Causal shadow and non-local modular flow

from degeneracy to perturbative genesis by correlation

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Background:

Understanding the quantum behavior of gravity is one of the most difficult problems in theoretical physics. Initiated by Juan Maldacena, scientists nowadays often use an non-perturbative method to study quantum gravity, that is the so-called AdS/CFT correspondence. However, it is still a conjecture although there are some evidences supporting it. The bulk reconstruction program is trying to complete the dictionary of AdS/CFT and prove it eventually.

The Entanglement Wedge (EW) reconstruction and Causal Wedge (CW) reconstruction are one of the most important questions in bulk reconstruction. Usually, EW is a larger spacetime subregion than CW. Theorists call the difference of them as Causal Shadow (CS)

On the other hand, the modular flow also plays important role in bulk reconstruction.
In our work, we have studied an interesting question that is the relationship between modular flow and CS.

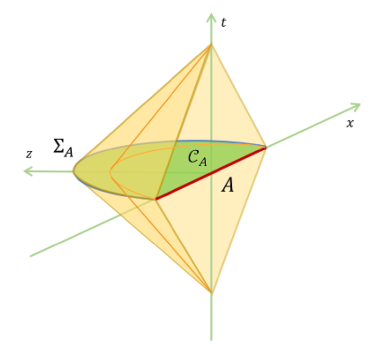

Figure 1: An illustration for Causal Wedge $\mathcal{C}_W(A)$ and Entanglement Wedge $\mathcal{E}_W(A)$.

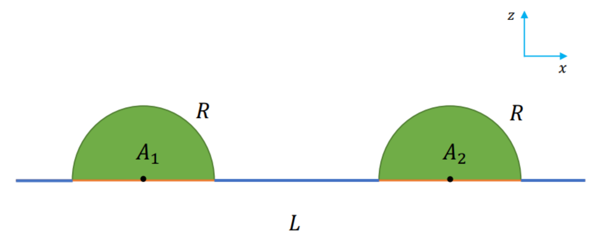

Figure 2: Two distant spheres in the conformal vacuum. The two spheres A_1 and A_2 with radii $R_1 = R_1 = R$ are separated by a large distance $L \gg R$.

Setup:

We study the generation of causal shadow in the context of perturbation theory. We focus on a particular type of deformation, that is correlations between two distant spheres in the conformal vacuum, see Figure 2. Where have adapted the Poincare coordinates of AdS, and the CFT lives on the boundary and is represented as a line in the figure 2.

The key ingredients of our proof and calculations:

1. The caustics (The absence of caustics will imply zero CS)
2. The shape perturbation theory (Convert the shape deformation to the change of metric)
3. The entanglement first law
4. The OPE of modular Hamiltonian

Modular Hamiltonian and Modular flow:

$$H = -\ln \rho_A\,, \qquad O_s(x) = e^{iHs} O(x) e^{-iHs}$$

EW $\geq$ CW (Classically):

Under the assumption that the bulk space-time satisfies the null energy condition (NEC) it was shown (by Aron C Wall etc.)that the entanglement wedge is always greater or equal to the causal wedge (classically).

$$\mathcal{C}_W(A) \subseteq \mathcal{E}_W(A).$$

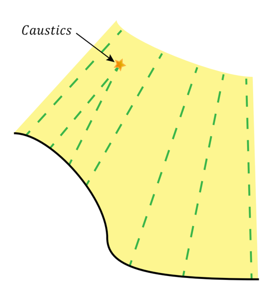

Figure 4: Caustics are formed on a null geodesic congruence when nearby null generators intersect.

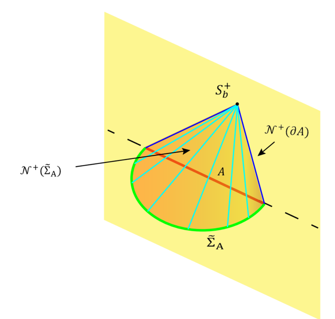

Figure 5: We are only showing the future components. In the absence of the bulk caustics on it, $\mathcal{N}^+\left(\tilde{\Sigma}_A\right)$ intersects with the asymptotic boundary at $\mathcal{N}^+(\partial A) \cup S_b^+$, with $S_b^+ = S_A^+$ in the form of a null-cone tip in our cases.

Local modular implies EW=CW (both classically and quantumly):

When the boundary modular flow is local, then there is a U(1) isometry (or conformal isometry) which will strongly constrain the geometry that is no caustics are allowed in the null surface except at the vertex.

$$\mathcal{N}^{\pm}(\partial A) = \left\{\lim_{x \to x'} x(s), x \in A, s \in \mathbb{R}^{\pm}, x' \in \partial A\right\} \qquad \mathcal{N}^{\pm}(\tilde{\Sigma}_A) = \left\{\lim_{x \to x'} x(s), s \in \mathbb{R}^{\pm}, x \in a, x' \in \tilde{\Sigma}_A\right\}$$

$$\tilde{\Sigma}_A = \mathcal{N}^+(\tilde{\Sigma}_A) \cap \mathcal{N}^-(\tilde{\Sigma}_A) \qquad \mathcal{C}_A = \dot{\mathcal{J}}^+\left(D(A)\right) \cap \dot{\mathcal{J}}^-\left(D(A)\right)$$

RT surface and Quantum Extremal Surface Q.E.S) :

The RT surface is defined by the minimal surface which anchored on the boundary of boundary subregion. And it relates the boundary entanglement entropy by the famous Ryu-Takayanagi (RT) formula.

$$S_A^{\psi} = \frac{\text{Area}(\Sigma_A)}{4G_N}$$

The quantum extremal surface is defined by:

$$S_{gen} = Ext_{\Sigma}\left(\frac{A(\Sigma)}{4G_N} + S(a)\right)$$

The shape dependence of Mutual Information:

To calculate the Q.E.S perturbatively, the shape dependence of mutual information is very useful, which has been calculated in our previous work.

$$\Sigma(A_1) \to \Sigma(A_1) + \eta \qquad \delta I(a_1, a_2) = \int_{\partial\mathcal{B}} d^dY \sqrt{h(Y)}\; n^{\mu}\eta^{\nu}(Y)\langle T^{\phi}_{\mu\nu}(Y)\Delta H_{a_1\cup a_2}\rangle$$

$$\frac{\delta I(a_1, a_2)}{\delta\eta(z, x^i)} = N_{d,\Delta} z^{2\Delta-d}(1 + z^2 + \sum_{i\geq 2} x_i^2)^{-2\Delta-1}$$

$$N_{d,\Delta} = \lambda^{2\Delta}\frac{\pi^{3/2}}{4^{2\Delta}\pi^{d/2}}\frac{\Gamma(2\Delta+1)^2}{\Gamma(\Delta)\Gamma(2\Delta+3/2)}\frac{1}{\Gamma(\Delta - d/2 + 1)}$$

Calculating Q.E.S perturbatively:

$$\Sigma_{QES} = \Sigma_{RT} + \zeta, \quad \Sigma_{RT} = \Sigma(A_1) \cup \Sigma(A_2) \qquad \frac{\delta S_{gen}\left(\Sigma_{RT} + \zeta\right)}{\delta\eta(x)} = 0, \;\; \forall\, x \in \Sigma_{RT} + \zeta$$

$$\left.\frac{\delta I_{a_1\cup a_2}}{\delta\eta(x)}\right|_{\Sigma'=\Sigma_{RT}} + \frac{1}{4G_N}\int_{\Sigma_{RT}} dy \left.\frac{\delta^2\,\text{Area}\left(\Sigma'\right)}{\delta\eta(y)\;\delta\eta(x)}\right|_{\Sigma'=\Sigma_{RT}} \zeta(y) + \mathcal{O}(\zeta^2) = 0, \quad \forall x \in \Sigma_{RT}$$

$$\zeta(x) = 4G_N\int_{\Sigma_{RT}} dy\, \mathcal{K}\left(x, y\right)\left[\frac{\delta I_{a_1\cup a_2}}{\delta\eta\left(y\right)}\right] + \mathcal{O}\left(G_N^2\right)$$

Conclusion:

1. Under some reasonable assumptions, we have proved:
   a. local modular flow ⇒ Zero CS (Classically)
   b. local modular flow ⇒ Zero CS (Quantumly)

   (The existence of the boundary conformal isometries extendable into the bulk severely constrains the caustics structure on the orthogonal null congruences, making it necessary for the RT surface and the causal horizon to coincide; it also constrains the quantum corrections so that it does not modify the location of the Q.E.S)
2. We have explicit calculated the profile of causal shadow from quantum correction for the first time.

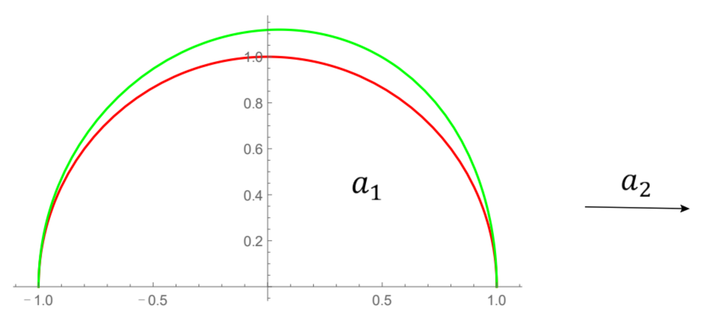

Figure 9: The profile of the causal shadow in terms of the causal horizon (red) against the Q.E.S (green) in AdS_3, with $\Delta = 2$. For illustration purpose we exaggerated the value of $G_N\lambda^{2\Delta}$ to be order 1.

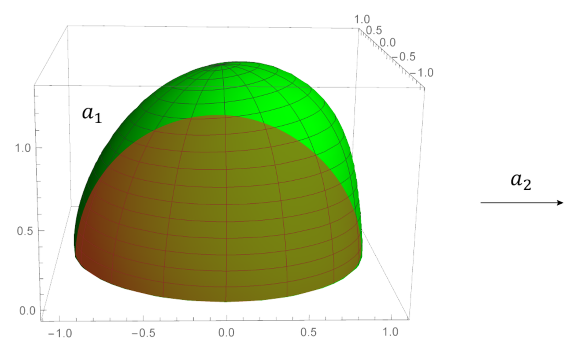

Figure 10: The profile of the causal shadow in terms of the causal horizon (red) against the Q.E.S (green) in AdS_4, with $\Delta = 2$. Again for illustration purpose we exaggerated the value of $G_N\lambda^{2\Delta}$ to be order 1

Outlook:

1. It is interesting to study other cases of perturbative causal shadows by allowing various types of perturbations, e.g. deforming the sub-region geometry or deforming the underlying states.
2. It is also important to understand this positivity from the boundary CFT perspective.
3. It is useful to devise quantitative measures for the extent of non-locality related to the modular flow, e.g. via the operator spreading, etc.

Main References:

[1]. T. Faulkner, A. Lewkowycz and J. Maldacena, Quantum corrections to holographic entanglement entropy, JHEP 11 (2013) 074, [1307.2892].
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