Gravitational wave analogues in spin nematics and cold atoms

L. Chojnacki[1], R. Pohle[2], H. Yan[3], Y. Akagi[2], N. Shannon[1]

[1]Okinawa Institute of Science and Technology, Okinawa, Japan

[2]The University of Tokyo, Japan

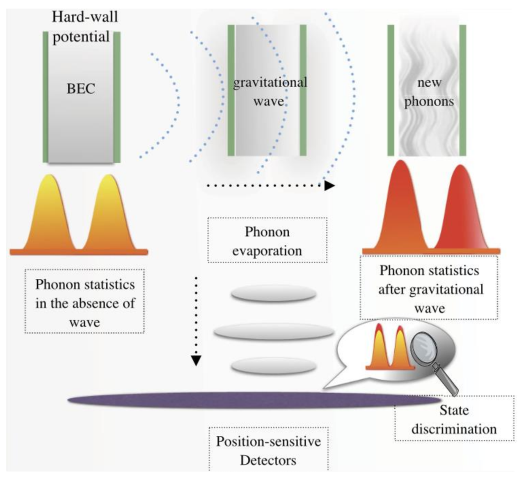
[3]Rice University, USA

Abstract: **Many large-scale phenomena in our Universe, such as gravitational waves, are challenging to reproduce in laboratory settings. However, parallels with condensed matter systems can provide alternative routes for experimental accessibility. Here we show how spin nematic phases provide a low-energy avenue for accessing the physics of linearized gravity, and in particular that their Goldstone modes are relativistically-dispersing massless spin-2 excitations, analogous to gravitational waves. Finally, we suggest a route to experimental observation.**

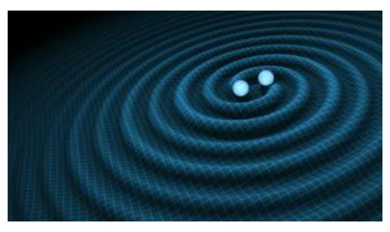
Context

- Development of QFT simulators within condensed matter and quantum fluids communities is attracting much interest.
- Currently no accessible realizations of spin-2 (quadrupolar) waves akin to gravitational waves.
- We here propose an avenue to realizing an analogue of spin-2 gravitational waves in condensed matter systems.

Ex. Proposed gravitational wave detector in BEC [1]



Gravitational waves in linearized gravity



Animation by R. Hurt, Caltech/JPL, LIGO (2015)

Gravitational action in curved space-time

$$\mathcal{S} = \frac{c^3}{16\pi G}\int dx^4 \sqrt{-g}R$$

Weak field limit (linearized gravity)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

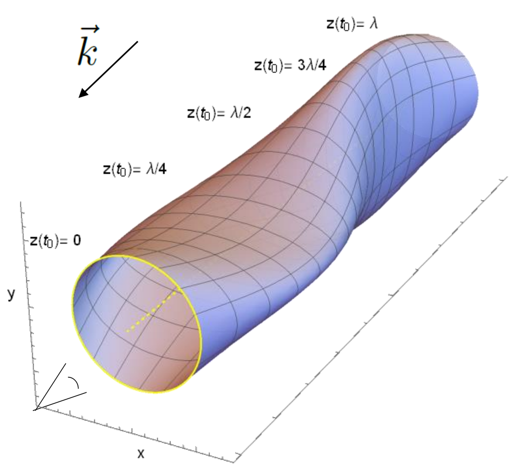
Effective field theory for linearized gravity

$$\mathcal{S}_{LGR} = -\frac{c^4}{16\pi G}\int dx^4 \left[\partial^\nu h^{\mu\rho}\partial_\nu h_{\mu\rho}\right]$$

Coordinate transformations act like the gauge freedom

$$x'^\mu = x^\mu + \xi^\mu(x)\,,$$
$$h'_{\mu\nu} = h_{\mu\nu} - \partial_\nu\xi_\mu - \partial_\mu\xi_\nu$$

Dynamics only occurs in spatial components

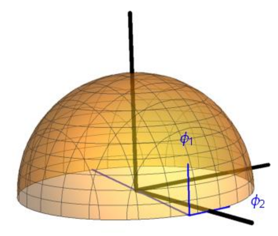


In transverse-traceless gauge, we see

$$h^\mu{}_\mu = 0\,,\ \text{[traceless]}$$
$$h_{0\mu} = 0\,,\ \text{[no scalar or vector components]}$$
$$\partial^n h_{nm}(x) = 0\,,\ \text{[no longitudinal dynamics]}$$

2 Dynamically non-trivial, relativistically dispersing, spin-2 modes

"Gravitational waves" in a spin nematic



2 Linearly dispersing Goldstone bosons Spin-2 modes

Quantum spin nematic order parameter

$$Q^{\alpha\beta} = S^\alpha S^\beta + S^\beta S^\alpha - \frac{2}{3}S(S+1)\delta^{\alpha\beta}$$

Promote by construction

$$Q^\mu_\mu = \tilde{Q}^\mu_\mu = 0\,,$$
$$Q_{0\mu} = \tilde{Q}_{0\mu} = 0$$

Effective low energy field theory

$$\mathcal{S}_{FQ} = -\frac{1}{2}\int dt dr^d \left[\chi_\perp(\partial^t Q^{\alpha\beta}\partial_t Q_{\alpha\beta}) - \rho_s(\partial^i Q^{\alpha\beta}\partial_i Q_{\alpha\beta})\right]$$

Dictionary for excitations

Gravitational wave polarizations for a wave propagating along z

$$\boldsymbol{h}^+ = e^{ik^\mu x_\mu}\begin{pmatrix}0&0&0&0\\0&h_+&0&0\\0&0&-h_+&0\\0&0&0&0\end{pmatrix}$$

$$\boldsymbol{h}^\times = e^{ik^\mu x_\mu}\begin{pmatrix}0&0&0&0\\0&0&h_\times&0\\0&h_\times&0&0\\0&0&0&0\end{pmatrix}$$

Ferroquadrupolar wave polarizations for a ground state ordered along z

$$\boldsymbol{Q}^1 = e^{ik^\mu x_\mu}\begin{pmatrix}0&0&0&0\\0&0&0&Q^1\\0&0&0&0\\0&Q^1&0&0\end{pmatrix}$$

$$\boldsymbol{Q}^2 = e^{ik^\mu x_\mu}\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&Q^2\\0&0&Q^2&0\end{pmatrix}$$

Mapping to a nematic basis for h

$$h_{\mu\nu} = R[\vec{v_k},\phi]^\alpha{}_\mu R[\vec{v_k},\phi]_\nu{}^\beta \Big[R[\vec{v_d},\theta]^\gamma{}_\rho R[\vec{v_d},\theta]_\sigma{}^\kappa\; Q_{\gamma\kappa}\left[\lambda_4\otimes\lambda_1 + \lambda_6\otimes\lambda_{3xy}\right]^{\rho\sigma}{}_{\alpha\beta}\Big]$$

Mapping to a strain-like basis for Q

$$Q_{\mu\nu} = R[\vec{v_d},\theta]^\alpha{}_\mu R[\vec{v_d},\theta]_\nu{}^\beta \Big[R[\vec{v_k},\phi]^\gamma{}_\rho R[\vec{v_k},\phi]_\sigma{}^\kappa\; h_{\gamma\kappa}\left[\lambda_1\otimes\lambda_4 + \lambda_{3xy}\otimes\lambda_6\right]^{\rho\sigma}{}_{\alpha\beta}\Big]$$

Surface of constant strain

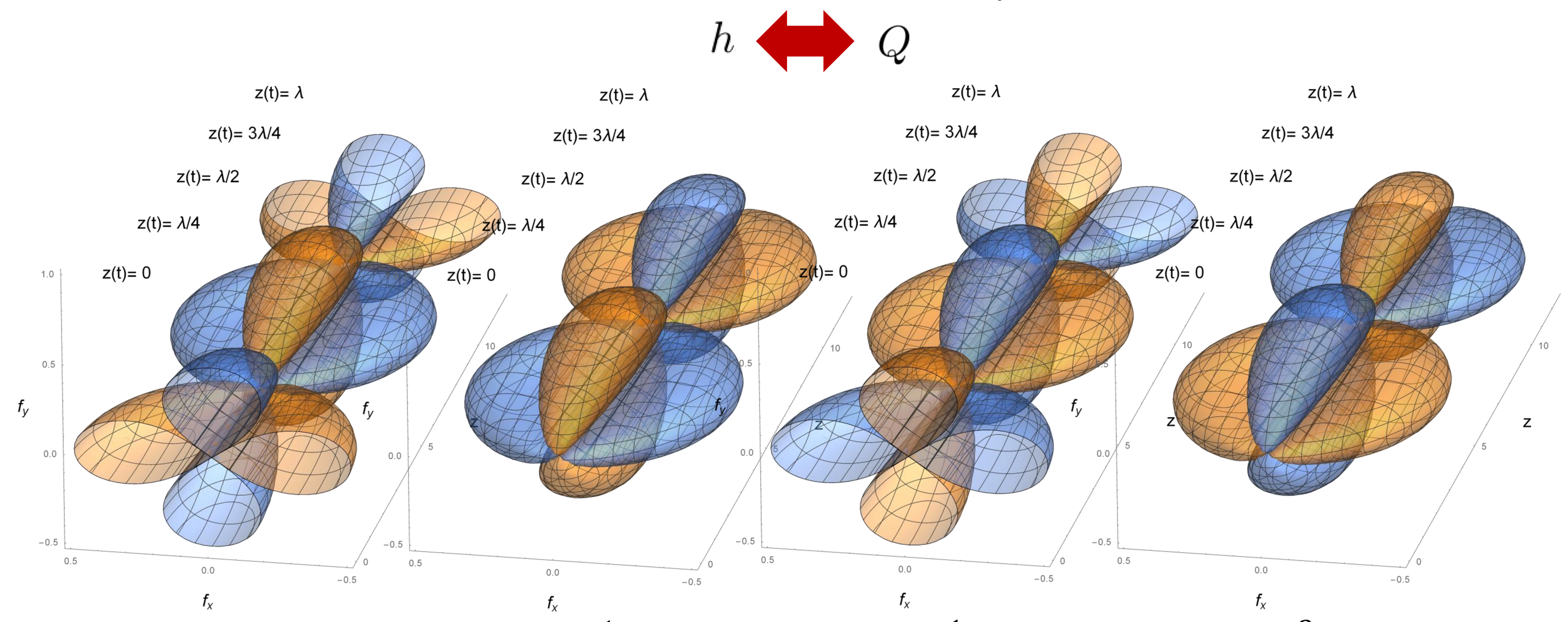
$$\pm\epsilon = \left(\frac{h_{\mu\nu}(x-x')^\mu(x-x')^\nu}{|(x-x')|^2}\right)^{\frac{1}{2}}$$

Surface of equal wavefunction amplitude

$$\pm\sqrt{p} = \frac{d^i\tilde{Q}_{ij}d^j}{|d|^2}$$

We can therefore identify

$$h \Longleftrightarrow Q$$



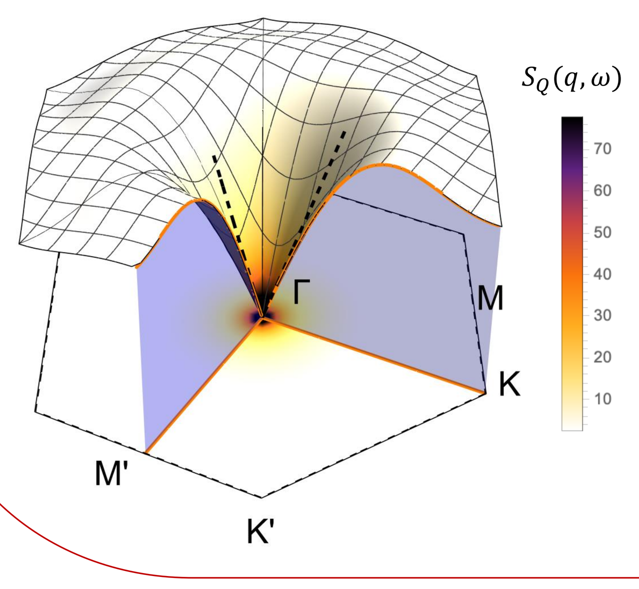
(a) $t = 0$ (b) $t = \frac{1}{4}\tau$ (c) $t = \frac{1}{2}\tau$ (d) $t = \frac{3}{4}\tau$

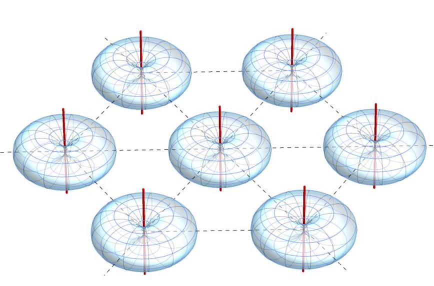
Microscopic model realization and experimental proposal

Microscopic model for ferroquadrupolar spin nematic, with candidate realizations in QLC materials [2] and cold atoms [3]

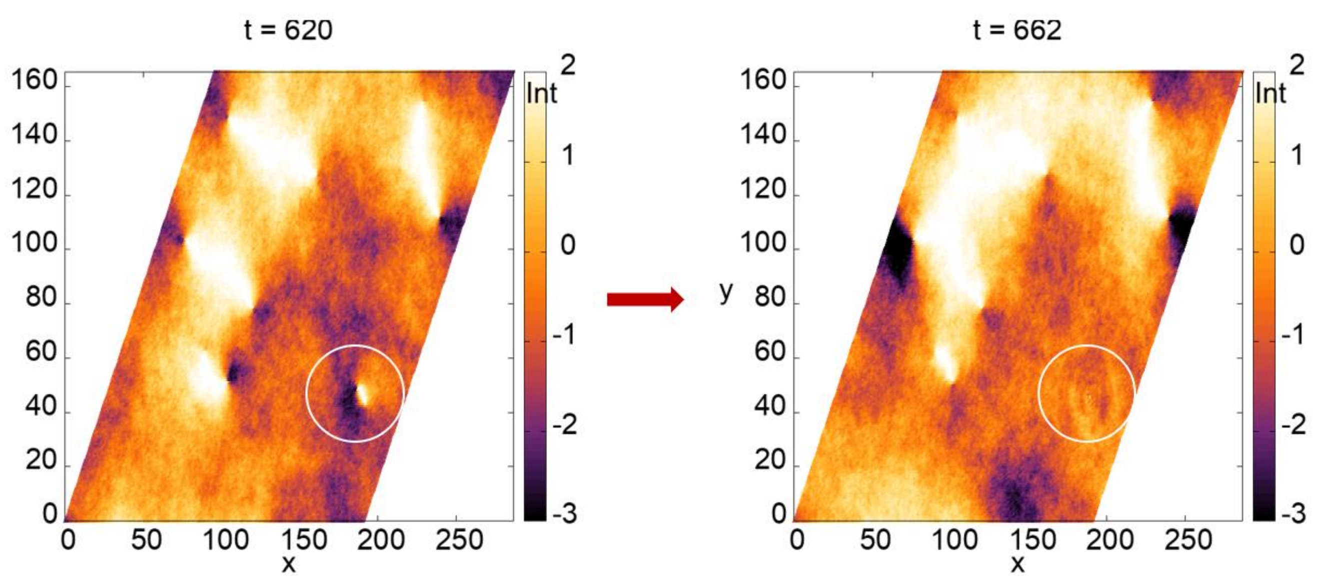
$$H_{BBQ} = \sum_{\langle ij\rangle} J_1 \vec{S}_i.\vec{S}_j + \sum_{\langle ij\rangle} J_2(\vec{S}_i.\vec{S}_j)^2$$

$$J_2 < 0$$
$$J_1 << J_2$$



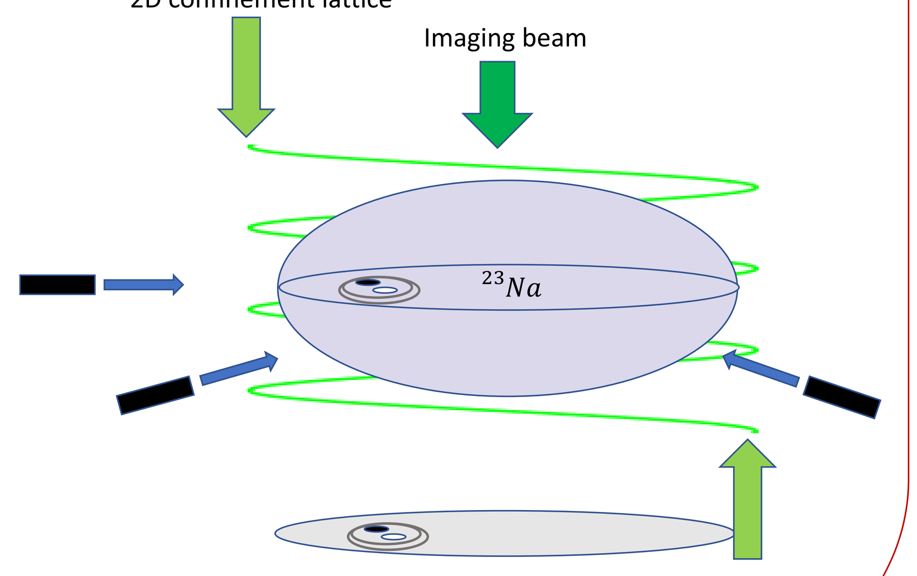


Simulating the FQ phase using method in [4]



$$Q(r) = \sum_{a,b} Q^{ab}(r)$$

Imaging quadrupolar waves in cold atoms using methods in [5]



References

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[6] L. Chojnacki, R. Pohle, H. Yan, Y. Akagi, N. Shannon *(In preparation)*; L. Chojnacki, H. Yan, N. Shannon *(In preparation)*

Conclusions

We have identified

- A one-to-one correspondence between Goldstone modes of a ferroquadrupolar ordered spin nematic and gravitational waves in linearized gravity.
- The SU(3) exchange model as a microscopic model to realize these waves in condensed matter.
- A route to their experimental production and observation using ^{23}Na spinor condensates.

Contact information: leilee.chojnacki@oist.jp