Predictions for Quantum Gravitational Signals from Inflation

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based on 2208.10514, 2208.11711, and WIP

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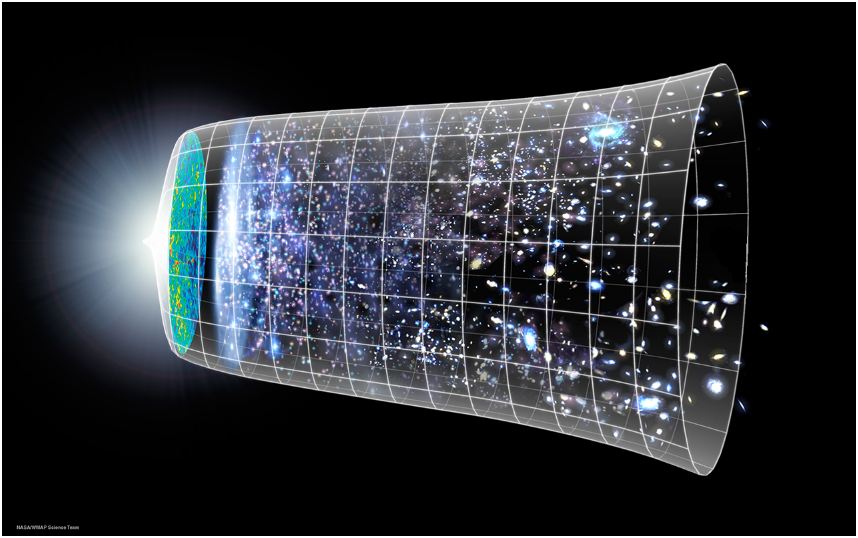
Outline

Quantum gravity is hard to detect experimentally

- extreme separation of scales
- for instance, $\ell_{\mathrm{Planck}}/\ell_{\mathrm{LHC}} \sim 10^{-15}$
- Planck-scale effects suppressed like $(\ell_{\mathrm{Planck}}/\ell_{\mathrm{LHC}})^{\#}$ ...ouch!

Scales are much closer in the **early universe**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Present-day signatures of QG from the early universe?

$\rightarrow$ Hopefully scales with some favourable power of $\ell_{\mathrm{Pl}}/\ell_H \sim 10^{-5}$

Well-motivated idea, but...

- What effect to look for, without specifying a theory of QG?
- What is the observational signature?

Summary of the basic idea

- GR+QFT works *really well* for cosmological perturbations

∴ minimally modify QFT on curved spacetime apparatus

- what are the most dominant corrections as the Planck scale is approached from below?

→ focus on breakdown of distance at short scales, i.e. *natural UV cutoff*

- model covariantly

Key Messages

- Signature of covariant natural UV cutoff in primordial power spectra
  - QG model-independent
  - inflation model-independent

- Cutoff scale is squeezed on both sides: $\ell_H > \ell_C \geq \ell_{\mathrm{Pl}}$
  - Precision cosmology can (already) bound ℓ_C

Outline

1. Introduction
2. Covariant Natural Ultraviolet Cutoffs
3. CMB Predictions
4. Conclusions and Outlook

Breakdown of distance

Generic expectation from most theories of quantum gravity:

Notion of distance breaks down at fine enough scales

A.K.A.

- finite minimal length scenarios
- natural UV cutoff

Want to model *covariantly*

How to make minimal length covariant?

$\rightarrow$ covariant generalization of maximum frequency, i.e. *bandlimit*

Ex: 1D function

$$f(x) = \int_{-\Omega}^{\Omega} \mathrm{d}k\ e^{ikx} F(k)$$

Notice: $-\partial_x^2(e^{ikx}) = k^2 e^{ikx}, \qquad k^2 \in [0, \Omega^2]$

- Lorentzian generalization: restrict spectrum of d'Alembertian, $\Box$

$$\phi(x) = \int_{\lambda \in [-\Omega^2, \Omega^2]} \mathrm{d}\mu(\lambda)\ u_\lambda(x)\ \Phi(\lambda)$$

where $\Box u_\lambda(x) = \lambda u_\lambda(x)$

[Kempf, Martin 0708.0062; ACD, Kempf, Martin 1210.0750]

Implement via the QFT path integral

Ex: Feynman propagator

The usual expression:

$$iG_F(x,x') = \frac{\int \mathcal{D}\phi\, \phi(x)\phi(x') e^{iS[\phi]}}{\int \mathcal{D}\phi\, e^{iS[\phi]}}$$

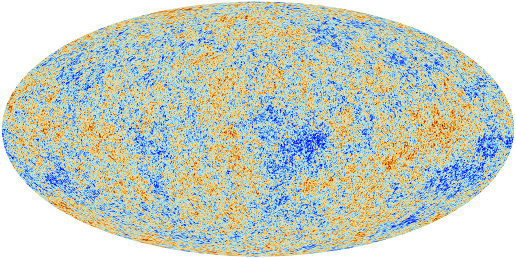
Discard trans-Planckian contributions:

$$iG_F^{\Omega}(x,x') = \frac{\int_{B_{\mathcal{M}}(\Omega)} \mathcal{D}\phi\, \phi(x)\phi(x') e^{iS[\phi]}}{\int_{B_{\mathcal{M}}(\Omega)} \mathcal{D}\phi\, e^{iS[\phi]}}$$

Outline

Goal

Look for signatures of Planckian physics in



Today:

- compute correction to *primordial power spectrum* (PPS)
- focus on *scalar perturbations*

PPS Served Three Ways

$$\Delta^2_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_\star}\right)^{n_s-1} \qquad \text{(1. observation)}$$

$$= \left.\frac{H^2}{\pi\epsilon M_{\mathrm{Pl}}^2}\right|_{aH=k} \qquad \text{(2. theory)}$$

$$= 4\pi k^3 |G_F(\eta_k, k)| \qquad \text{(3. useful here)}$$

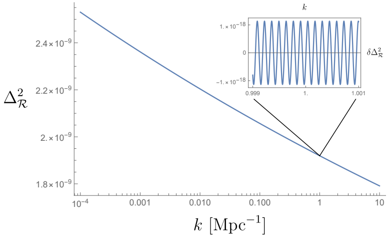
(1) and (2): $H(k) = M_{\mathrm{Pl}}\sqrt{\pi A_s \epsilon_\star}\left(\frac{k}{k_\star}\right)^{-\epsilon_\star}$

(3) : Correction $\delta\Delta^2_{\mathcal{R}}(k) \equiv 4\pi k^3 |G_F^\Omega(\eta_k, k) - G_F(\eta_k, k)|$

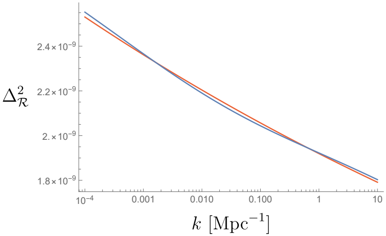
- Ω is an **unknown parameter** to be constrained by data

Visualization

$\Omega = M_{\text{Pl}}$



$\Omega = (0.5 \times 10^{-4}) M_{\text{Pl}}$



Conclusion: left is probably imperceptible, right is probably too drastic

$(A_s = 2 \times 10^{-9}$, $\epsilon = 0.003$, $n_s = 0.97$, $k_\star = 0.05\ \text{Mpc}^{-1})$

Signature in the PPS

Small oscillations superimposed on the conventional PPS

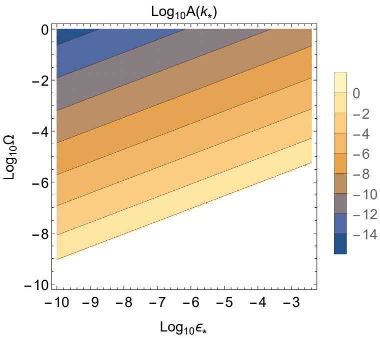
Sharp cutoff:

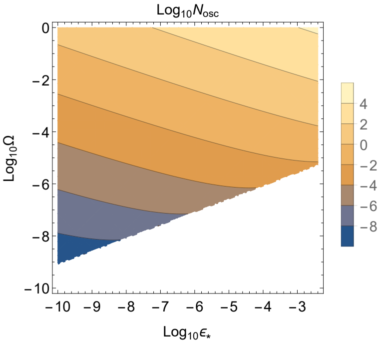
$$\frac{\delta\Delta_{\mathcal{R}}^2}{\Delta_{\mathcal{R}}^2} = \mathcal{C}\,\frac{\sigma(k)^{3/2}}{\ln(\sigma(k)/2)}\,\sin\left(\omega(k)\,\sigma(k)\right)$$

- $\mathcal{C} = 0.8796...$
- $\sigma(k) \equiv \dfrac{H(k)}{\Omega}$
- $\omega(k) \equiv \dfrac{1}{\sigma(k)^2}\ln\dfrac{e\sigma(k)}{2}$

Observational prospects

- In practice, need to fit Ω and $\epsilon_\star \leq 0.0037$





- Amplitude $A \sim \Omega^{-3/2}\epsilon_\star^{3/4}$, # Oscillations $N_{\rm osc} \sim \Omega\epsilon_\star^{1/2}$
- Allowed region: $H < \Omega < M_{\rm Pl}$

Outline

Key Messages

- Signature of covariant natural UV cutoff in primordial power spectra
  - one-parameter pattern of superposed oscillations
- Cutoff scale is squeezed on both sides: $\ell_H > \ell_C \geq \ell_{\rm Pl}$
  - Precision cosmology can (already) bound $\ell_C \sim 1/\Omega$

Next Steps

- Fit to CMB data (WIP)
- Other observational imprints, e.g., primordial non-gaussianity

Extra junk

Some things I didn't talk about

- tensor power spectrum
- information theoretic interpretation of Ω
- softened cutoffs
- EFT of inflation
- self-adjoint realizations of $\Box$ in FLRW (initial conditions)

Some comments

- Fully covariant: $\mathrm{spec}\,\Box$ is just a list of numbers
- Ω is info-theoretic
- operational interpretation:
  - discarding most off-shell (i.e. quantum) contributions to P.I.

Ex: massless scalar field

$$\Box\phi = 0 \qquad \text{on-shell}$$

$$\Box\phi = \lambda\phi \qquad \text{off-shell}$$

Path integrals are unwieldy

$$iG_F^\Omega(x,x') = \frac{\int_{B_\mathcal{M}(\Omega)} \mathcal{D}\phi\,\phi(x)\phi(x')e^{iS[\phi]}}{\int_{B_\mathcal{M}(\Omega)} \mathcal{D}\phi\,e^{iS[\phi]}}$$

Equivalent definition via projectors:

$$G_F^\Omega = P_\Omega G_F P_\Omega$$

where, acting on a test function $u(x)$,

$$P_\Omega u(x) \equiv \sum_{\lambda\in\mathrm{spec}\Box} \theta(\Omega^2 - |\lambda|)\ \langle\psi_\lambda, u\rangle\psi_\lambda(x)$$

Remark: soften the *sharp* cutoff by smoothing the Heaviside step function

Calculation overview

Inputs:

- FLRW scale factor, $a(\eta)$

$$\mathrm{d}s^2 = a^2(\eta)\left[-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2\right]$$

- assumption: single-field inflation

Compute:

- $G_F^\Omega = P_\Omega G_F P_\Omega$

$\Rightarrow$ $\delta\Delta^2_{\mathcal{R}}(k) \equiv 4\pi k^3 |G_F^\Omega(\eta_k, k) - G_F(\eta_k, k)|$

Ω is an **unknown parameter**

$\rightarrow$ to be fixed by comparing with data

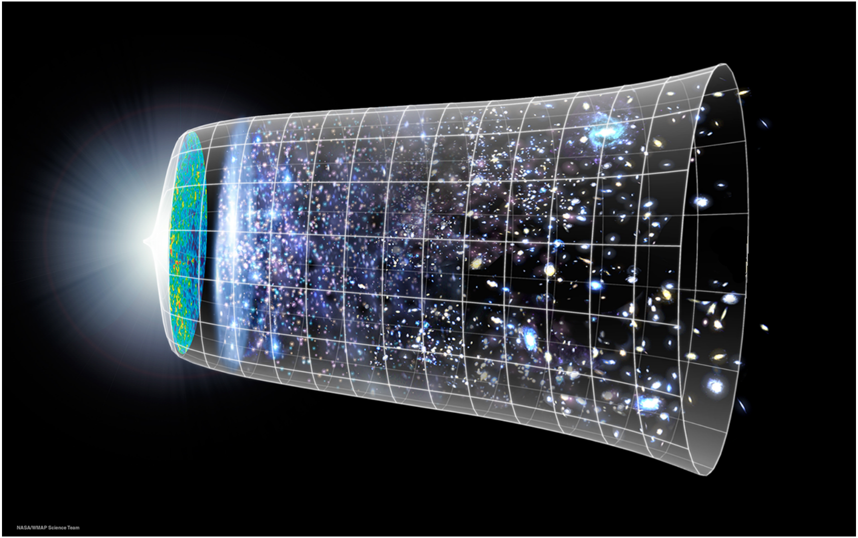
$\rightarrow$ expect $H < \Omega < M_{\mathrm{Pl}}$

Some subtleties

- Exact FLRW computations are intractable
  - "adiabatic" de Sitter approximation
  - schematically, let $a(\eta) = (-H\eta)^{-1}$ with slowly-varying H
  - error suppressed by slow-roll parameters, non-oscillatory

- Choice of vacuum state $\leftrightarrow$ choice of self-adjoint realization of $\Box$
  - i.e. need to specify (generalized) boundary conditions for $\Box$ for a well-posed Sturm-Liouville eigenvalue problem
  - here assume Bunch-Davies
  - deduce by comparing to textbook definition

$$G_F(x,x') = \langle 0|\mathcal{T}\hat{\phi}(x)\hat{\phi}(x')|0\rangle \overset{!}{=} \sum_{\lambda\neq 0} \frac{1}{\lambda}\psi_\lambda^*(x)\psi_\lambda(x') + \text{(homog.)}$$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Crash course on inflation

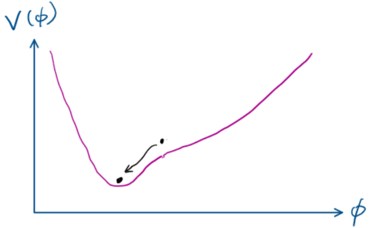
$\equiv$ period of extreme accelerated expansion in early universe

Good (geometric) model: *Friedmann-Lemaître-Robertson-Walker*

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\mathbf{x}^2$$

scale factor: $a(t)$ Hubble parameter: $H(t) \equiv \frac{\dot{a}(t)}{a(t)}$

- Simplest models: driven by scalar *inflaton* field in an excited state

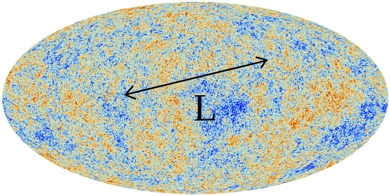


Perturbations

- Write $\phi(t, \mathbf{x}) = \phi_0(t) + \delta\phi(t, \mathbf{x})$, $g_{\mu\nu}(t, \mathbf{x}) = g^{(0)}_{\mu\nu}(t) + \delta g_{\mu\nu}(t, \mathbf{x})$
- Quantize the perturbations

Fluctuations of scalar d.o.f. $\mathcal{R} \Leftrightarrow$ Fluctuations of CMB

$\mathcal{R} \equiv$ *comoving curvature perturbation*



$\sim$ Correlation function

$\langle \mathcal{R}(t, \mathbf{x}) \mathcal{R}(t, \mathbf{x} + \mathbf{L}) \rangle$

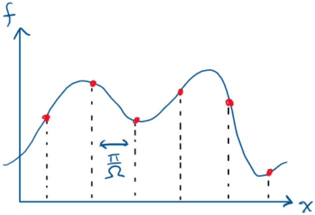
scale $k \sim L^{-1}$

Fourier transform of $\langle \mathcal{R}\mathcal{R} \rangle$: **Primordial Power Spectrum,** $\Delta^2_{\mathcal{R}}(k)$

Shannon-Nyquist sampling

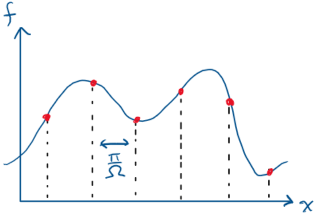
Theorem (Nyquist-Shannon): If $f(x) = \int_{-\Omega}^{\Omega} \mathrm{d}k \; e^{ikx} F(k)$, then

$$f(x) = \sum_{n=-\infty}^{\infty} f(x_n) \frac{\sin(\Omega(x - x_n))}{\Omega(x - x_n)}, \qquad x_n = \frac{n\pi}{\Omega}$$



Interpretation

$$f(x) = \int_{-\Omega}^{\Omega} \mathrm{d}k \; e^{ikx} F(k)$$



- Bandlimited signals are "continuous and discrete at the same time"
  - Continuous signal, discrete density of information in space
- Density set by **bandlimit**, Ω
  - high-frequency / short-distance / ultraviolet (UV) cutoff

Q: What is frequency anyways, abstractly?

$$f(x) = \int_{-\Omega}^{\Omega} e^{ikx}\, F(k)\, dk$$

Answer:

A frequency is an eigenvalue of a derivative operator, such as $-\frac{\partial^2}{\partial x^2}$, and a bandlimit is a cutoff on its spectrum!

$$-\frac{\partial^2}{\partial x^2}\left(e^{ikx}\right) = k^2 e^{ikx} \qquad k^2 < \Omega^2$$

Why is this a useful observation?

- Natural generalization for scalar fields is the d'Alembertian

$$\Box = \frac{1}{\sqrt{-g}}\partial_\mu \left(g^{\mu\nu}\sqrt{-g}\partial_\nu\right)$$

- Idea: covariant bandlimit $\Omega \Leftrightarrow \mathrm{spec}\{\Box\} \subset [-\Omega^2, \Omega^2]$
- Derivative operator $\Box$ is covariant $\Rightarrow$ covariant notion of frequency and UV cutoff

Example: Minkowski Space

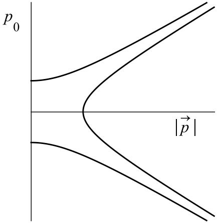
Let $\mathcal{M}$ denote flat, $1+3$ dimensional spacetime.

- Consider the scalar field $\phi(t, \boldsymbol{x})$
- The d'Alembertian: $\Box = -\frac{\partial^2}{\partial t^2} + \triangle$
- Eigenfunctions: $e^{i(p^0 t - \boldsymbol{p}\cdot\boldsymbol{x})}$
- Eigenvalues: $(p^0)^2 - |\boldsymbol{p}|^2$

$$\therefore \quad B(\Omega) = \operatorname{span}\left\{ e^{i(p^0 t - \boldsymbol{p}\cdot\mathbf{x})} \;\middle|\; \left|(p^0)^2 - |\boldsymbol{p}|^2\right| \leq \Omega^2 \right\}$$

What eigenvalues are allowed?

$$\lambda = (p^0)^2 + |\boldsymbol{p}|^2$$



The set $S = \left\{ (p^0, \boldsymbol{p}) \;\middle|\; \left|(p^0)^2 - |\boldsymbol{p}|^2\right| \leq \Omega^2 \right\}$

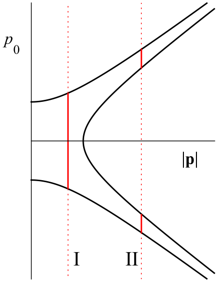
- Observation: arbitrarily large p^0, $\boldsymbol{p}$ contained in S
- What sort of UV cutoff is Ω, then?

Rather, consider the field $\phi(t, \boldsymbol{p})$ in *momentum space*

$$\left|(p^0)^2 - |\boldsymbol{p}|^2\right| \leq \Omega^2$$

$$\Downarrow$$

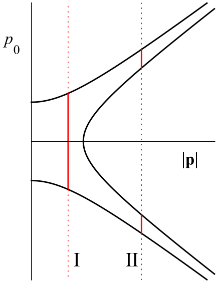
$$r_1 \equiv \mathrm{Re}\left\{\sqrt{|\boldsymbol{p}|^2 - \Omega^2}\right\} \leq |p^0| \leq \sqrt{|\boldsymbol{p}|^2 + \Omega^2} \equiv r_2$$



$$S_{\boldsymbol{p}} = \begin{cases} [-r_2, r_2] & |\boldsymbol{p}| \leq \Omega \\ [-r_2, -r_1] \cup [r_1, r_2] & |\boldsymbol{p}| > \Omega \end{cases}$$

- Each fixed spatial mode $\boldsymbol{p}$, by which we mean $\phi(t, \boldsymbol{p})$ with $\boldsymbol{p}$ fixed, is temporally bandlimited

Mode freezing



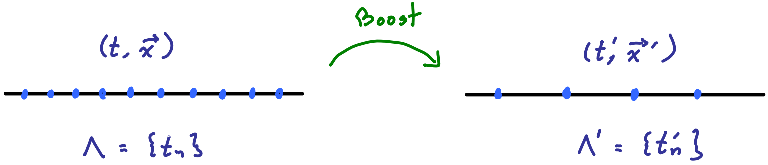
$$S_{\boldsymbol{p}} = \begin{cases} [-r_2, r_2] & |\boldsymbol{p}| \leq \Omega \\ [-r_2, -r_1] \cup [r_1, r_2] & |\boldsymbol{p}| > \Omega \end{cases}$$

1. There are still arbitrarily large spatial modes $|\boldsymbol{p}|$, but they have very small bandwidth in time.
   - *i.e.*, need very few sample points to reconstruct $\phi(t, \boldsymbol{p})$ for large, fixed $\boldsymbol{p}$

$\Rightarrow$ Density of degrees of freedom in time $\to$ 0; modes freeze out

Covariance is maintained

2 Lorentz covariance is respected, as it must be.



- Time dilation $\Rightarrow \Lambda \to \Lambda'$ where $D(\Lambda') < D(\Lambda)$
- Length contaction $\Rightarrow \boldsymbol{p} \to \boldsymbol{p}'$ where $|\boldsymbol{p}'| > |\boldsymbol{p}|$
  - Bandwidth of $\boldsymbol{p}'$ mode lower
  - Density of sample points required lower