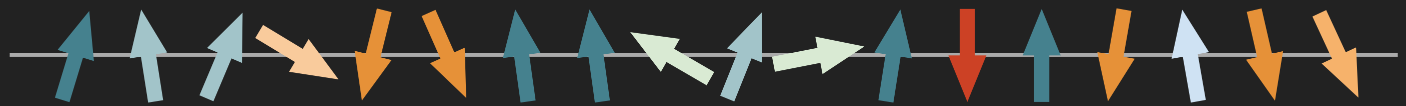
EXTREME VALUE THEORY
AND
LOCALIZATION IN RANDOM SPIN CHAINS



Jeanne Colbois
Nicolas Laflorencie
LPT | CNRS & Toulouse University | France

arXiv:2305.10574

Soon to appear in PRB

Quantum information, quantum matter and quantum gravity | Kyoto | 27.09.2023

TOSSING A COIN

HTTTHTHH

THTHHT

$$L = 176$$

M. F. Schilling, The College Mathematics Journal 21(3), 196-207 (1990)
P. Révész, Proc. 1978 Int'l Cong. of Mathematicians, 749-754 (1980)

TOSSING A COIN

HTTHTHTTHTHHHTHTTHHTTHHTHTTTHHHTTHTHTHTHHTHT
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THTHTTHHTHTHTHTHHTHHHTTTHHTHTHTHTHTHTHHTHTHT
HHTHTHHTTHTHHTHTHTHHTHTHHTHTHTHTHTHHTHTHHTHT

176 (81 T / 95 H)

HTTTHTTTHTHTHHTHHHHHHTTTTHHHHHHHTHTHHHTTHTHH
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TTHTTHHHHTHTHHHTTTTTHTHTTHHTHTTHHTHHHHTHHTHT

176 (83 T / 93 H)

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TOSSING A COIN

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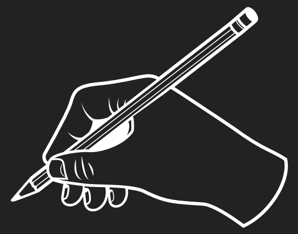

176 (81 T / 95 H)

$$\mathcal{P}(\ell) \sim 2^{-\ell} \qquad \mathcal{P}(\ell_{\max}) \sim \frac{1}{L} \qquad \Rightarrow \ell_{\max} \sim \ln L / \ln 2 \sim 7.45$$

HTTTHTTTHTHTHHTHHHHHHTTTTHHHHHHHTHTHHHTTHTHH
THHTTTHHHTHHHTTHHHHTHHTHHHTTTHTHTTHTHTTHHTHT
THTHTTTTTTTHHTHTHHHTHHTTHHTTTTTHHHTTHTHTHHTH
TTHTTHHHHTHTHHHTTTTTHTHTTHHTHTTHHTHHHHTHHTHT

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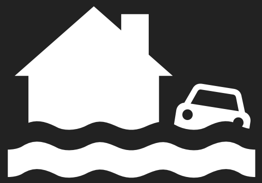
EXTREME VALUE THEORY



Extreme floods

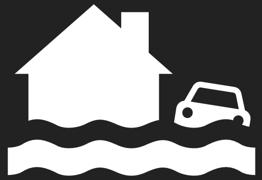
Market risks

Athletic records

Large wildfires

E. J. Gumbel, *Statistics of Extremes*, Dover, (1958, 2004)
S. N. Majumdar, A. Pal, G. Schehr, Physics Reports, **840**, 1 (2020)

EXTREME VALUE THEORY



Extreme floods

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Condensed matter

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EXTREME VALUE THEORY

Extreme floods

Market risks

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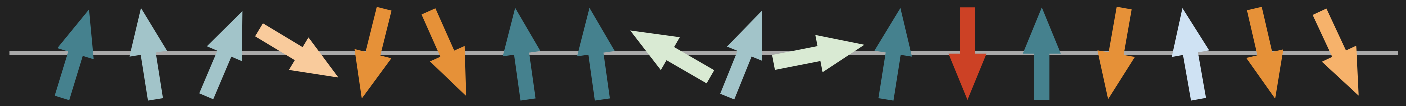
Condensed matter

Disordered spin chains

R. Juhász, Y,C. Lin, and F, Iglói, Phys. Rev. B 73, 224206 (2006)
N. Pancotti, M. Knap, D. A. Huse, J. I. Cirac, and M. C. Bañuls, Phys. Rev. B 97, 094206 (2018)
I. A. Kovács, T.Pető, and F.Iglói, Phys. Rev. Res. 3, 033140 (2021)
W.-H. Kao and N, B. Perkins, Phys. Rev. B 106, L100402 (2022)
J. C., N. Laflorencie, arXiv:2305.10574

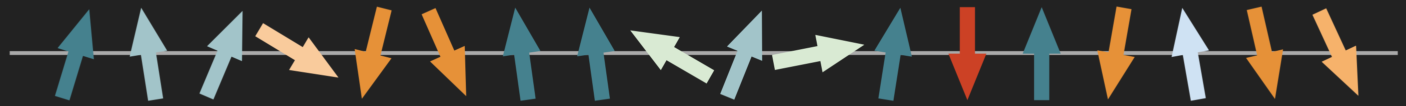
E. J. Gumbel, *Statistics of Extremes*, Dover, (1958, 2004)
S. N. Majumdar, A. Pal, G. Schehr, Physics Reports, **840**, 1 (2020)

SUMMARY : SPIN-1/2 CHAIN IN RANDOM FIELD



Spin-1/2 $W = h =$ disorder strength for random fields along S^z

SUMMARY : SPIN-1/2 CHAIN IN RANDOM FIELD



Spin-1/2 $W = h =$ disorder strength for random fields along S^z

XX chain ("many-body Anderson")

Heisenberg chain

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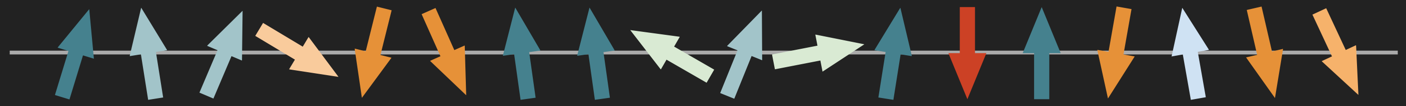

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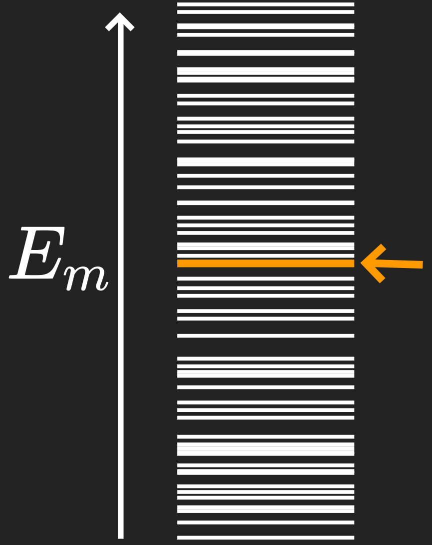
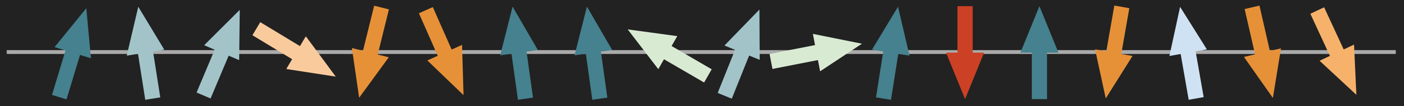


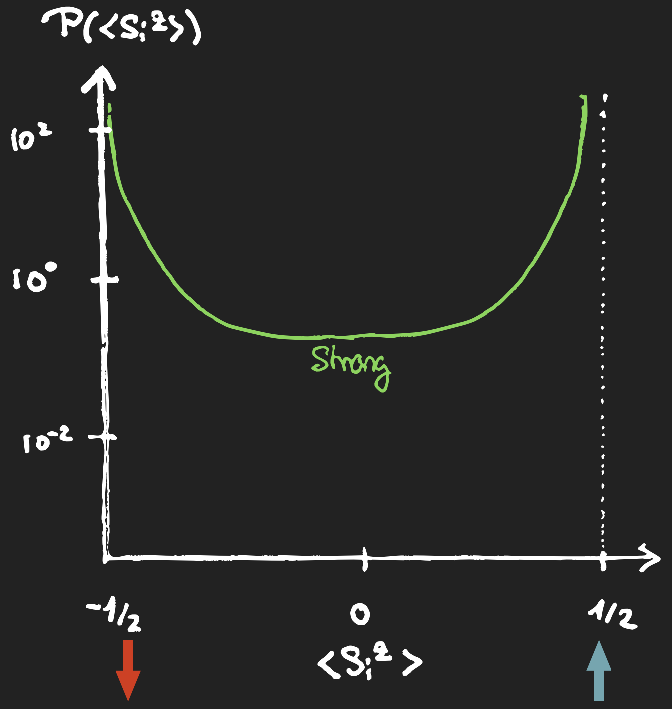
Eigenstates in the **middle** of the many-body spectrum

SUMMARY : SPIN-1/2 CHAIN IN RANDOM FIELD



Distribution over disorder realizations and high-energy eigenstates

Anderson chain / XX chain



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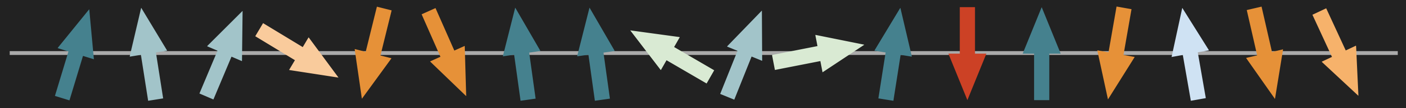

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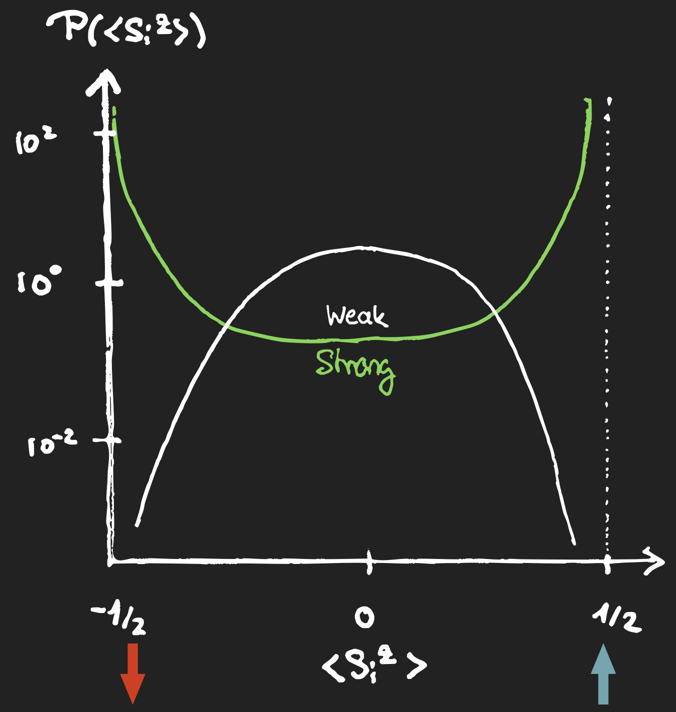
Anderson chain / XX chain



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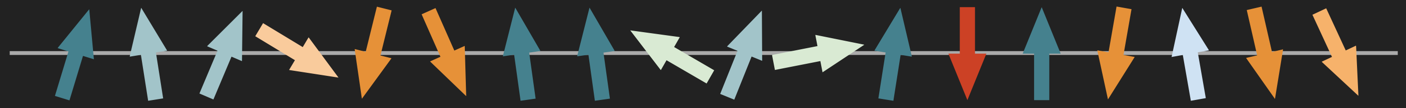

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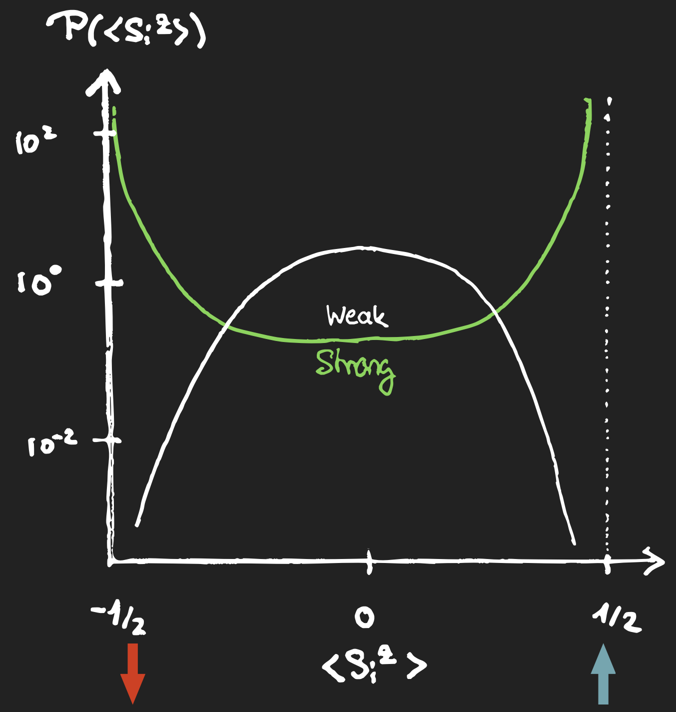
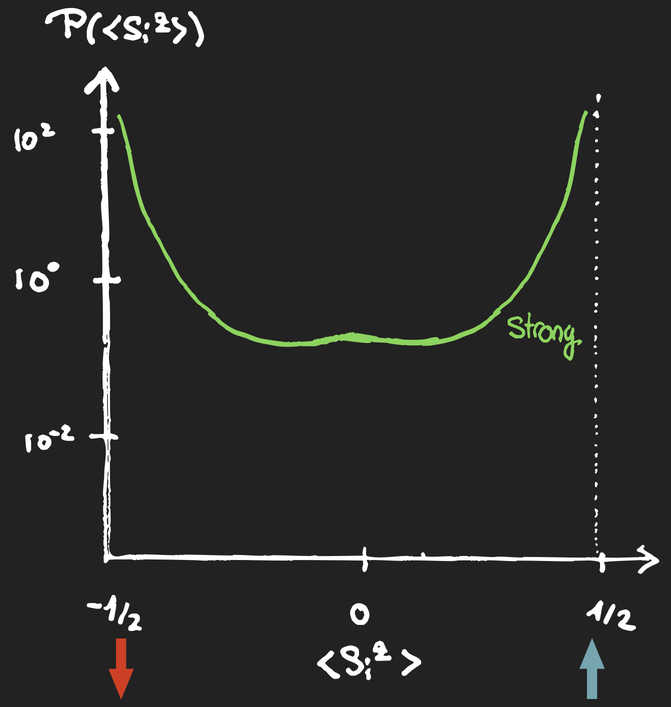


Heisenberg chain



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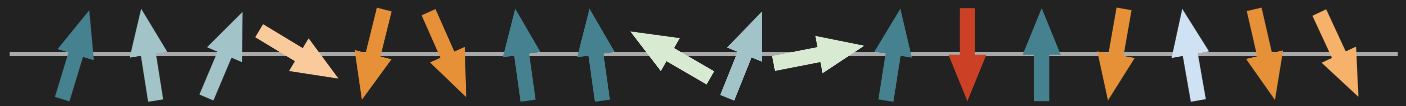
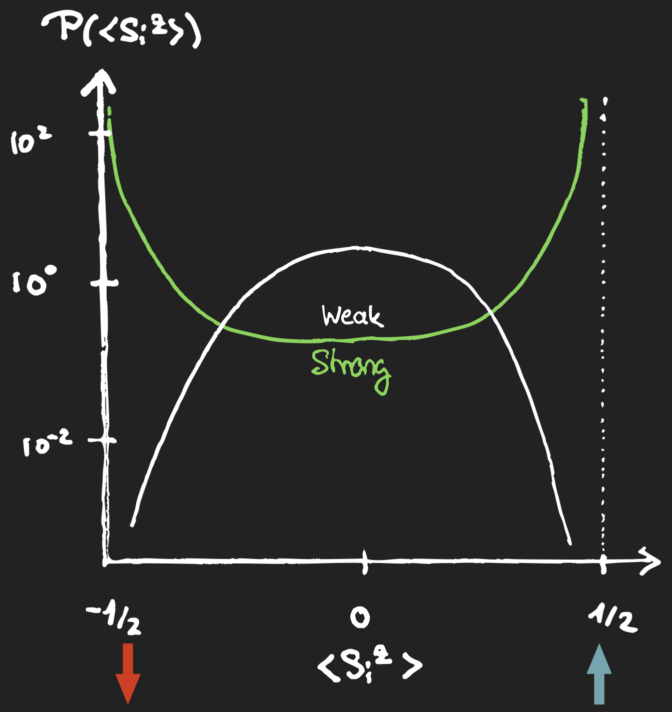
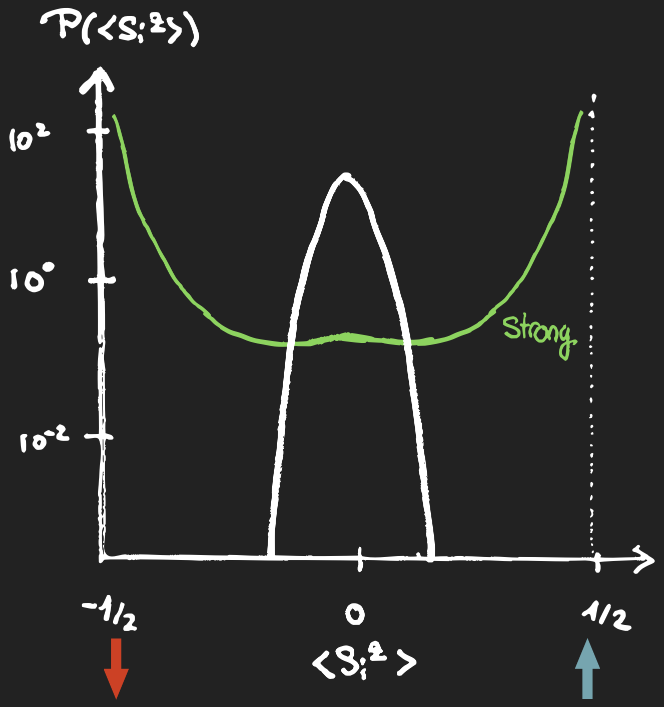


Distribution over disorder realizations and high-energy eigenstates

Anderson chain / XX chain



Heisenberg chain



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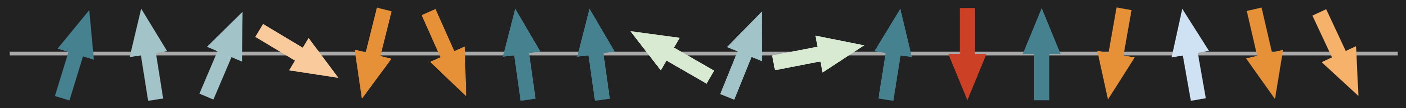
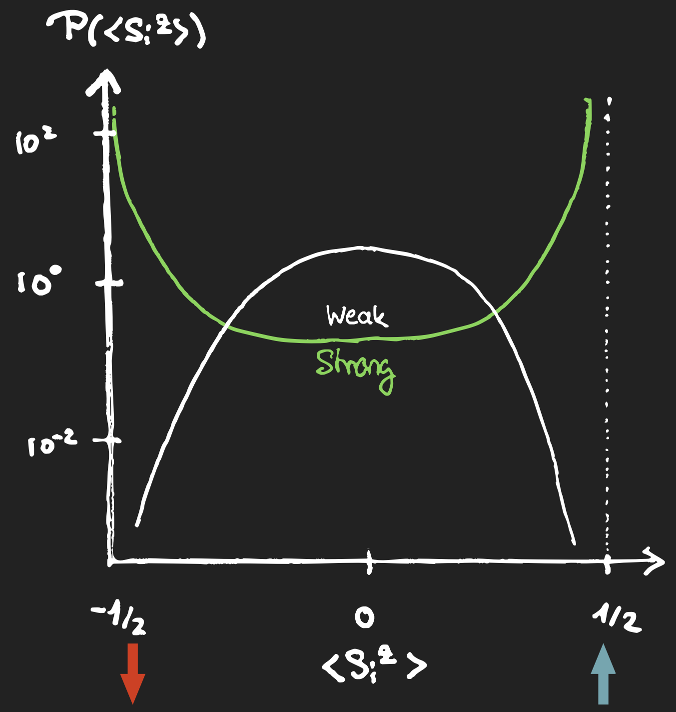
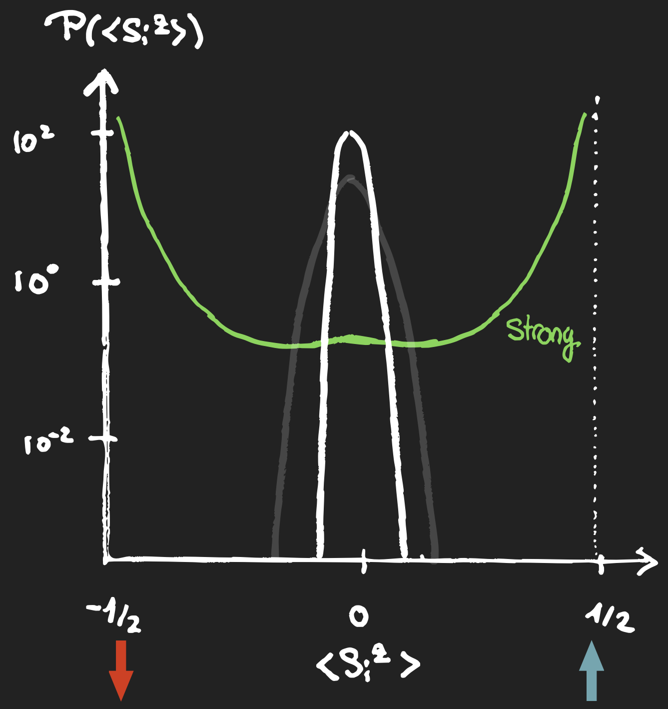


Distribution over disorder realizations and high-energy eigenstates

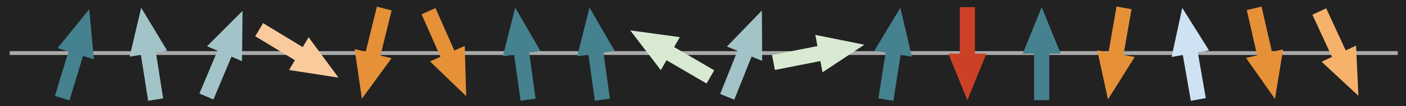
Anderson chain / XX chain



Heisenberg chain



QUESTIONS



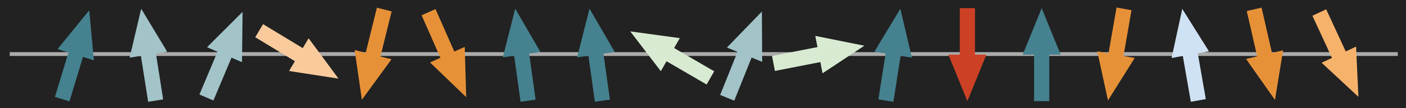
Spin-1/2 $W = h =$ disorder strength for random fields

→ W

Ergodic | MBL regime(s)

?

QUESTIONS



Spin-1/2 $W = h =$ disorder strength for random fields

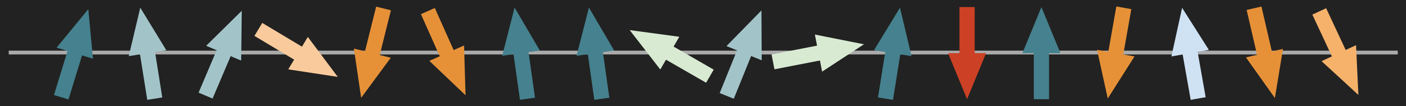
$\longrightarrow W$

Ergodic | MBL regime(s)

?

Fate of isolated quantum systems?

QUESTIONS



Spin-1/2 $W = h =$ disorder strength for random fields

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Ergodic | MBL regime(s)

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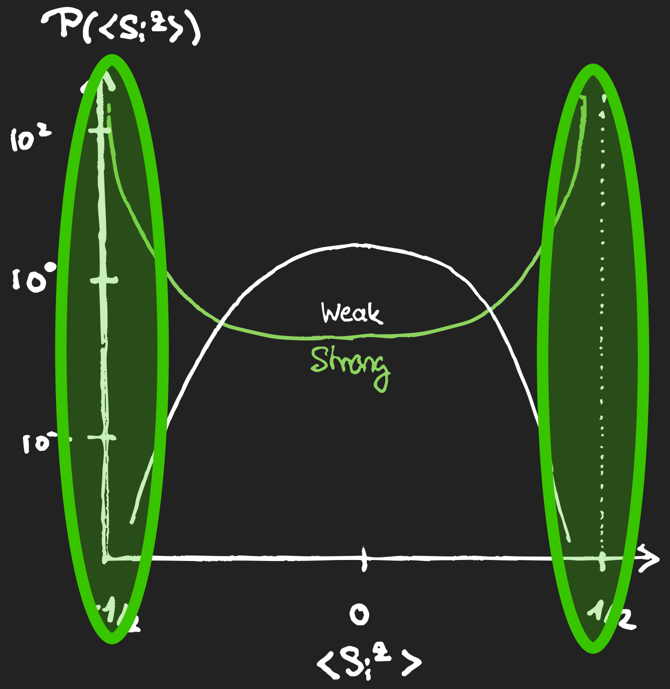
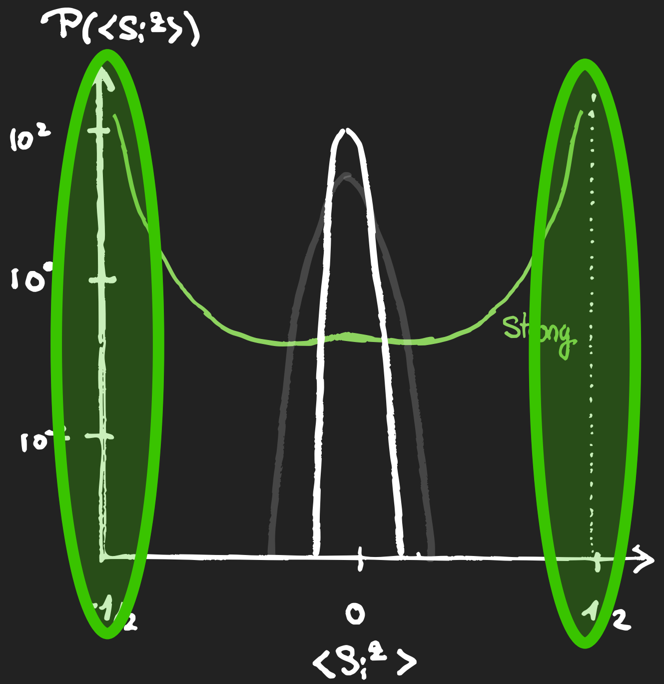
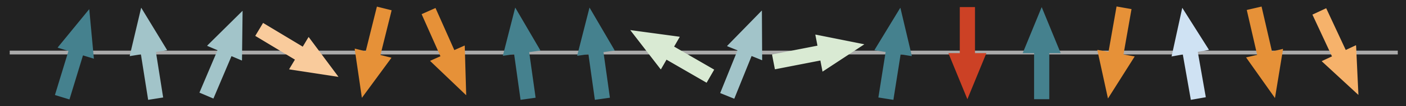


- **Toy model?**



QUESTIONS



Spin-1/2 $W = h =$ disorder strength for random fields

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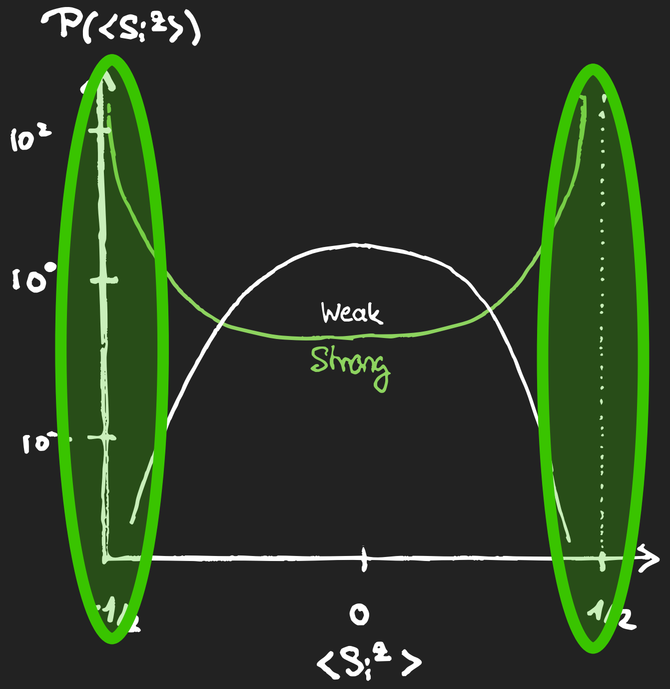
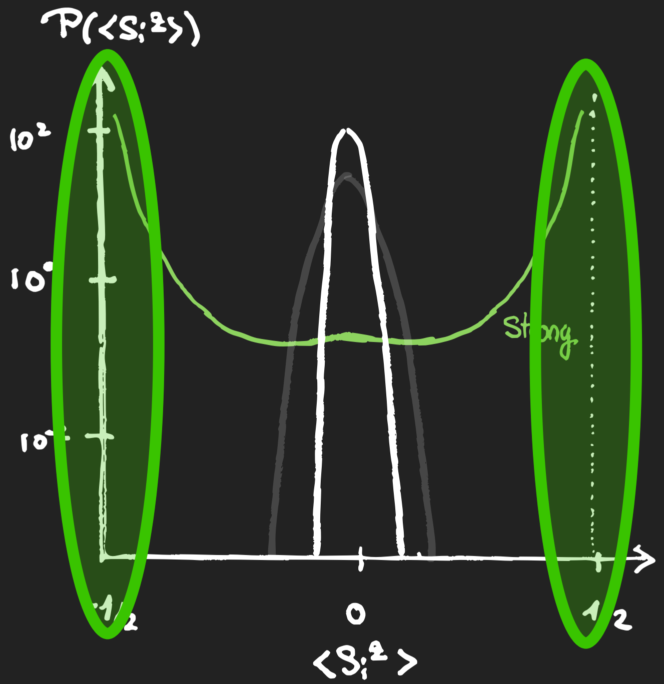
Ergodic | MBL regime(s)

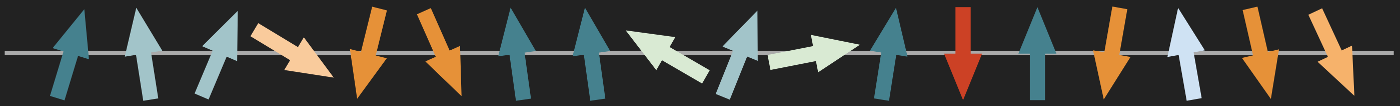
?



- **Toy model?**
- **Quantitative description?**



QUESTIONS

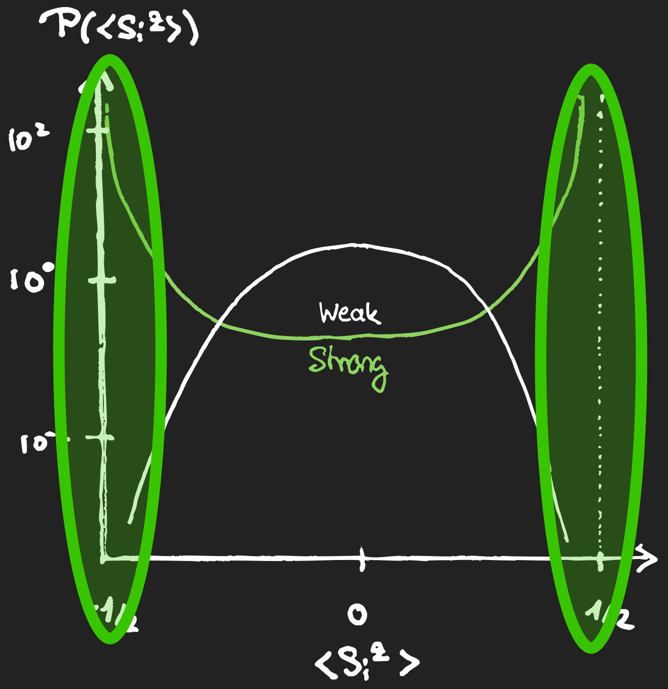


Spin-1/2 $W = h =$ disorder strength for random fields

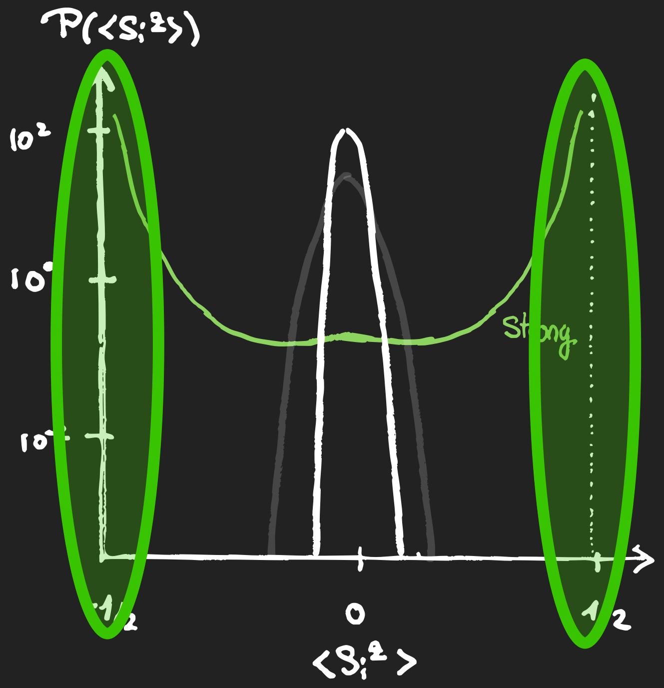
$\longrightarrow W$

Ergodic | MBL regime(s)

?



- **Toy model?**
- **Quantitative description?**
- **Consequences?**



SCOPE

1. Spin chains in random field and localization : introduction

2. Exact diagonalization

3. Minimal deviations in the XX chain

4. Quantitative analysis: extreme value theory

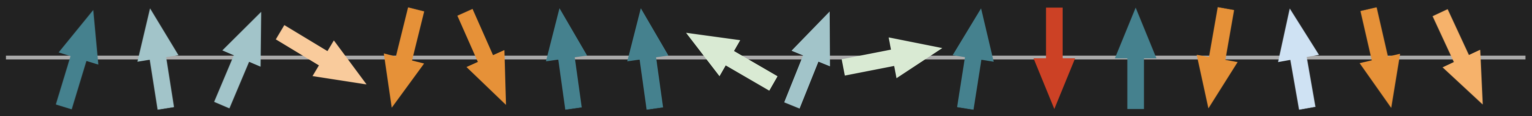
5. Consequences

SPIN CHAINS IN RANDOM FIELD AND LOCALIZATION

SPIN-1/2 CHAIN IN A RANDOM FIELD

$$\mathcal{H} = \sum_i \frac{J}{2}\left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ + 2\Delta S_i^z S_{i+1}^z\right) - \sum_i h_i S_i^z$$

$$S^{x,y,z} = \frac{1}{2}\sigma^{x,y,z}$$



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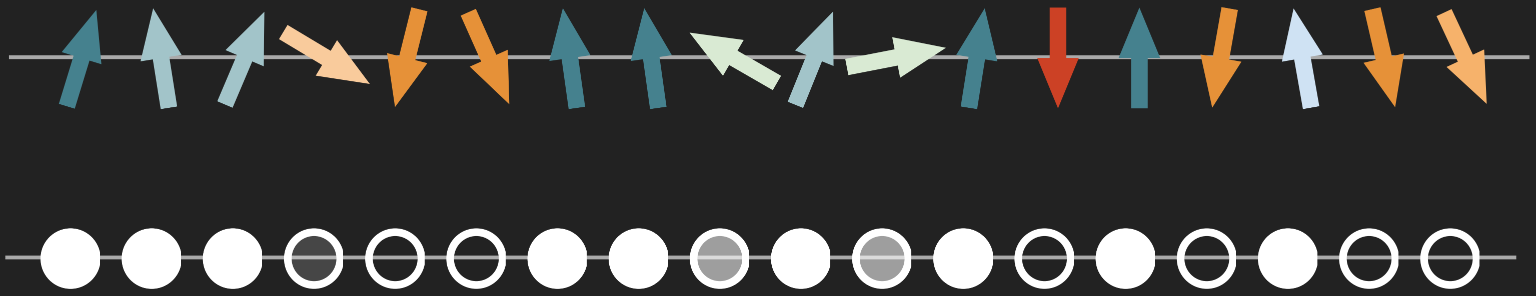

Jordan-Wigner

Spinless fermions

(hardcore bosons)

$$\mathcal{H}_f = \sum_i \left[\frac{J}{2}\left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i + 2\Delta n_i n_{i+1}\right) - h_i n_i\right]$$

P. Jordan and E. Wigner, Z. Physik **47**, 631–651 (1928)

SPIN-1/2 CHAIN IN A RANDOM FIELD

$$\mathcal{H} = \sum_i \frac{J}{2}\left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ + 2\Delta S_i^z S_{i+1}^z\right) - \sum_i h_i S_i^z$$

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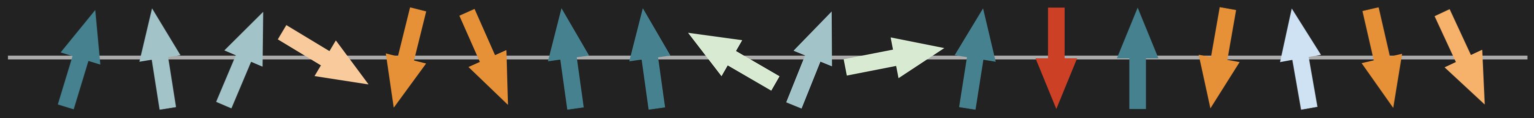
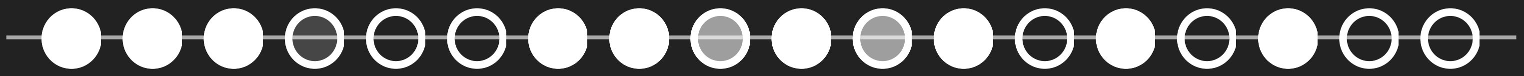
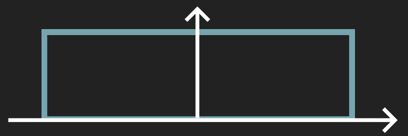
Spin-flip | Ising interaction | Magnetic field





$\mathcal{P}(h_i) =$



Attraction/ repulsion

$$\mathcal{H}_f = \sum_i \left[\frac{J}{2}\left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i + 2\Delta n_i n_{i+1}\right) - h_i n_i\right]$$

Jump | On-site energy

P. Jordan and E. Wigner, Z. Physik **47**, 631–651 (1928)

ANDERSON LOCALIZATION

$$\mathcal{H} = \sum_i \frac{J}{2} \left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ + 2\Delta S_i^z S_{i+1}^z \right) - \sum_i h_i S_i^z$$

1 particle

Spin-flip — Ising interaction — Magnetic field

P. W. Anderson, Phys. Rev. **109,** 1492 (1958); N.F. Mott & W.D. Twose, Advances in Physics **10**, 107-163, (1961)
B. A. Van Tiggelen, In: J. P. Fouque (eds), *Diffuse Waves in Complex Media*, NATO Science Series, **531,** Springer, Dordrecht, (1999)

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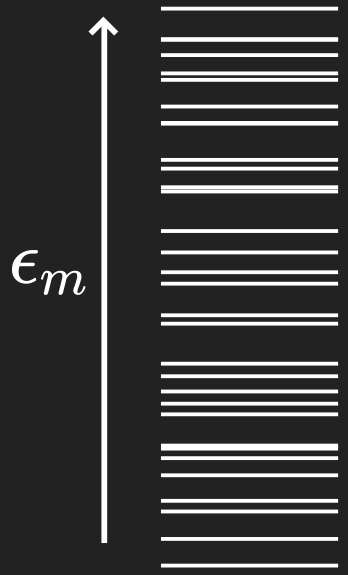
$$\mathcal{H}_f = \sum_m \epsilon_m b_m^\dagger b_m$$ 1 particle



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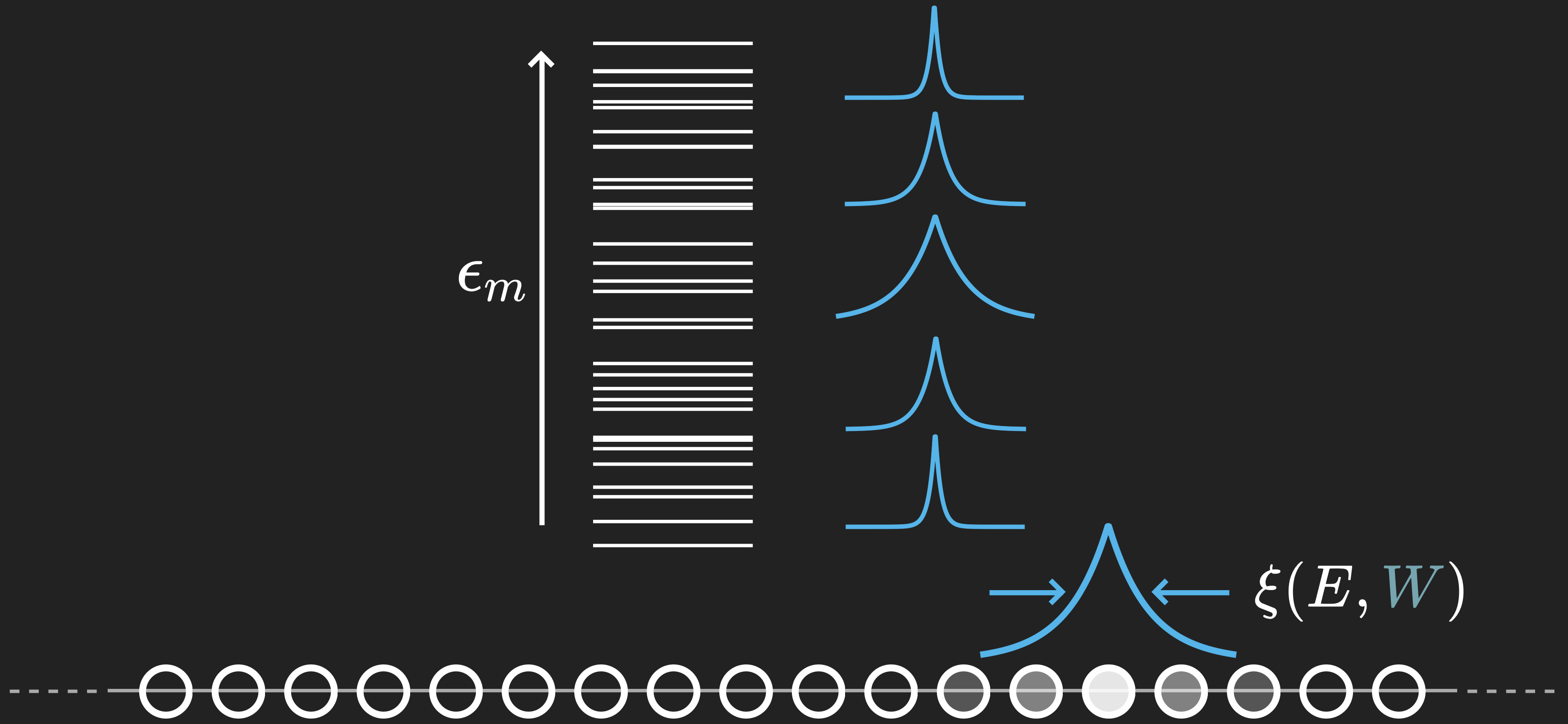

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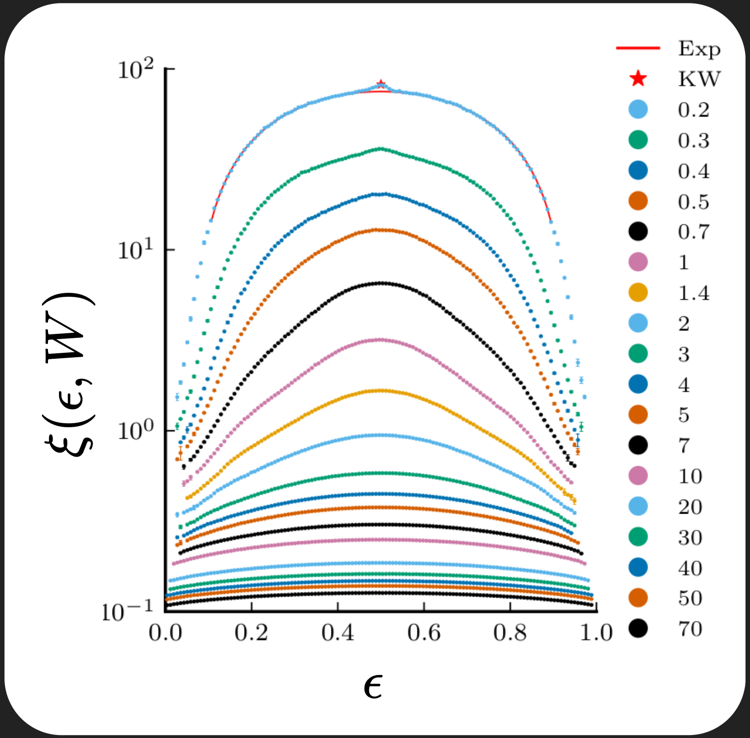
LOCALIZATION LENGTH



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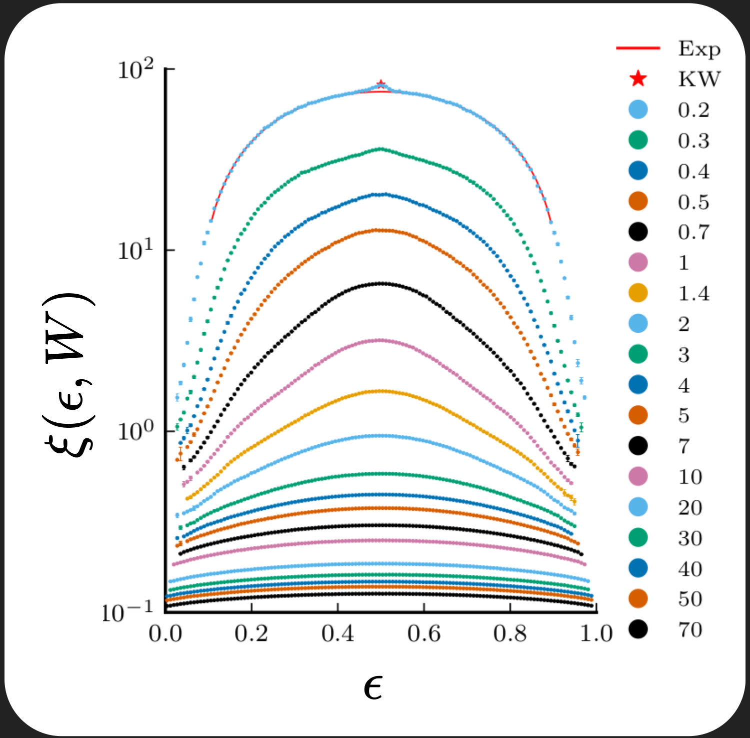

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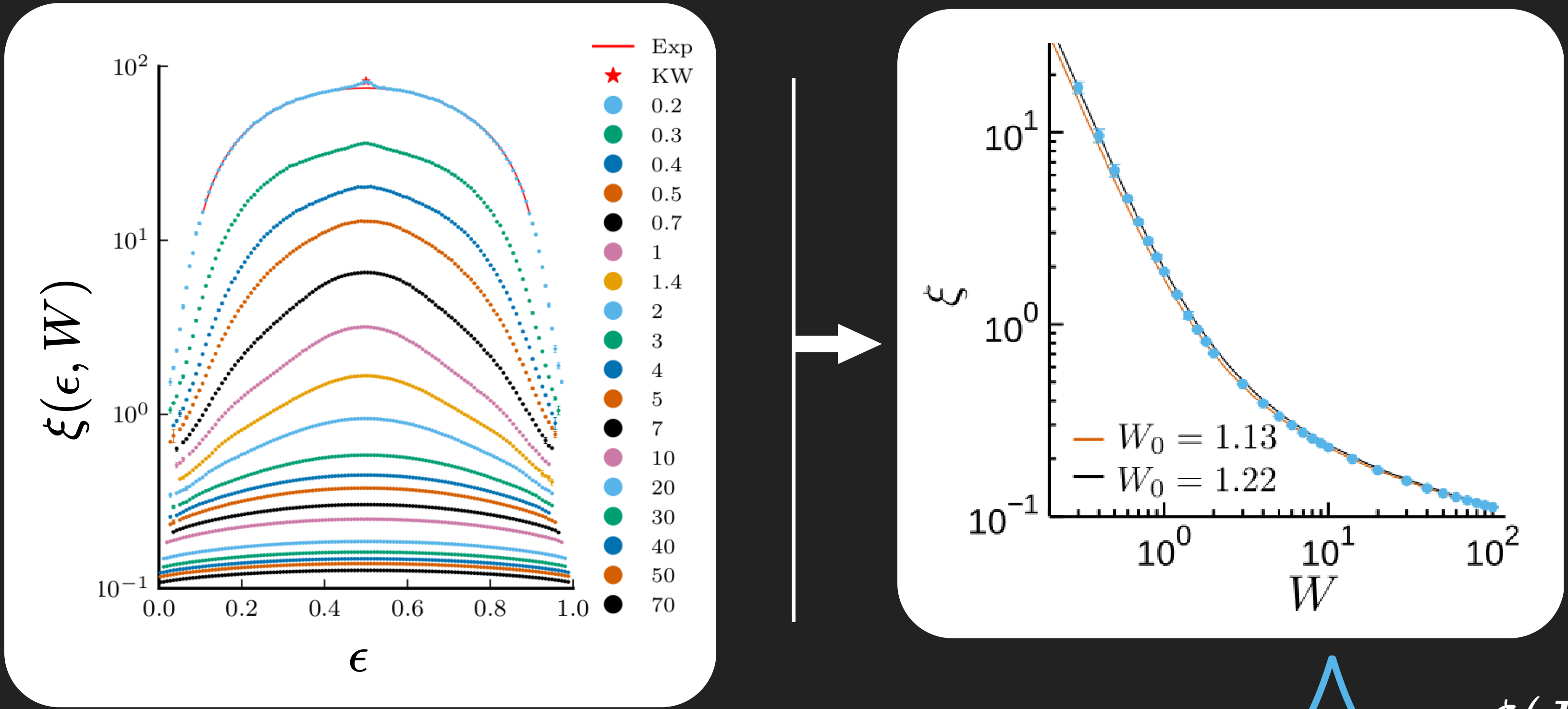

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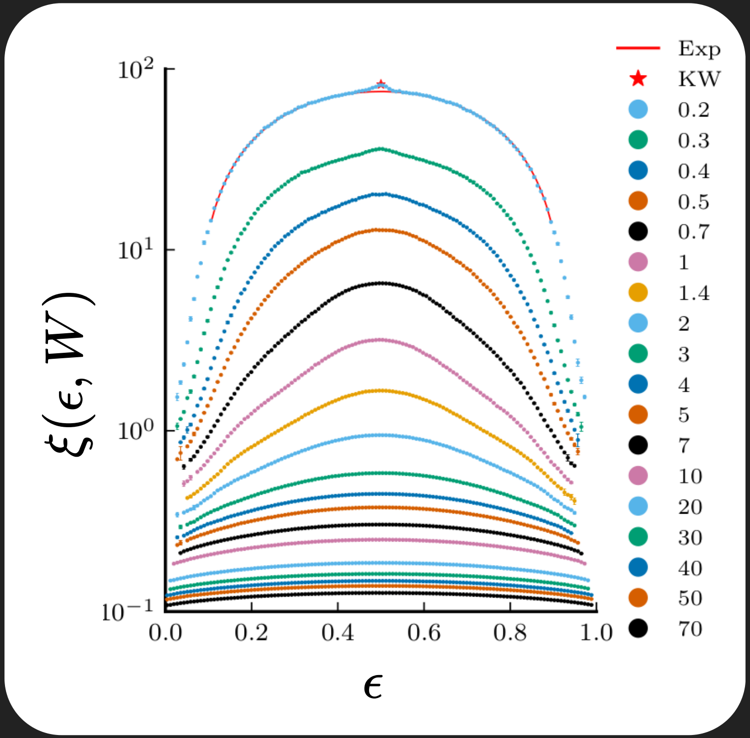
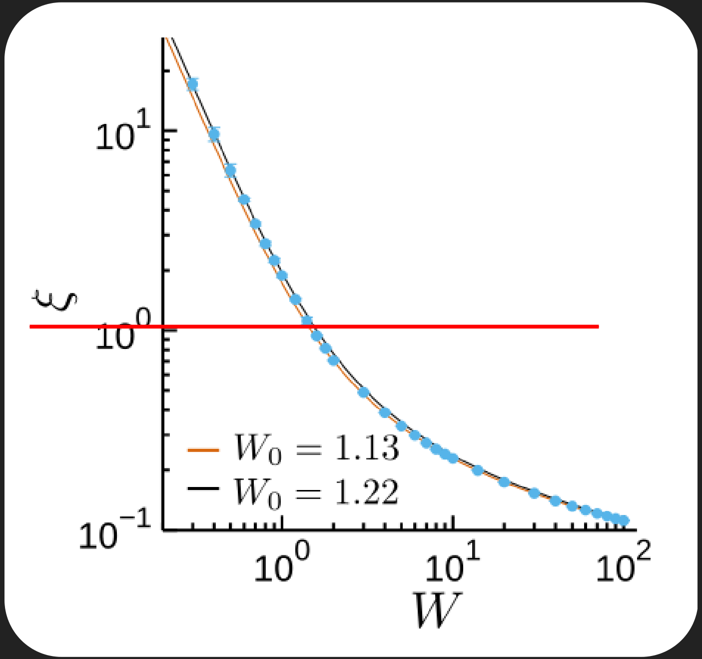
LOCALIZATION LENGTH





A. C. Potter, R. Vasseur, and S. A. Parameswaran, PRX 5, 031033 (2015)

$$\xi = \frac{1}{\ln\left(1 + \left(\frac{W}{W_0}\right)^2\right)}$$

$$W \gg 2$$

$$\xi \ll 1$$

P. W. Anderson, Phys. Rev. **109,** 1492 (1958); N.F. Mott & W.D. Twose, Advances in Physics **10**, 107-163, (1961)

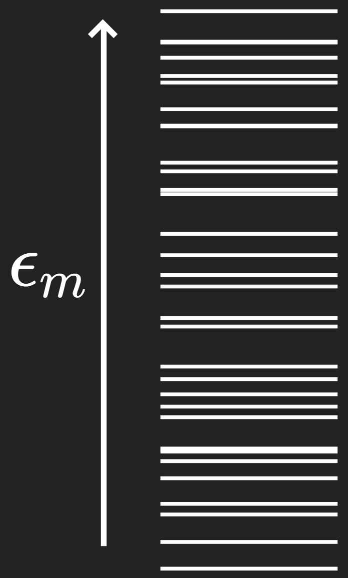
B. A. Van Tiggelen, In: J. P. Fouque (eds), *Diffuse Waves in Complex Media*, NATO Science Series, **531,** Springer, Dordrecht, (1999)

"MANY-BODY" ANDERSON INSULATOR (= XX CHAIN)

$$\mathcal{H}_f = \sum_i \left[\frac{J}{2} \left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i \right) - h_i n_i \right]$$

$L/2$ fermions

$S_z = 0$



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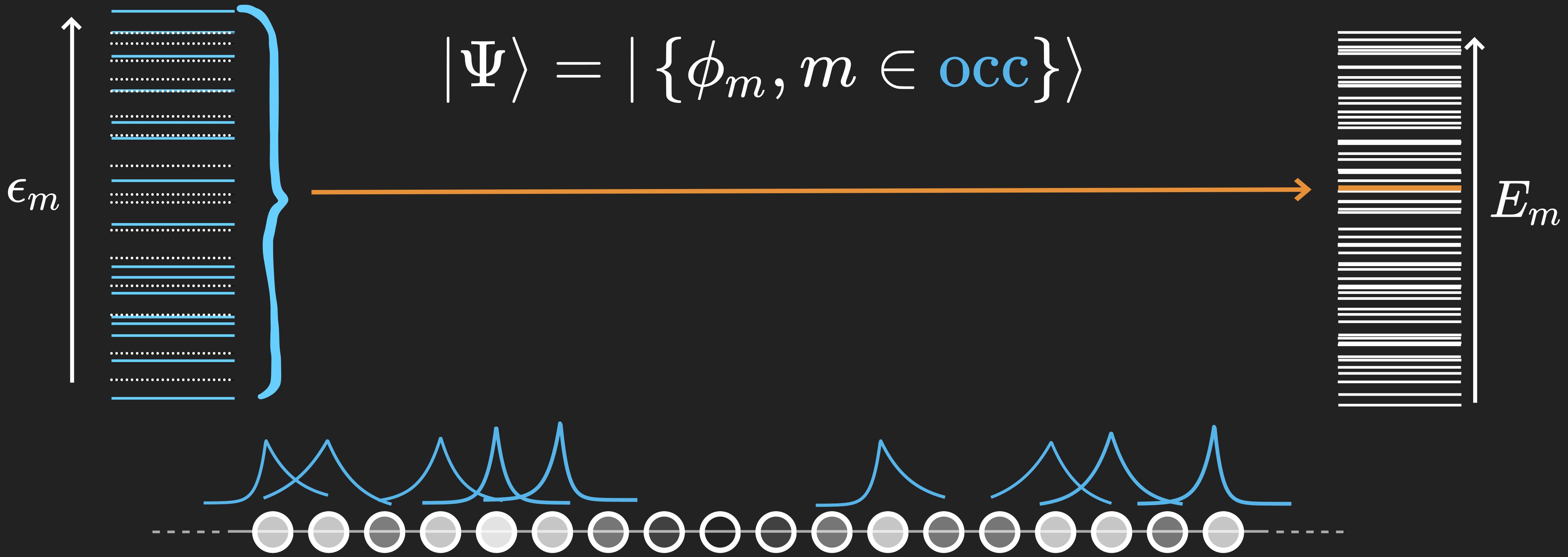
$$|\Psi\rangle = |\{\phi_m, m \in \text{occ}\}\rangle$$



HEISENBERG: INTRODUCING INTERACTIONS

$$\mathcal{H} = \sum_i \frac{J}{2}\left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ + 2\Delta S_i^z S_{i+1}^z\right) - \sum_i h_i S_i^z$$

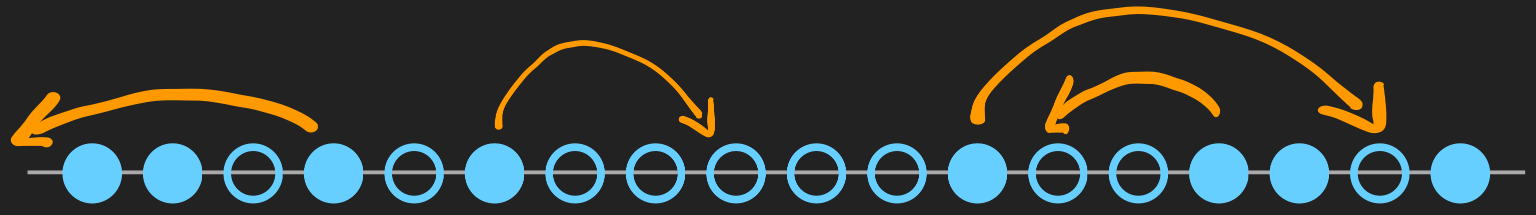
$$\mathcal{H}_f = \sum_i \left[\frac{J}{2}\left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i + 2\Delta n_i n_{i+1}\right) - h_i n_i\right]$$

EFFECT OF INTERACTIONS?

$$\mathcal{H} = \sum_i \frac{J}{2} \left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ + 2\Delta S_i^z S_{i+1}^z \right) - \sum_i h_i S_i^z$$

In the Anderson basis:

$$\mathcal{H} = \sum_m \epsilon_m b_m^\dagger b_m + \sum_{j,k,l,m} V_{j,k,l,m} b_j^\dagger b_k^\dagger b_l b_m$$



Anderson orbitals m

$$\mathcal{H}_f = \sum_i \left[\frac{J}{2} \left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i + 2\Delta n_i n_{i+1} \right) - h_i n_i \right]$$

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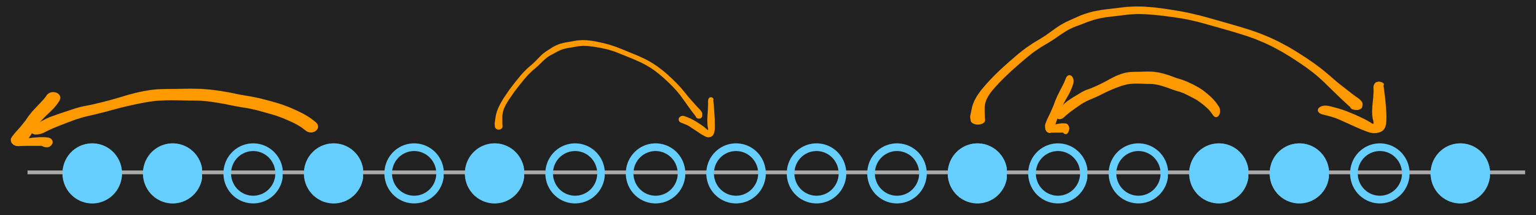

Anderson orbitals m

Interactions favor delocalization. Do they fully destroy localization?

$$\mathcal{H}_f = \sum_i \left[\frac{J}{2}\left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i + 2\Delta n_i n_{i+1}\right) - h_i n_i\right]$$

P. W. Anderson, Phys. Rev. **109,** 1492 (1958)

EFFECT OF INTERACTIONS? GROUND STATE

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Do interactions destroy localization?

$$\mathcal{H}_f = \sum_i \left[\frac{J}{2}\left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i + 2\Delta n_i n_{i+1}\right) - h_i n_i\right]$$

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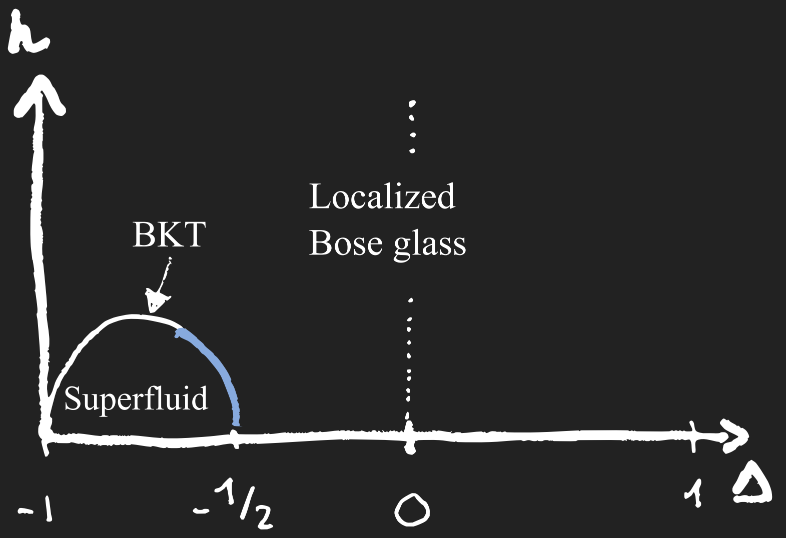

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T. Giamarchi and H. J. Schulz, EPL **3** 1287 (1987); PRB **37**, 325 (1988)
Z. Ristivojevic, et al PRL **109**, 026402 (2012);
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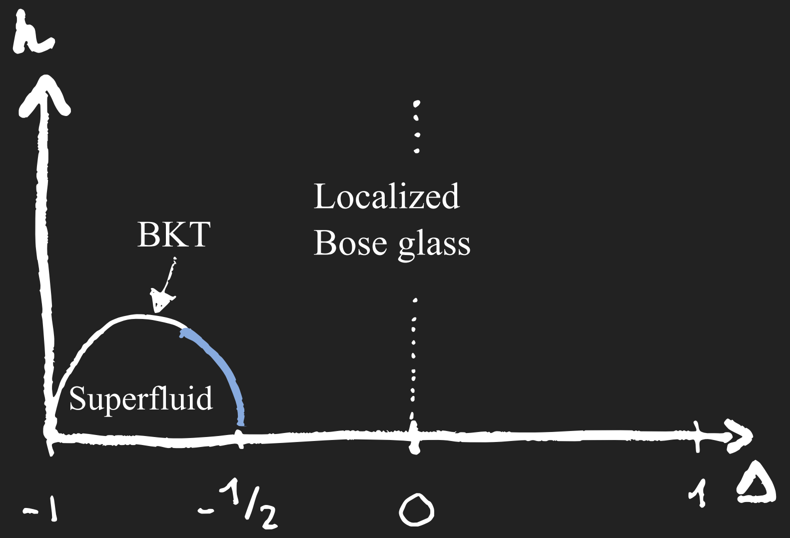
Do interactions destroy localization?

What about high temperatures / high energy eigenstates?



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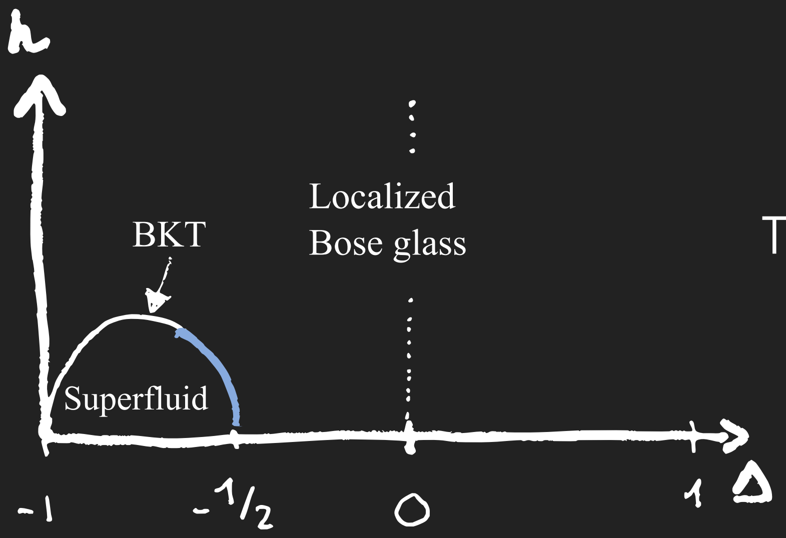

What about high temperatures / high energy eigenstates?

Thermal average ⟷ Time average
?
ETH

Do isolated quantum systems thermalize?

$$\mathcal{H}_f = \sum_i \left[\frac{J}{2}\left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i + 2\Delta n_i n_{i+1}\right) - h_i n_i\right]$$

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J. M. Deutsch , PRA. **43,** 2046–2049, (1991) ,
M. Srednicki, PRE **50,** 888–901, (1994)
L. D'Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol, Adv. Phys. **65**, 239 (2016)

WEAK INTERACTIONS AND DISORDER

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Analytical, general picture:

L. Fleischman, P. W. Anderson, PRB **2**, 2336 (1980) → single-particle excitations and conditions for Anderson transition
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P. Jacquod, D. L. Shepelyansky, PRL **79**, 1837 (1997) → Gap ratio statistics, finite systems
I. V. Gornyi, A. D. Mirlin, D. G. Polyakov, PRL **95**, 206603 (2005) → zero conductivity at low temperature
*D. M. Basko, I. L. Aleiner, B. L. Altschuler, Annals of Physics **321**, 1126 (2006) → metal-insulator transition, localization in Fock space
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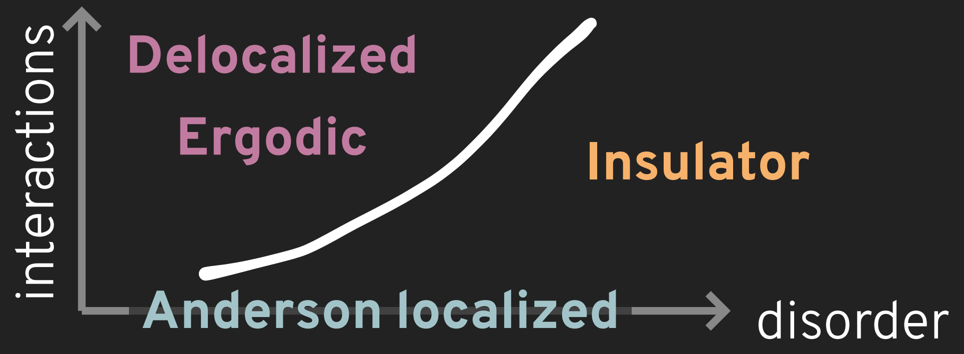
Analytical, general picture:

Interactions $\Rightarrow$ transition between weak and strong disorder



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HEISENBERG IN RANDOM FIELD : A PARADIGMATIC EXAMPLE

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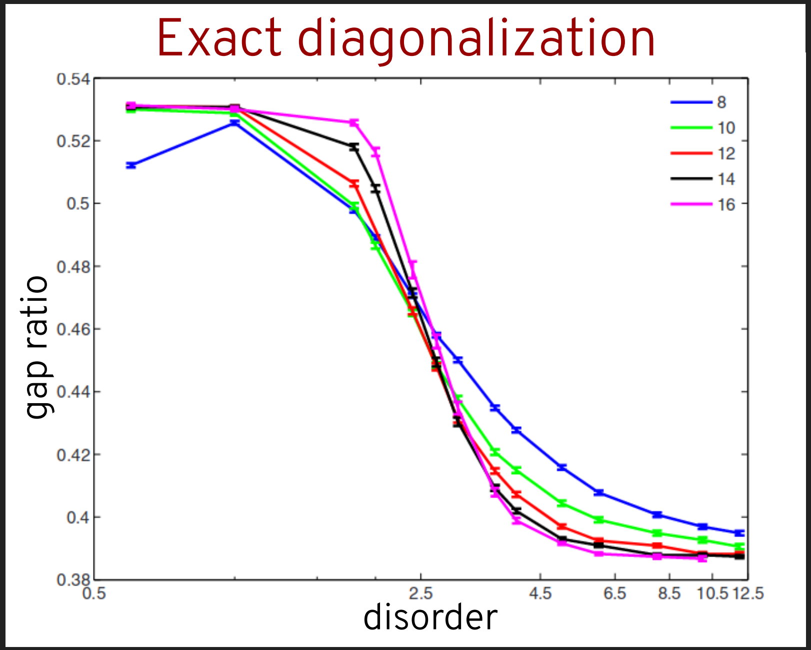
A few among many…



A. Pal, D. Huse, PRB **82**, 174411 2010

(See series of works by V. Oganesyan, A. Pal, D. Huse, 2007-2010)

Probes: gap ratio

D. A. Abanin, E. Altman, I. Bloch, and M. Serbyn, Rev. Mod. Phys. **91**, 021001 (2019)

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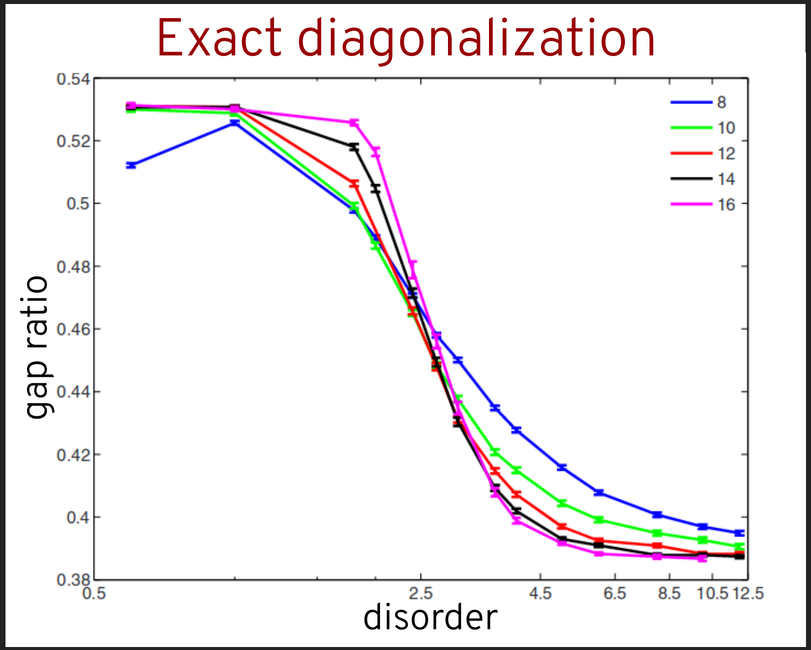
A few among many...

Gaussian orthogonal ensemble statistics = random matrix

level repulsion

$\leftrightarrow$ ergodic



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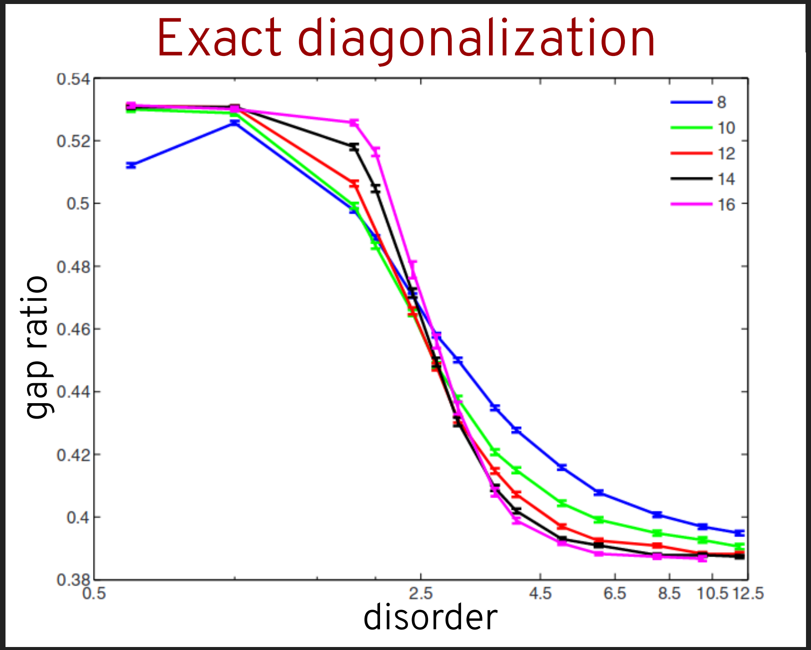

A. Pal, D. Huse, PRB **82**, 174411 2010

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Poisson statistics

non-ergodic

Probes: gap ratio

D. A. Abanin, E. Altman, I. Bloch, and M. Serbyn, Rev. Mod. Phys. **91**, 021001 (2019)

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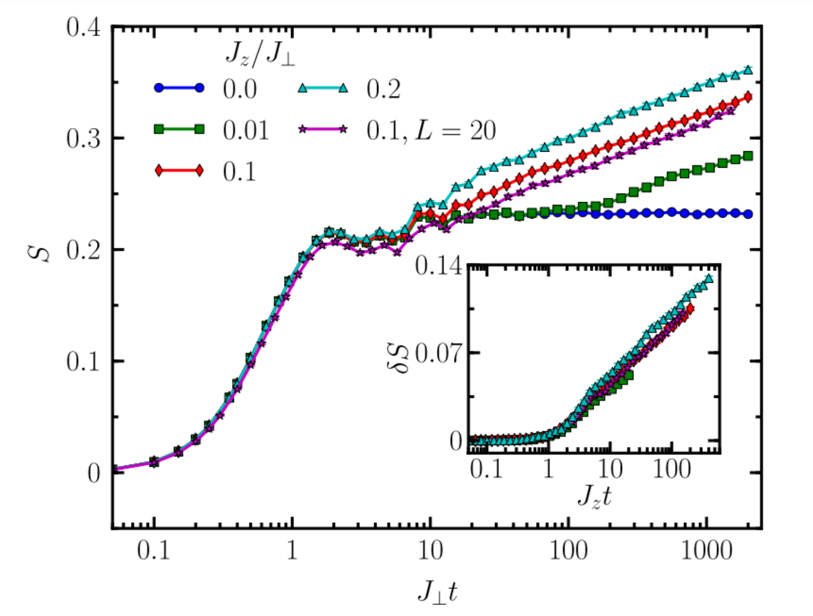
A few among many…

Initial S^z basis random product state

+

TEBD

W = 5



Anderson
No growth of entanglement

J. H. Bardarson, F. Pollmann, and J. E. Moore, PRL **109**, 017202 (2012)
M. Znidaric, T. Prosen, and P. Prelovsek PRB 77, 064426 (2008)

Probes: gap ratio | entanglement entropy

D. A. Abanin, E. Altman, I. Bloch, and M. Serbyn, Rev. Mod. Phys. **91**, 021001 (2019)   F. Alet, N. Laflorencie, C. R. Phys. **19**,498 (2018)

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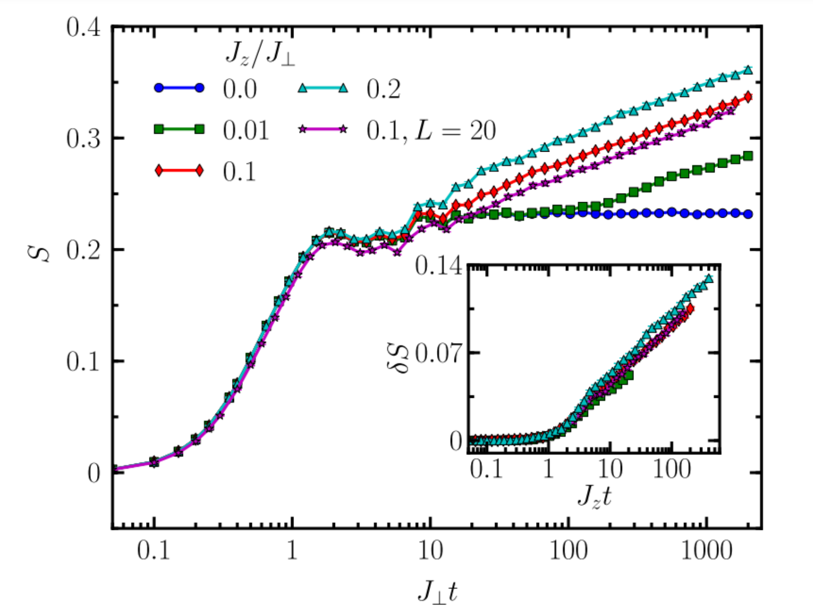

J. H. Bardarson, F. Pollmann, and J. E. Moore, PRL **109**, 017202 (2012)
M. Znidaric, T. Prosen, and P. Prelovsek PRB 77, 064426 (2008)

Many body
Log growth of entanglement

Anderson
No growth of entanglement

Probes: gap ratio | entanglement entropy

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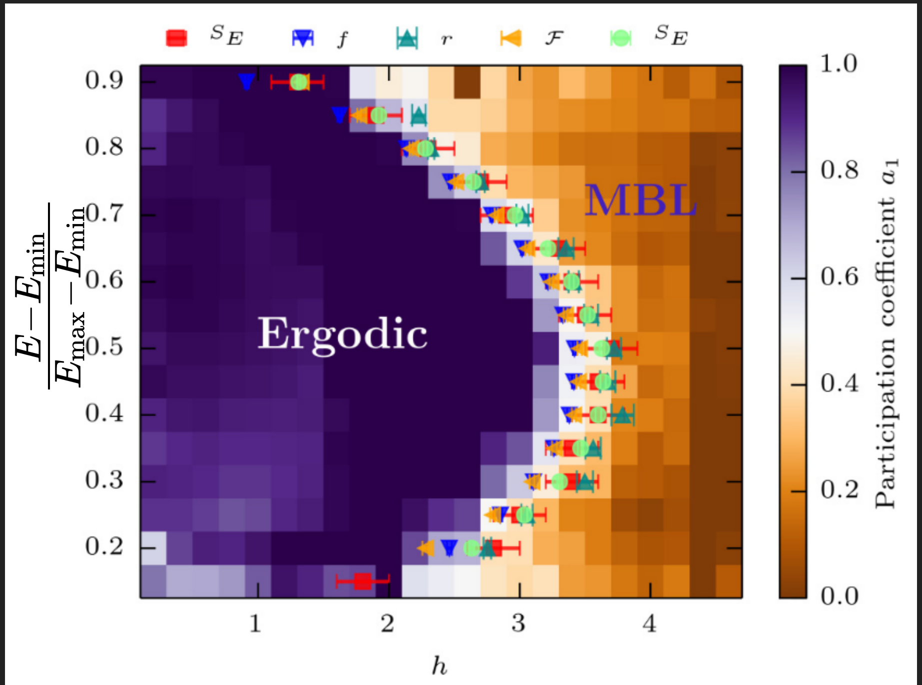
A few among many...

$$|\Psi\rangle = \sum_{\alpha=1}^{\mathcal{N}} \psi_\alpha |\alpha\rangle$$

$$S_q = \frac{1}{1-q} \ln\left(\sum_{\alpha=1}^{\mathcal{N}} |\psi_\alpha|^{2q}\right)$$

$$S_q = a_q \ln(\mathcal{N})$$

Configuration space



D. J. Luitz, N. Laflorencie, F. Alet, PRB **91**, 081103(R) (2015)

Probes: gap ratio | entanglement entropy | participation entropy

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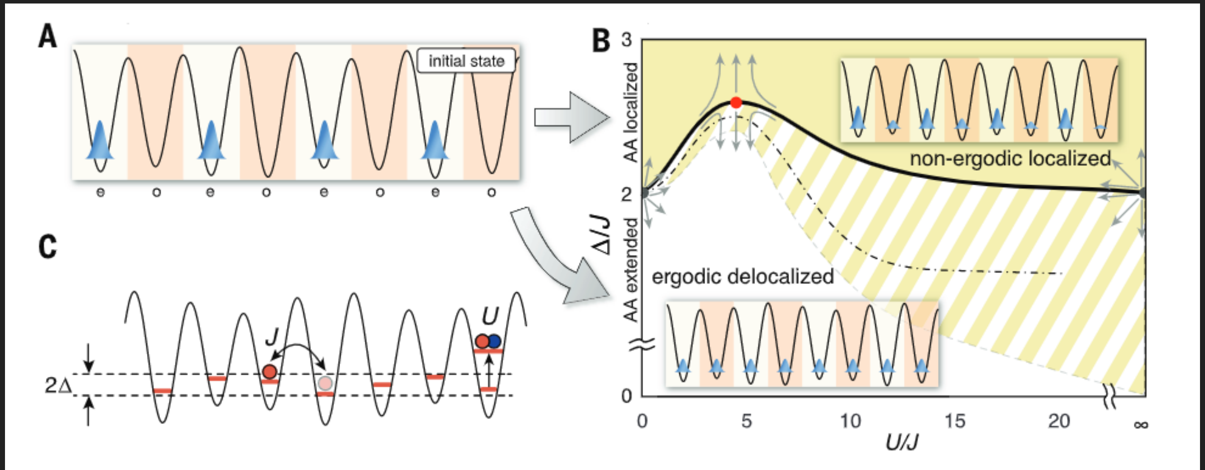
A few among many...

$$\hat{H} = -J\sum_{i,\sigma}(\hat{c}^\dagger_{i,\sigma}\hat{c}_{i+1,\sigma} + \text{h.c.}) + \Delta\sum_{i,\sigma}\cos(2\pi\beta i + \phi)\hat{c}^\dagger_{i,\sigma}\hat{c}_{i,\sigma} + U\sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$

$$\mathcal{I} = \frac{N_e - N_o}{N_e + N_o}$$



M. Schreiber *et al.* (I. Bloch) , Science **349**, 842 (2015)

Probes: gap ratio | entanglement entropy | participation entropy | imbalance [...]

D. A. Abanin, E. Altman, I. Bloch, and M. Serbyn, Rev. Mod. Phys. **91**, 021001 (2019)   F. Alet, N. Laflorencie, C. R. Phys. **19**,498 (2018)

DEBATE

2016
Finite L

$\longrightarrow W$

Ergodic | MBL

DEBATE

2016 → W

Finite L: Ergodic | MBL

2023 → W

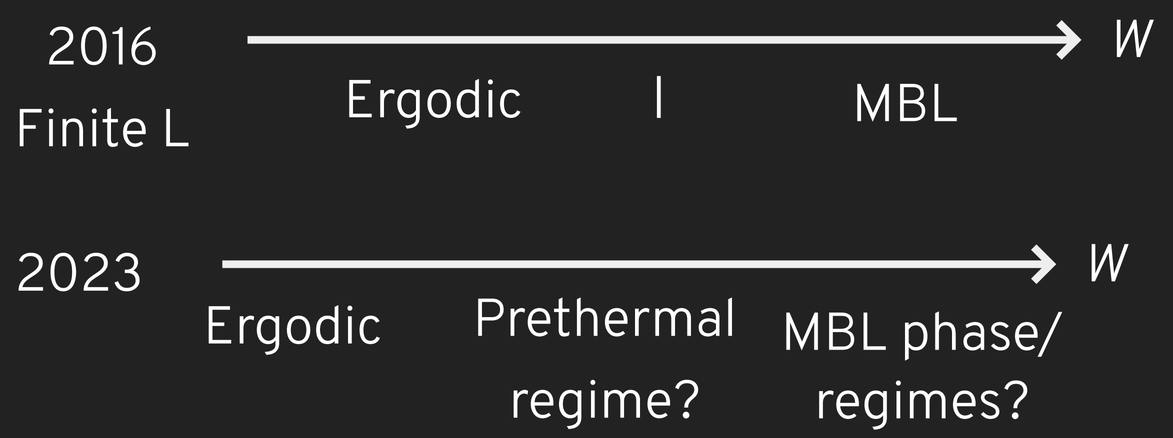
Ergodic — Prethermal regime? — MBL phase/ regimes?

- Finite-size scaling? Location of the transition?
- Destabilization by ergodic bubbles even at strong disorder?
- Immediate onset of quantum chaos? Intermediate phase(s)?

J. Šuntajs, J. Bonča, T. Prosen, and L. Vidmar, PRE **102**, 062144 (2020); D.A. Abanin, et al, Annals of Physics **427**, 168415, (2021);

D. Sels, A. Polkovnikov, JCCM January 2023_1 (2023); Tyler LeBlond, Dries Sels, Anatoli Polkovnikov, and Marcos Rigol, PRB **104**, L201117 (2021); A. Morningstar et al, PRB **105**, 174205 (2022); L. Colmenarez, D. Luitz, W. De Roeck, arXiv:2308.01350 (2023); P, Sierant and J. Zakrzewski, PRB 105, 224203 (2022)...

DEBATE



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For today : **Magnetization**, ED data and comparison to the **Anderson** line

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EXACT DIAGONALIZATION

CHALLENGE

- Many-body $\mathcal{N} = \frac{L!}{\left(\frac{L}{2}!\right)^2}$

Simulations of Many-Body Localizable (MBL) lattices models | Fabien Alet | Cargese

CHALLENGE

- Many-body $\mathcal{N} = \frac{L!}{\left(\frac{L}{2}!\right)^2}$
- Disorder $\rightarrow$ translation invariance, high number of realisations

Simulations of Many-Body Localizable (MBL) lattices models | Fabien Alet | Cargese

CHALLENGE

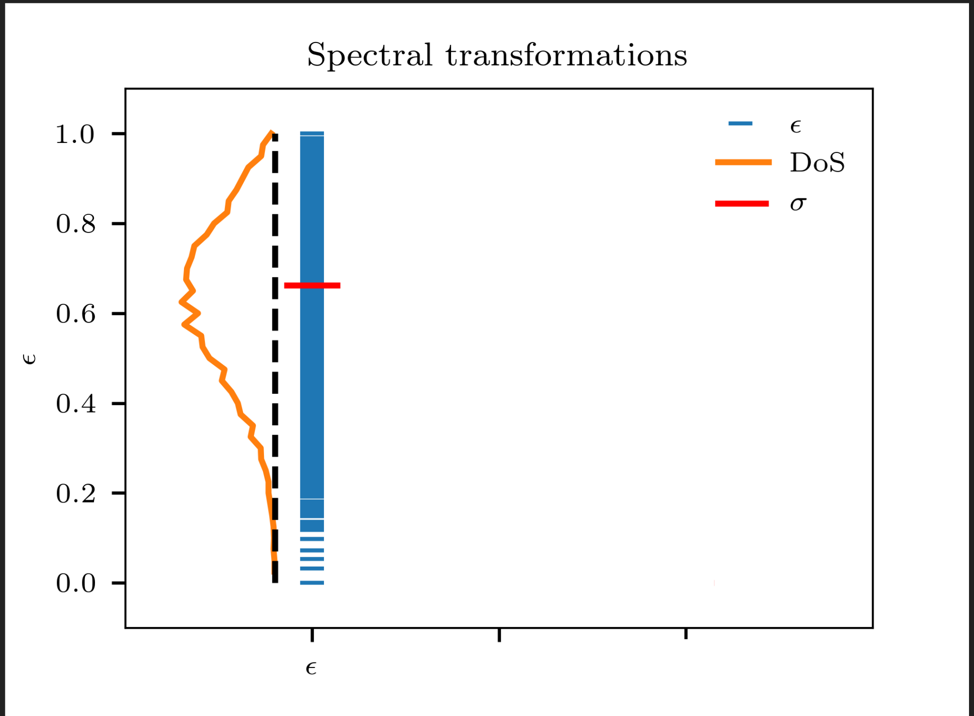
- Many-body $\mathcal{N} = \frac{L!}{\left(\frac{L}{2}!\right)^2}$
- Disorder $\rightarrow$ ~~translation invariance~~, high number of realisations
- High-energy eigenstates
- High density of eigenstates
- Potential absence of thermalization

▶ Simulations of Many-Body Localizable (MBL) lattices models | Fabien Alet | Cargese

SPECTRAL TRANSFORMATION

$$\mathcal{N} = \frac{L!}{\left(\frac{L}{2}!\right)^2}$$

High energy eigenstates close to $\epsilon = \sigma$, high dos



$L = 14, S^z_{\mathrm{tot}} = 0$, clean case

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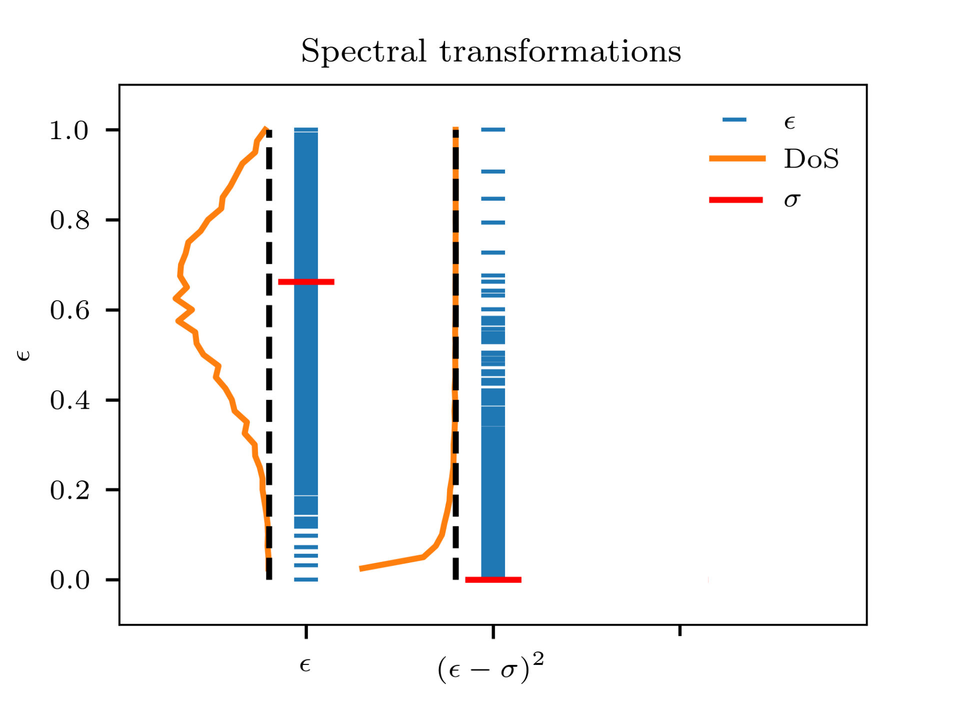

$L = 14, S^z_{\text{tot}} = 0$, clean case

Idea : transform the spectrum!

$$F = (H - \sigma)^2$$

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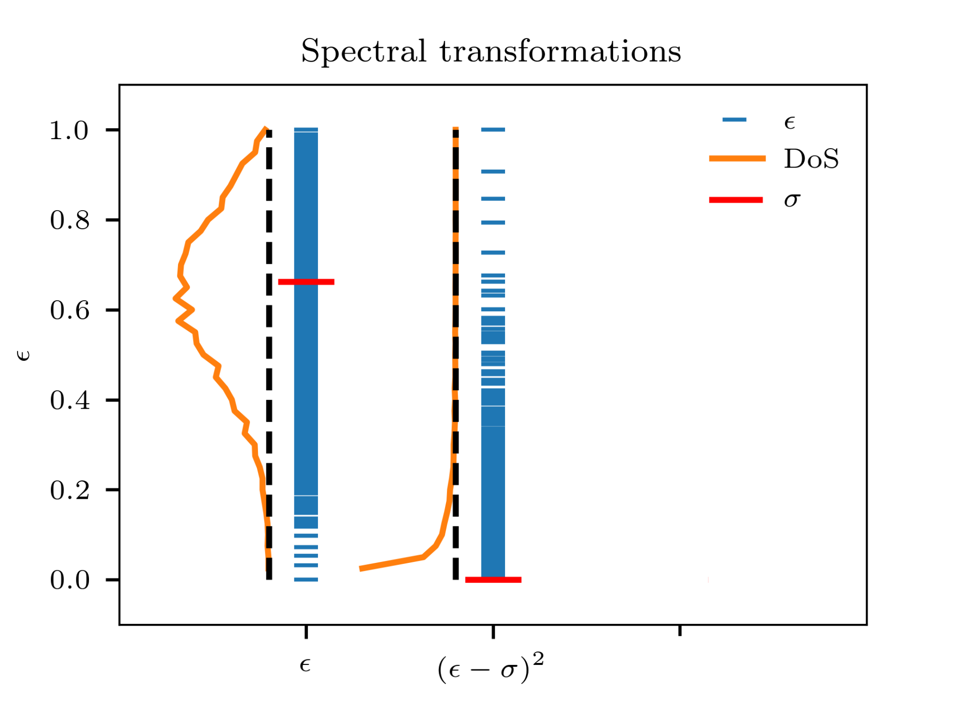
High energy eigenstates close to $\epsilon = \sigma$, high dos



$L = 14, S^z_{\mathrm{tot}} = 0$, clean case

Idea : transform the spectrum!

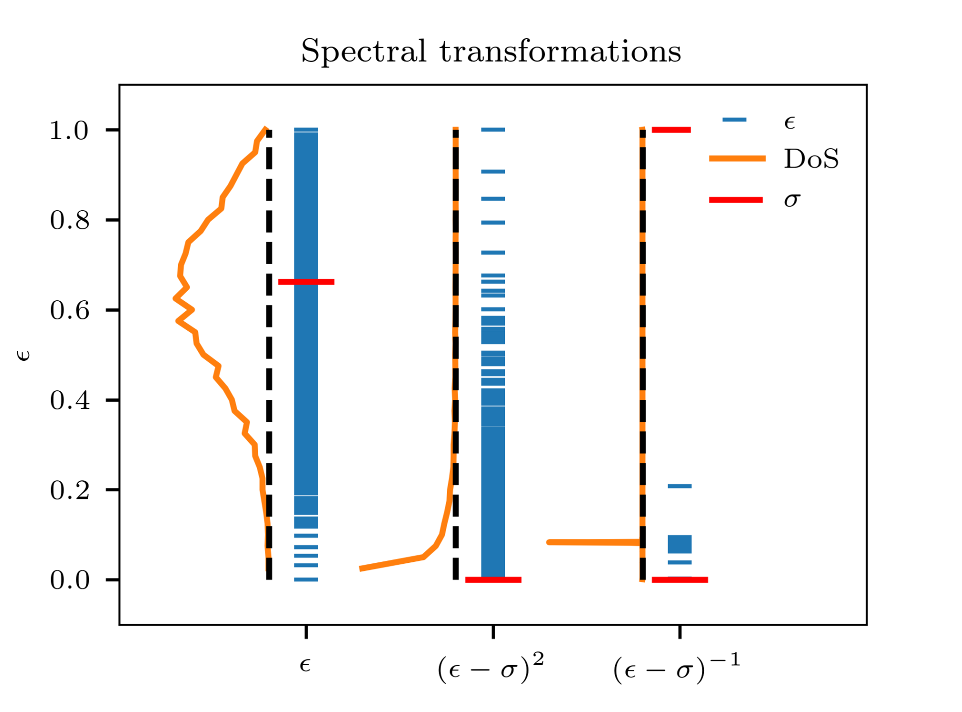
$$F = (H - \sigma)^2$$

D. Luitz, N. Laflorencie, F. Alet, PRB **91,** 081103(R) (2015)
F. Pietracaprina, et al, .SciPost Phys. **5**, 045 (2018)
P. Sierant et al, PRL **125**, 156601 (2020)

SPECTRAL TRANSFORMATION

$$\mathcal{N} = \frac{L!}{\left(\frac{L}{2}!\right)^2}$$

High energy eigenstates close to $\epsilon = \sigma$, high dos



$L = 14, S^z_{\text{tot}} = 0$, clean case

Idea : transform the spectrum!

$$F = (H - \sigma)^2$$

$$G = (H - \sigma)^{-1}$$

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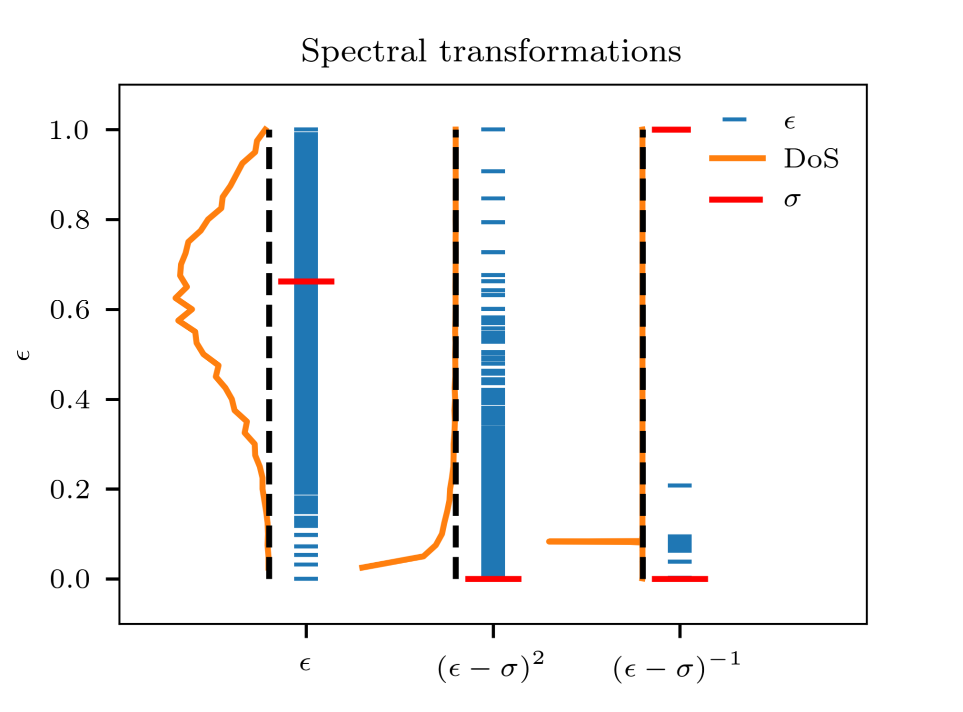

$L = 14, S^z_{\mathrm{tot}} = 0$, clean case

Idea : transform the spectrum!

$$F = (H-\sigma)^2$$

$$G = (H-\sigma)^{-1}$$

Do not invert!

Solve $(H-\sigma)\vec{y} = \vec{x}$

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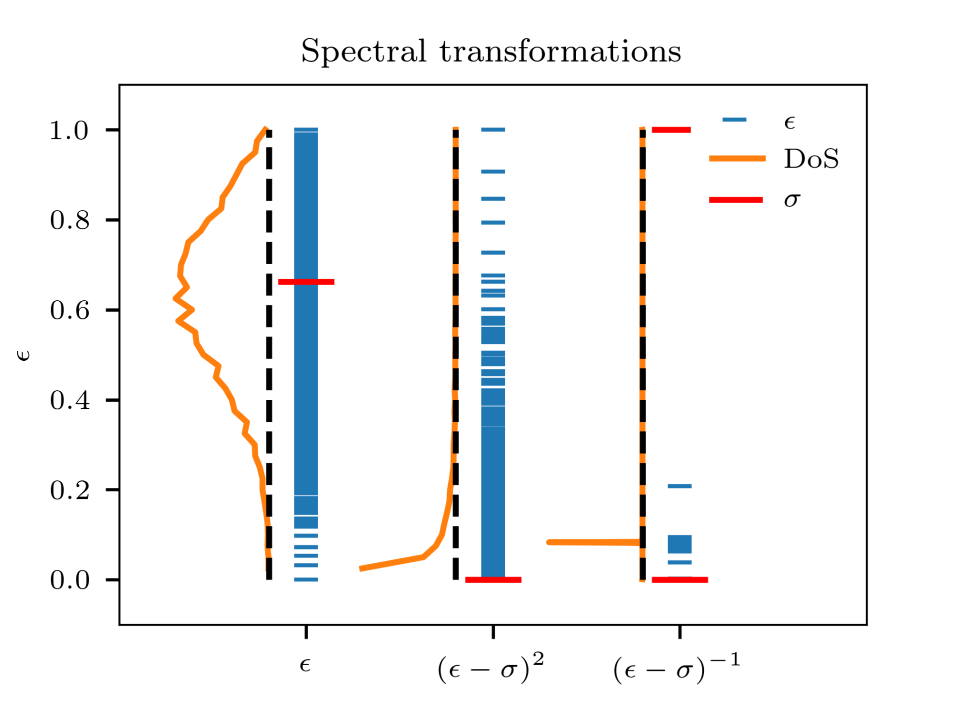
High energy eigenstates close to $\epsilon = \sigma$, high dos



$L = 14, S^z_{\mathrm{tot}} = 0$, clean case

Idea : transform the spectrum!

$$F = (H - \sigma)^2$$ ✗

$$G = (H - \sigma)^{-1}$$

$$G = \text{"} \sum_k \alpha_k H^k \text{"}$$

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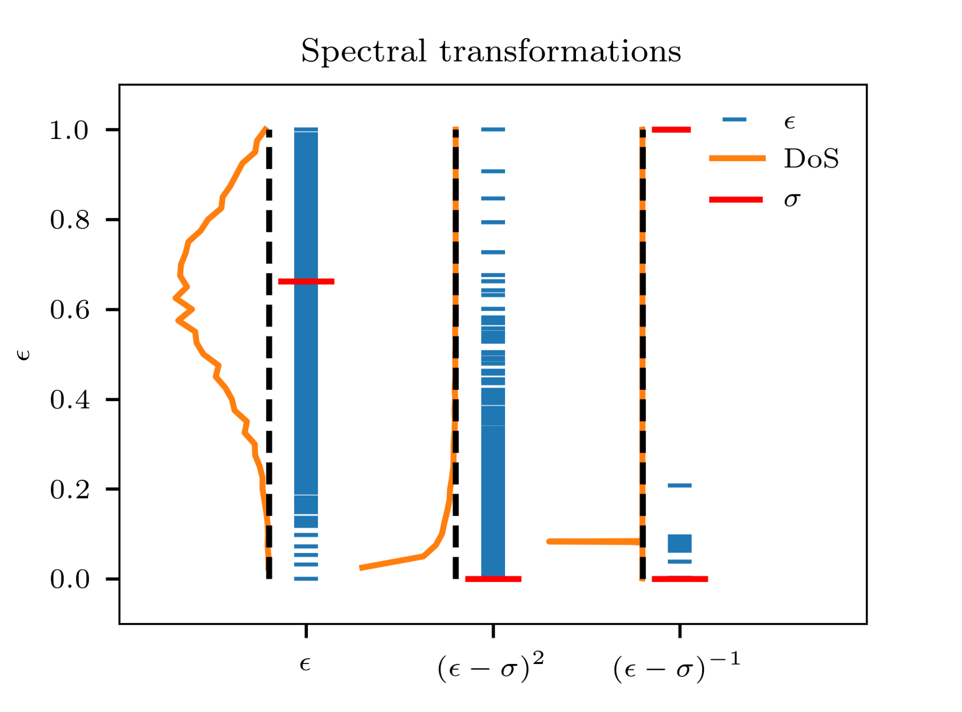
High energy eigenstates close to $\epsilon = \sigma$, high dos



$L = 14, S^z_{\text{tot}} = 0$, clean case

Up to 22, 24 sites

$\mathcal{N} > 2 \cdot 10^6$

non-zero el. $> 3 \cdot 10^7$

$$G = (H - \sigma)^{-1}$$

$$G = \text{"} \sum_k \alpha_k H^k \text{"}$$

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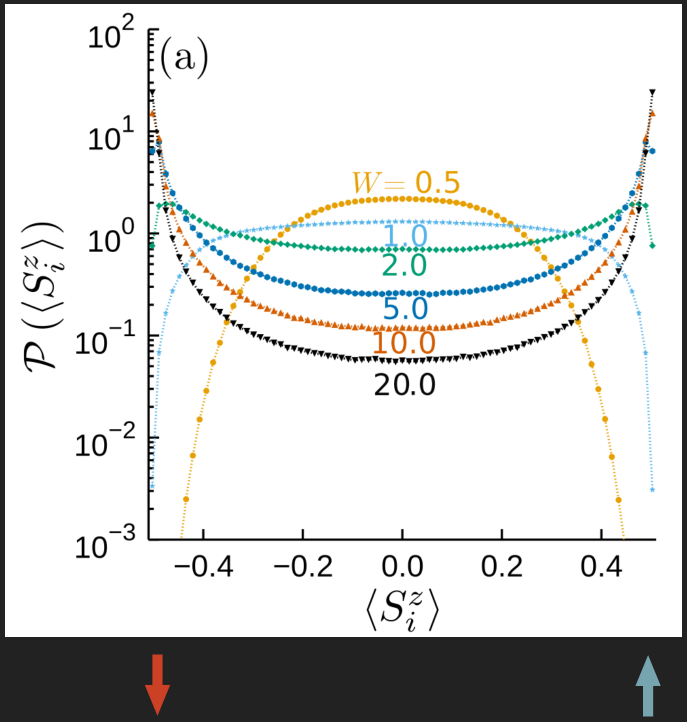
MAGNETIZATION DISTRIBUTIONS

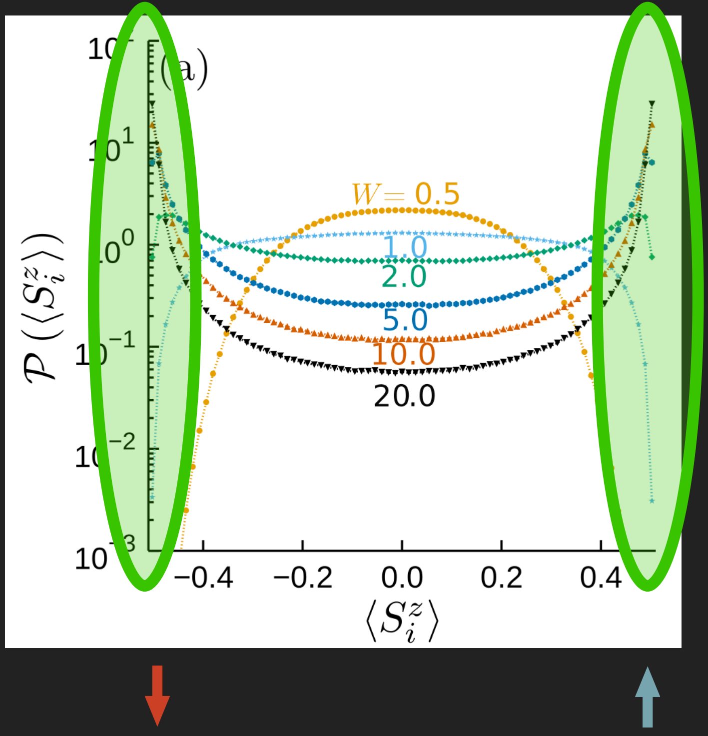
Anderson chain / XX chain



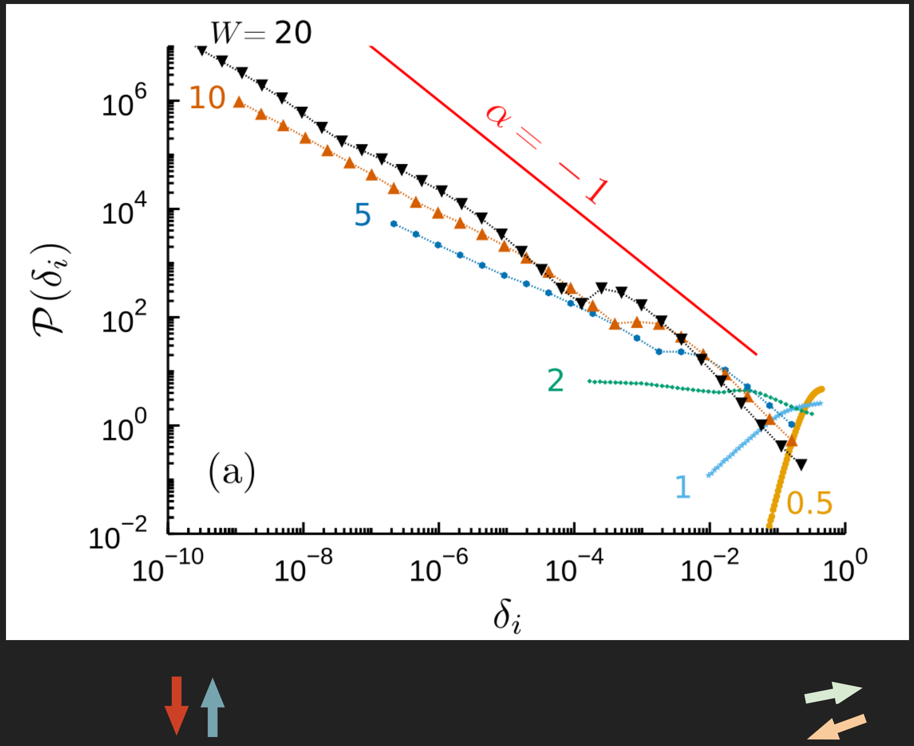
N. Laflorencie, G. Lemarié, N. Macé, PRR **2**, 042033(R) (2020)
J. C., N. Laflorencie, arXiv:2305.10574

MAGNETIZATION DISTRIBUTIONS

Anderson chain / XX chain



$$\delta_i = 1/2 - |\langle S_i^z \rangle|$$



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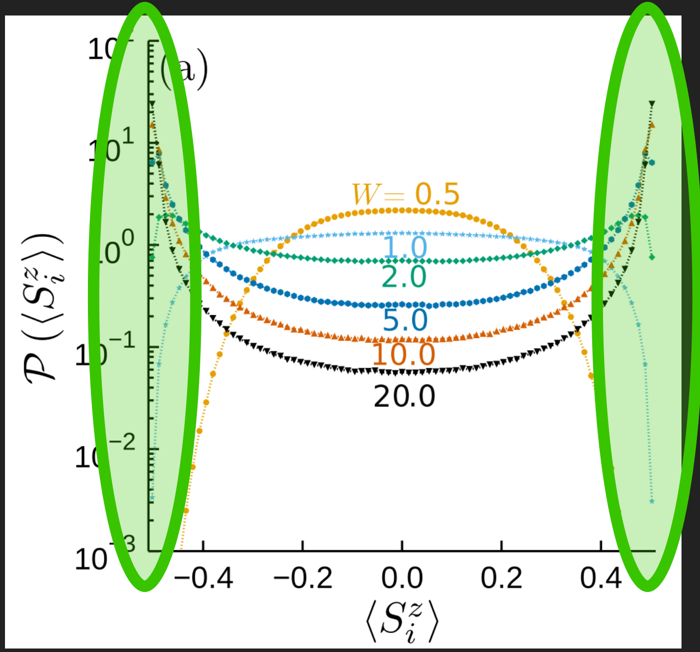
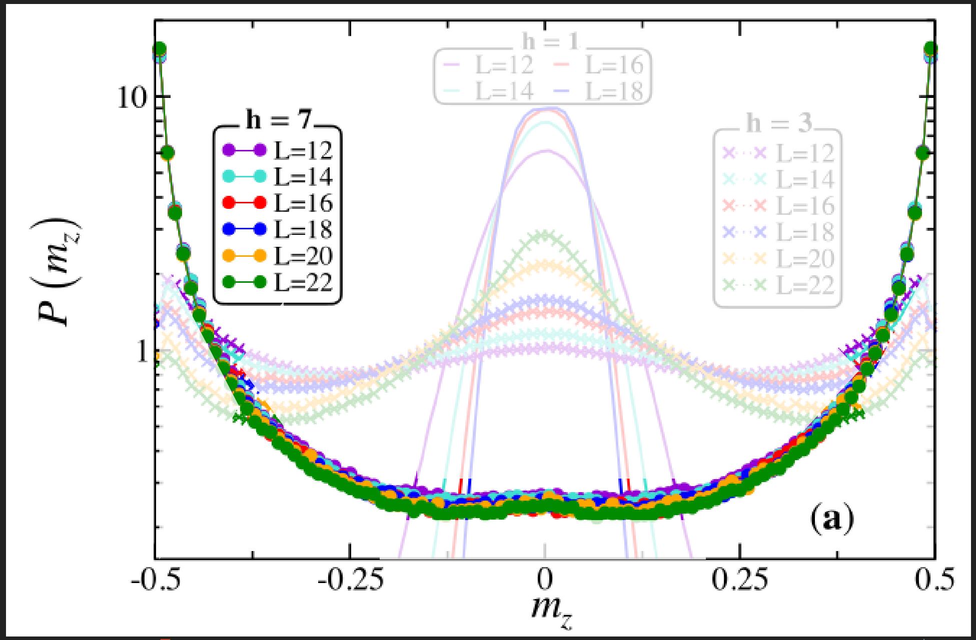


$$\delta_i = 1/2 - |\langle S_i^z \rangle|$$

Heisenberg chain



V. Khemani, F. Pollmann, and S. L. Sondhi, PRL 116, 247204 (2016)
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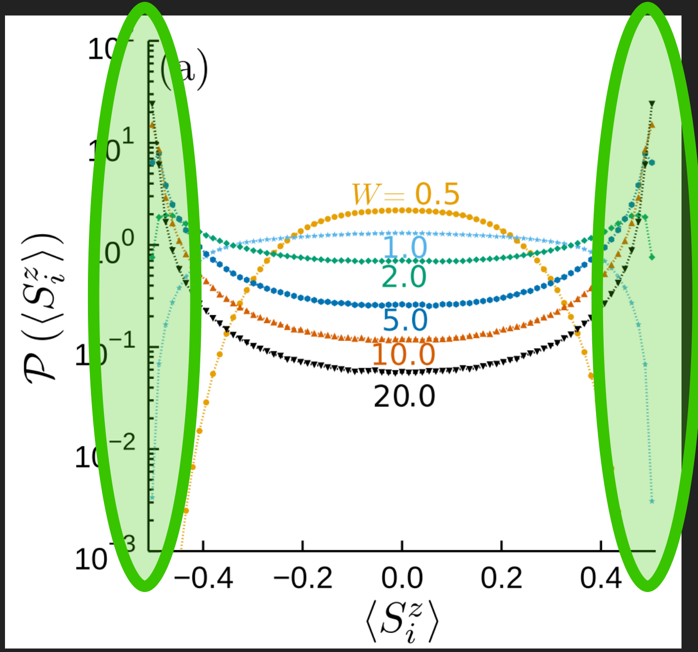
MAGNETIZATION DISTRIBUTIONS

Anderson chain / XX chain



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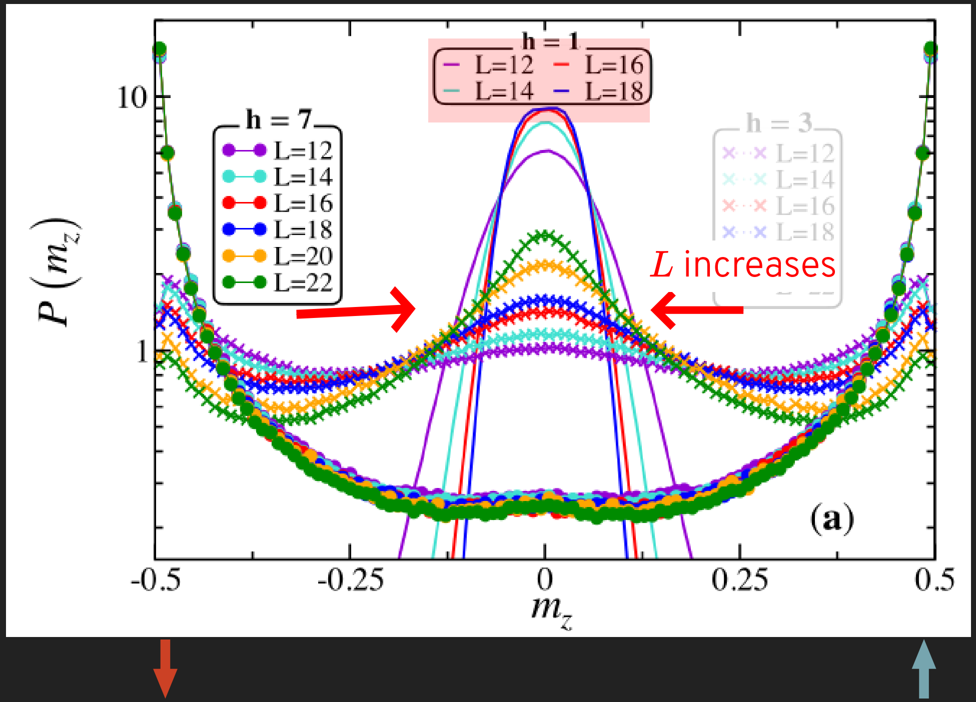
Heisenberg chain



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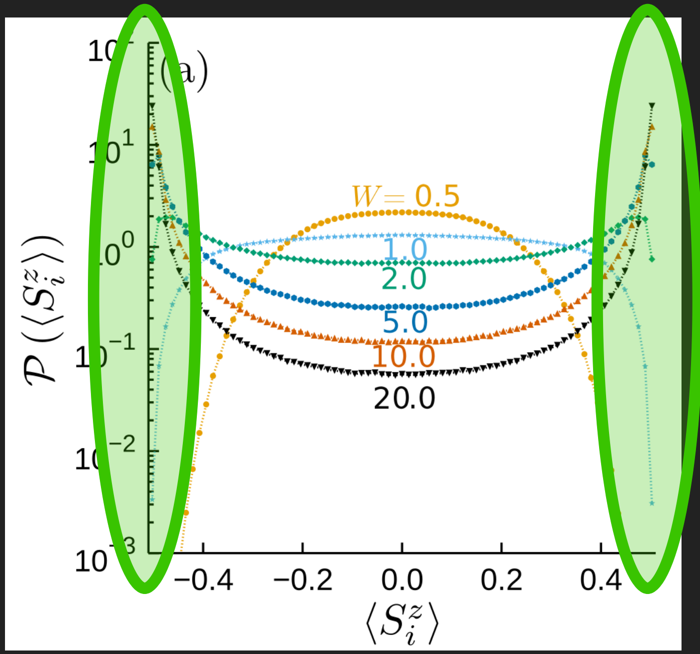
MAGNETIZATION DISTRIBUTIONS

Anderson chain / XX chain



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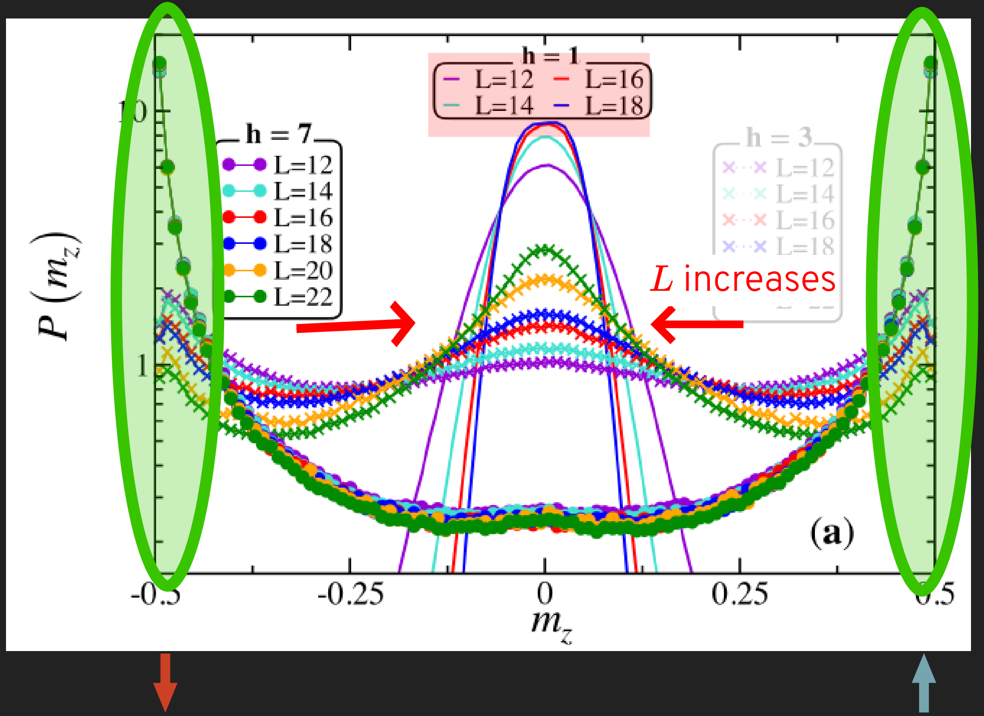
Heisenberg chain



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MAGNETIZATION DISTRIBUTIONS

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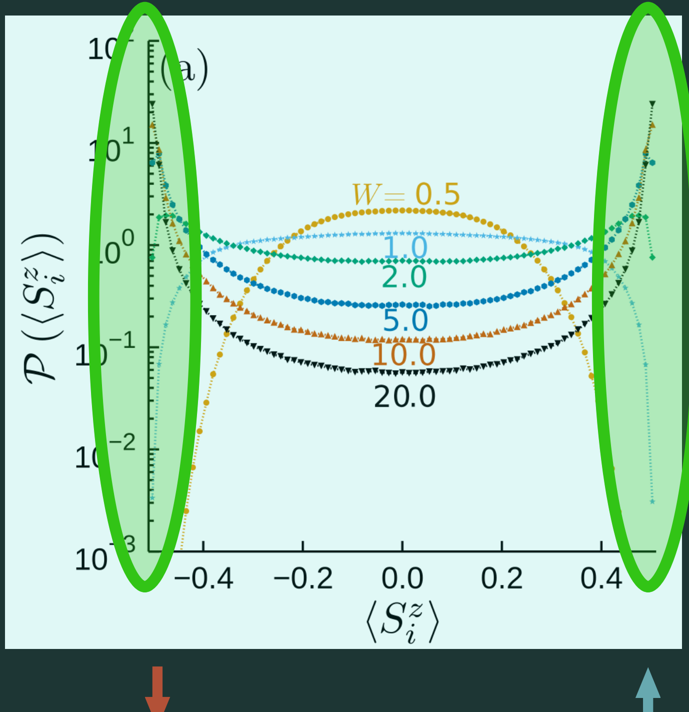

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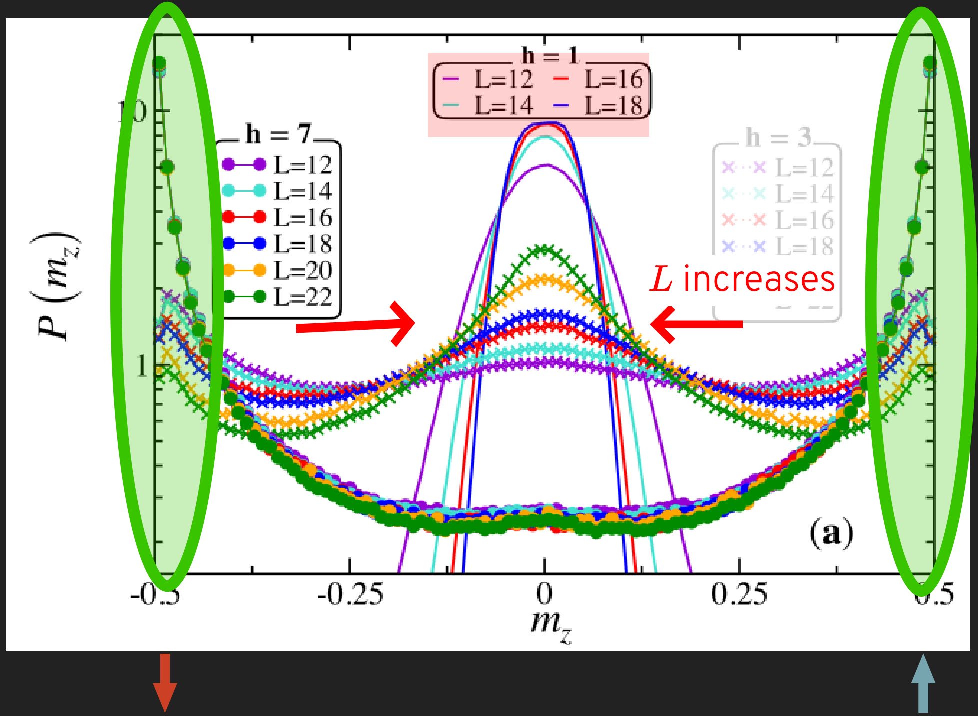
Heisenberg chain



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MINIMAL DEVIATIONS IN THE XX CHAIN - TOY MODEL

ED ON ONE SAMPLE - XX (ANDERSON) CHAIN

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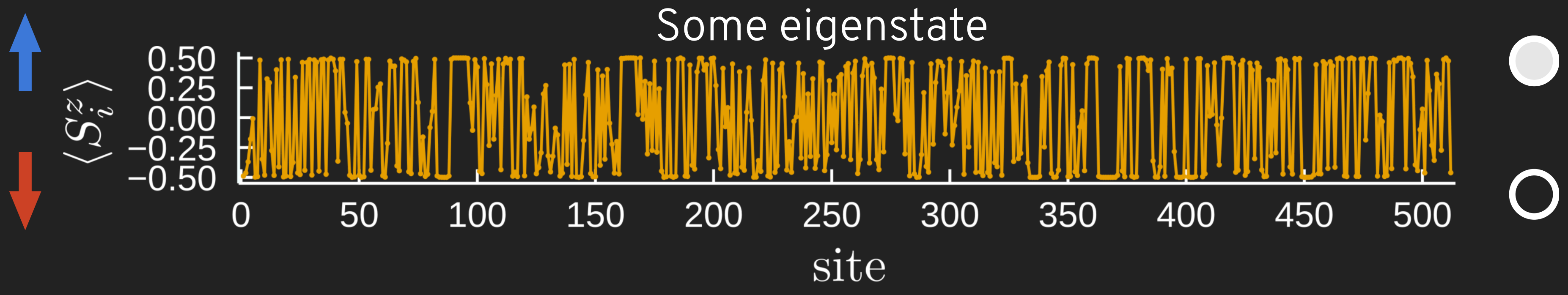

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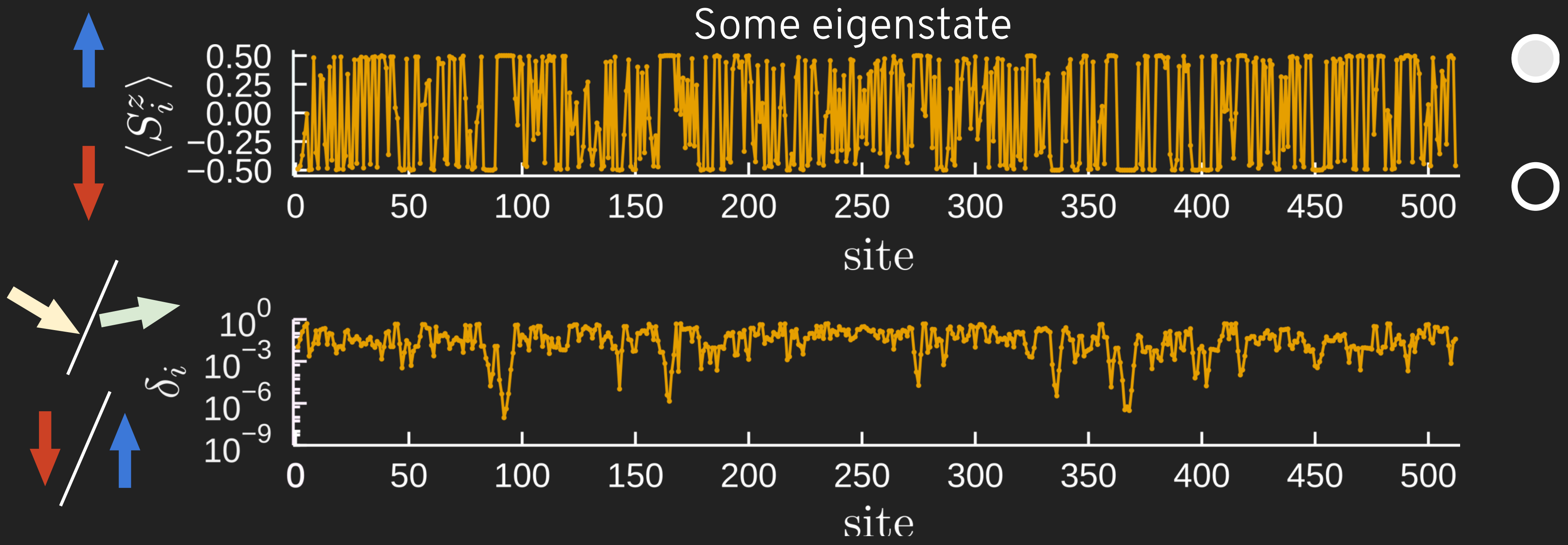
$$\delta_i = 1/2 - |\langle S_i^z \rangle|$$

ED ON ONE SAMPLE - XX (ANDERSON) CHAIN



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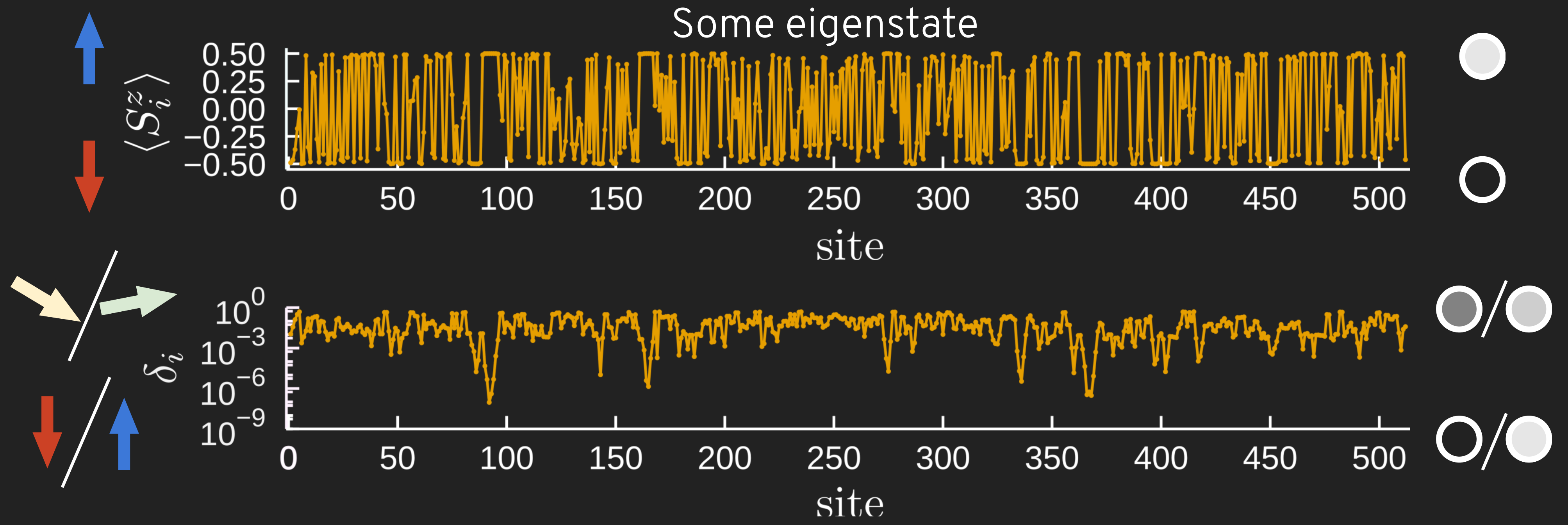

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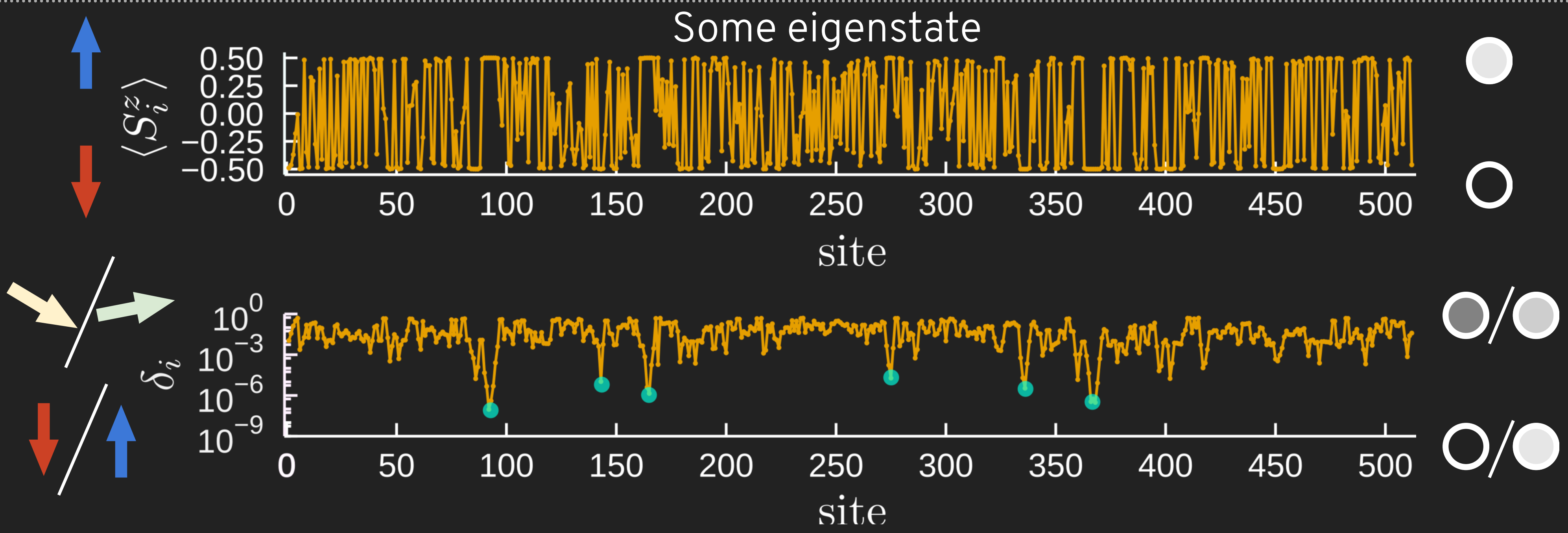

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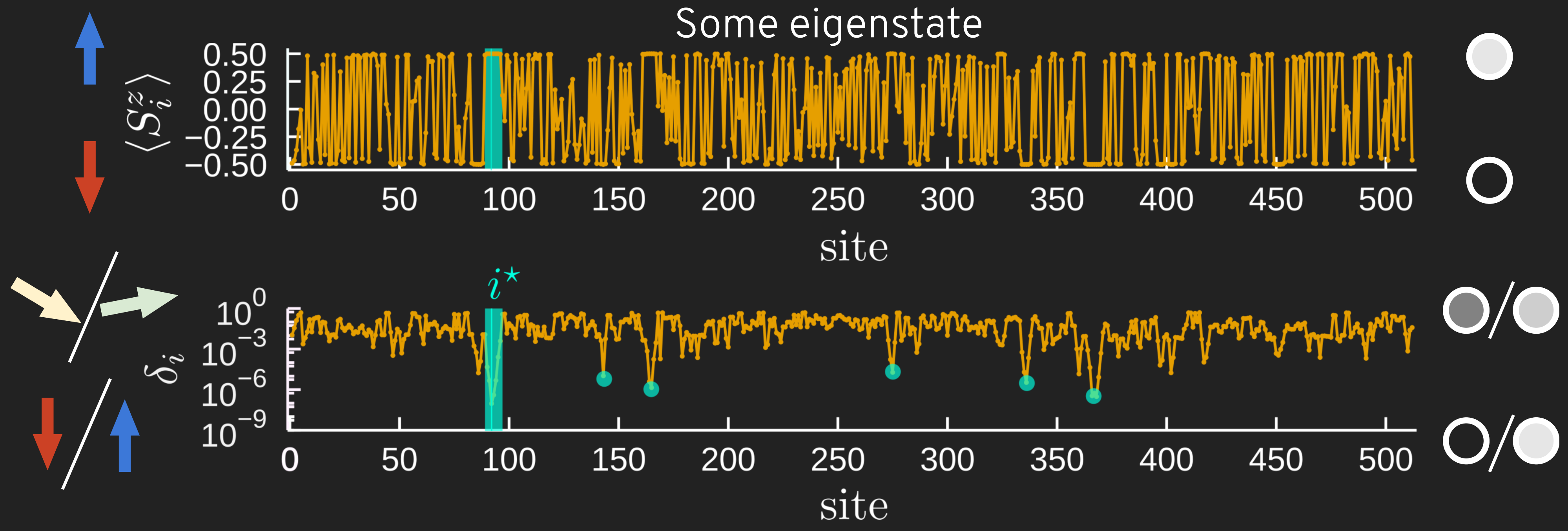
SPIN FREEZING



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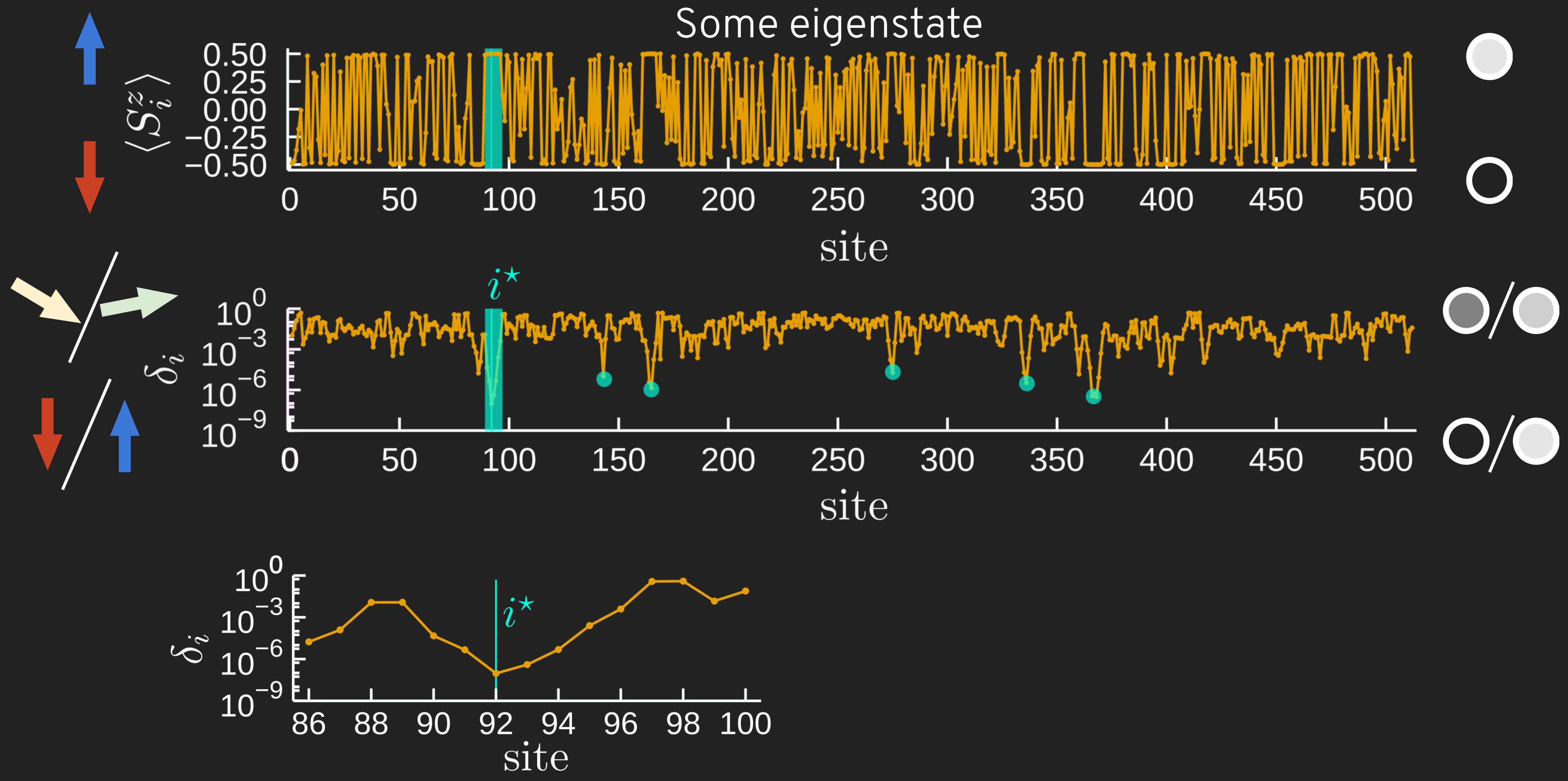

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J. C., N. Laflorencie, arXiv:2305.10574

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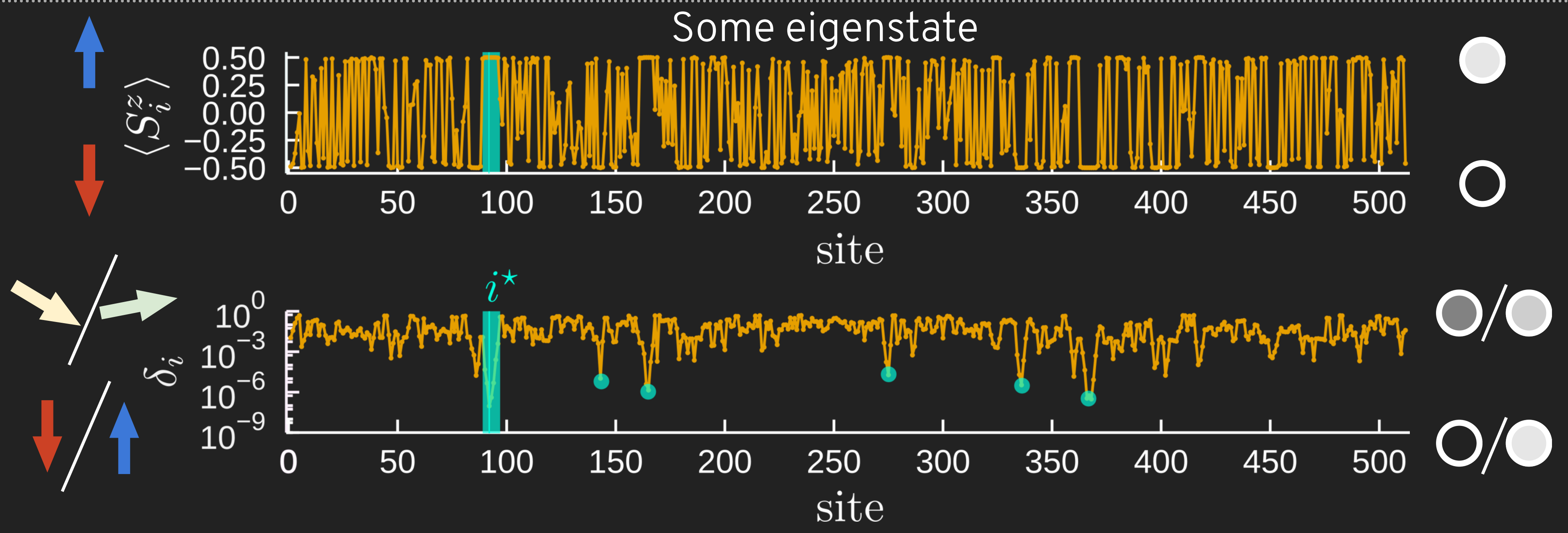

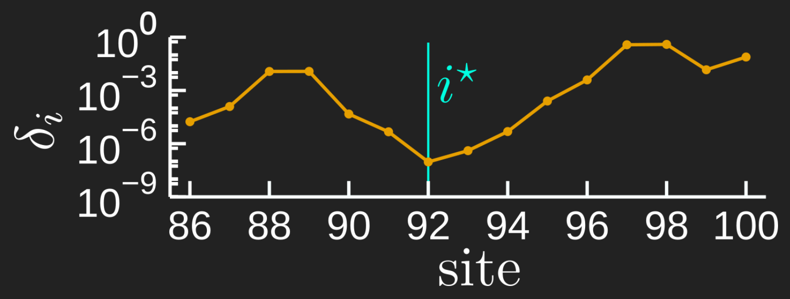

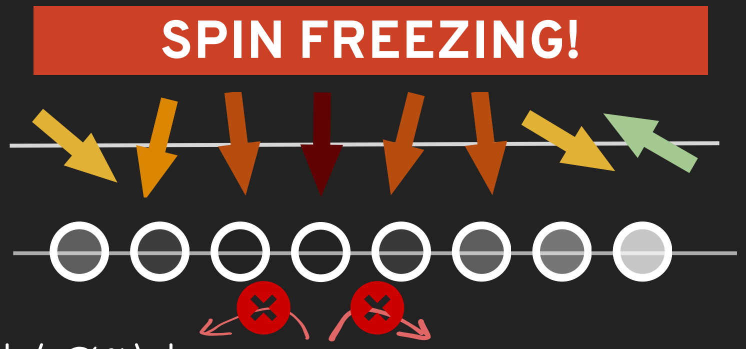

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TOY MODEL : ANALYTICAL DESCRIPTION

Dupont, Macé, Laflorencie, PRB **100**, 134201, (2019)
Laflorencie, Lemarié, Macé, PRR **2**, 042033(R), (2020)
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TOY MODEL : ANALYTICAL DESCRIPTION

$$|\phi_m(i)|^2 \propto \exp\left(-\frac{|i-i_0^m|}{\xi}\right)$$

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TOY MODEL : ANALYTICAL DESCRIPTION

$$|\phi_m(i)|^2 \propto \exp\left(-\frac{|i-i_0^m|}{\xi}\right)$$

$$\Rightarrow \langle n_i \rangle = \langle S_i^z \rangle + 1/2 = \sum_{m \in \mathrm{occ}} |\phi_m(i)|^2$$

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MINIMAL DEVIATION?

$$\delta_i = 1/2 - |\langle n_i \rangle - 1/2|$$

$$|\phi_m(i)|^2 \propto \exp\left(-\frac{|i-i_0^m|}{\xi}\right)$$

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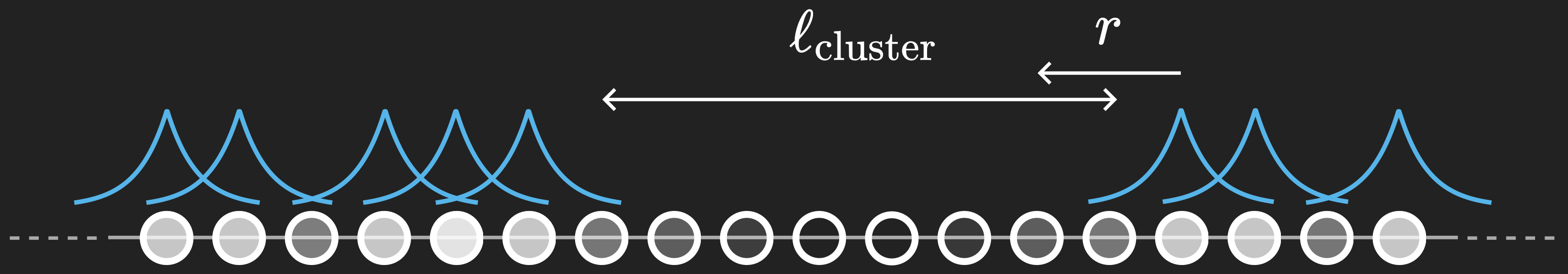

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MINIMAL DEVIATION?

$$\delta_i = 1/2 - |\langle n_i \rangle - 1/2|$$

$$\bigcirc : \quad \delta_i = \langle n_i \rangle \approx e^{-\frac{r}{\xi}} + \ldots$$

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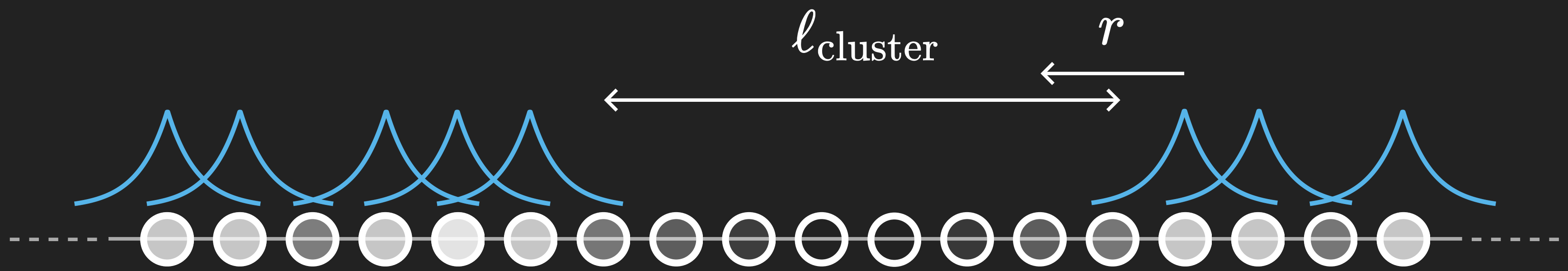

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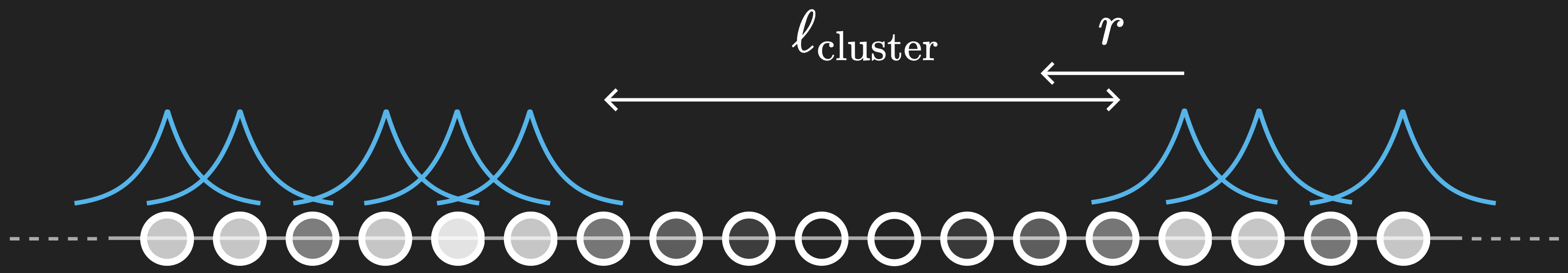
$$|\phi_m(i)|^2 \propto \exp\left(-\frac{|i-i_0^m|}{\xi}\right) \quad \Rightarrow \quad \delta_{\min} \approx e^{-\frac{\ell_{\text{cluster}}}{2\xi}}$$



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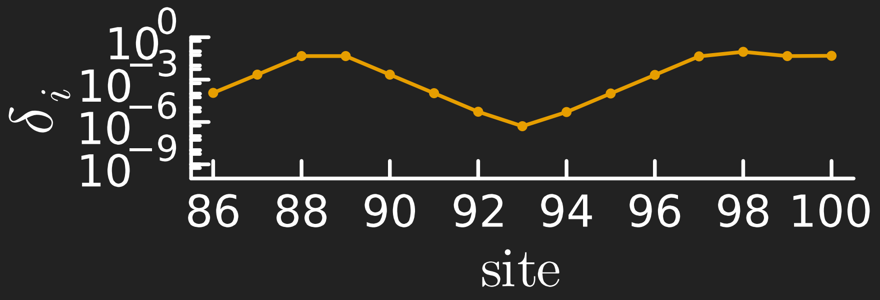

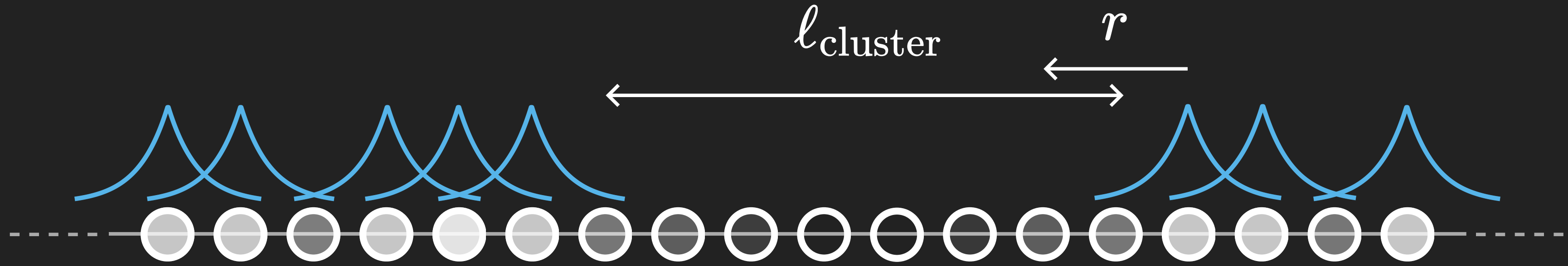

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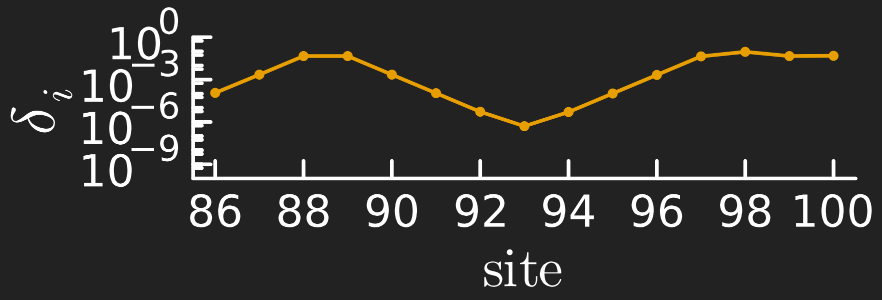
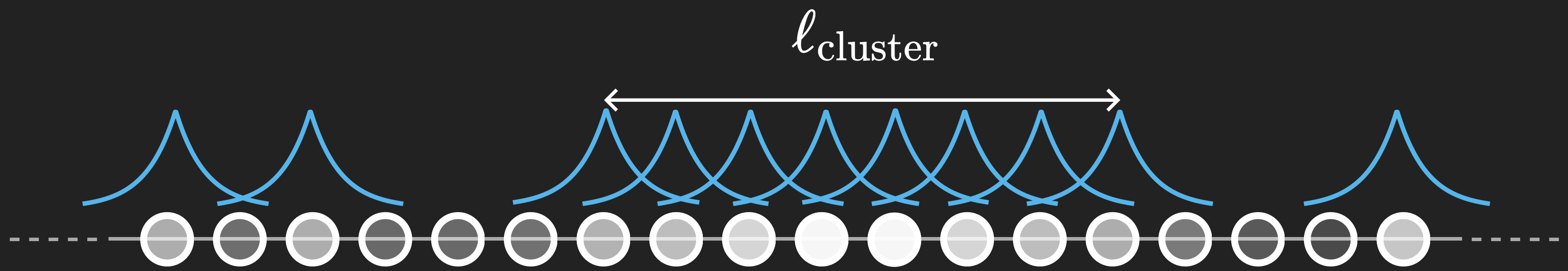
$$\Rightarrow \quad \delta_{\min} \approx e^{-\frac{\ell_{\text{cluster}}}{2\xi}}$$





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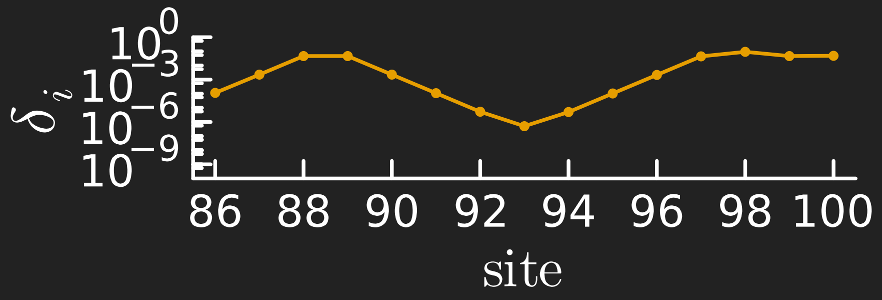
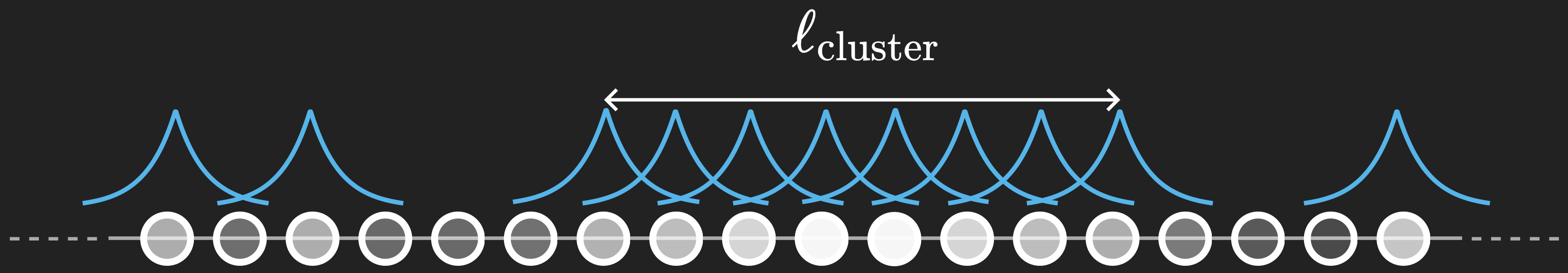
MINIMAL DEVIATION?

$$\delta_i = 1/2 - |\langle n_i \rangle - 1/2|$$

$$\bigcirc : \quad \delta_i = \langle n_i \rangle \approx e^{-\frac{r}{\xi}} + \dots$$

$$\Rightarrow \quad \delta_{\min}^{\text{typ}} = e^{\overline{\ln \delta_{\min}}} \approx e^{-\frac{\overline{\ell_{\text{cluster}}}}{2\xi}}$$





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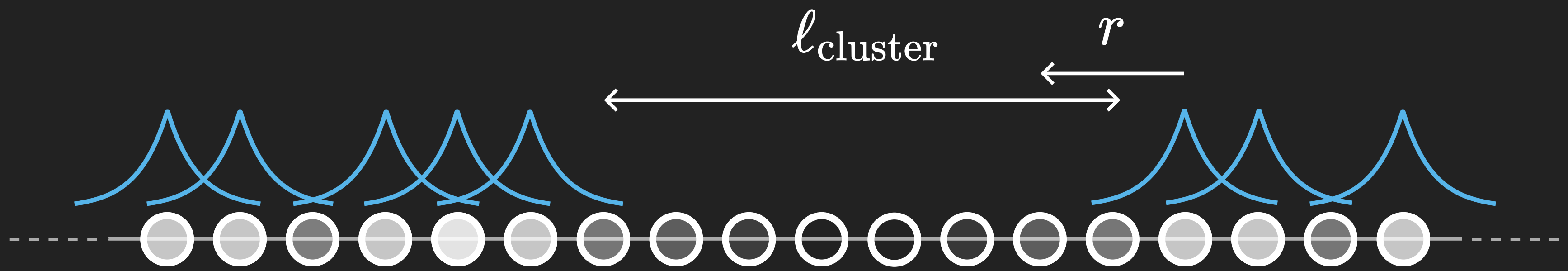
MINIMAL DEVIATION?

$$\delta_{\min}^{\text{typ}} \approx e^{-\frac{\overline{\ell_{\text{cluster}}}}{2\xi}}$$



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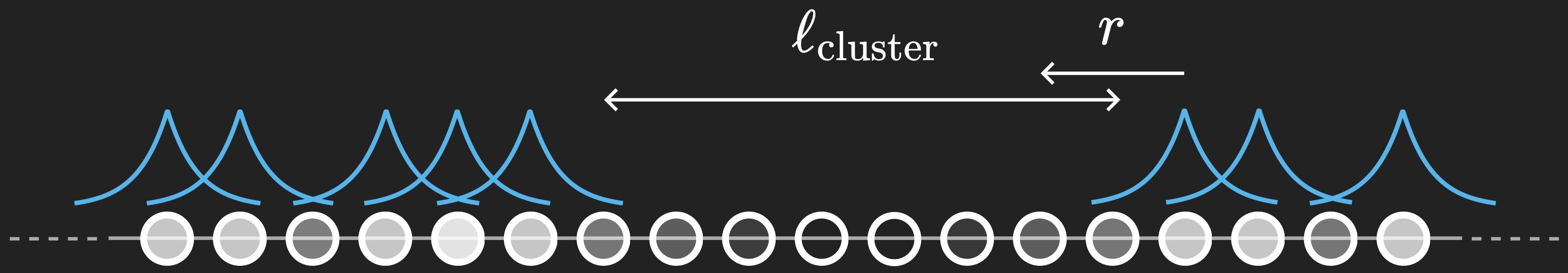
MINIMAL DEVIATION?

$$\delta_{\min}^{\mathrm{typ}} \approx e^{-\frac{\overline{\ell_{\mathrm{cluster}}}}{2\xi}}$$

$$\overline{\ell_{\mathrm{cluster}}} \approx \frac{\ln L}{\ln 2}$$



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JC, N. Laflorencie, arXiv:2305.10574

MINIMAL DEVIATION?

$$\delta_{\text{min}}^{\text{typ}} \approx e^{-\frac{\overline{\ell_{\text{cluster}}}}{2\xi}}$$

$$\overline{\ell_{\text{cluster}}} \approx \frac{\ln L}{\ln 2}$$

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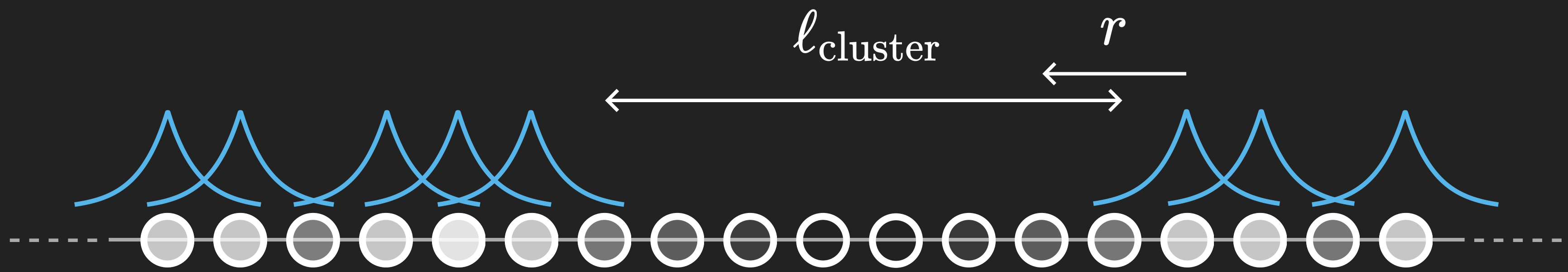

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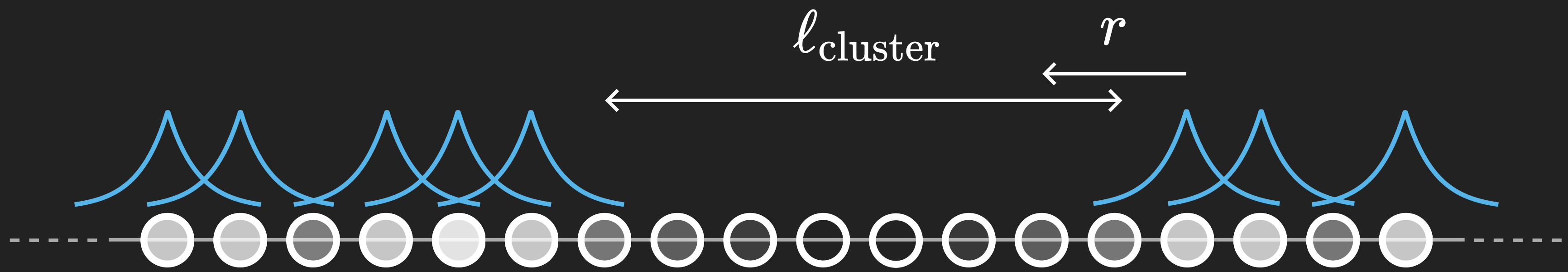

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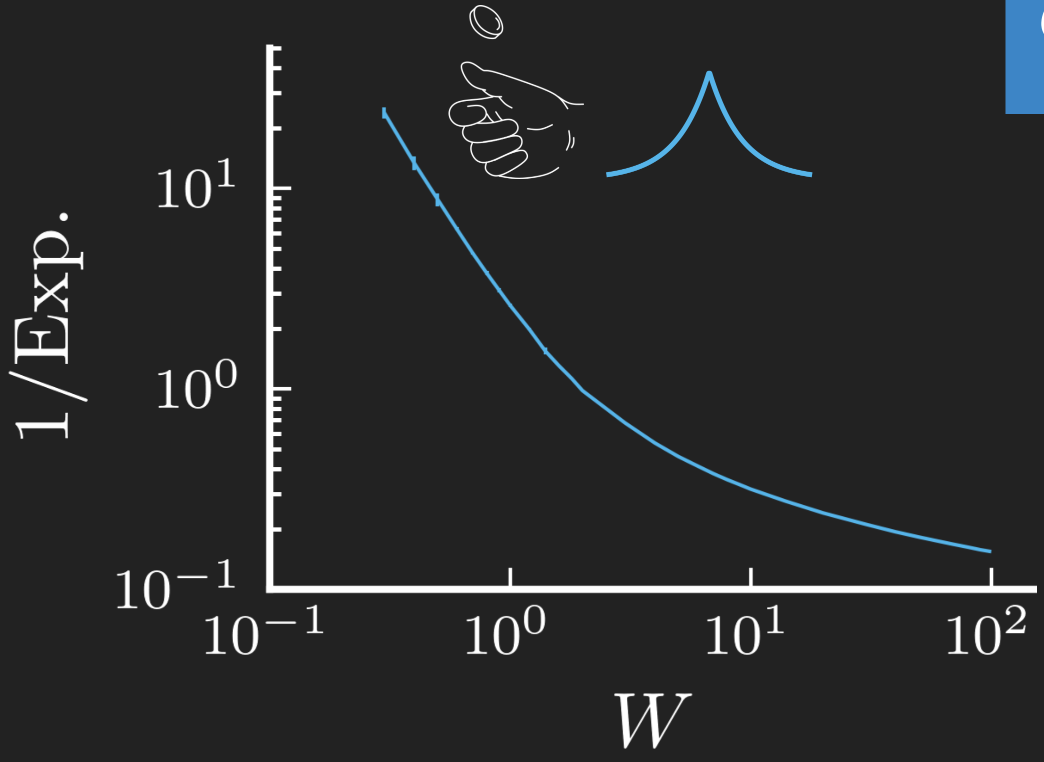

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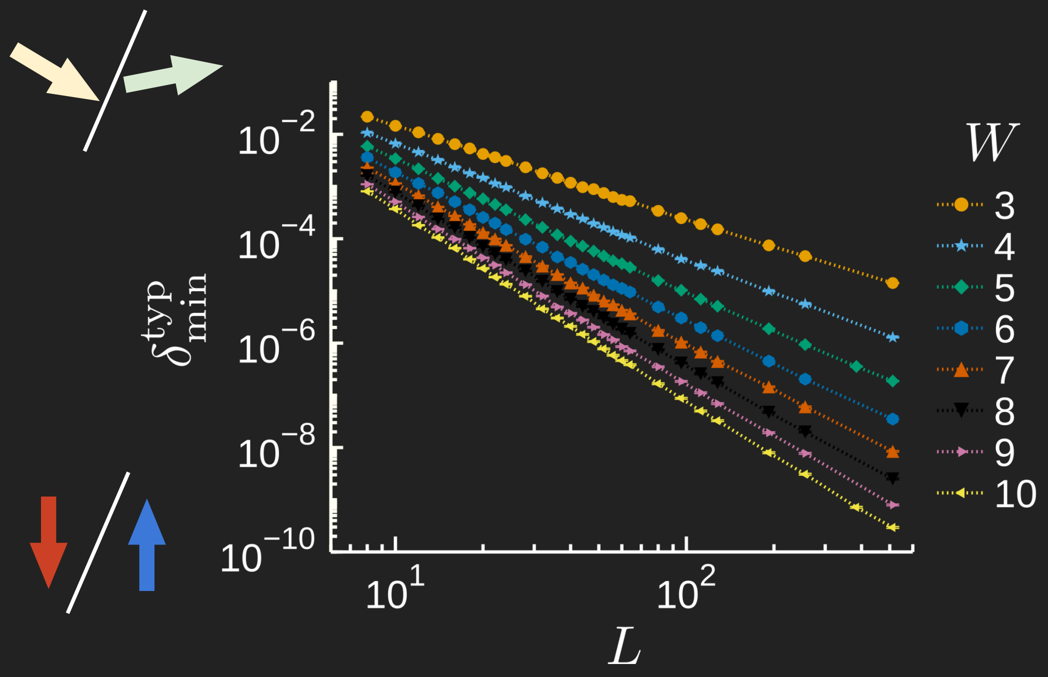

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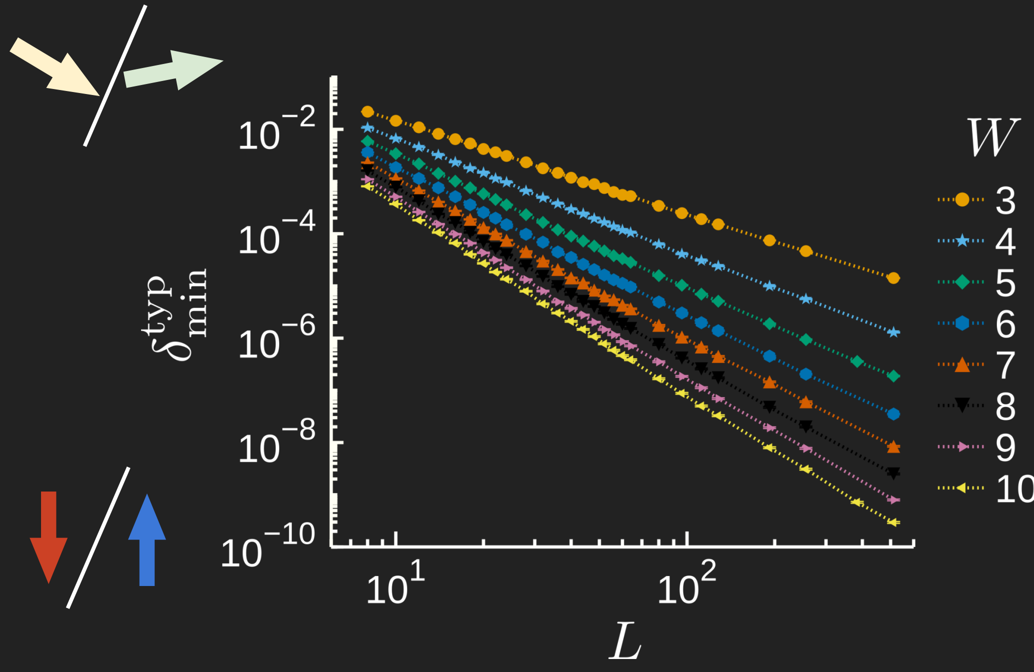

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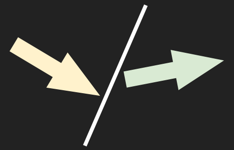

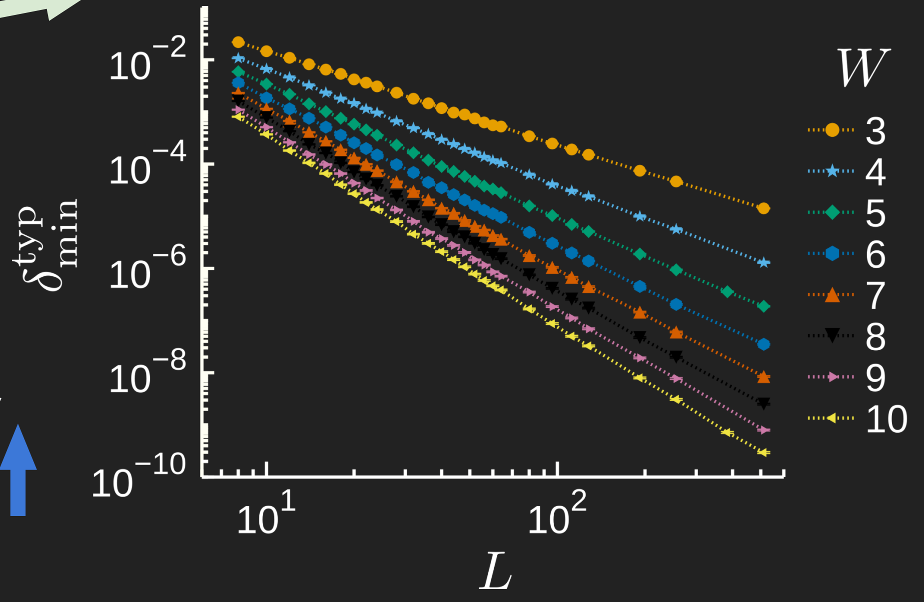

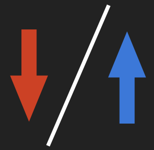

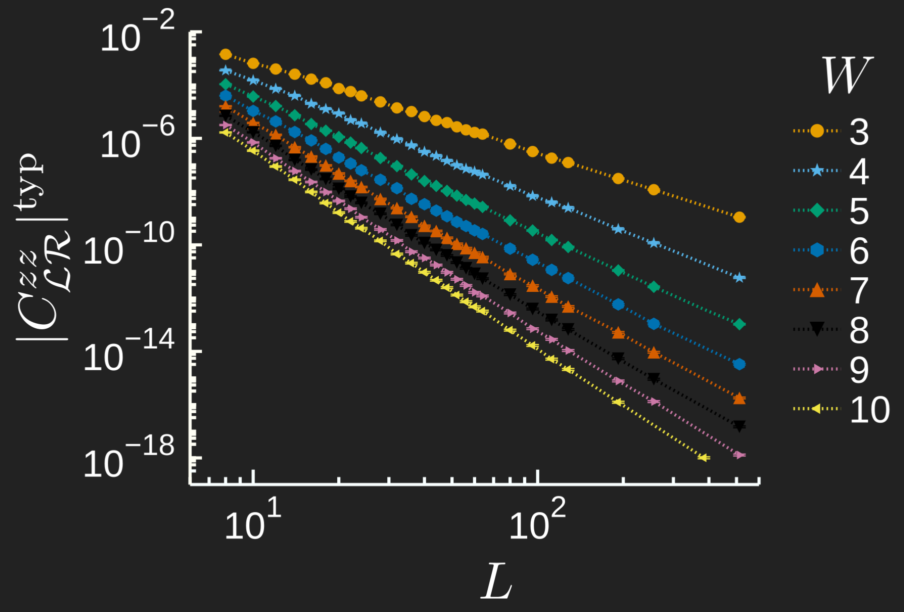

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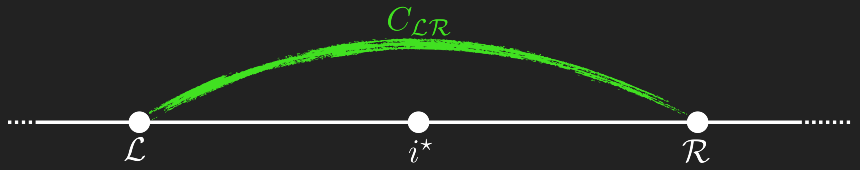

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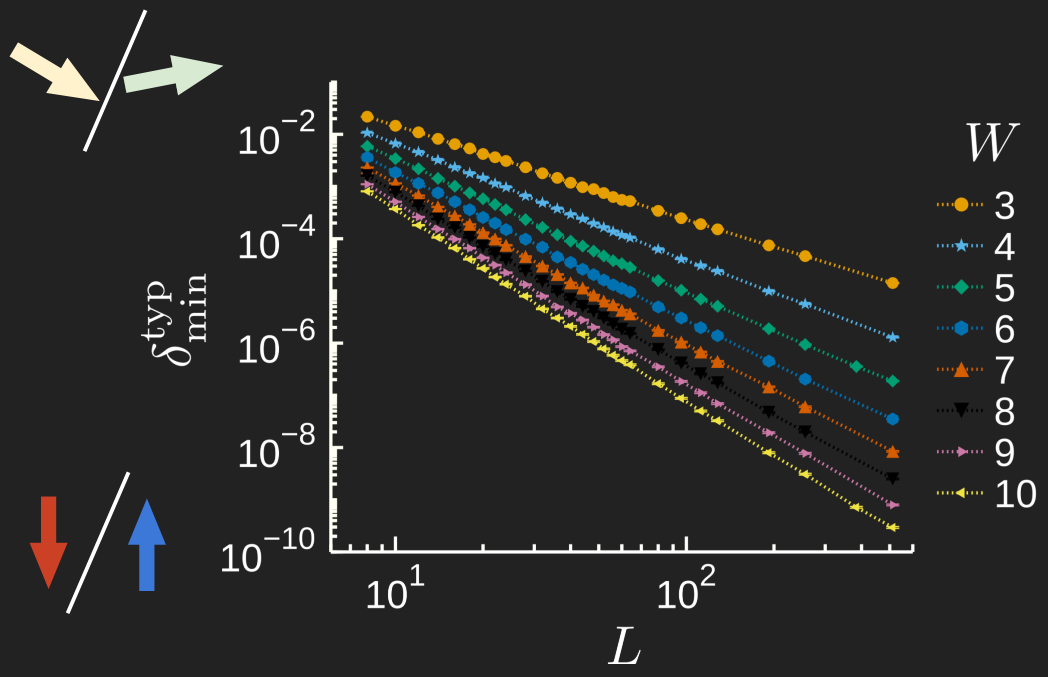

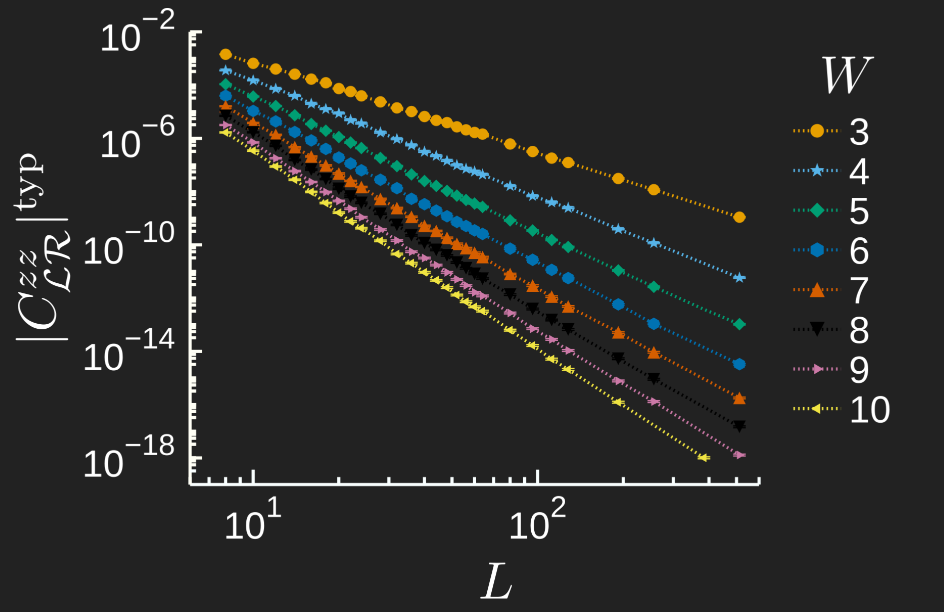

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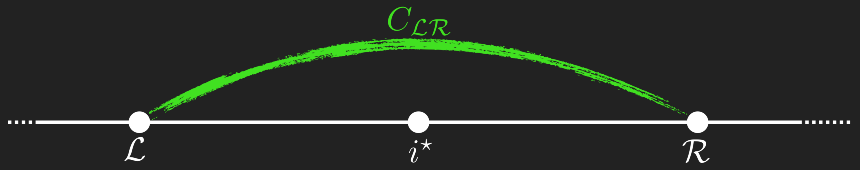

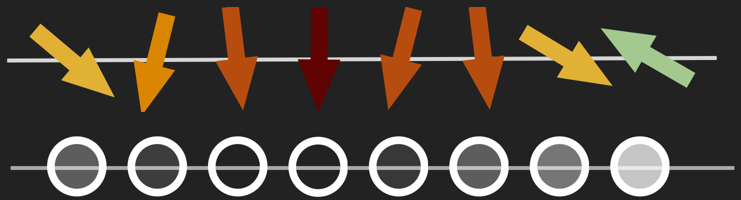

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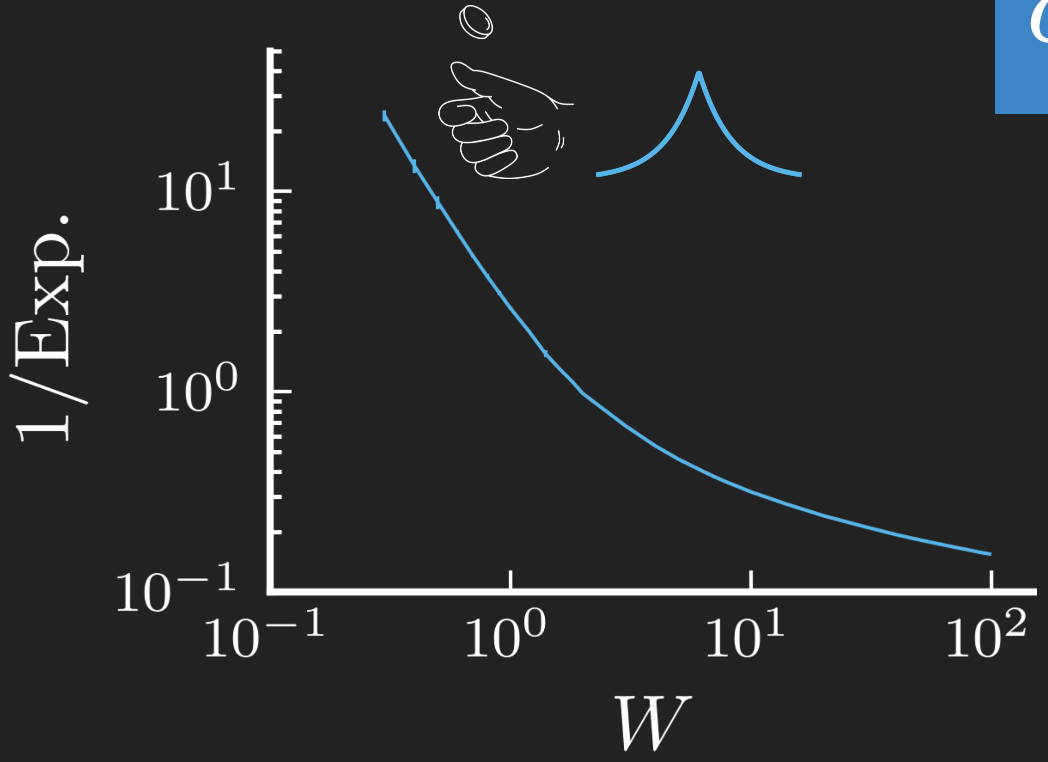

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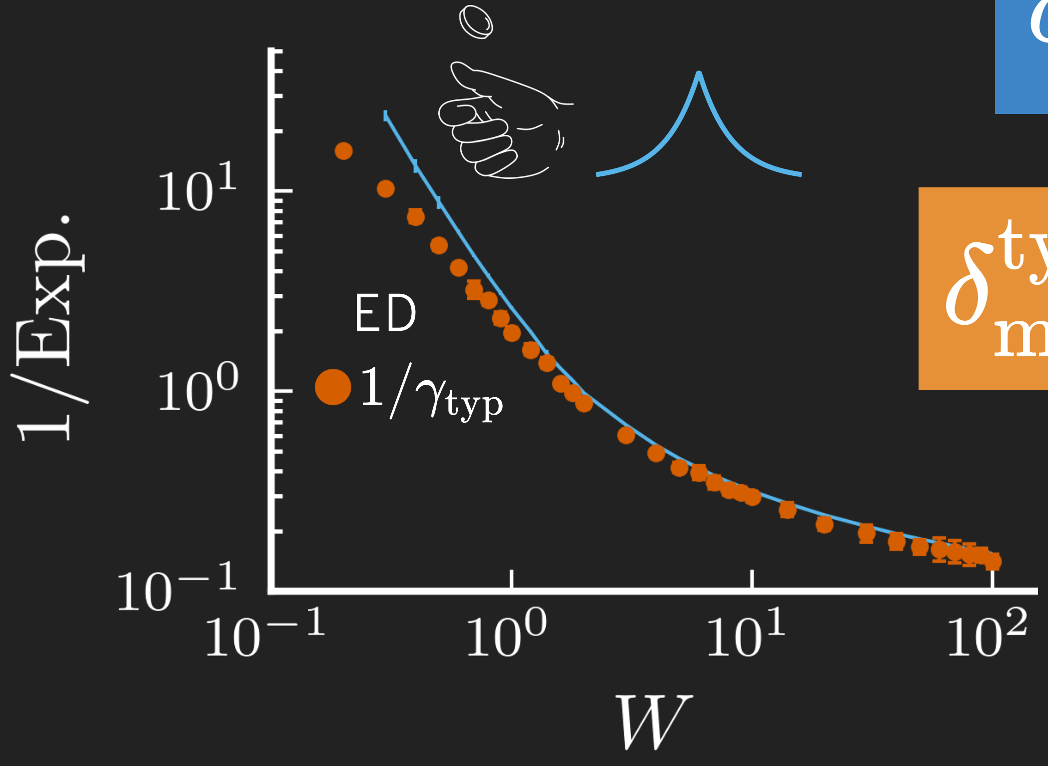

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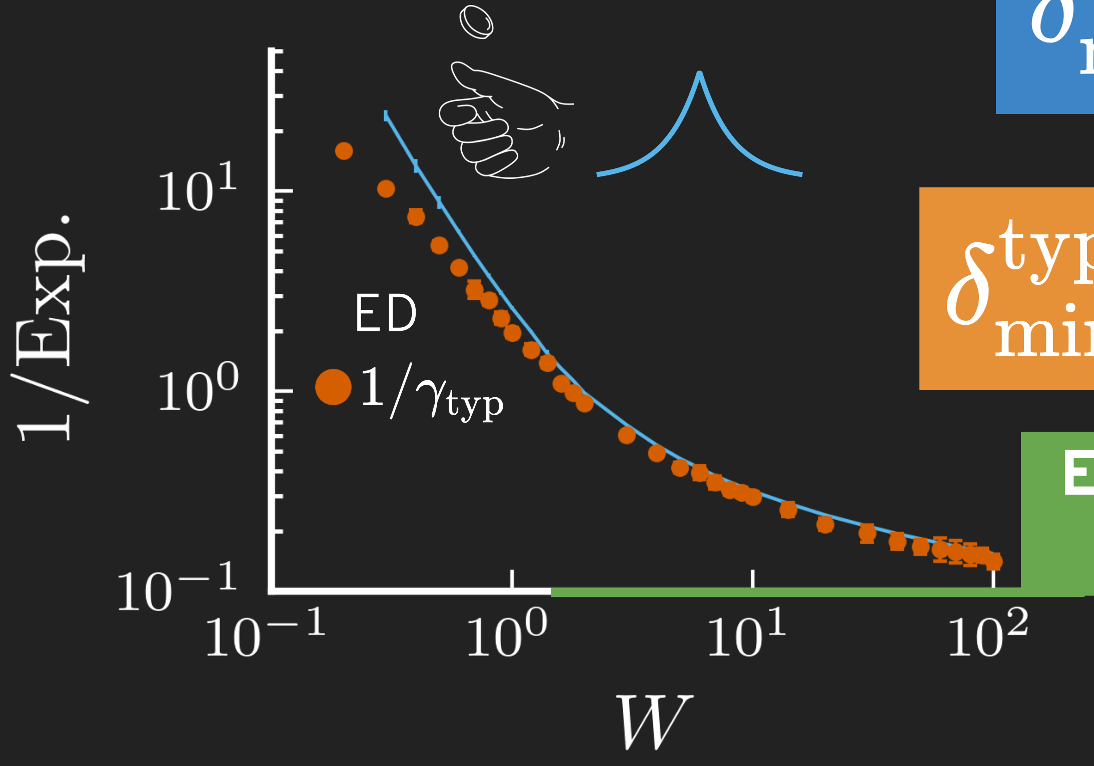

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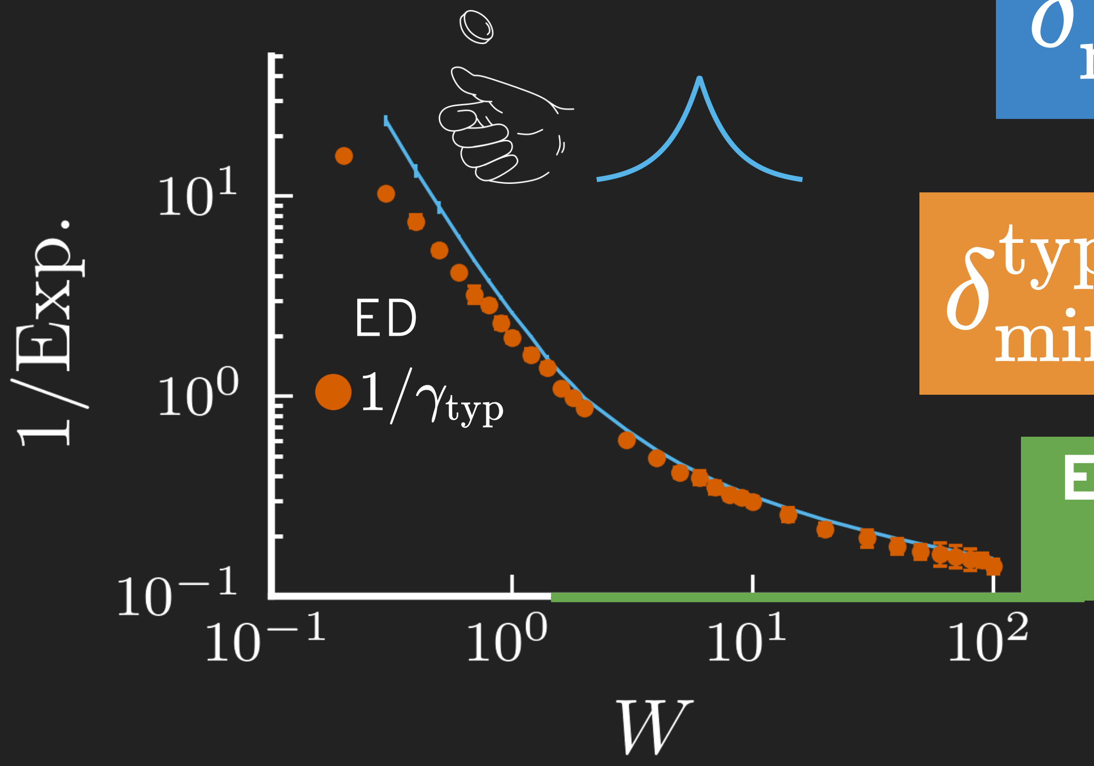

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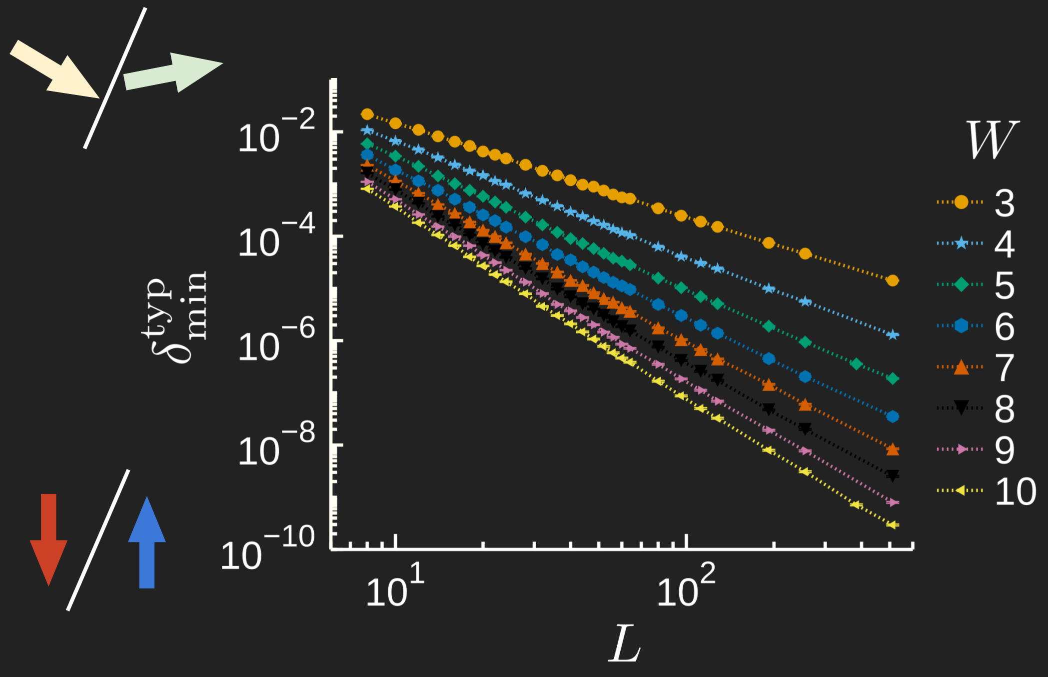
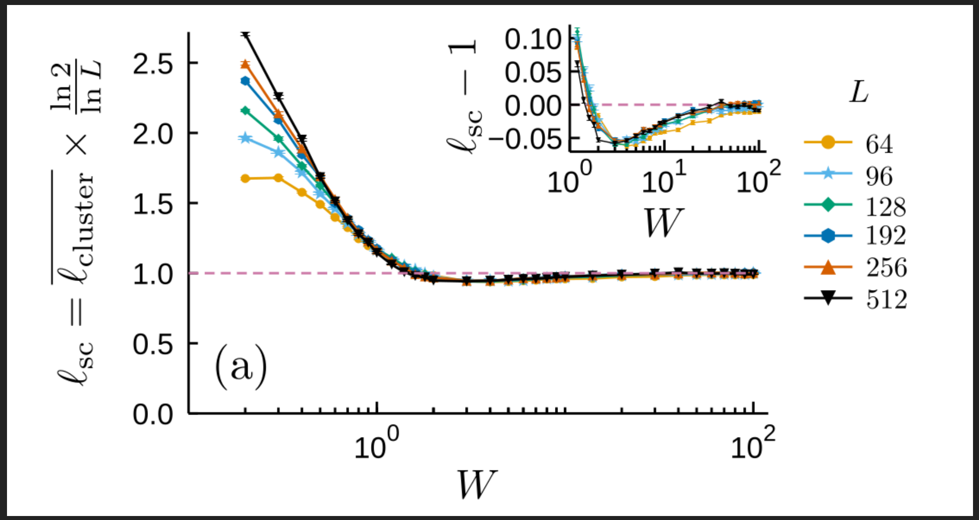
SCALING





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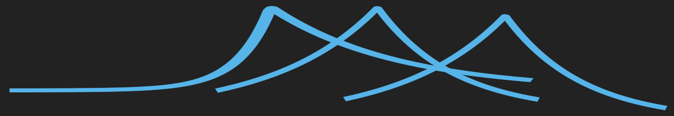

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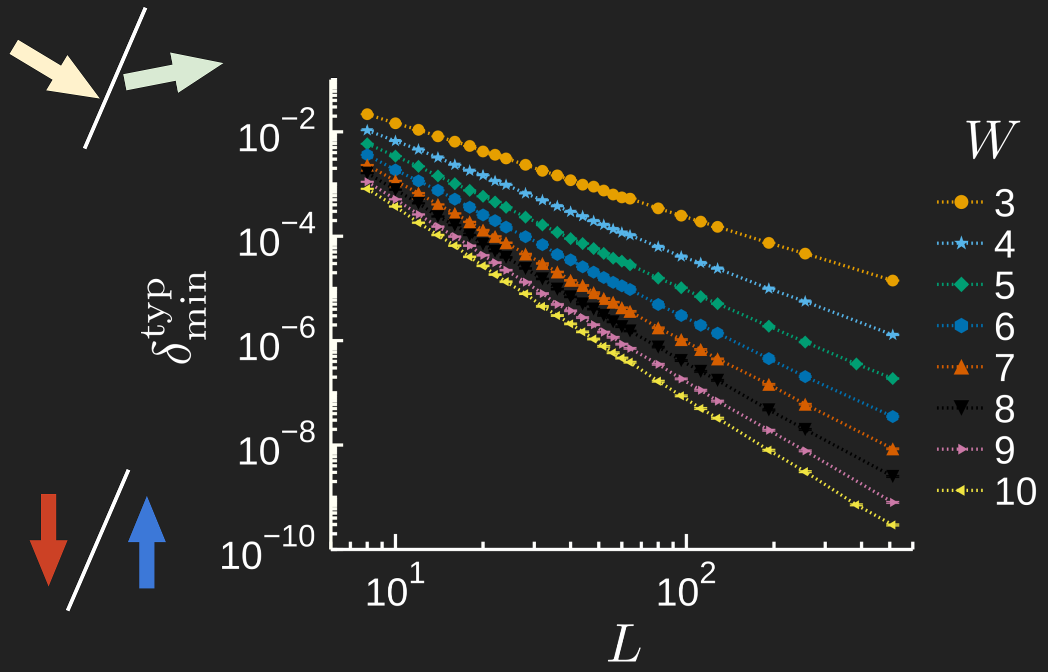

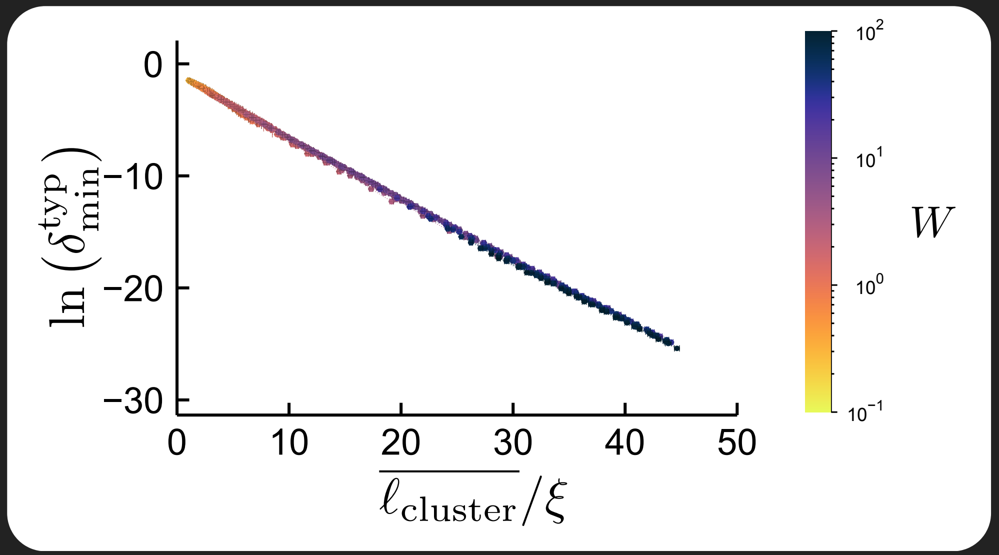

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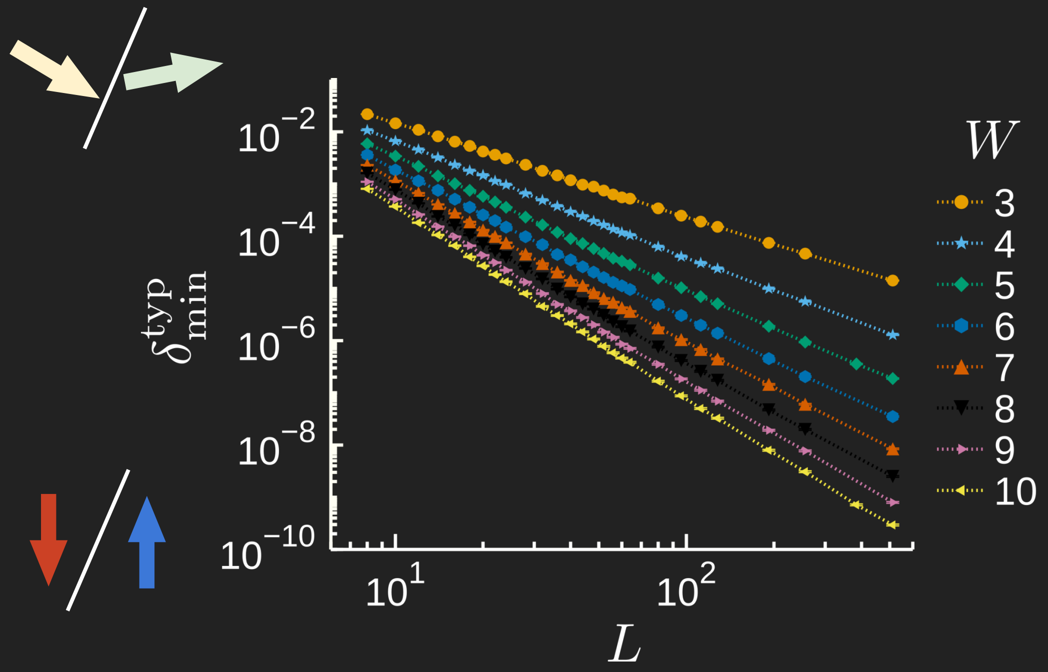

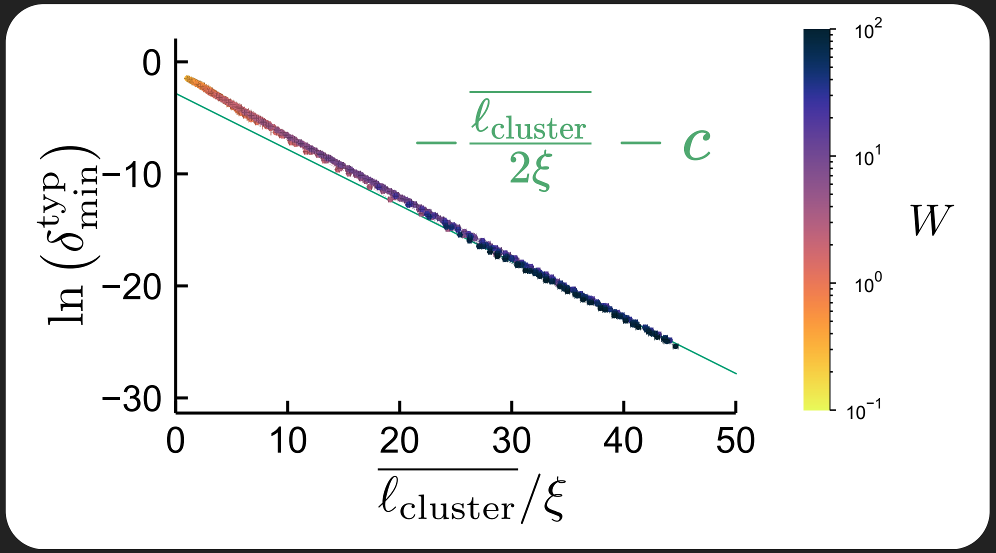

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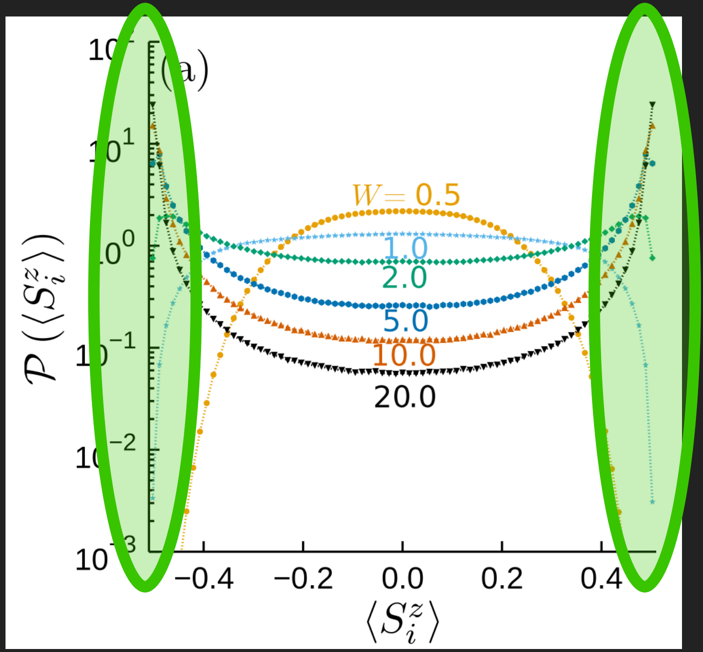
QUANTITATIVE DESCRIPTION : EXTREME VALUE STATISTICS

TAILS AND EXTREMES

Tails



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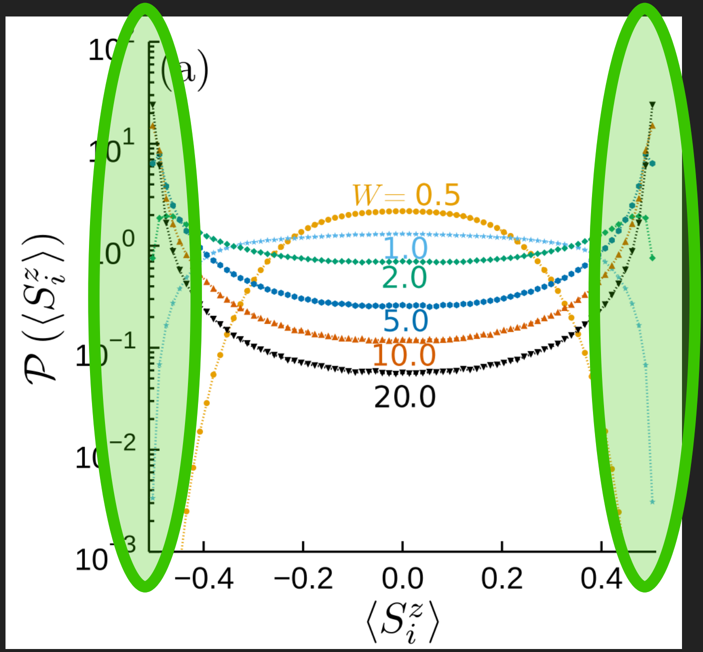
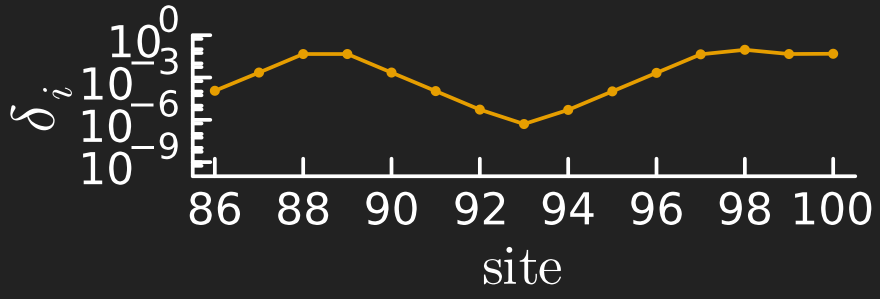


So far :

Extreme value



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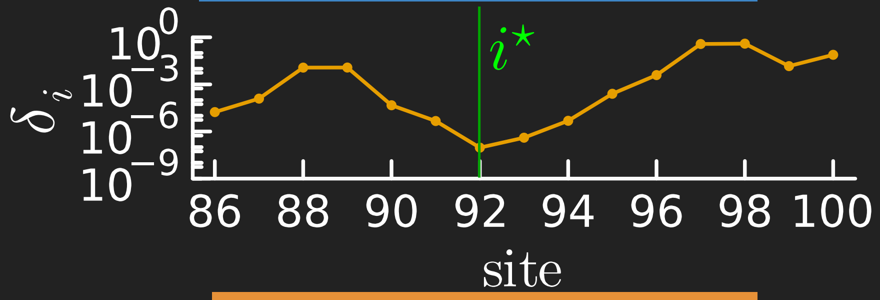

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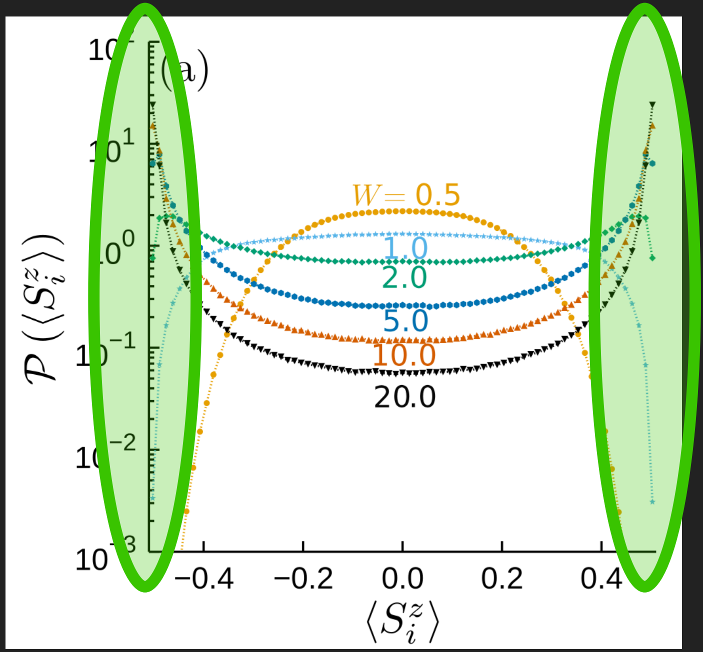
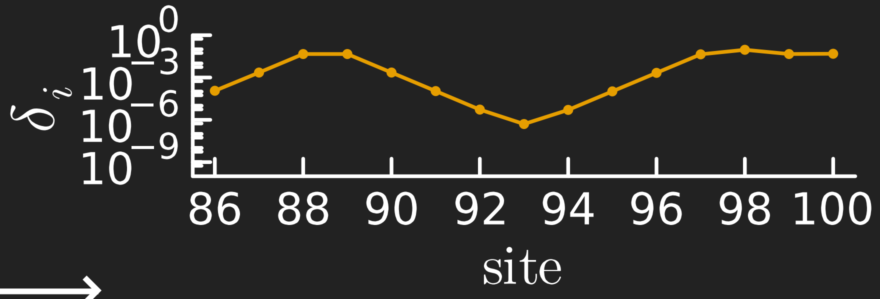


Extreme value theory

Extreme value



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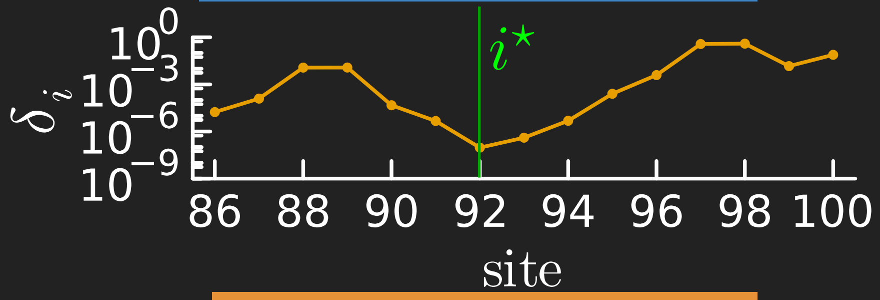

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TAILS AND EXTREMES

Tails

Extreme value

← Extreme value theory →

$$\{X_i\}_{i=1,2,...,L} \sim p(x) \longrightarrow$$

$$Y = \max(X_i)$$
$$Z = \text{rescaled}(Y)$$

E. J. Gumbel, *Statistics of Extremes*, Dover, (1958, 2004)
S. N. Majumdar, A. Pal, G. Schehr, Physics Reports, **840**, 1 (2020)

TAILS AND EXTREMES

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Power-law tail

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Fréchet law

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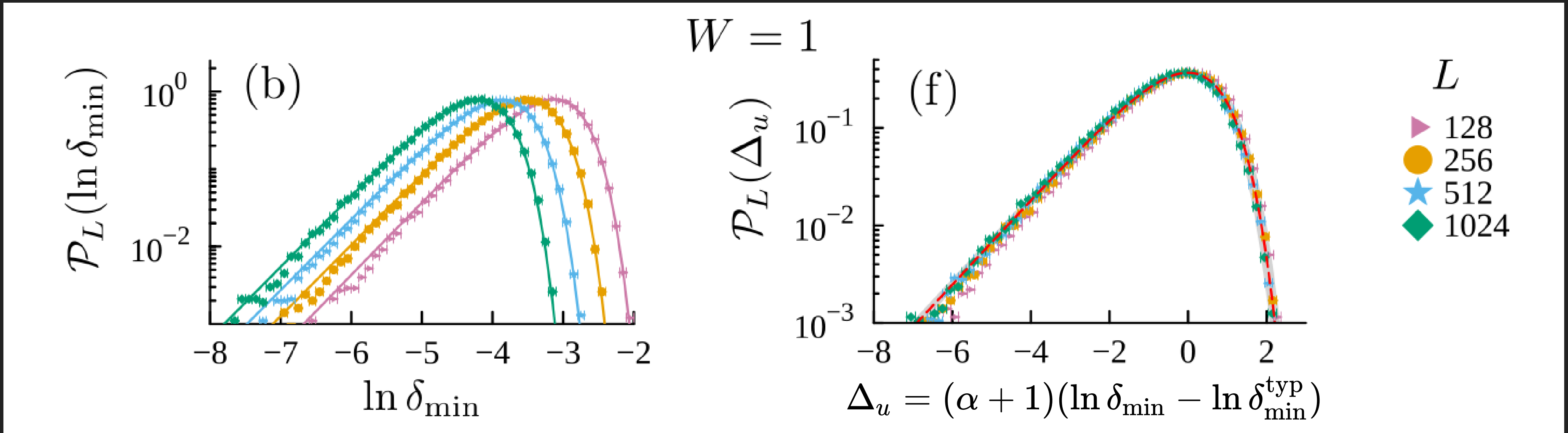

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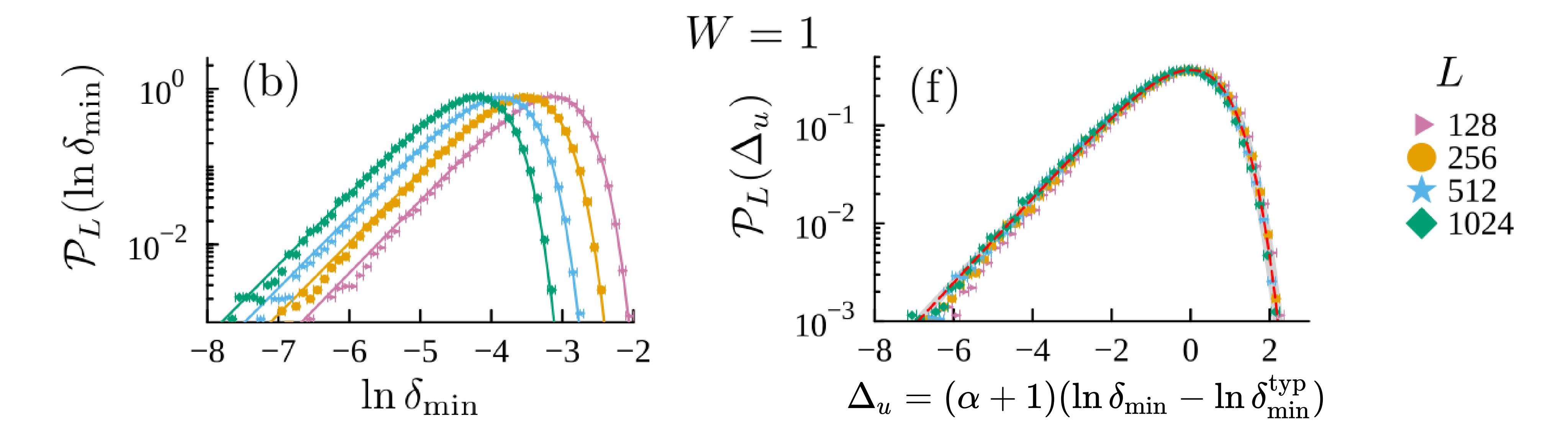

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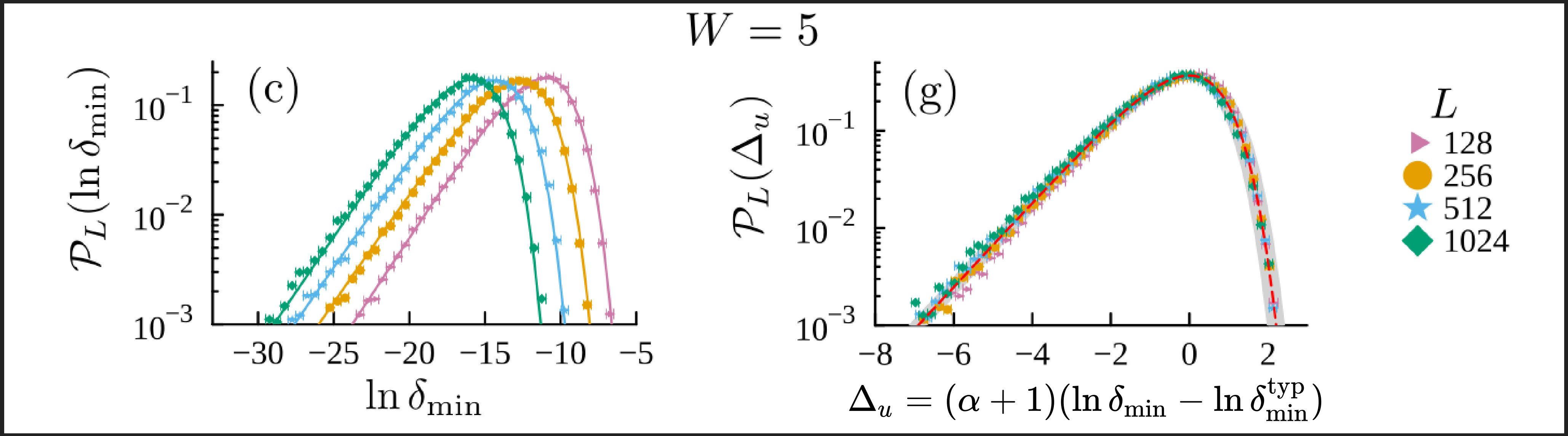

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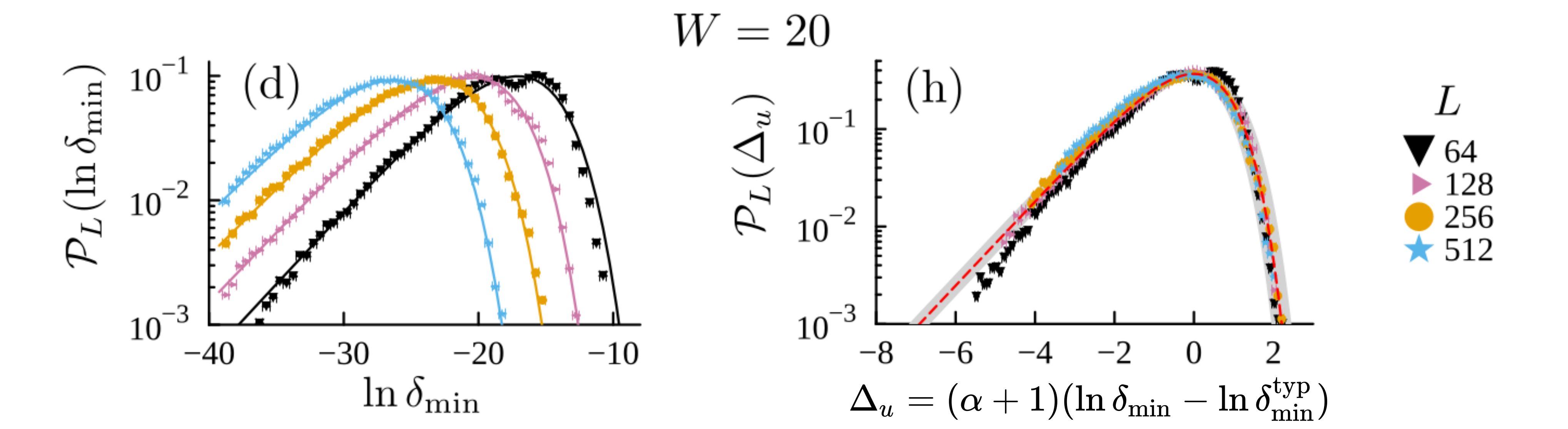

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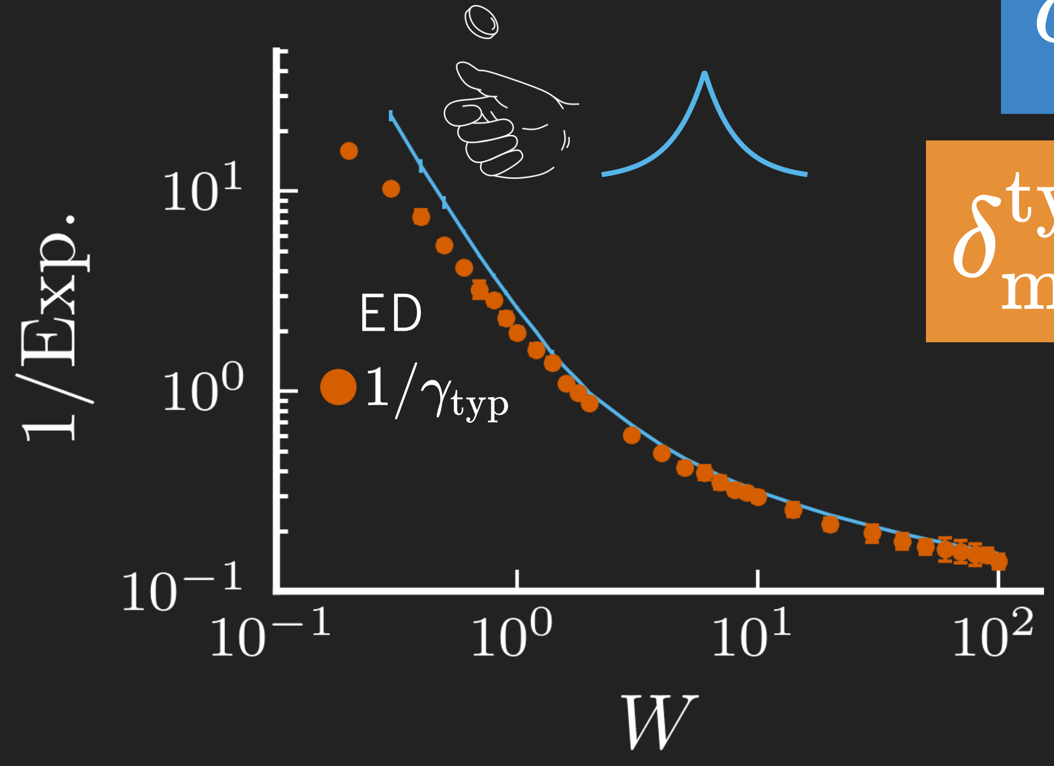

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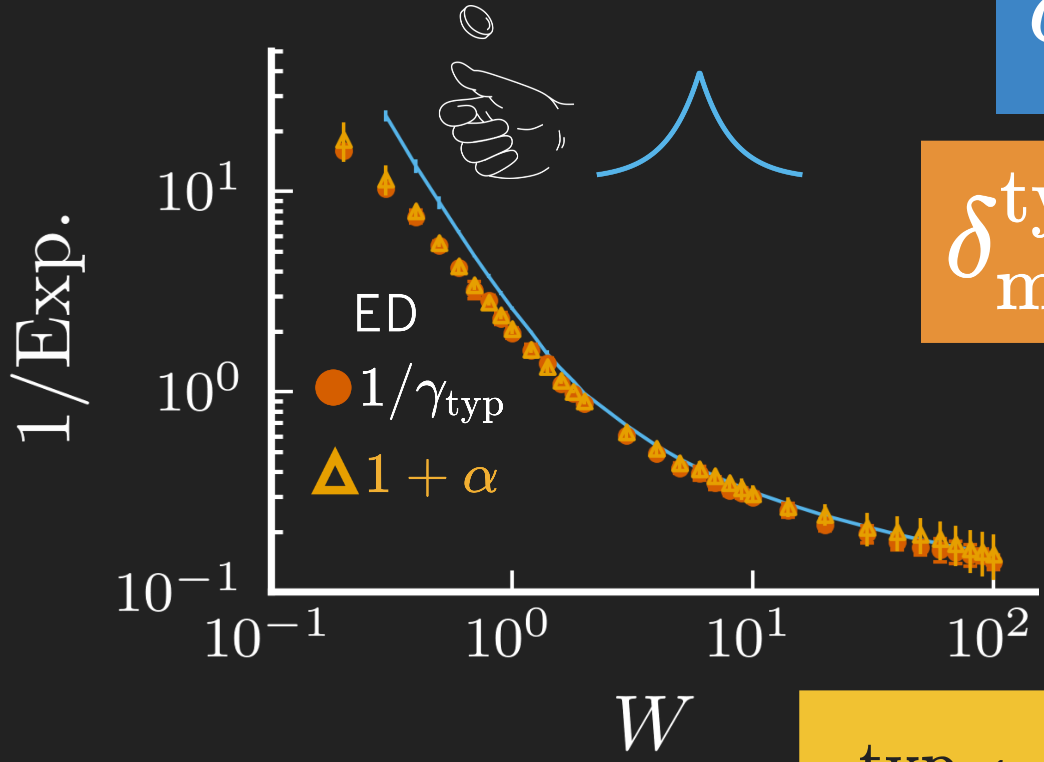

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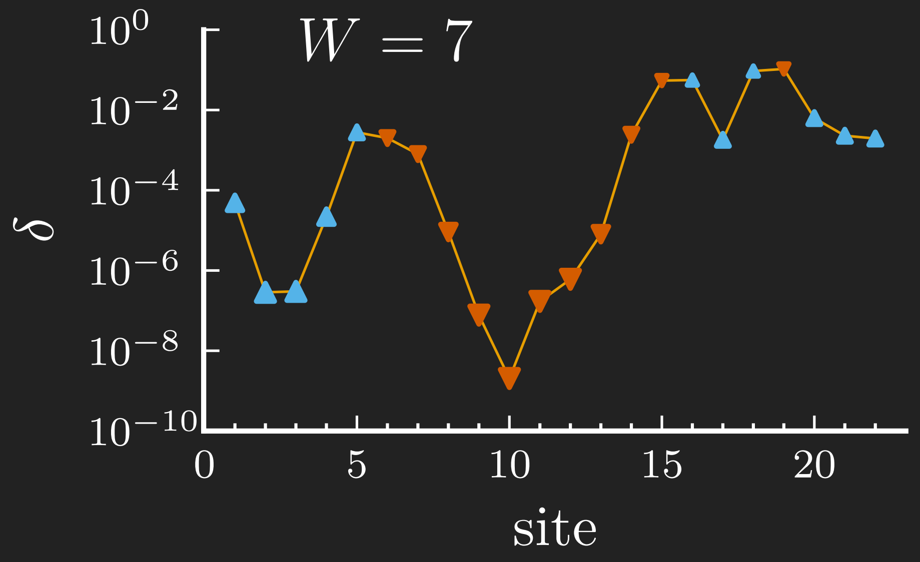
CONSEQUENCES : INTERACTING SYSTEM

EFFECT OF INTERACTIONS?

$$\mathcal{H} = \mathcal{H}_{XX} + \Delta \sum_i S_i^z S_{i+1}^z$$

"Stability" of the cluster with respect to the interactions?



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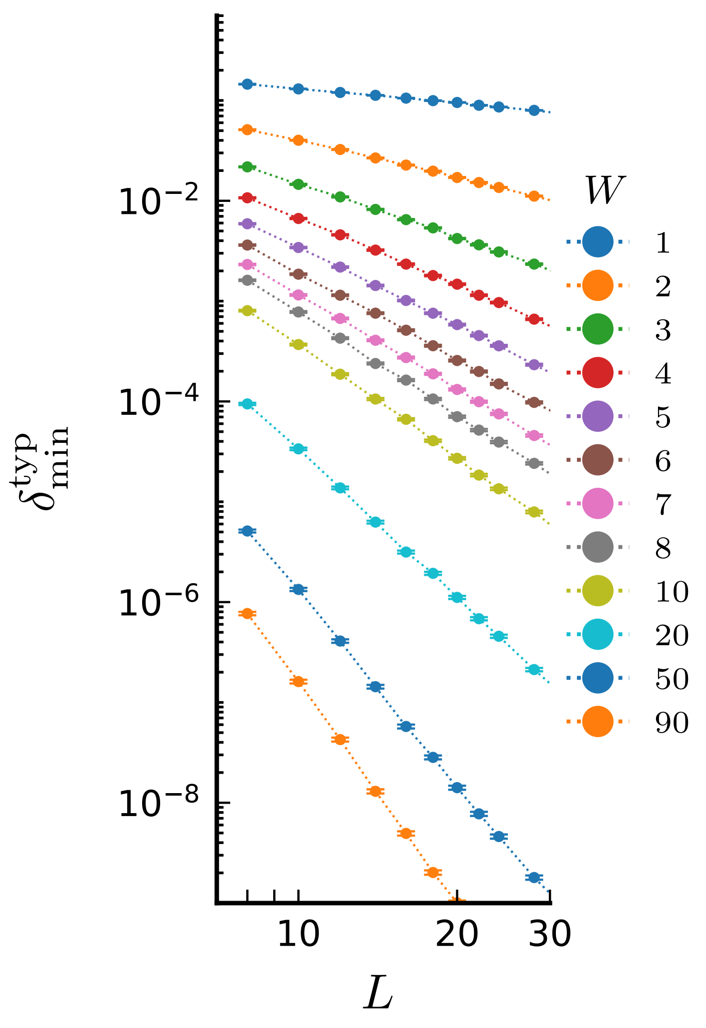

disorder increases

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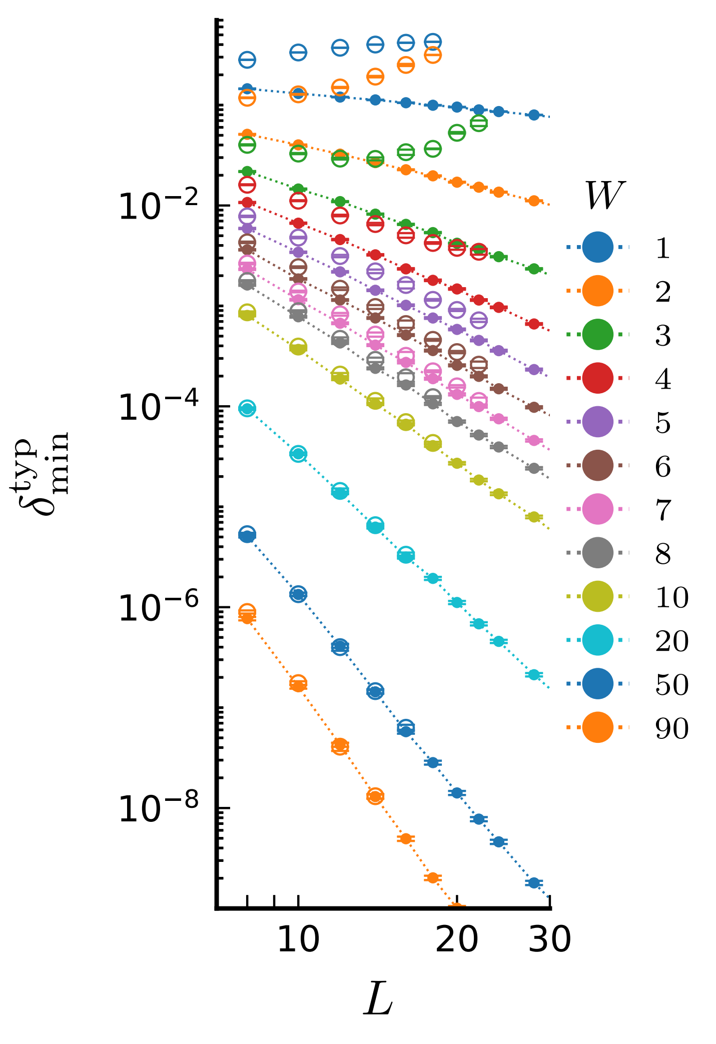
EFFECT OF INTERACTIONS?

Empty circles: Heisenberg



$\delta_i \to 1/2$ ($\langle S_i^z \rangle \to 0$)

disorder increases

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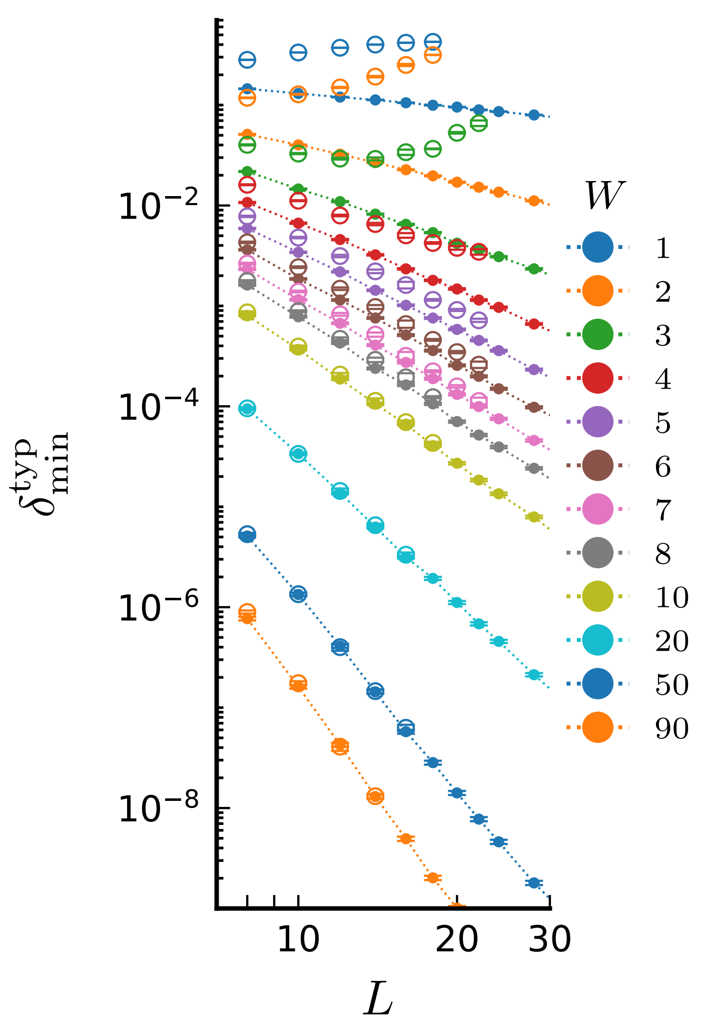
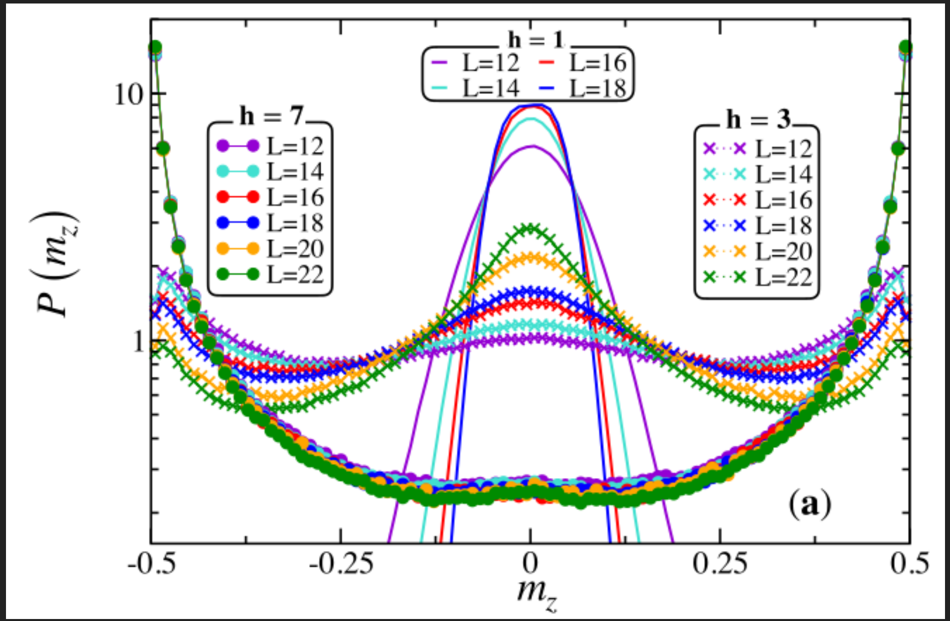
Empty circles: Heisenberg



$\delta_i \to 1/2$ ($\langle S_i^z \rangle \to 0$)

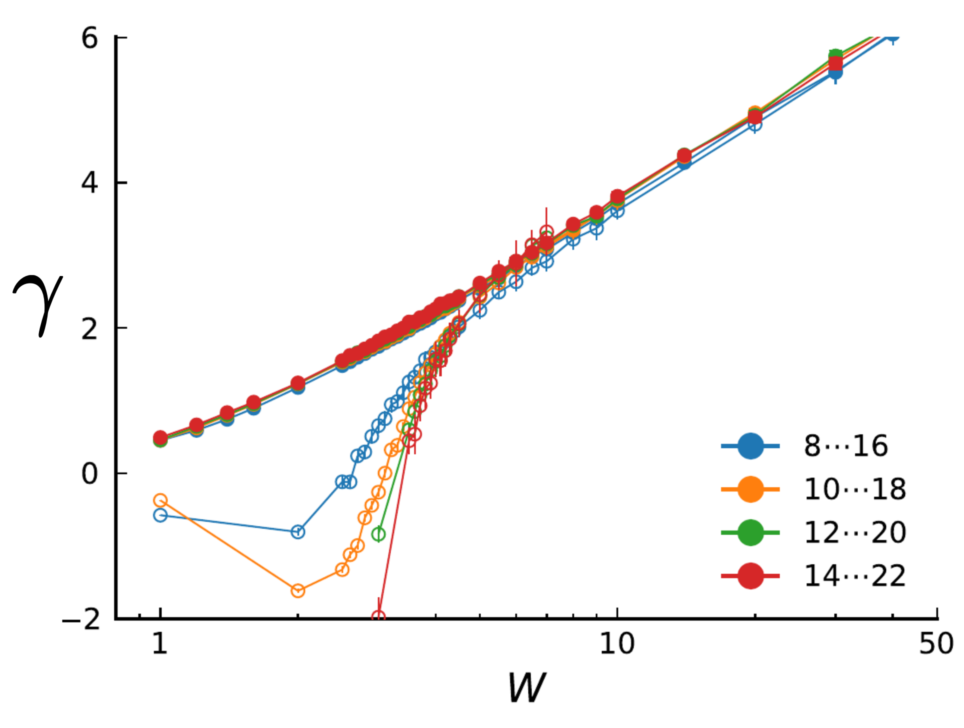
disorder increases



Laflorencie, Lemarié, Macé, PRR **2**, 042033(R), (2020)
JC, N. Laflorencie, arXiv:2305.10574

EXPONENT

At strong disorder, $\gamma \sim 1/\xi$



$\Rightarrow$ Interpretation of the exponent as related to a many-body localization length

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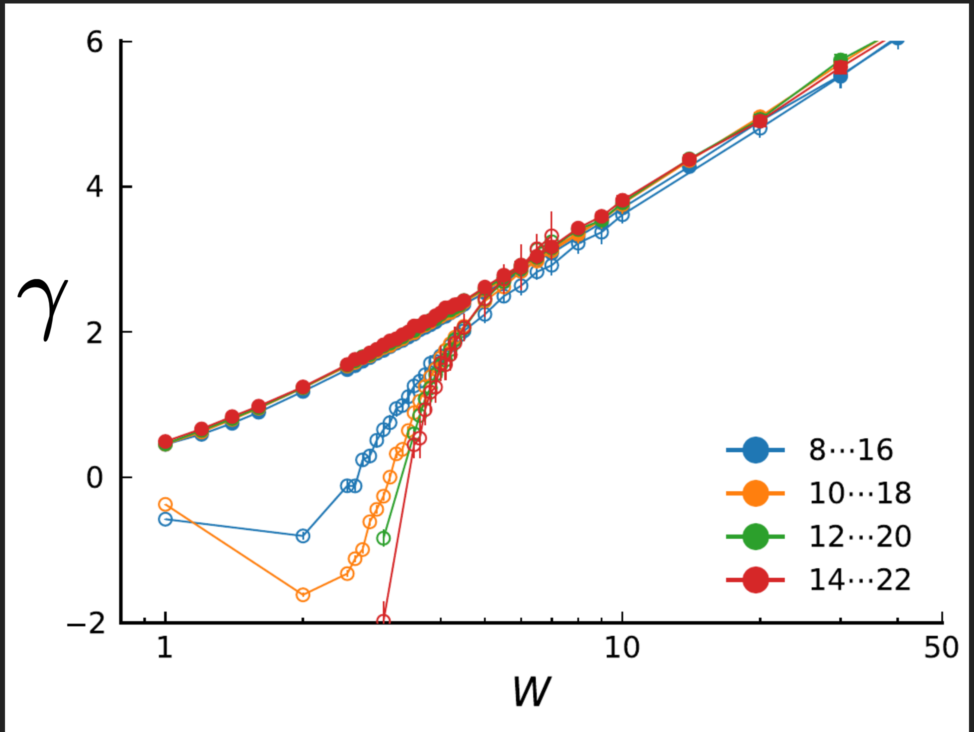

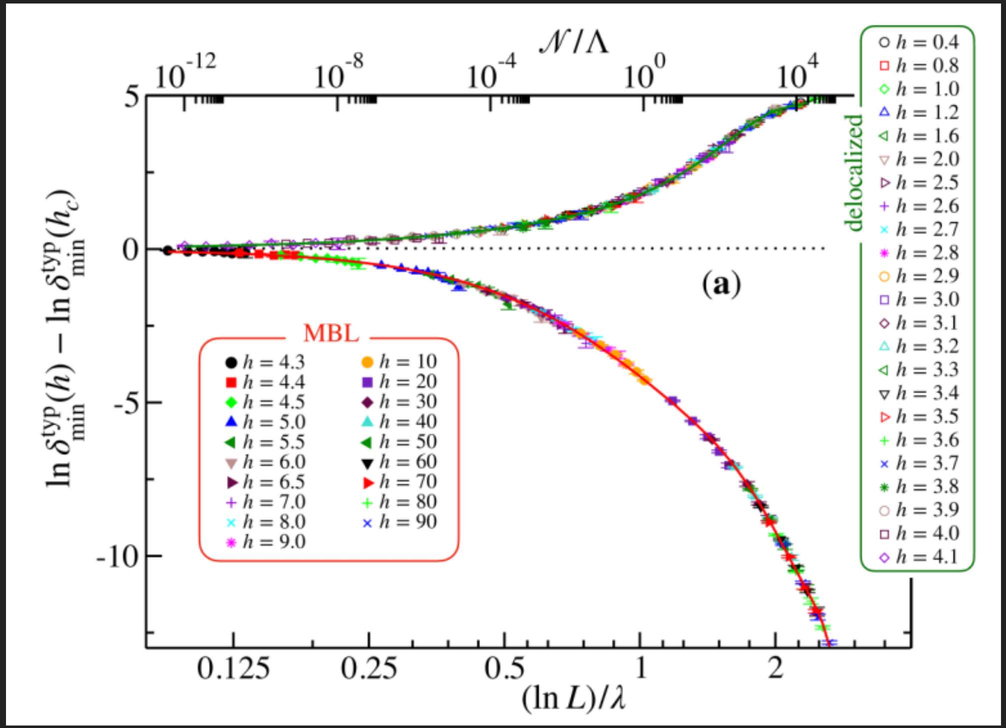

$\Rightarrow$ Interpretation of the exponent as related to a many-body localization length

Λ : disorder-dependent non-ergodicity volume

λ : interpreted as a localization length

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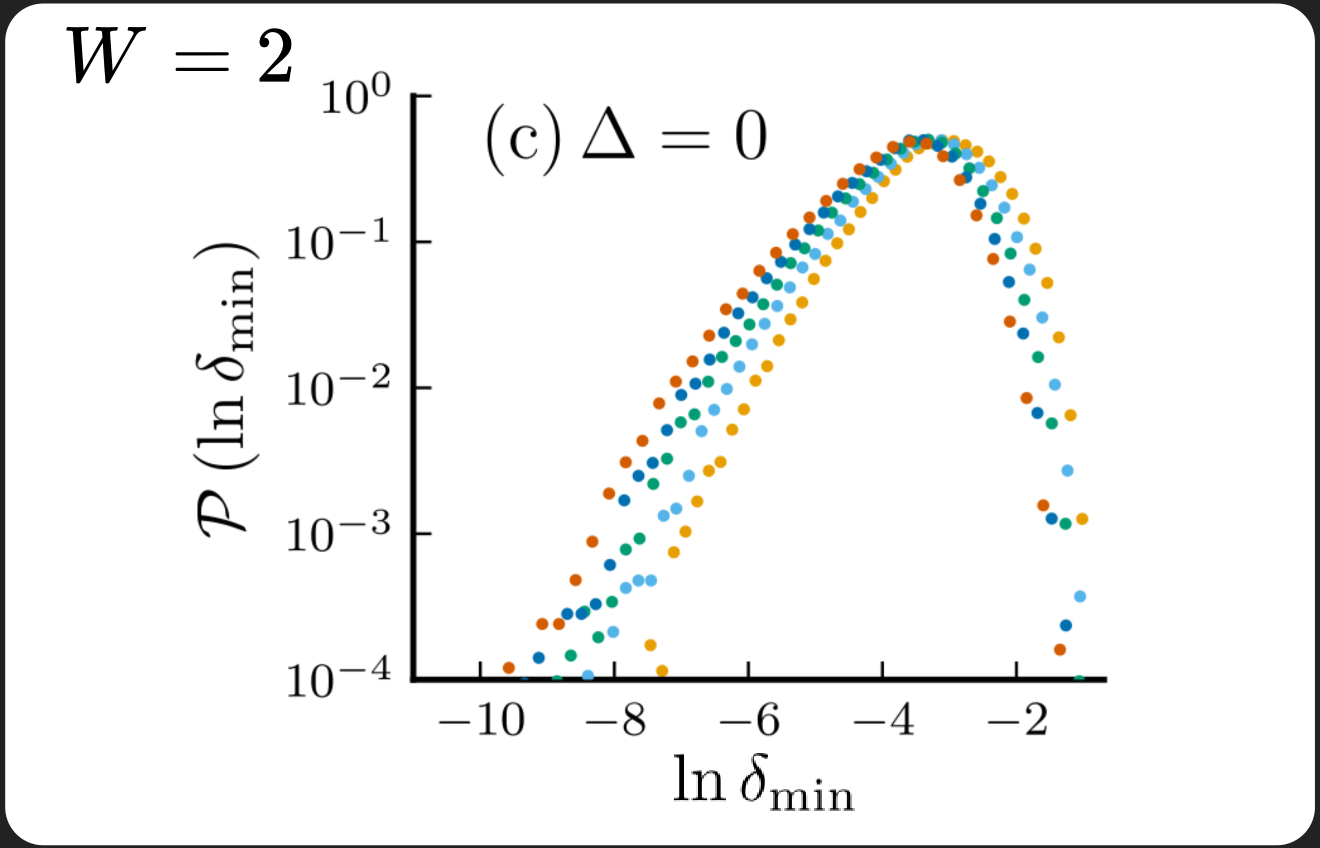
EXTREME VALUE DISTRIBUTIONS

Conjecture : Gumbel (?) on the Ergodic side, Fréchet on the MBL side.



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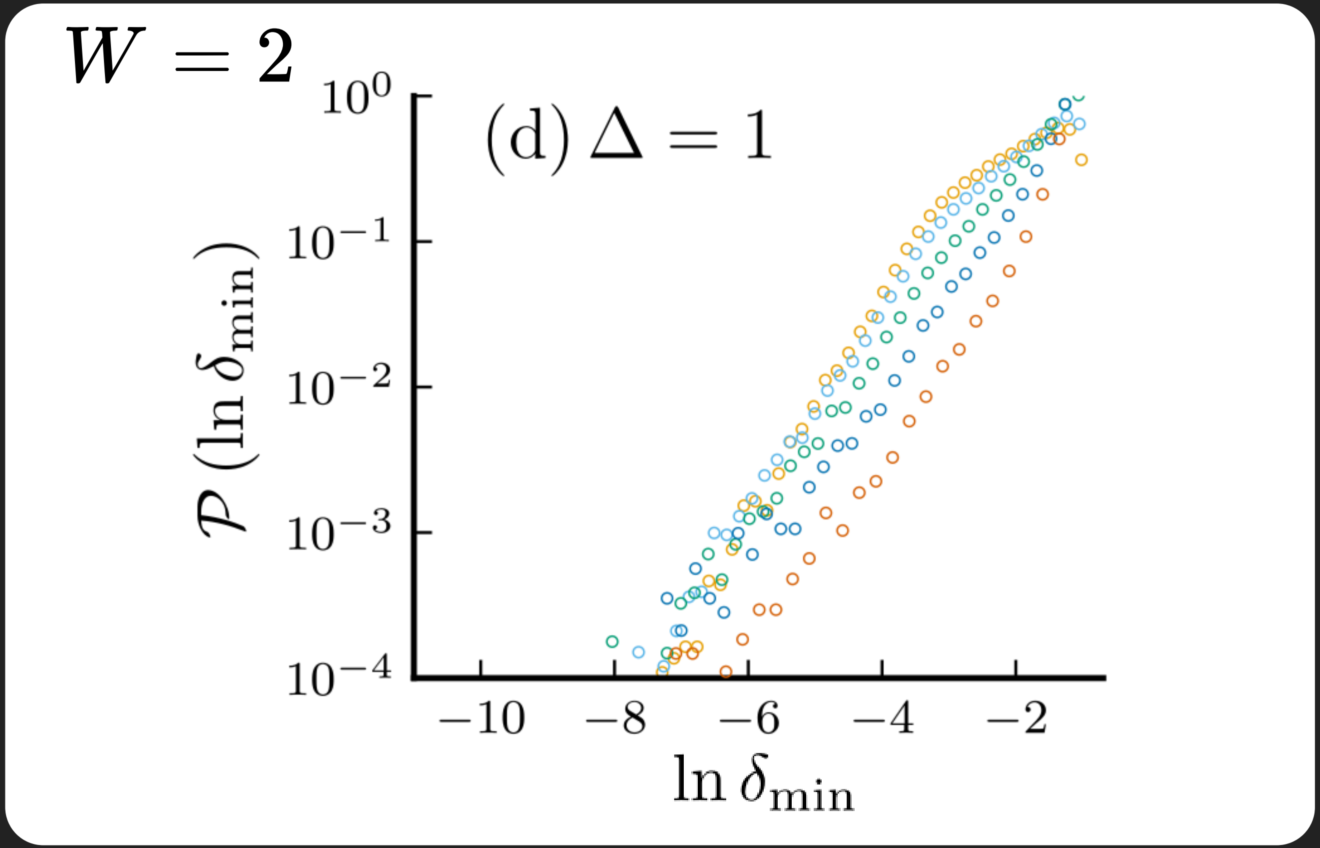

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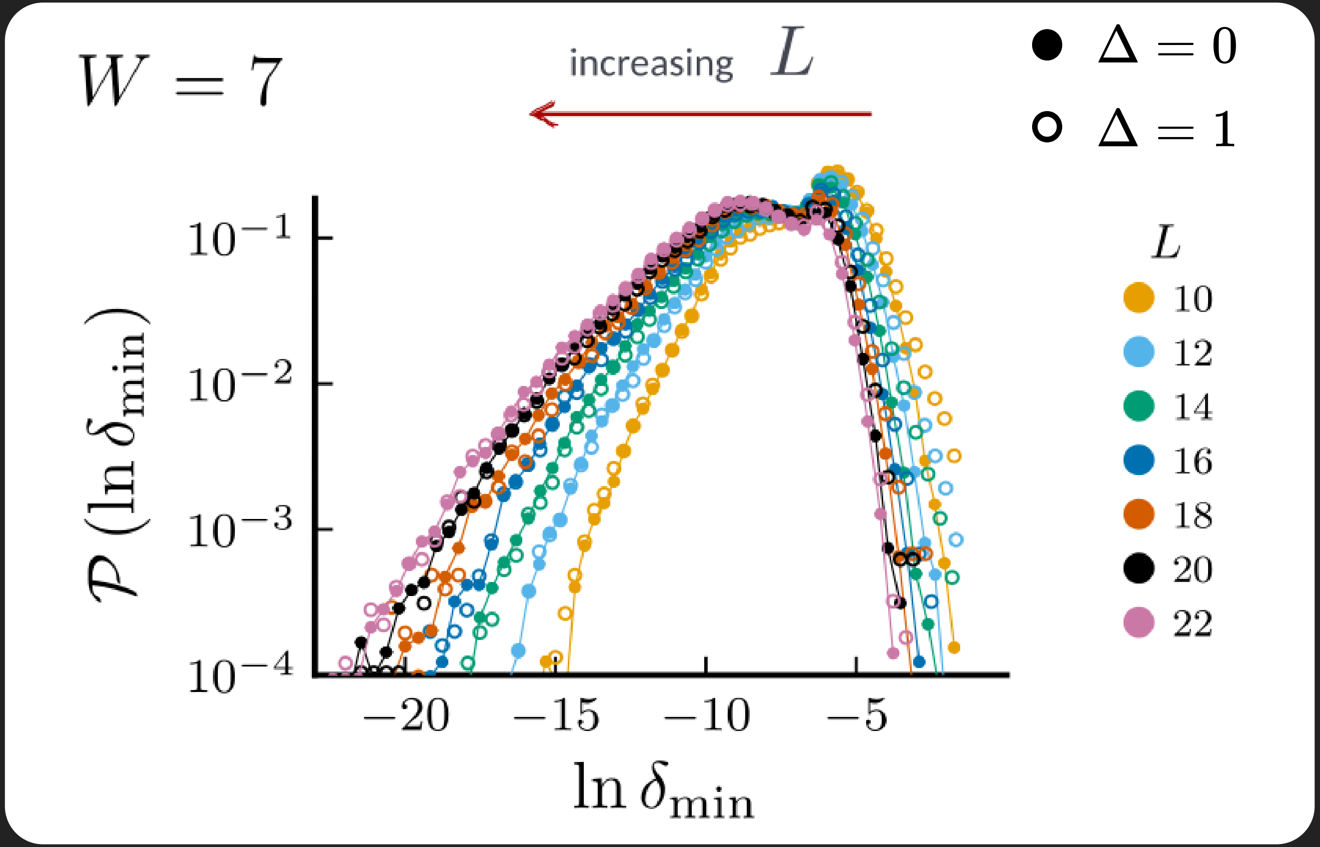

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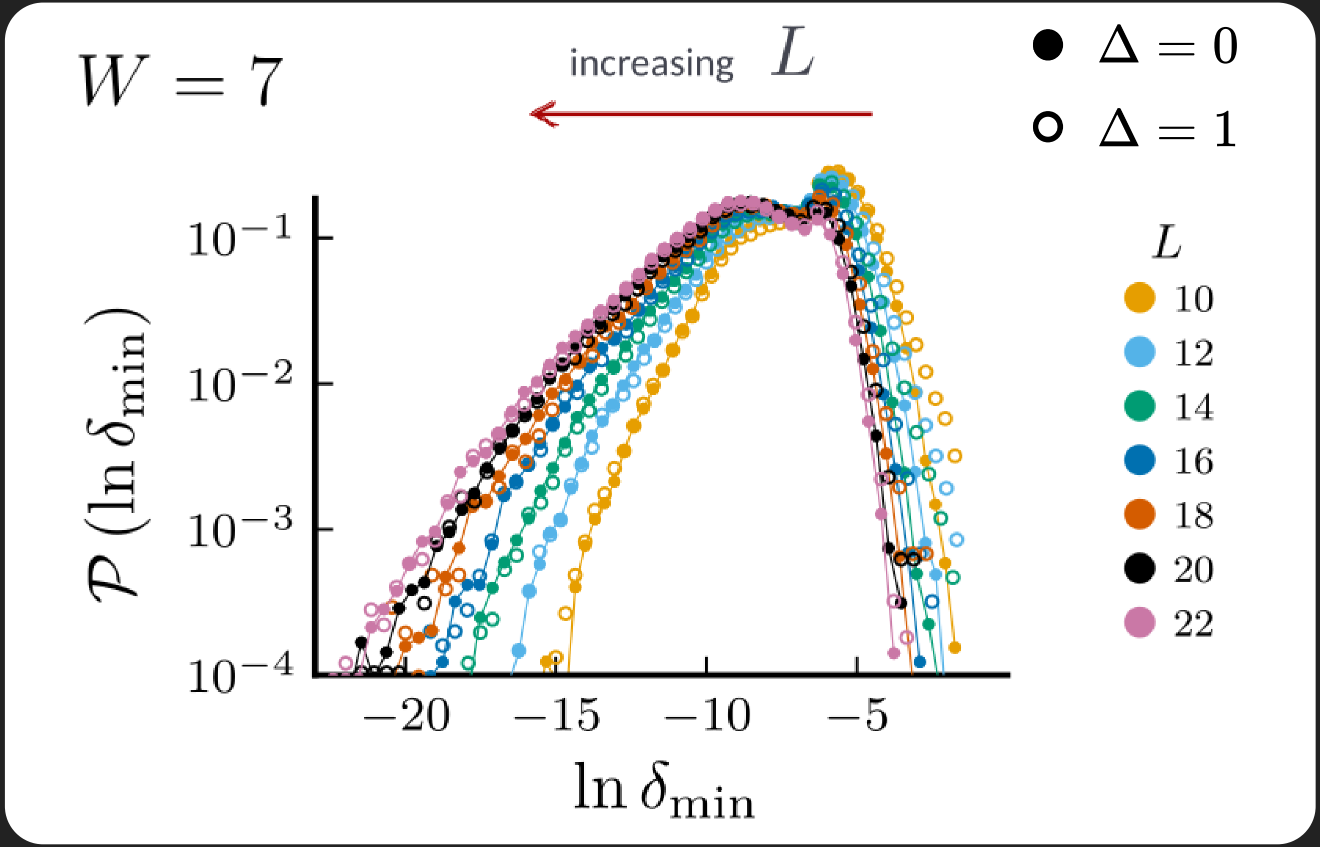

Kullback-Leibler divergence :

$$\mathrm{KL}(p|q) = \sum_i q_i \ln \frac{q_i}{p_i}$$

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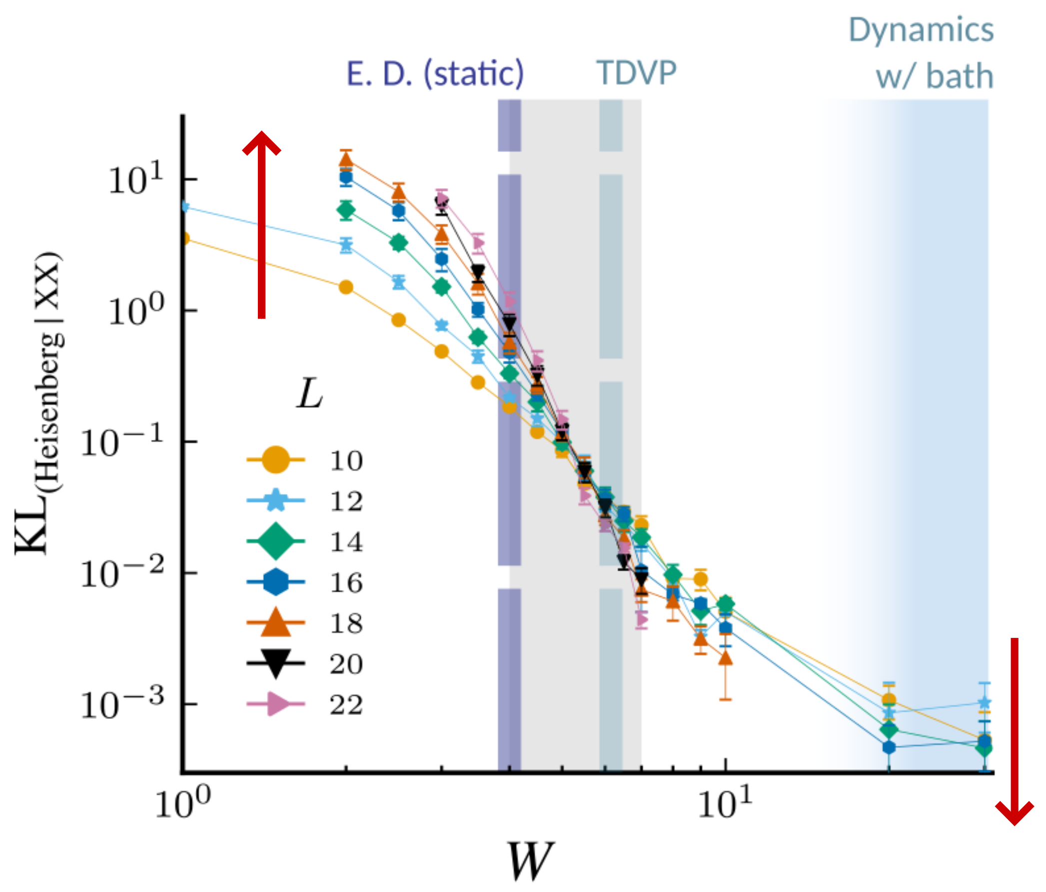

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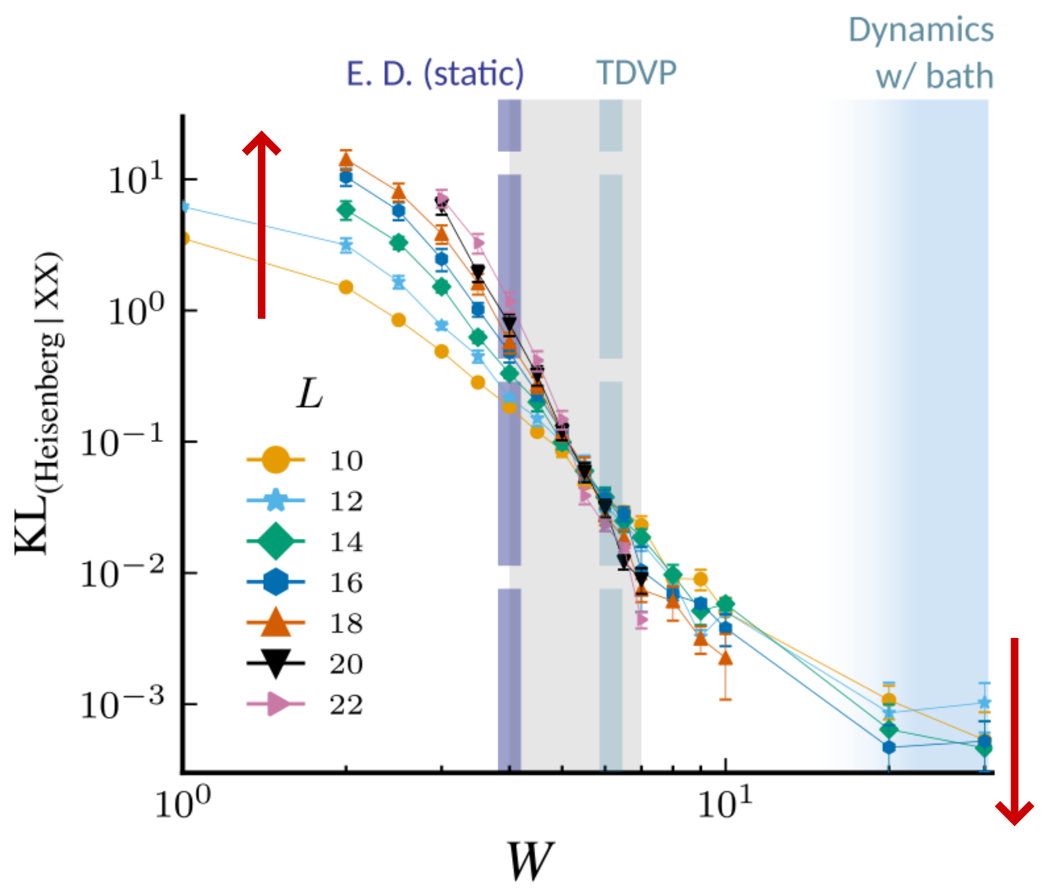
E. H. V. Doggen et al., PRB 98, 174202 (2018)
See e.g. D. Sels, PRB 106 , L020202 (2022)

CONSEQUENCES : HEISENBERG



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Transition in the extreme value distributions

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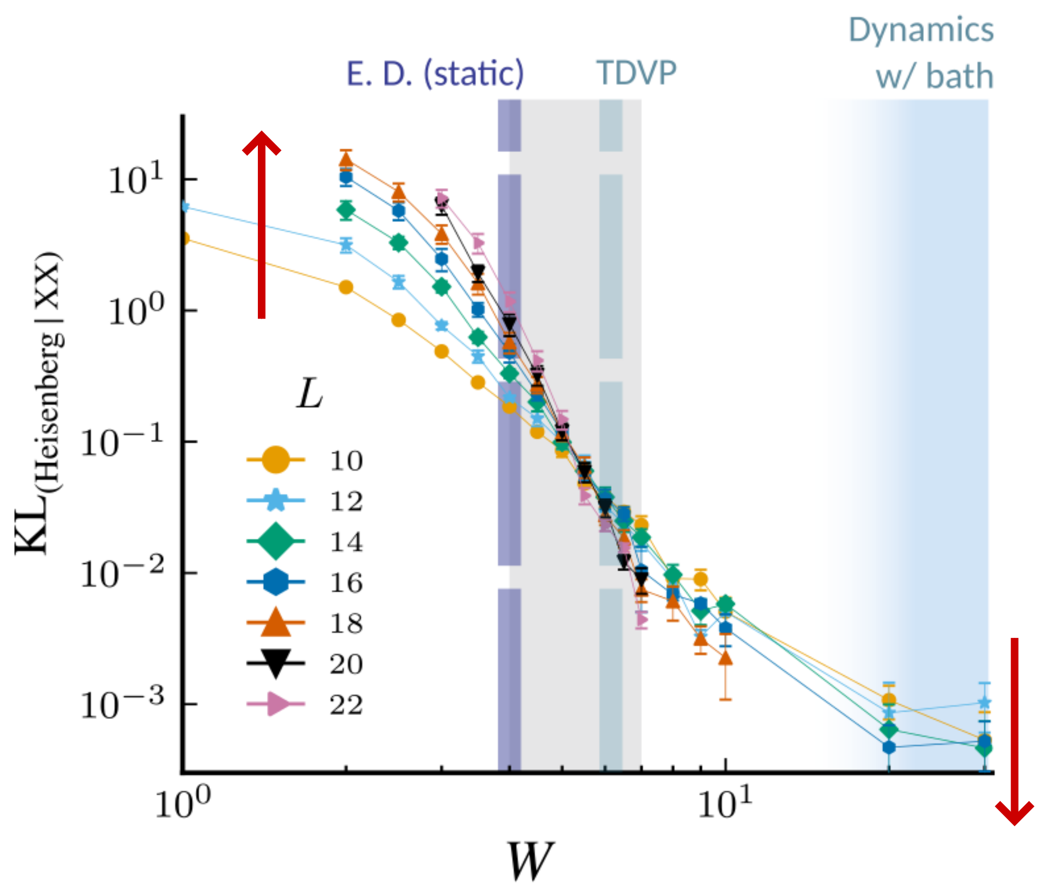

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Transition in the extreme value distributions

Coinciding with the MBL transition?

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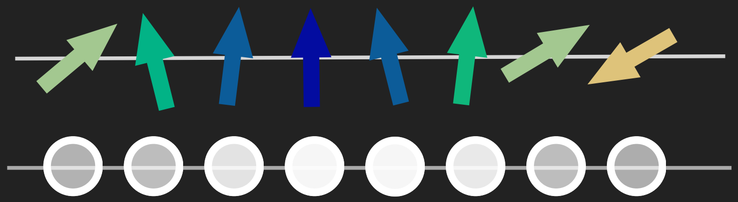
TAKE HOME MESSAGE

SPIN FREEZING!



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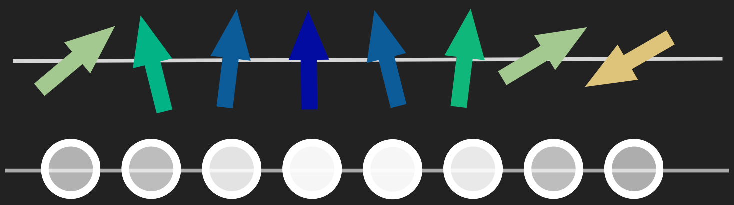
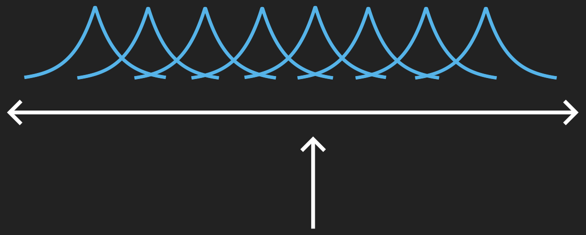


XX chain :

- controlled by largest cluster of occupied orbitals



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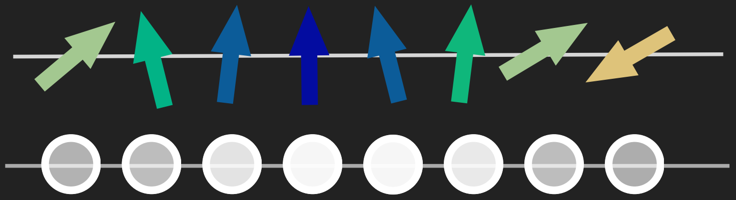

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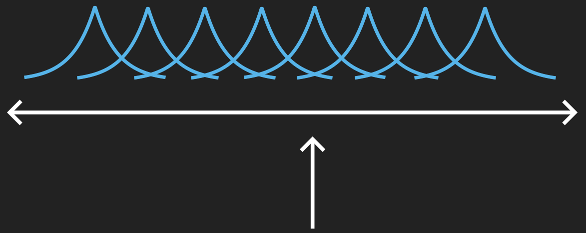

- Excellent fits & collapses with a Fréchet Law

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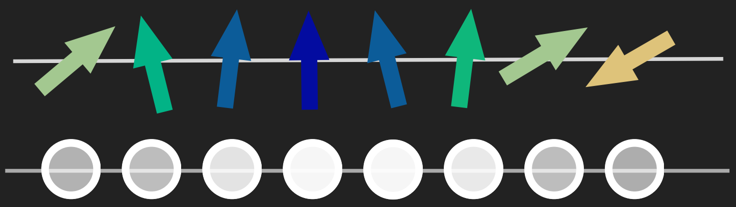

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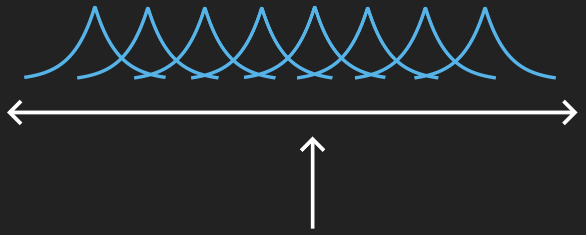

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Heisenberg chain at strong disorder:

Chain breaks!

→ W

No chain breaks | Chain breaks

TAKE HOME MESSAGE

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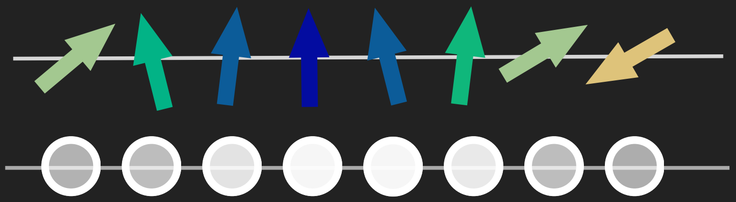

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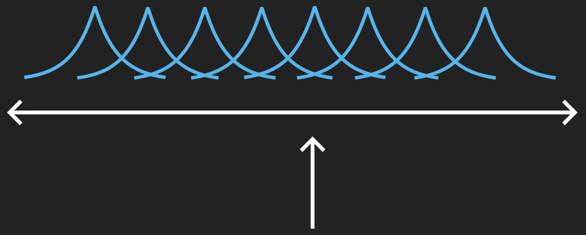

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Comparing δ_{min} deviation in Heisenberg vs Many-body Anderson:

Extreme value transition characterized by the **KL divergence.**

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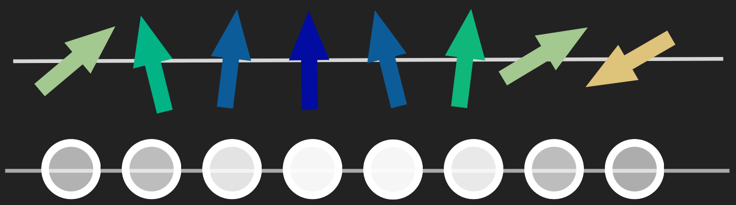

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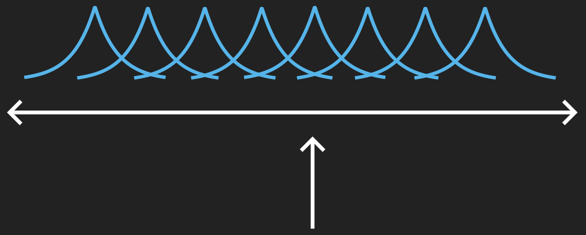

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Thank you for your attention!

arXiv:2305.10574

GOOD QUESTION!

PROBES

Increasing number of involved states

"Dynamical" probes

two-eigenstates correlation functions

KL divergence between eigenstates

Many-body resonances between eigenstates

Minimal gap

Spectral repulsion

Distribution of matrix elements?

Gap ratio

Level compressibility

Spectral form factor

Imbalance

Out-of-Time-Order Correlator (OTOC)

Evolution / autocorrelation of a prepared state
and these probes -->

"Static" probes

Multifractality (participation entropy)

Magnetization & extremal magnetization

-> Lcluster, delta min

Entanglement entropy (EE)

Entanglement spectrum (ES)

(minimal correlator)

Dream: LIOMs