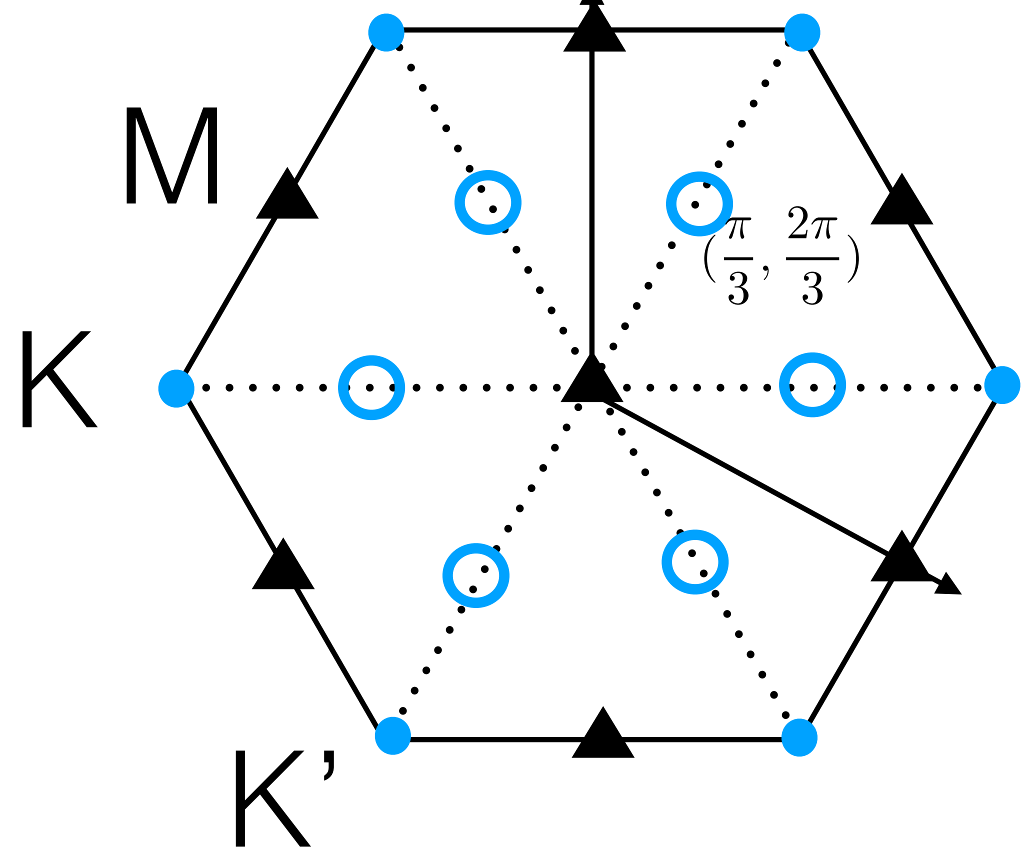
In a theory of massless Dirac fermions, there are monopoles and fermion zero modes

Table 2 Triangular lattice: fermion bilinears and monopole symmetries

	T_1	T_2	R	C_6	$\mathcal{T}$
M_{00}	$+$	$+$	$-$	$+$	$-$
M_{i0}	$+$	$+$	$+$	$-$	$+$
M_{01}	$-$	$-$	$-M_{03}$	$-M_{02}$	$+$
M_{02}	$+$	$-$	M_{02}	M_{03}	$+$
M_{03}	$-$	$+$	$-M_{01}$	M_{01}	$+$
M_{i1}	$-$	$-$	M_{i3}	M_{i2}	$-$
M_{i2}	$+$	$-$	$-M_{i2}$	$-M_{i3}$	$-$
M_{i3}	$-$	$+$	M_{i1}	$-M_{i1}$	$-$
$\Phi_1^\dagger$	$e^{-i\frac{\pi}{3}}\Phi_1^\dagger$	$e^{i\frac{\pi}{3}}\Phi_1^\dagger$	$-\Phi_3^\dagger$	Φ_2	Φ_1
$\Phi_2^\dagger$	$e^{i\frac{2\pi}{3}}\Phi_2^\dagger$	$e^{i\frac{\pi}{3}}\Phi_2^\dagger$	$\Phi_2^\dagger$	$-\Phi_3$	Φ_2
$\Phi_3^\dagger$	$e^{-i\frac{\pi}{3}}\Phi_3^\dagger$	$e^{-i\frac{2\pi}{3}}\Phi_3^\dagger$	$-\Phi_1^\dagger$	$-\Phi_1$	Φ_3
$\Phi_{4/5/6}^\dagger$	$e^{i\frac{2\pi}{3}}\Phi_{4/5/6}^\dagger$	$e^{-i\frac{2\pi}{3}}\Phi_{4/5/6}^\dagger$	$\Phi_{4/5/6}^\dagger$	$-\Phi_{4/5/6}$	$-\Phi_{4/5/6}$

The $M_{ij} = \overline{\psi}\sigma^i\tau^j\psi$ denotes the 16 fermion mass terms. Their transformation under lattice and time reversal symmetry are shown followed by the corresponding table for the six magnetic monopoles Φ_i. Symmetries $T_{1/2}$, R, C_6 denote translation and reflection marked in Fig. 1, and six-fold rotation around a site, respectively

Quantum numbers are known !



Clear signatures of a Dirac spin liquid in neutron scattering exp.

https://doi.org/10.1038/s41467-019-11727-3 OPEN

Unifying description of competing orders in two-dimensional quantum magnets

Xue-Yang Song[1], Chong Wang[1,2], Ashvin Vishwanath[1] & Yin-Chen He[1,2]

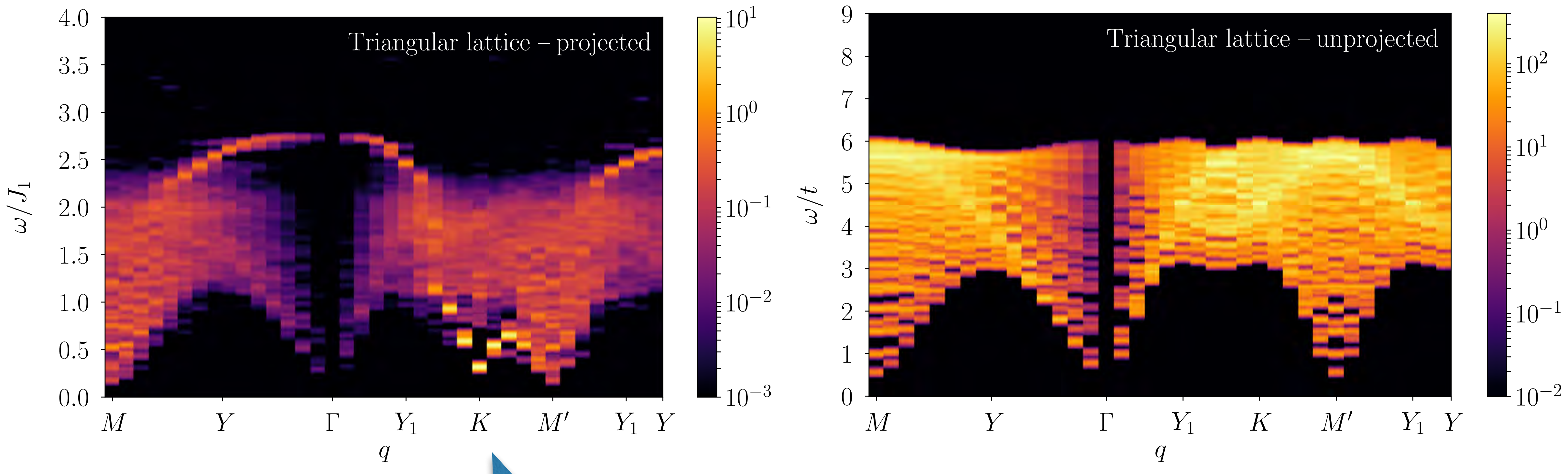
PHYSICAL REVIEW X **10**, 011033 (2020)

From Spinon Band Topology to the Symmetry Quantum Numbers of Monopoles in Dirac Spin Liquids

Xue-Yang Song,[1] Yin-Chen He,[2,1] Ashvin Vishwanath,[1] and Chong Wang[2,1]

Dynamical correlations

Variational Monte-Carlo



Low-energy monopoles ?

PHYSICAL REVIEW X **9**, 031026 (2019)

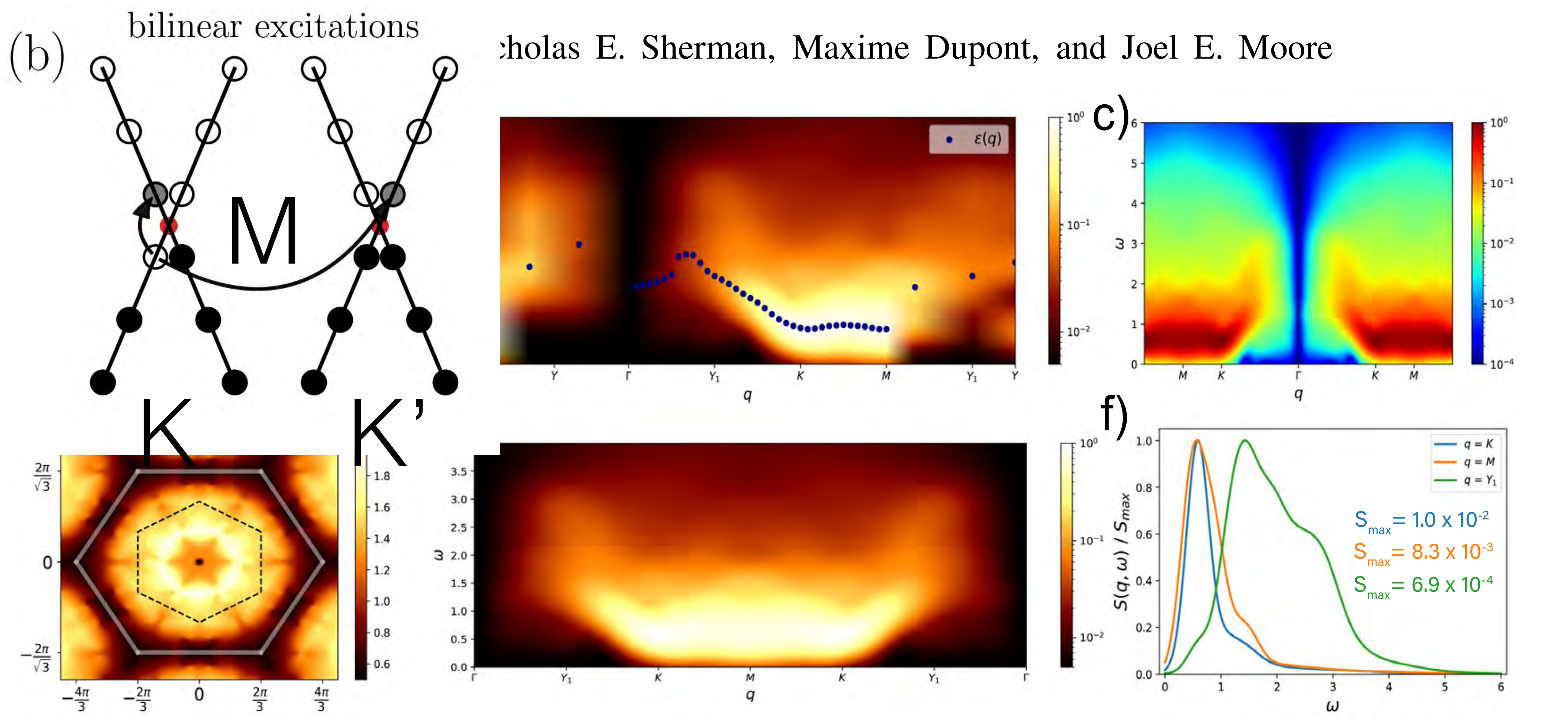
Dynamical Structure Factor of the J_1-J_2 Heisenberg Model on the Triangular Lattice: Magnons, Spinons, and Gauge Fields

Francesco Ferrari[1] and Federico Becca[2]

DMRG

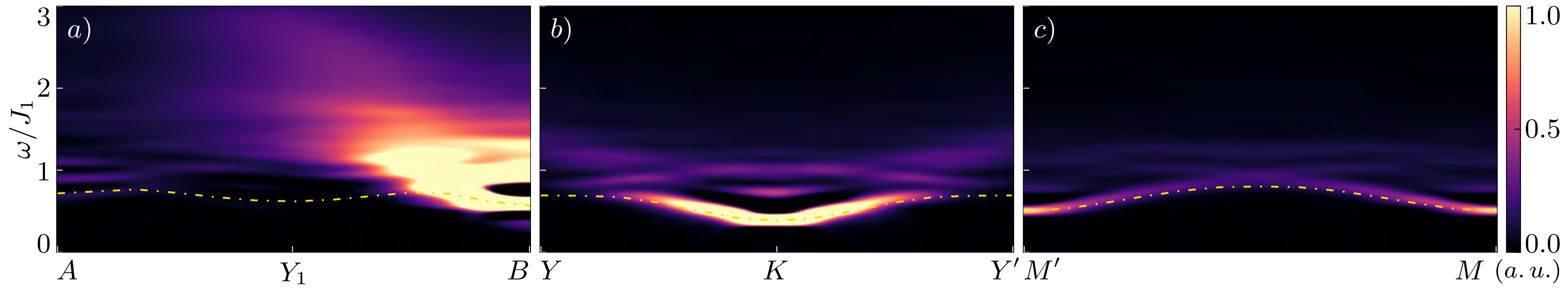
Spectral function of the $J_1 - J_2$ Heisenberg model on the triangular lattice

…cholas E. Sherman, Maxime Dupont, and Joel E. Moore



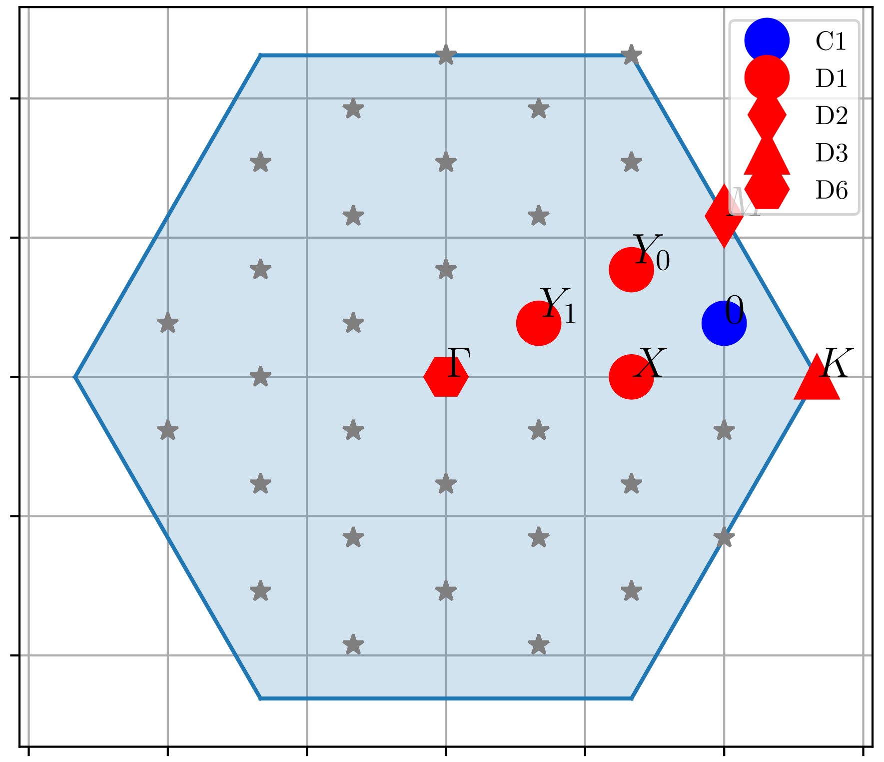
Dynamical Signatures of Symmetry Broken and Liquid Phases in an $S = 1/2$ Heisenberg Antiferromagnet on the Triangular Lattice

Markus Drescher,[1] Laurens Vanderstraeten,[2] Roderich Moessner,[3] and Frank Pollmann[1,4]

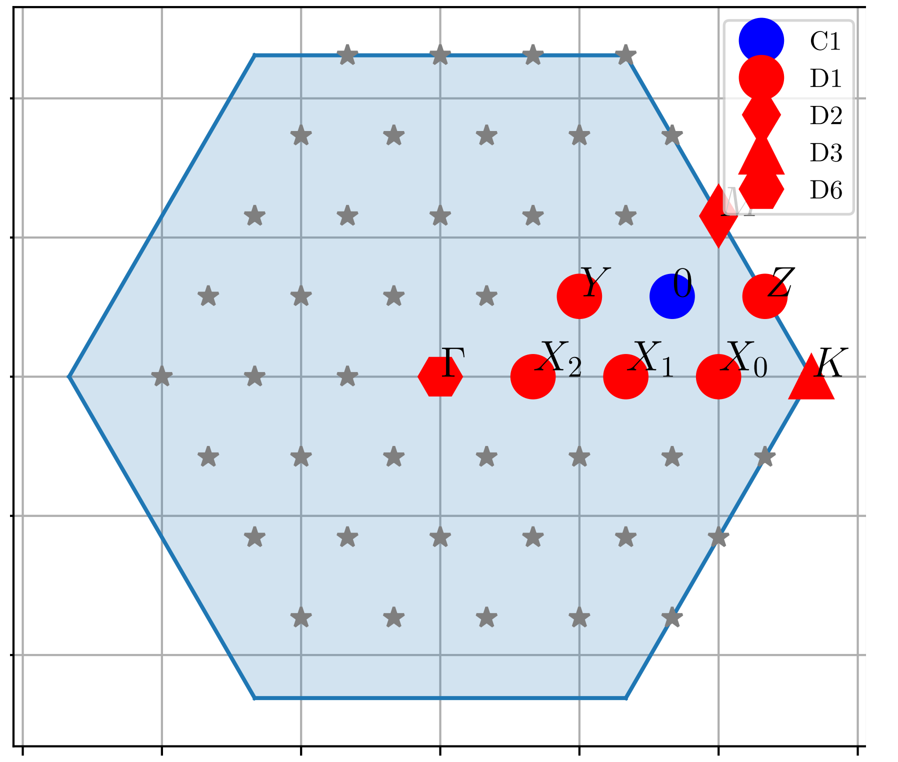


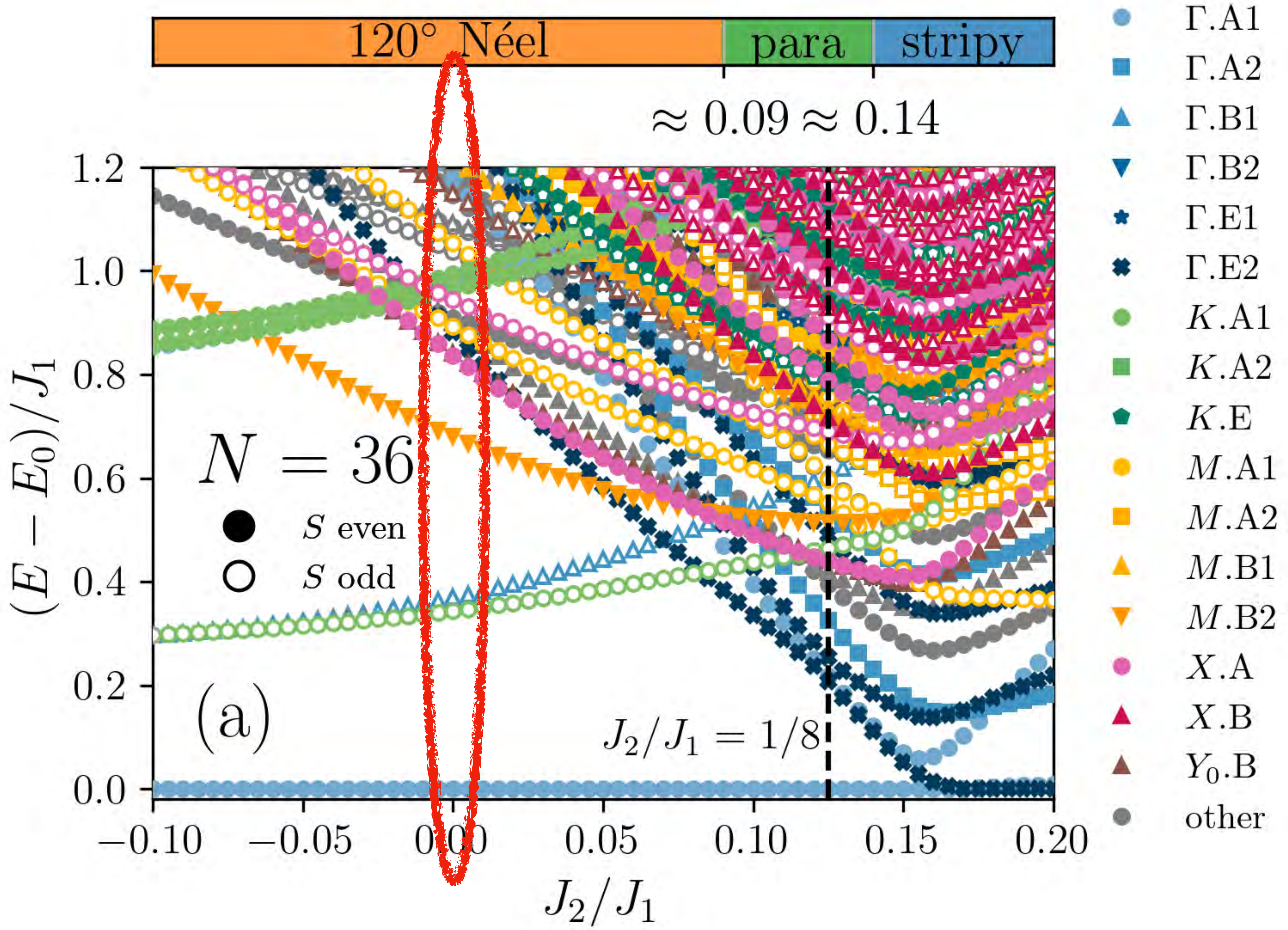
Dirac spin liquid ? or else ?

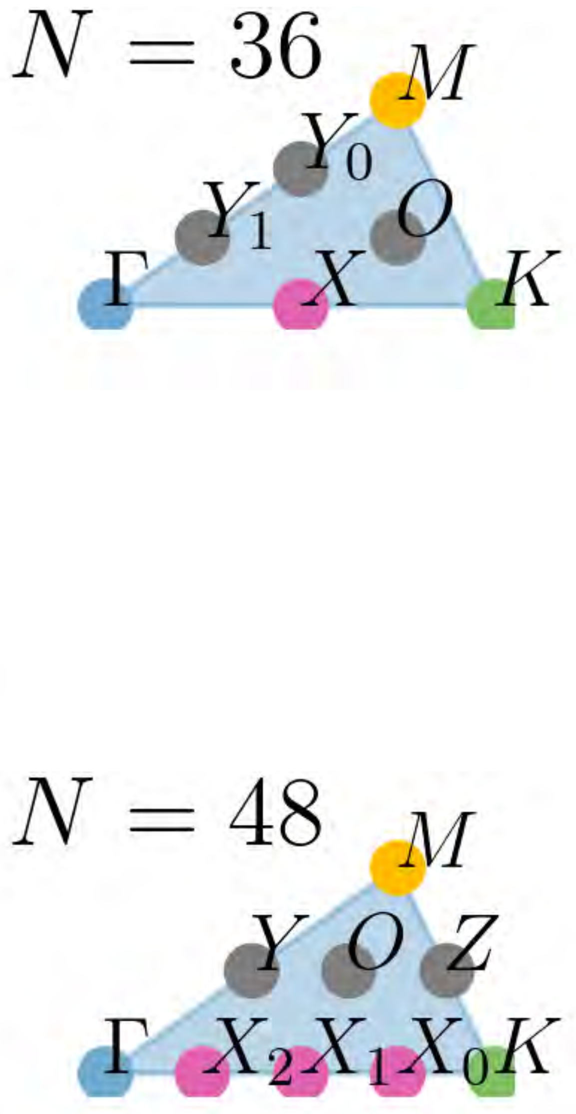
Exact diagonalization spectrum

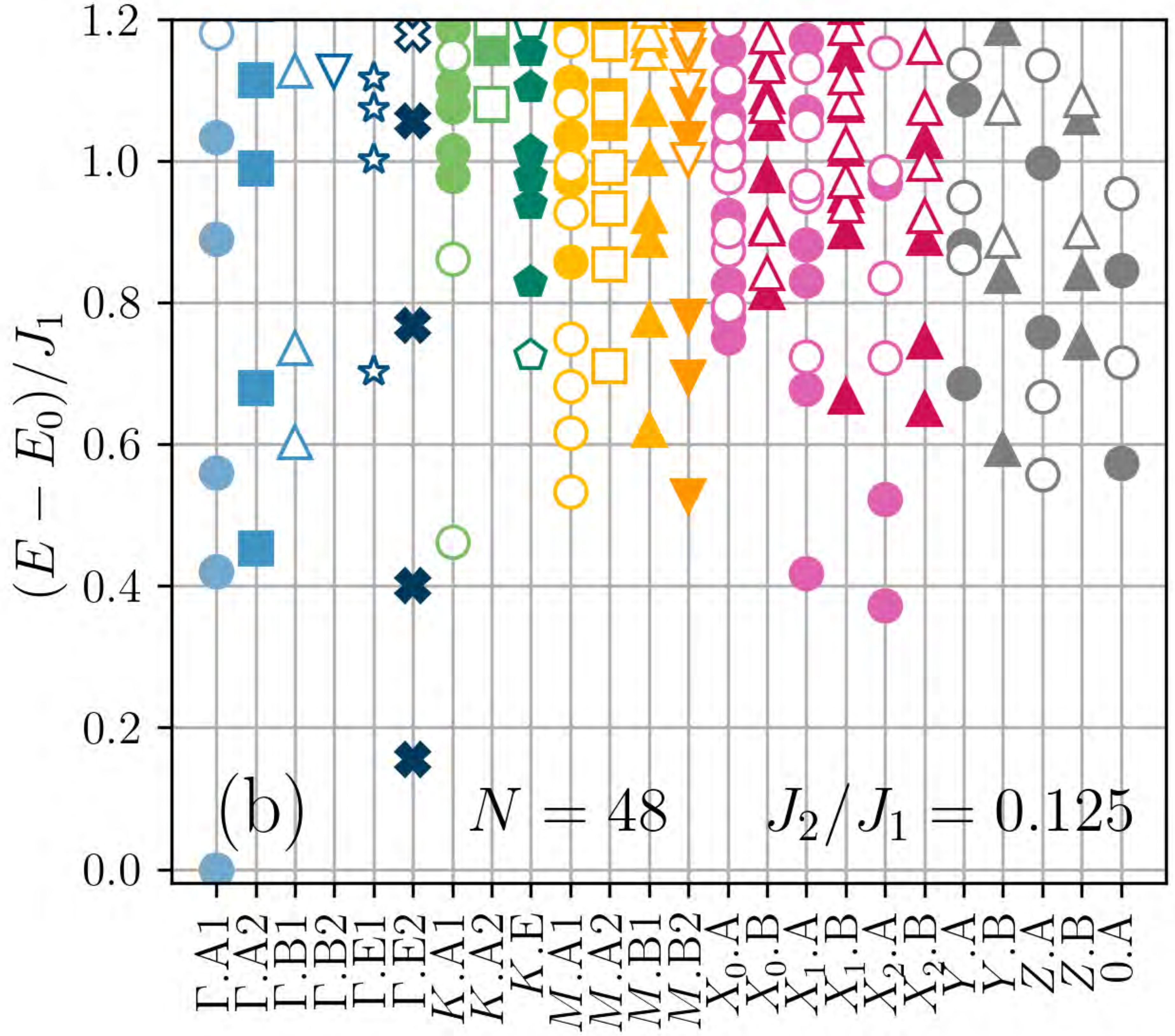


$$H = J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i \cdot \vec{S}_j$$

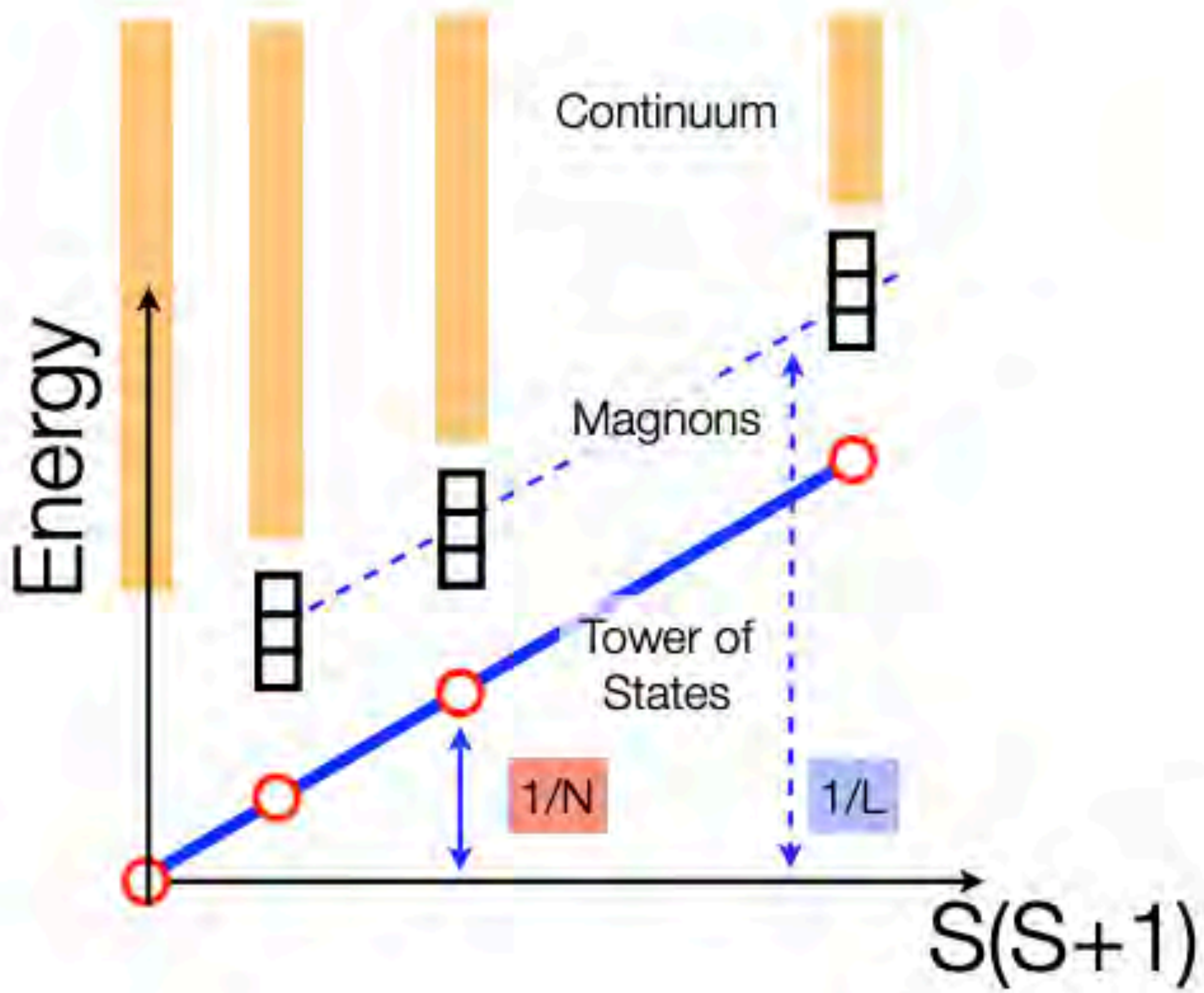


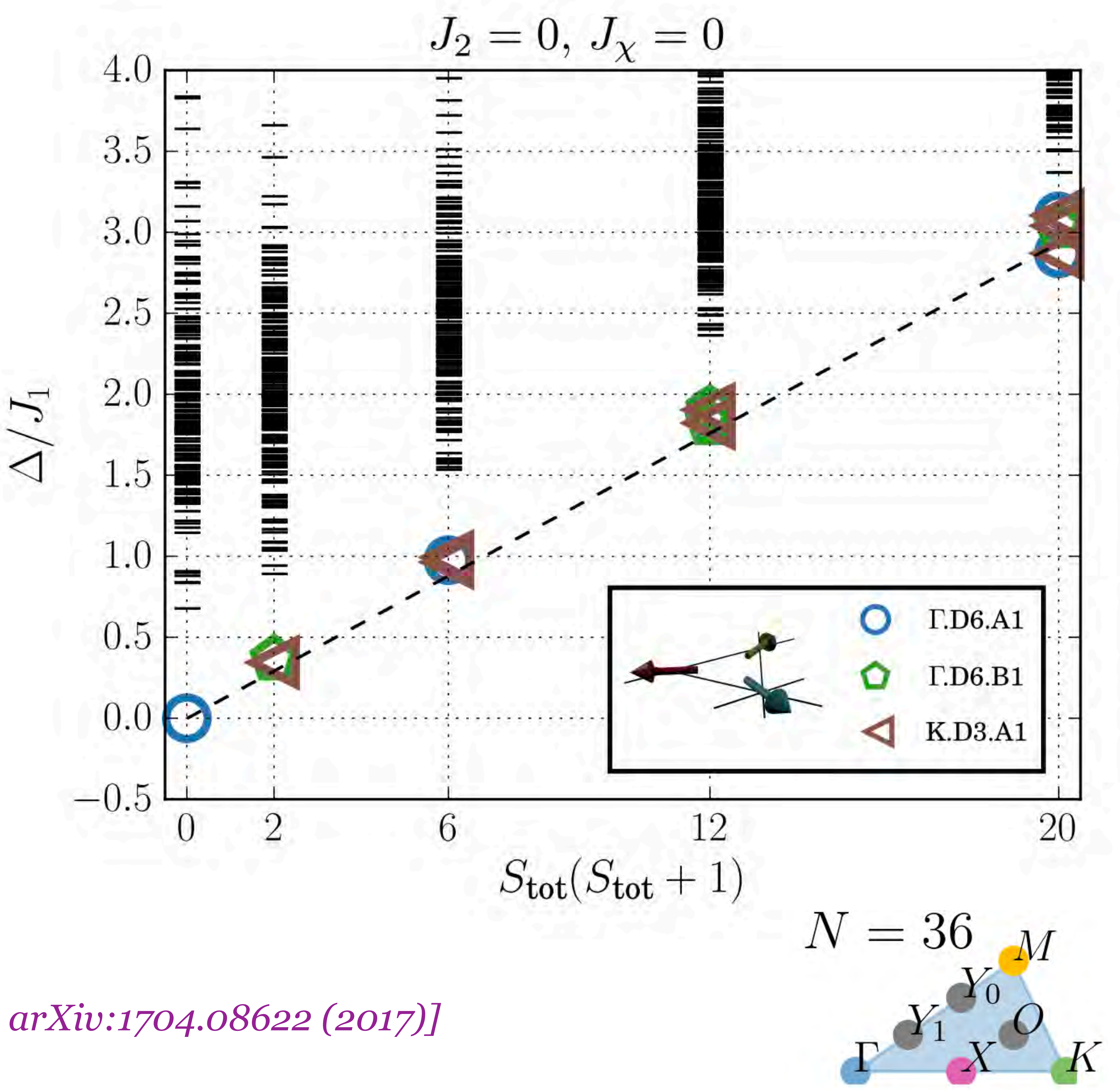






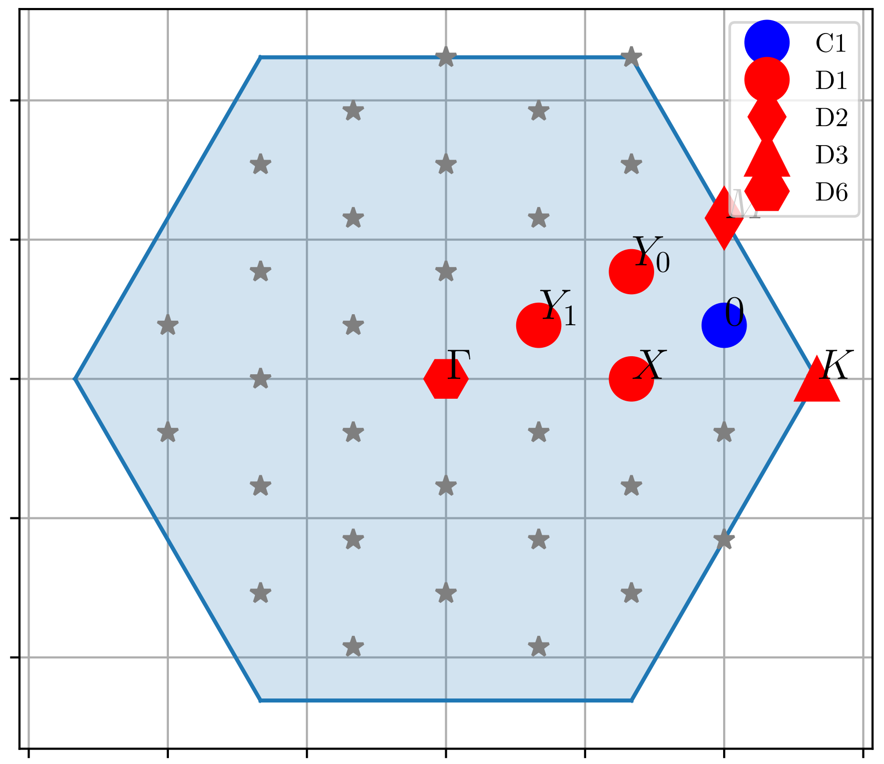
Anderson's tower of states: Néel order



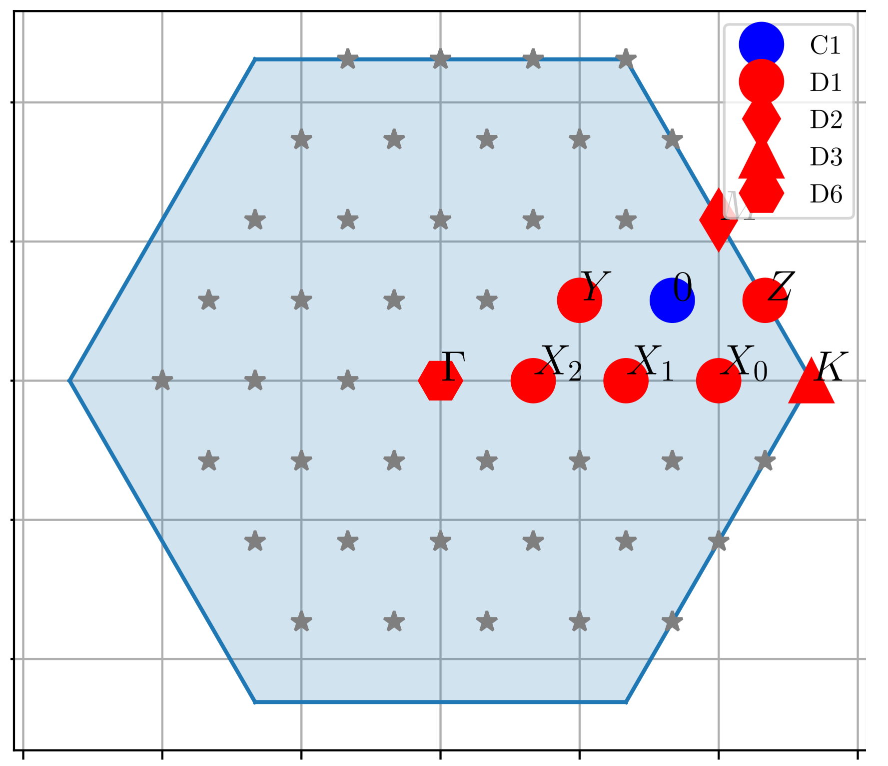


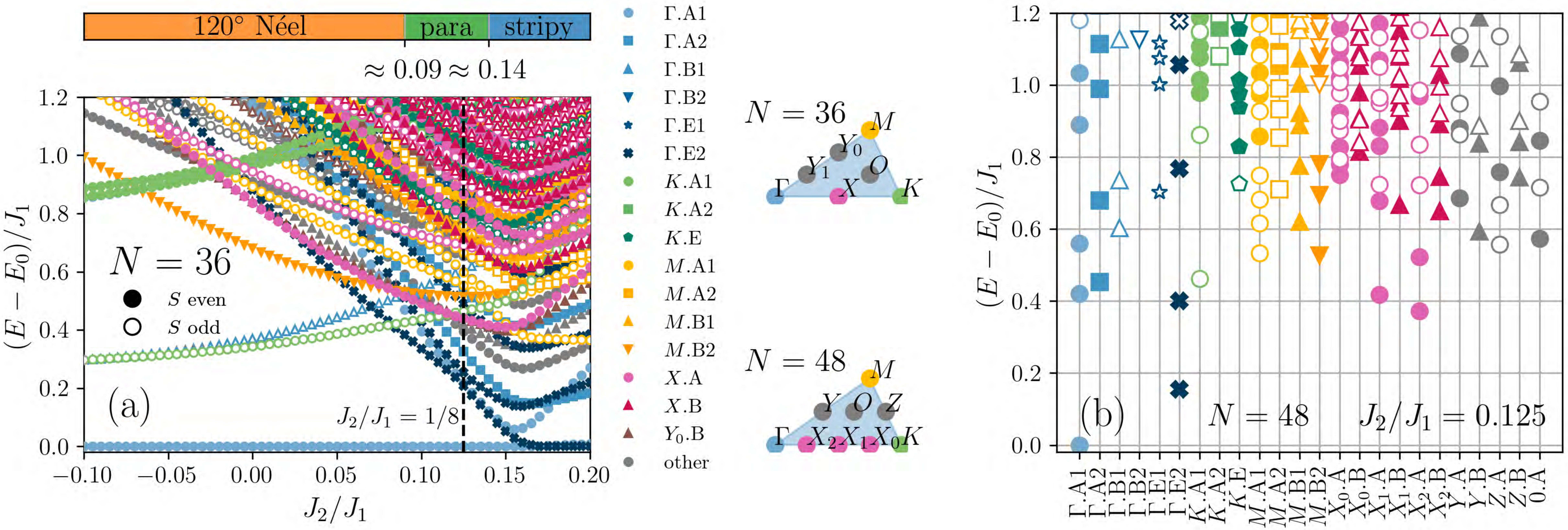
[A, Wietek, M. Schuler, A. M. Läuchli, arXiv:1704.08622 (2017)]

Low-energy excitations: ED results

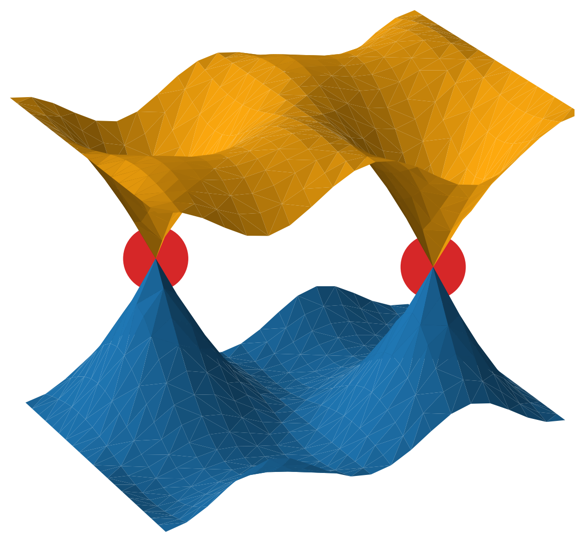


$$H = J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i \cdot \vec{S}_j$$

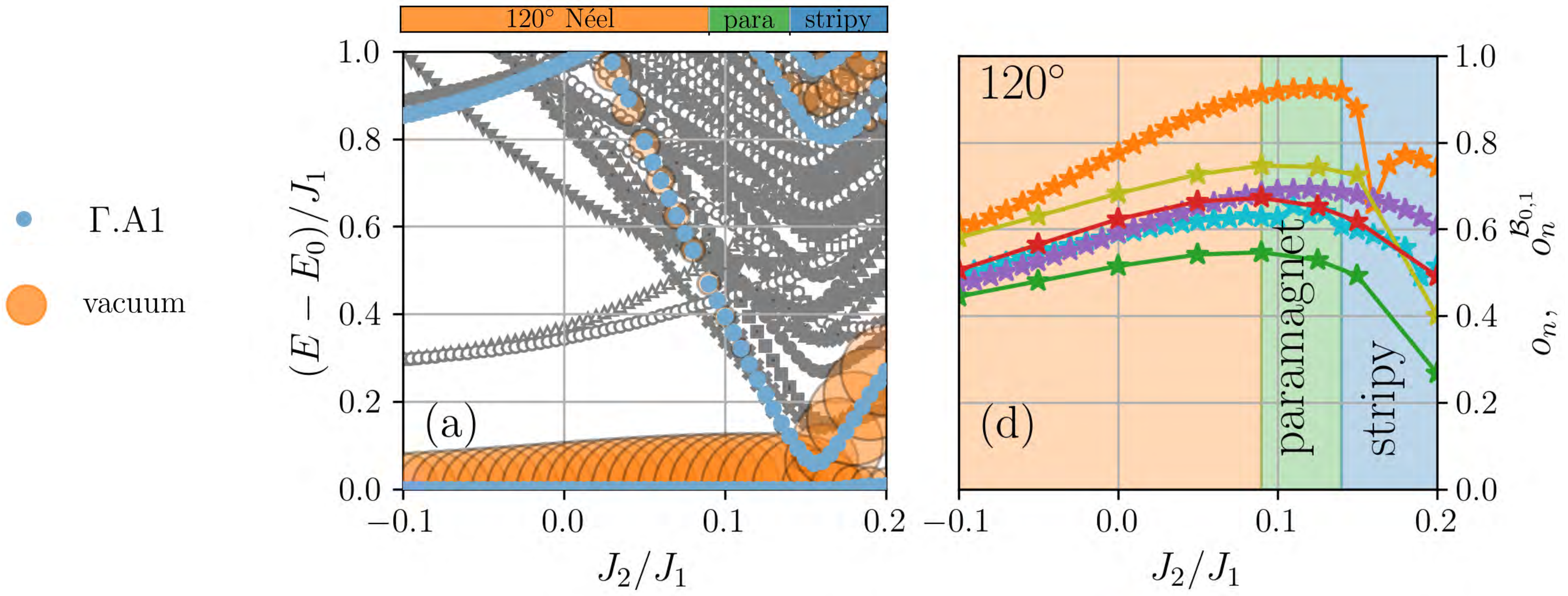




Filled Dirac sea = QED3 vacuum

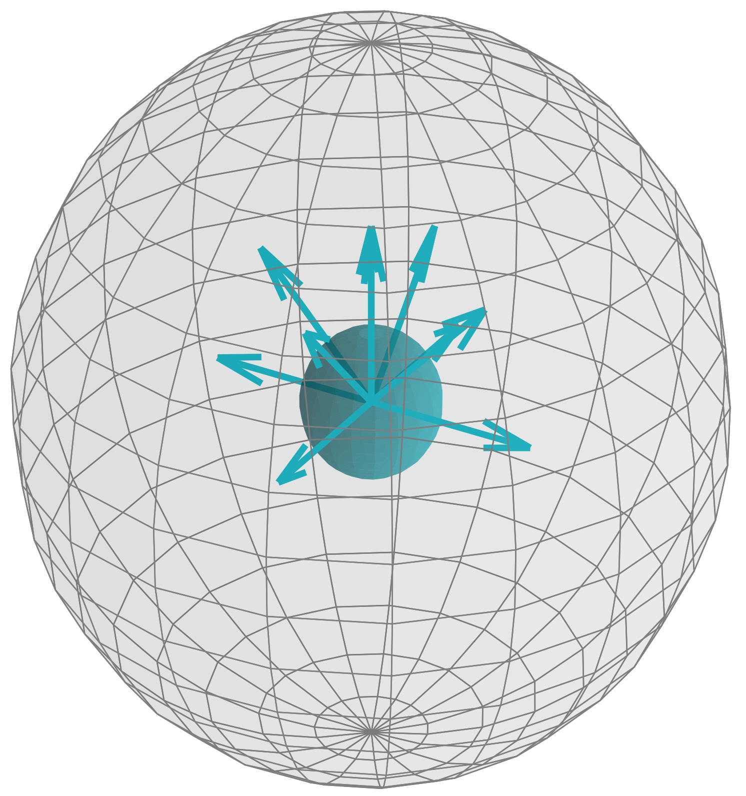


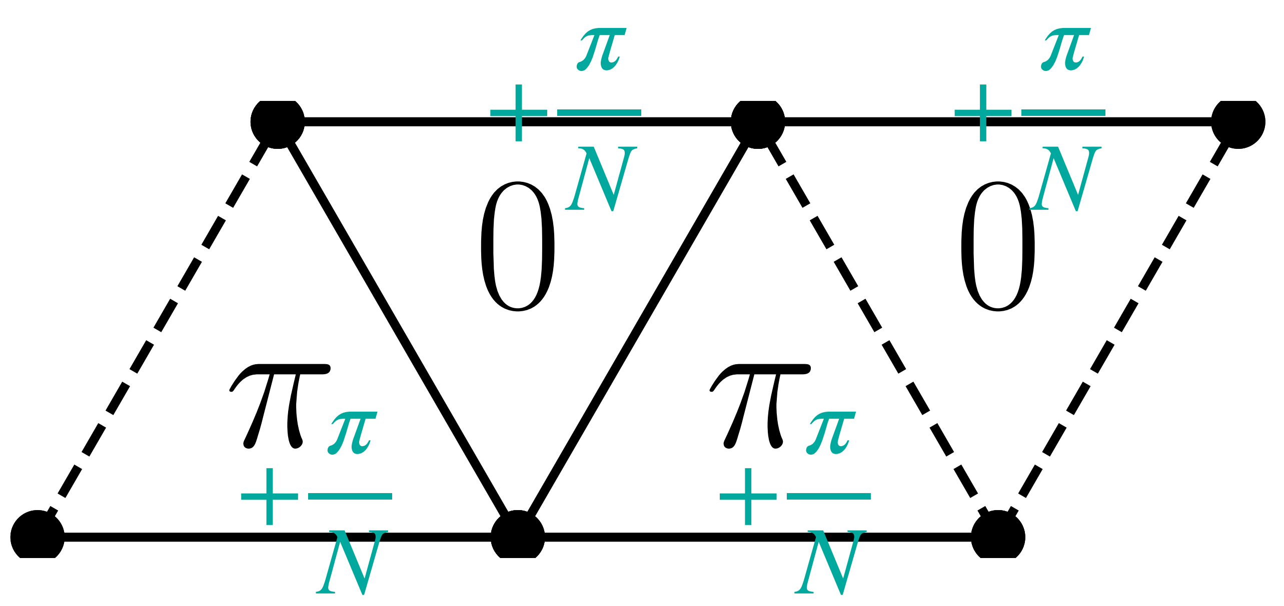
- Construct "vacuum" state by filling the Dirac sea with ↑ / ↓ fermions
- Perform Gutzwiller projection numerically to compute a *spin* wavefunction $|\psi_{GW}\rangle$
- Compute overlaps $o_n = |\langle\psi_{GW}|\psi_n\rangle|$ with exact eigenstates from exact diagonalization $|\psi_n\rangle$

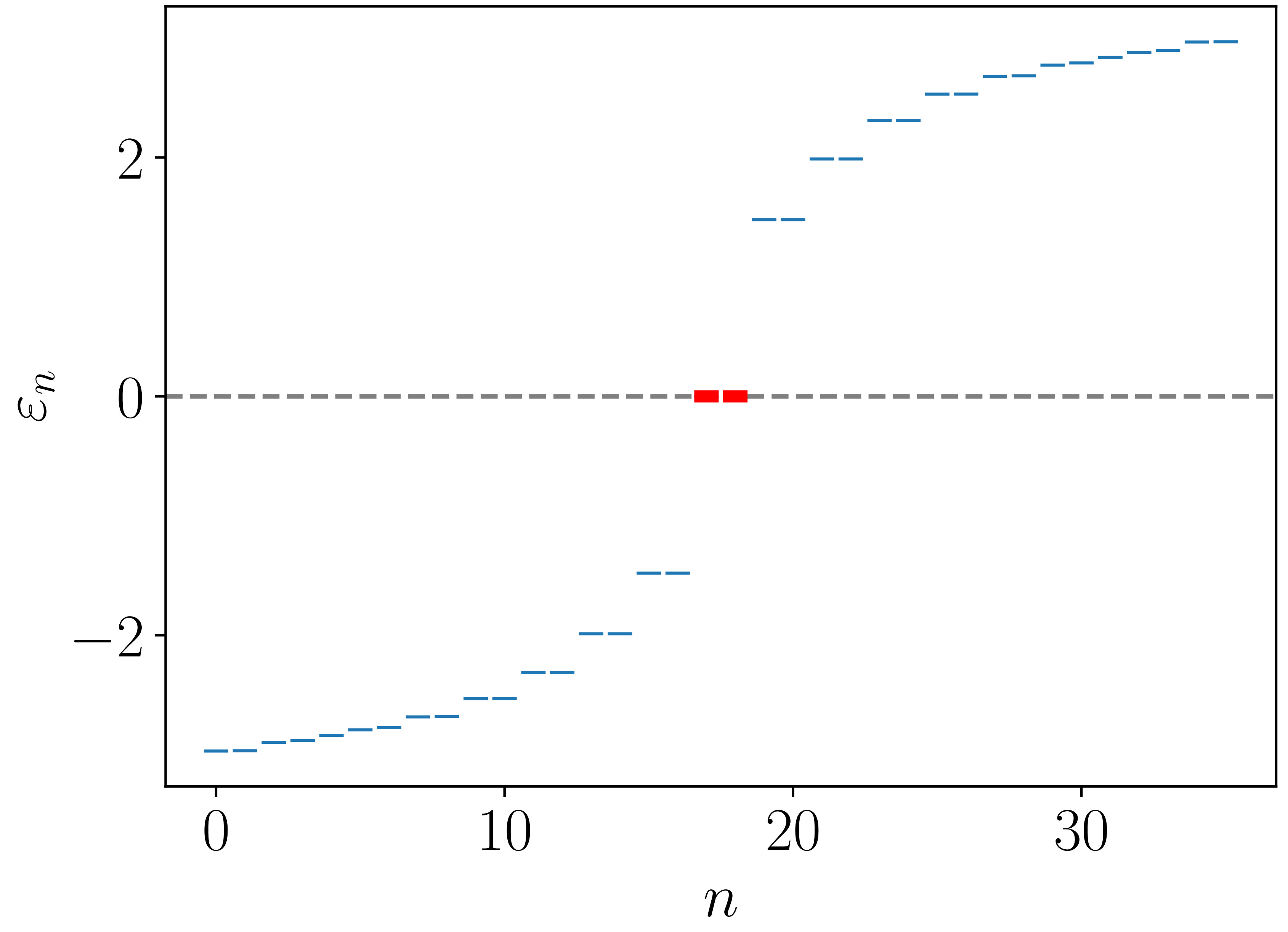


Monopole excitations

- Excitation by adding a 2π flux to the gauge field through the torus





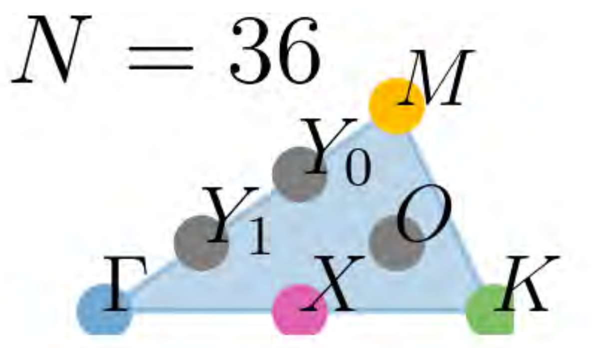


- Energy spectrum of ansatz has two-degenerate zero modes

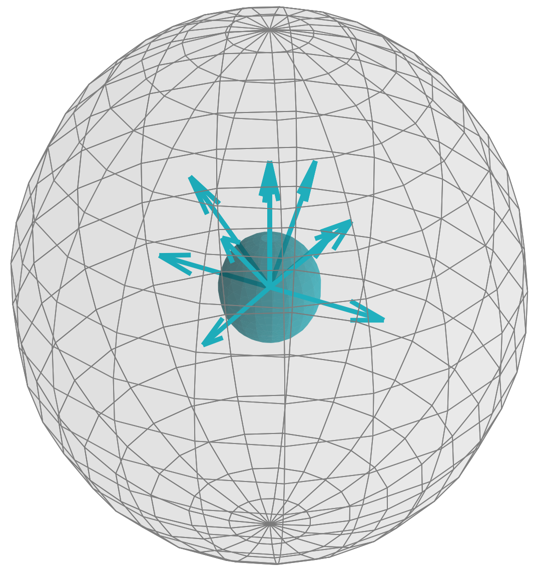
- Filling all modes below zero, 3 choices to zero modes as singlet, 3 choices as triplet → 6 monopole excitations

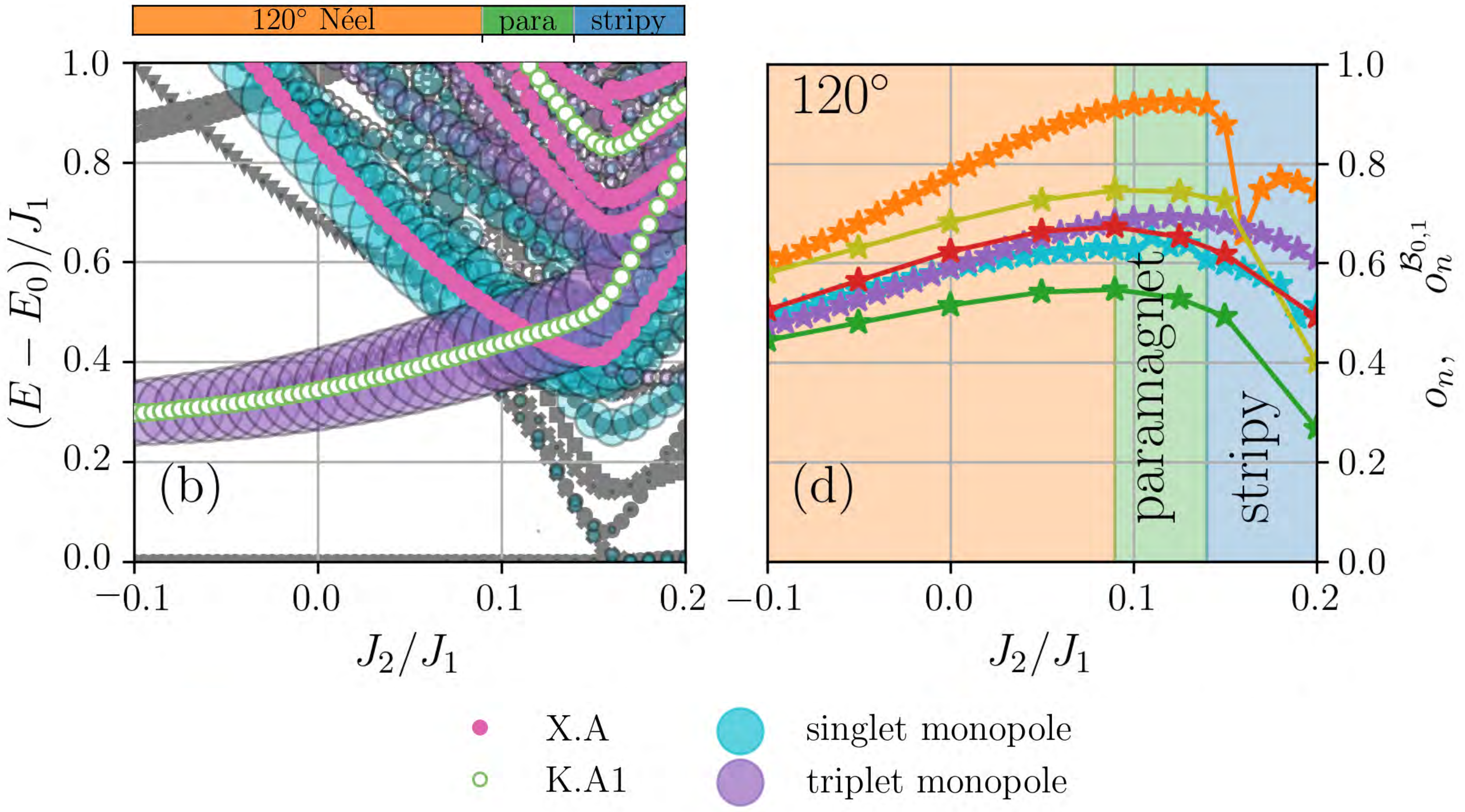
- Highly non-trivial prediction from [X.-Y. Song, C. Wang, A. Vishwanath, Y.-C. He, Nat. Commun. 10, 4254 (2019)]

Singlet monopoles $\rightarrow \mathbf{k} = X$ Triplet monopoles $\rightarrow \mathbf{k} = K$



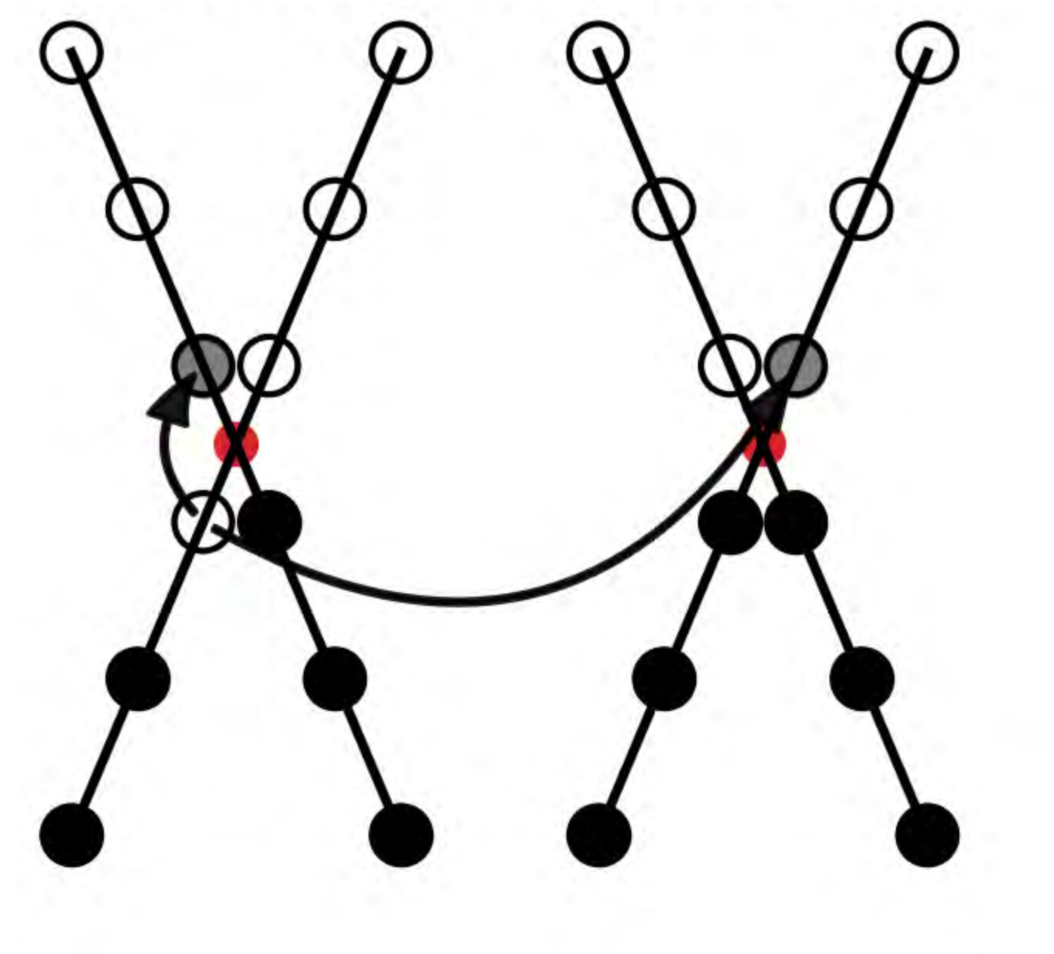
Monopole excitations

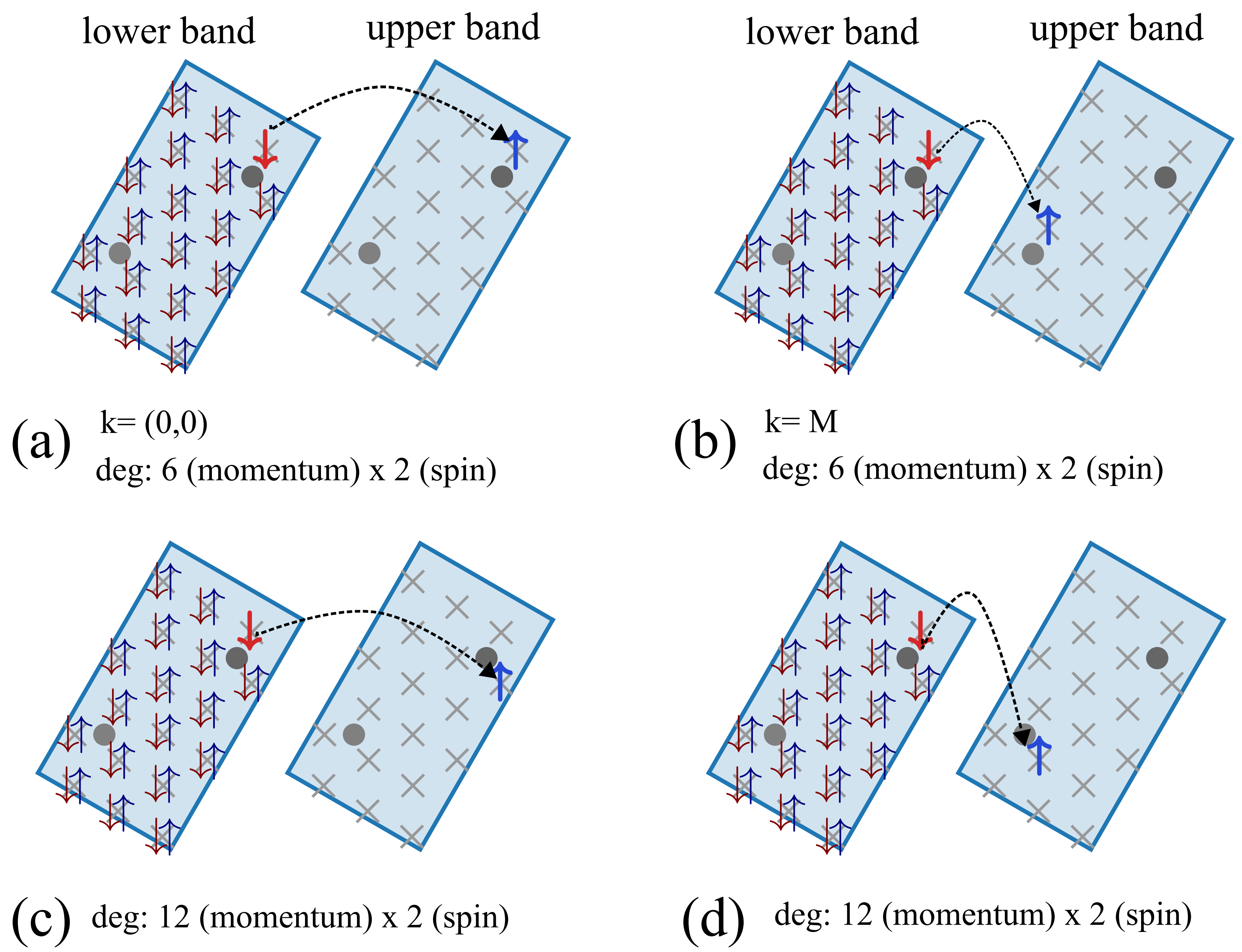




Bilinear excitations

- Particle-hole excitations out of the Dirac sea implement "bilinear" excitations $\bar{\Psi}\mathbf{M}\Psi$

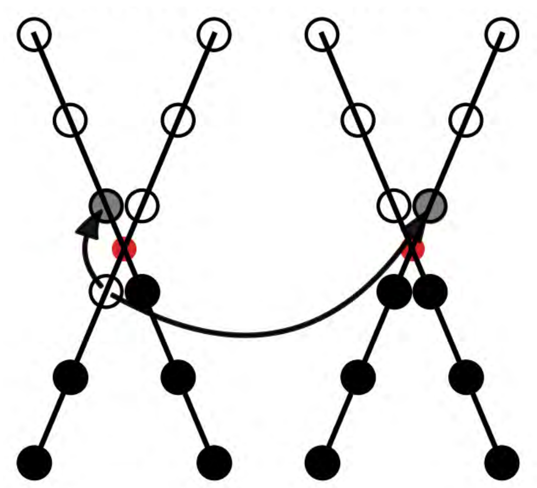


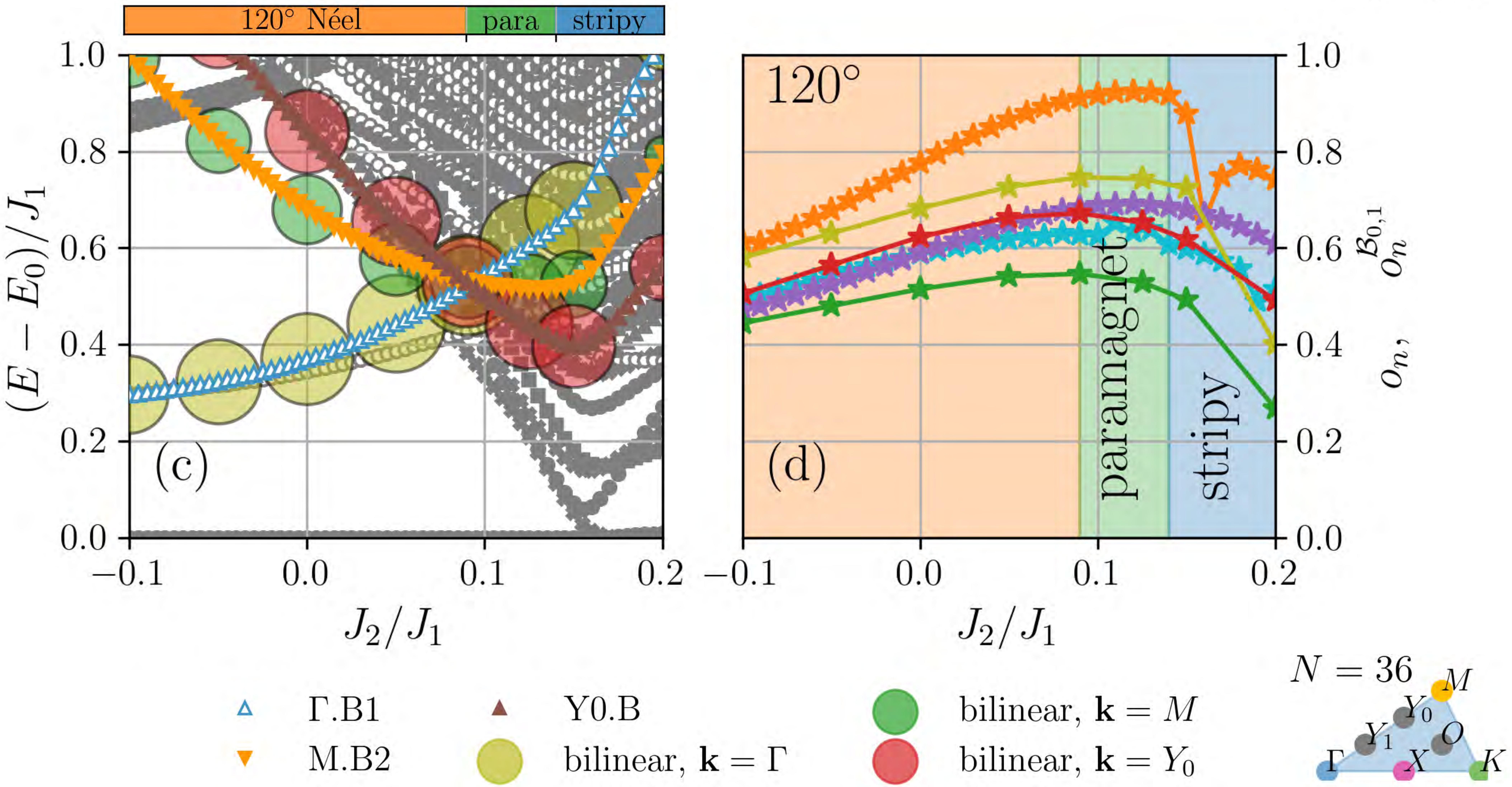


- In total, this yields 144 excited states, Gutzwiller projection conserves momentum (much larger than the expected $N_f^2=16$ on the sphere)

- Overlaps with bilinear space: $\left(o_n^{\mathscr{B}}\right)^2 \equiv \sum_{\alpha=1}^{\mathrm{dim}(\mathscr{B})} |\langle \phi_\alpha | \psi_n \rangle|^2$

Bilinear excitations





Low-energy excitations

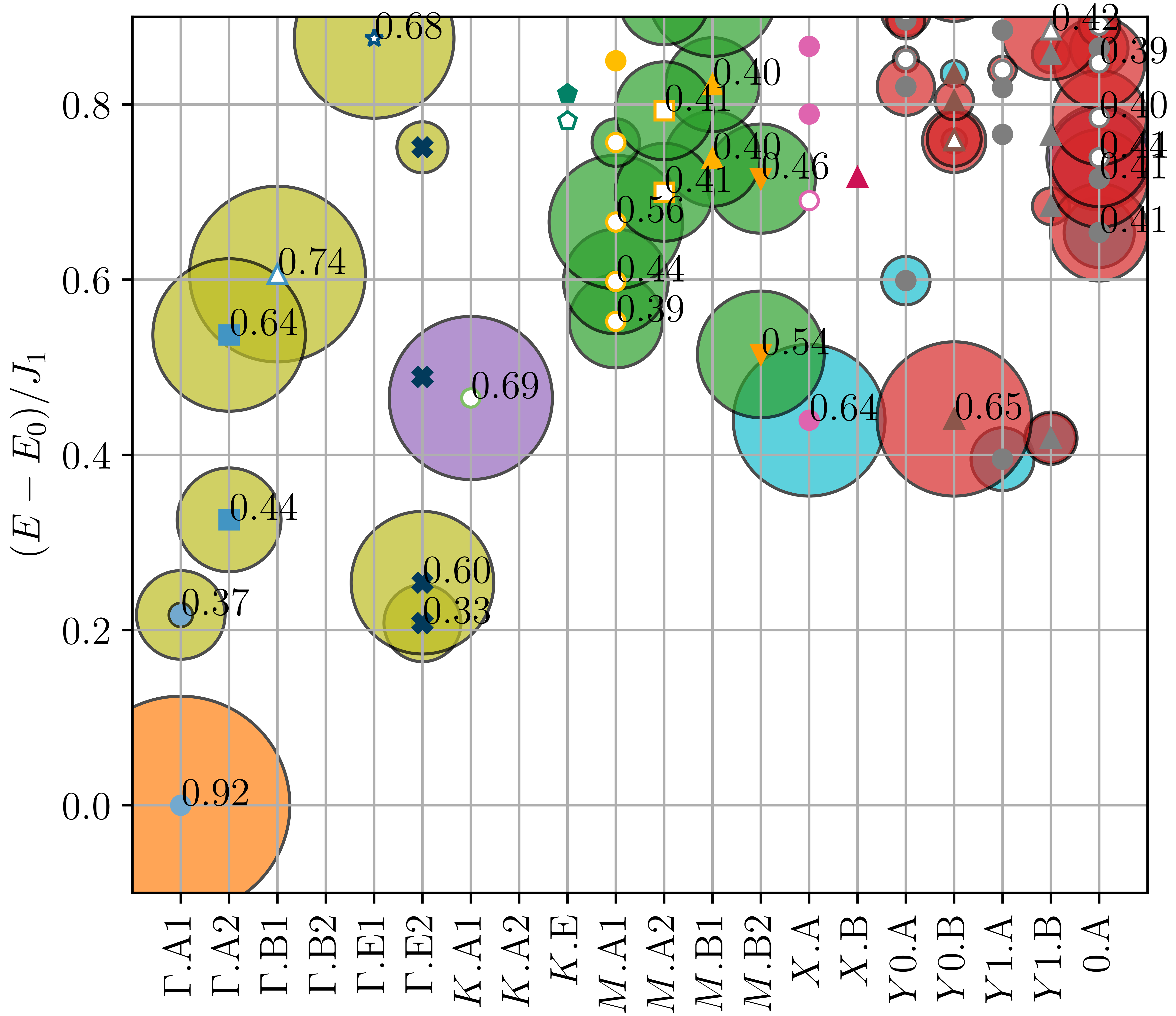
Overlaps between variational states and exact eigenstates on N=36

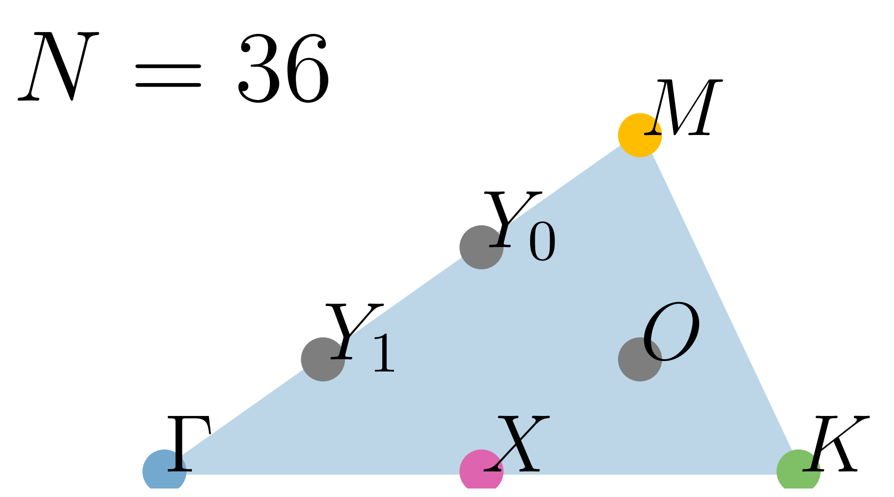
Note that all variational states have **no free parameters** !

Example: ground-state overlap reaches 0.92; moreover quantum numbers are in agreement

Low-energy excitations : summary

Overlaps $|\langle\psi|\psi_{ED}\rangle|$ on low-energy states for $J_2/J_1=1/8$ (N=36)





singlet monopole
triplet monopole
bilinear $\mathbf{k} = \Gamma$
bilinear $\mathbf{k} = M$
bilinear $\mathbf{k} \neq \Gamma, M$
vacuum

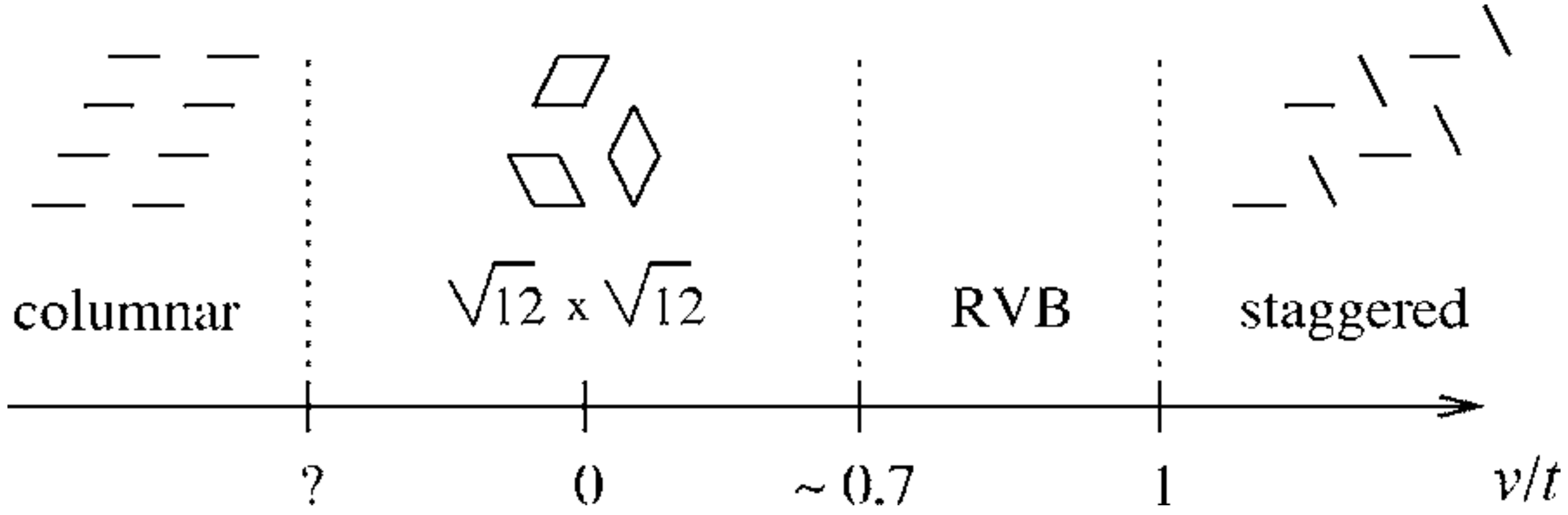
Gauge field ? Look at the quantum dimer model !

$$H = -t \sum \left(|//\rangle\langle \_\overline{\ \ }| + \text{H.c.} \right) + V \sum \left(|//\rangle\langle //| + |\_\overline{\ \ }\rangle\langle \_\overline{\ \ }| \right)$$

Rokhsar-Kivelson quantum dimer model

Gauss' law ~ dimer constraint

This represents a lattice gauge theory, but symmetry is only Z2



Crystalline phases / Z2 topological spin liquid

VOLUME 86, NUMBER 9 PHYSICAL REVIEW LETTERS 26 FEBRUARY 2001

Resonating Valence Bond Phase in the Triangular Lattice Quantum Dimer Model

R. Moessner and S. L. Sondhi

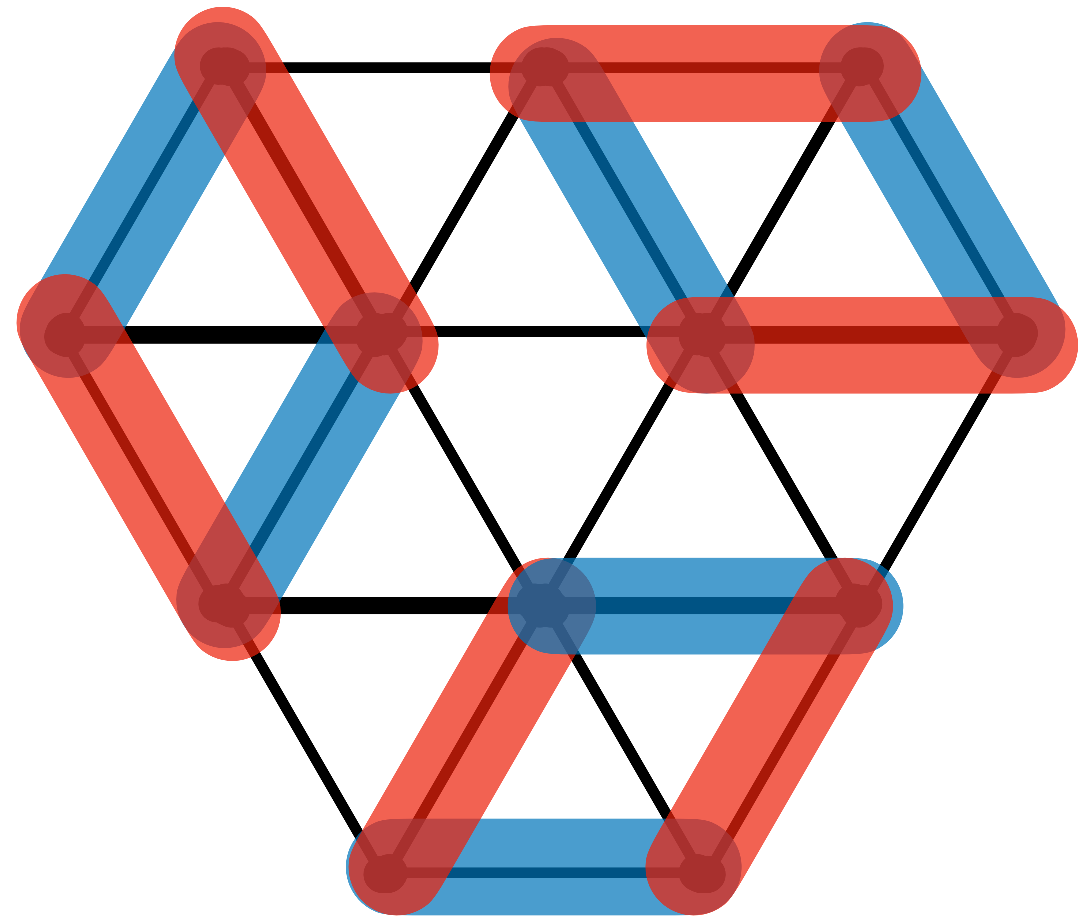
PHYSICAL REVIEW B **71**, 224109 (2005)

Zero-temperature properties of the quantum dimer model on the triangular lattice

Arnaud Ralko,[1] Michel Ferrero,[2] Federico Becca,[2] Dmitri Ivanov,[1] and Frédéric Mila[1]

Possible valence-bond crystal

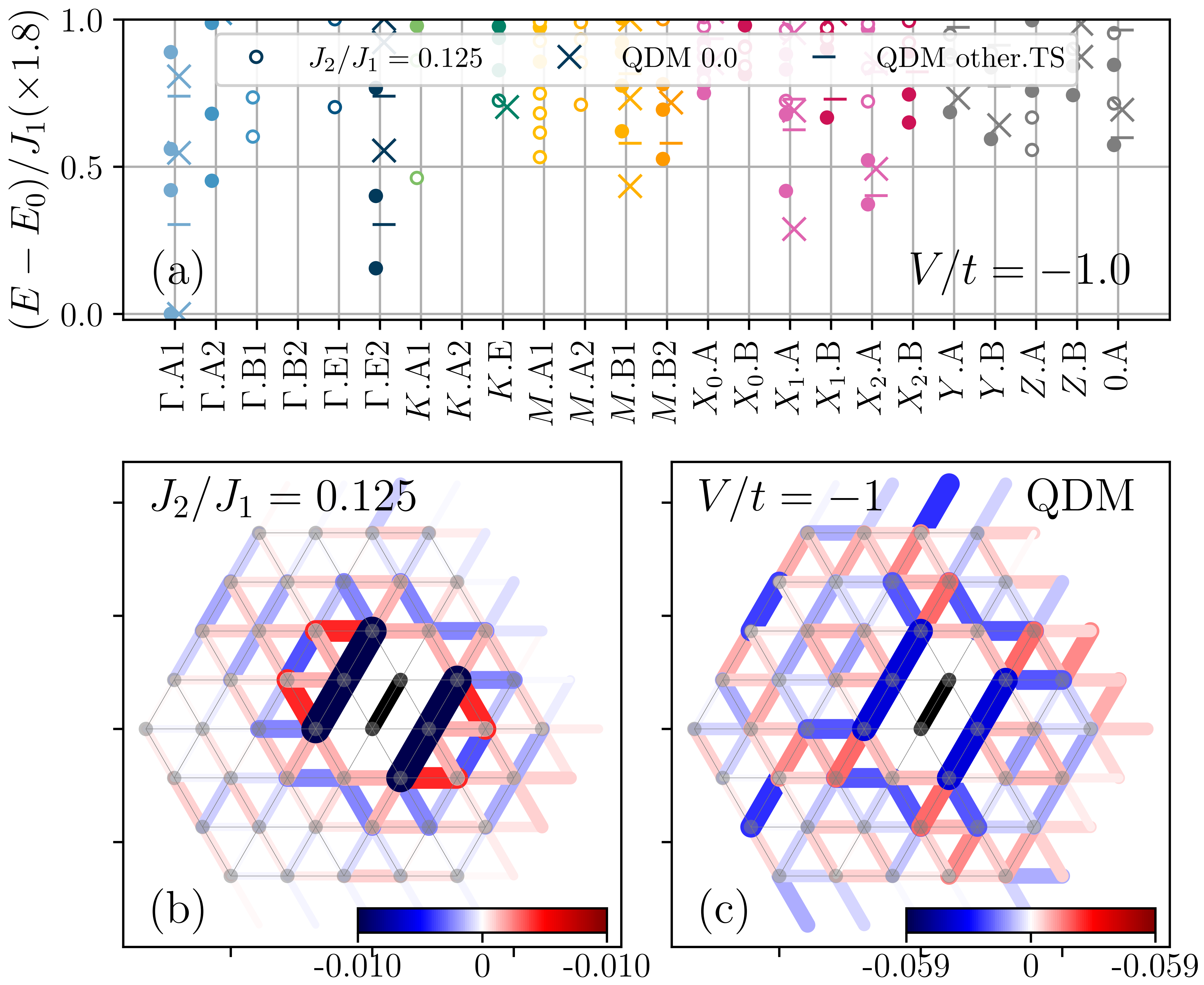
- Strong similarities between the paramagnetic regime and a VBC phase observed in the quantum dimer model (many refs.)



- Ground state dimer correlations have same sign structure

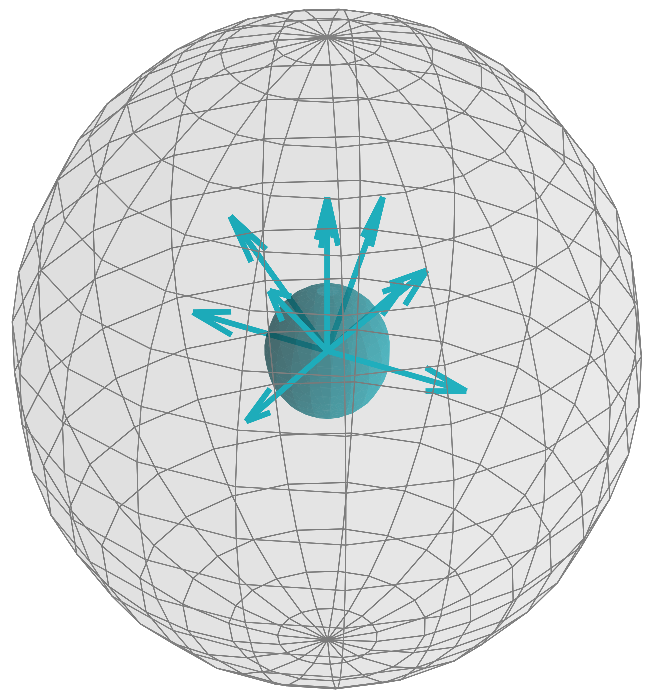
$$\mathscr{D} = \langle(\mathbf{S}_0 \cdot \mathbf{S}_1)(\mathbf{S}_i \cdot \mathbf{S}_j)\rangle_c$$

- Key features of many body spectrum agree (Including singlets at Γ, K, M, X)



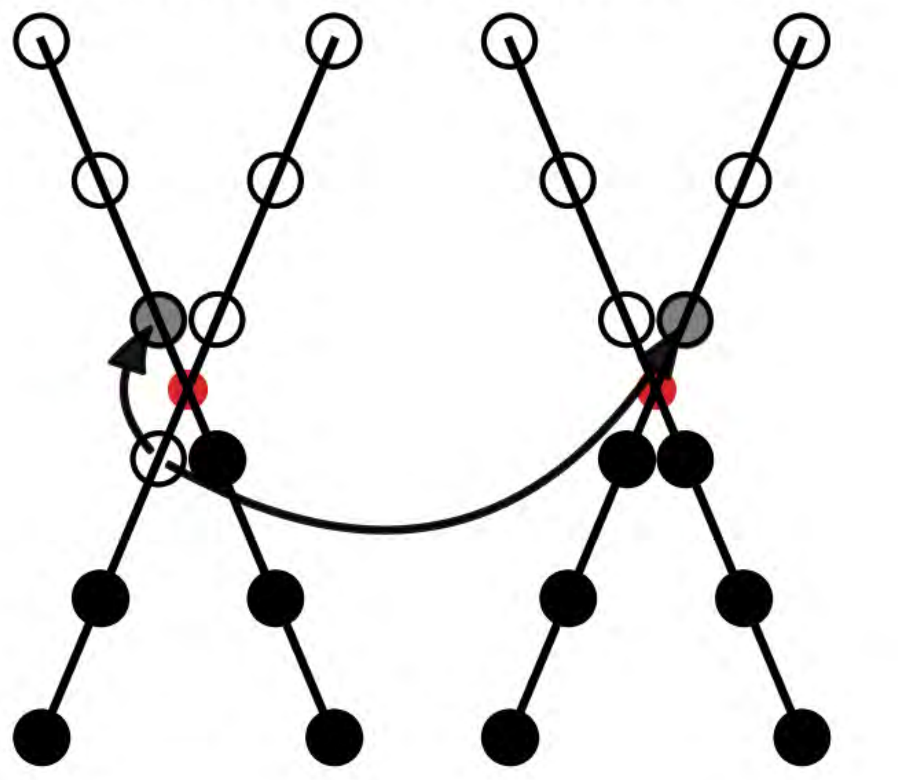
See also the Peierls instability with phonons: U. Seifert et al. preprint arXiv:2307.12295

Conclusion and outlook



- J_1-J_2 spin-1/2 triangular Heisenberg has a rich phase diagram
- Most of it can be understood via the Dirac spin liquid on the triangular lattice
- DSL can serve as a parent state of several orders on the triangular lattice
- We have discovered a one-to-one relation between excitations of J_1-J_2 Heisenberg model and excitations of QED_3
- Indications of a possible valence bond crystal in the paramagnetic regime
- Deconfined quantum critical point between the $120°$ Néel phase and a VBS phase?

[C.M. Jian, A. Thomson, A. Rasmussen, Z. Bi, C. Xu, Phys. Rev. B **97** 195115 (2018)]



- Similar ideas could be used in other numerical methods (VMC, DMRG…)

[A. Wietek, S. Capponi, A. M. Läuchli, arXiv:2303.01585 (2023)]

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