ExU-YITP Workshop on Holography, Gravity and Quantum Information

RG flows of SYK (and a bit of holography & cosmology)

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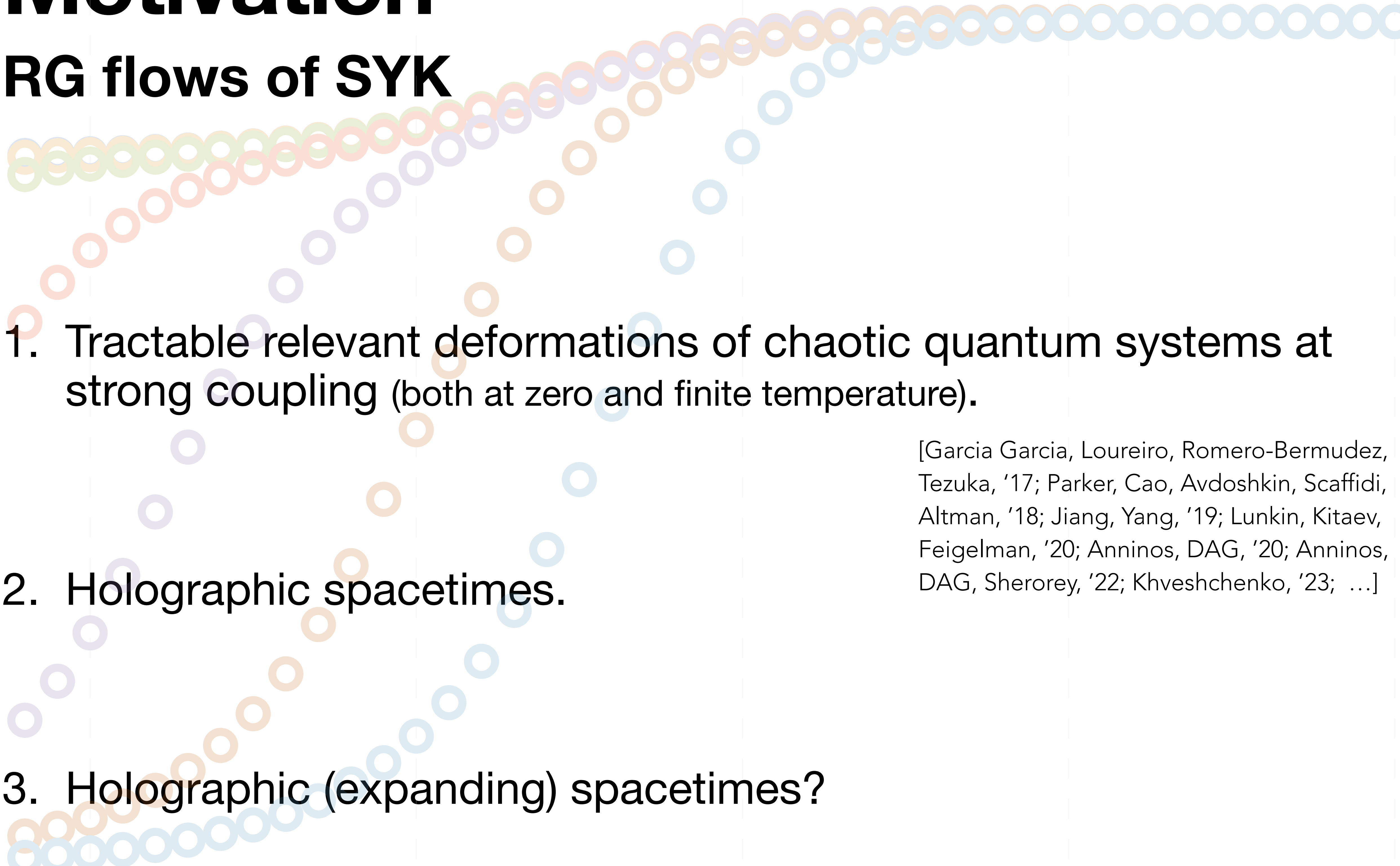
Based on 2011.01944, 2212.04944 and w.i.p. in collaboration with D. Anninos and S. Sheorey

Motivation

RG flows of SYK



1. Tractable relevant deformations of chaotic quantum systems at strong coupling (both at zero and finite temperature).

[Garcia Garcia, Loureiro, Romero-Bermudez, Tezuka, '17; Parker, Cao, Avdoshkin, Scaffidi, Altman, '18; Jiang, Yang, '19; Lunkin, Kitaev, Feigelman, '20; Anninos, DAG, '20; Anninos, DAG, Sherorey, '22; Khveshchenko, '23; …]

2. Holographic spacetimes.

3. Holographic (expanding) spacetimes?

Holographic spacetimes

- Emergent holographic direction associated with the change in the energy scale of the boundary theory.

[de Boer, Verlinde Verlinde, '99]

- Precise in the context of 2d dilaton-gravity.

- Consider the following (Euclidean) action,

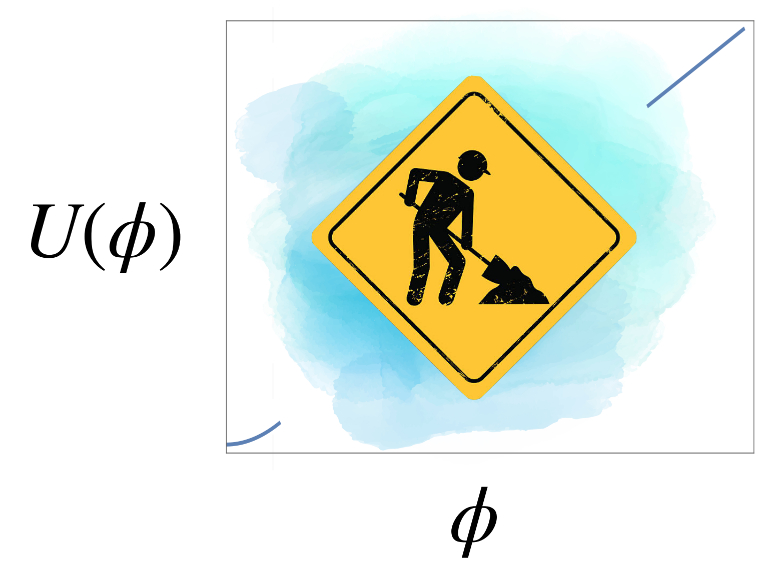
$$I = I_0 - \frac{1}{16\pi G_N}\int d^2x\sqrt{g}\,\left(\phi R + U(\phi)\right) - \frac{1}{8\pi G_N}\int du\sqrt{h}\phi_b K$$

- Saddle point solutions are given by

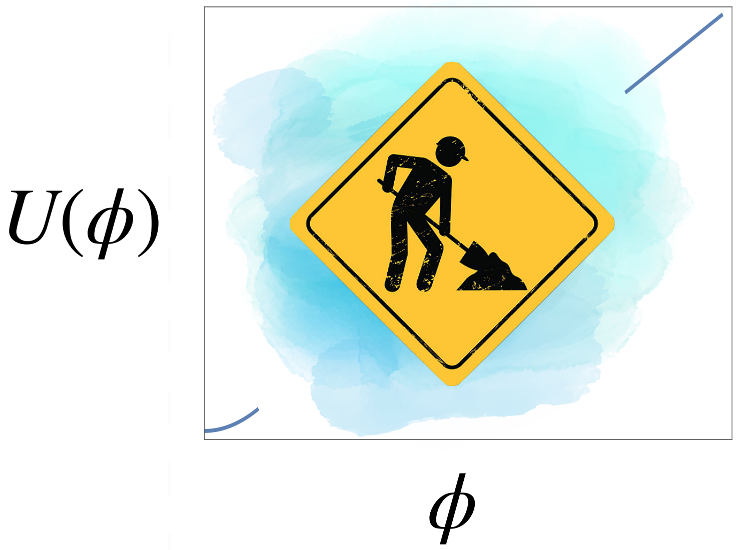
$$ds^2 = f(r)dt^2 + \frac{dr^2}{f(r)}\;, \qquad \phi = r\,,$$

$$f(r) = \int_{r_h}^{r} U(\rho)d\rho$$

[Cavaglia, '99; Grumiller, McNees, '07; Anninos-DAG-Hofman, '18; Witten '20, …]



Question: How to microscopically construct the dilaton potential?



Constructing the dilaton potential

1. One possibility is to deform the dual matrix model.

[Saad, Shenker, Stanford, '19; Maxfield, Turiaci,'20; Witten, '20; Turiaci, Usatyuk, Weng, '21; Eberhardt, Turiaci, '23]

2. In this talk, we will stay in the saddle-point approximation.

- Consider the thermodynamics of the dilaton-gravity theory:

$$T = \frac{U(r_h)}{4\pi}, \qquad S = S_0 + \frac{r_h}{4G_N}$$

$$U(\phi) \approx T(S - S_0)$$

Thermodynamics of quantum mechanical large *N* model $\equiv$ Dilaton potential of 2d dilaton gravity theory

- Reminiscent of spacetime from thermodynamic arguments

[Jacobson, '95]

Constructing the dilaton potential II

Some examples:

- JT gravity:

$$U(\phi)_{JT} = 2\phi \rightarrow S = S_0 + cT$$

- Flow geometries:

$$U(\phi)_{flow} \sim \begin{cases} 2\phi \ , \\ 2\alpha\,\phi + \alpha_0 \ . \end{cases}$$

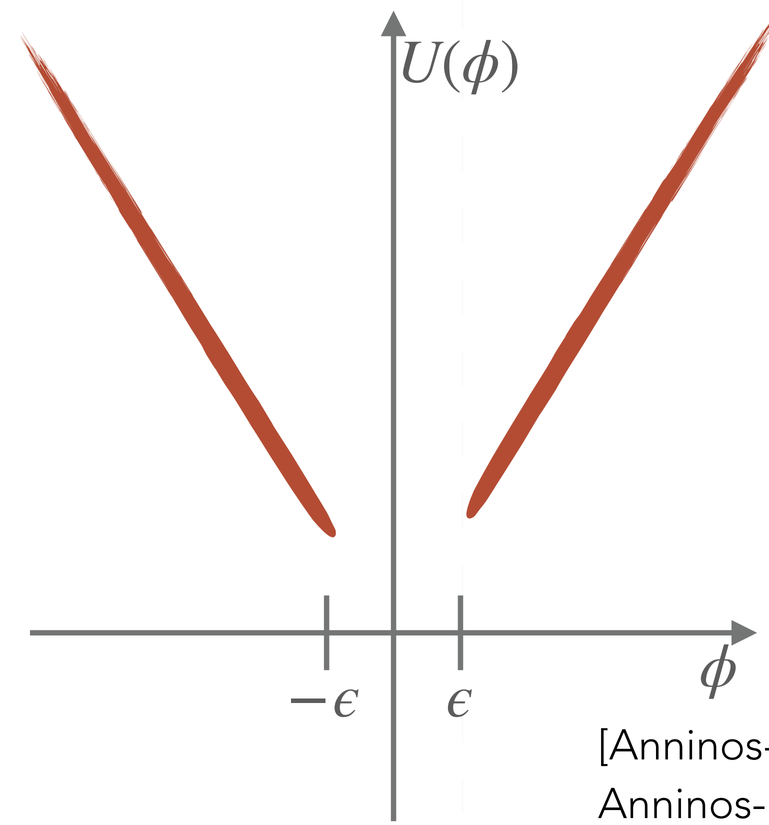
- dS JT gravity:

$$U(\phi)_{dS\,JT} = -2\phi \rightarrow S = S_0 - cT$$

[Svesko, Verheijden, Verlinde, Visser, '22]

- Centaur geometries:

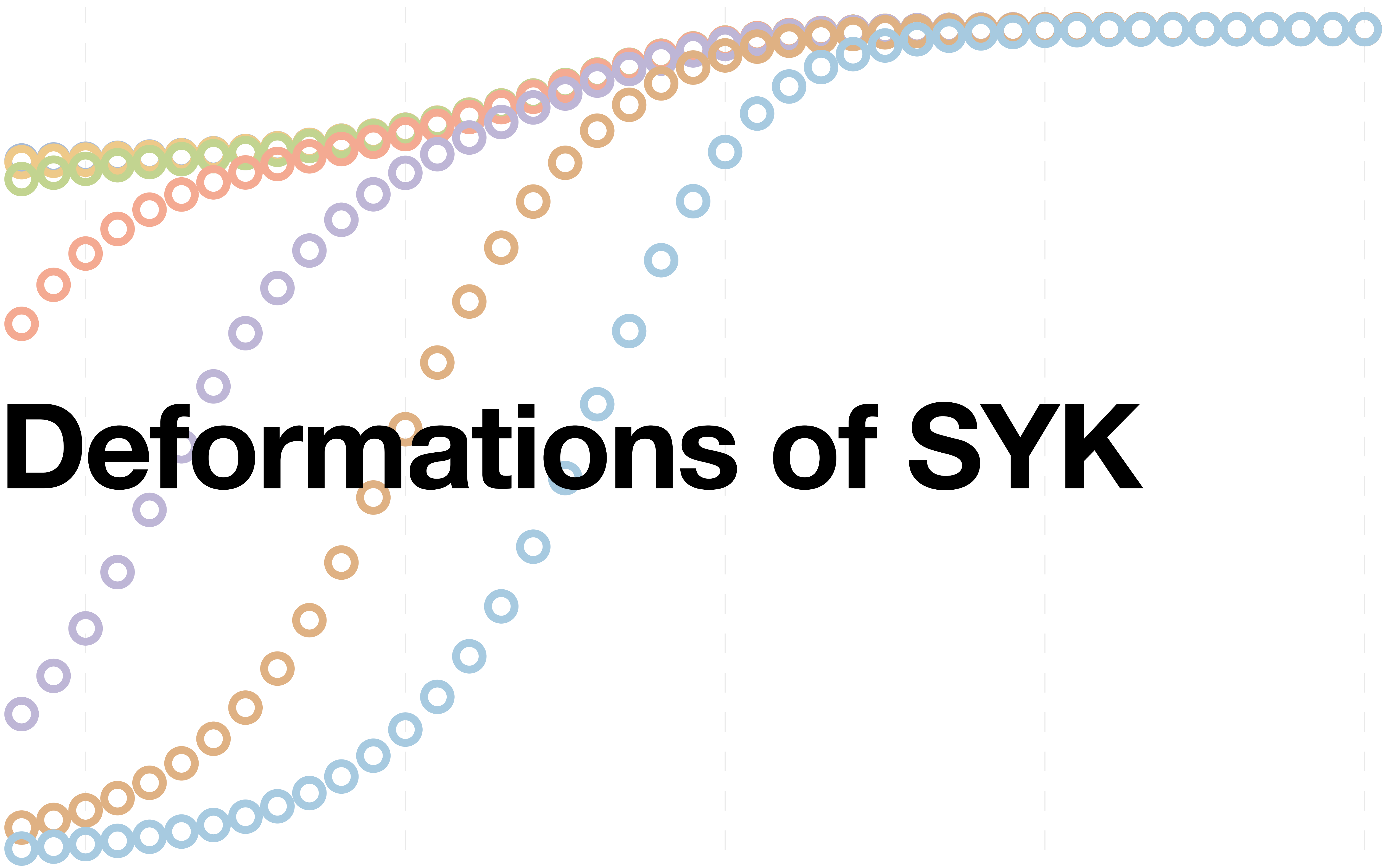
$$U(\phi)_{centaur} \sim \begin{cases} 2\phi \ , \\ -2\phi \ , \end{cases}$$



[Anninos-Hofman '17; Anninos-DAG-Hofman, '18]

Infrared microscopic deformations related to holographic expansion

(or at least, the limitations of building it)



Deformations of SYK

The SYK model

- N Majorana fermions

$$\{\psi_i, \psi_j\} = \delta_{ij}\,.$$

- Random all-to-all interactions

$$H^{(q)}_{\mathrm{SYK}} = (i)^{q/2} \sum J_{i_1 \cdots i_q} \psi_{i_1} \psi_{i_2} \cdots \psi_{i_q}\,.$$

- The large N saddle is given by Schwinger-Dyson equations

$$G = (\partial_\tau - \Sigma)^{-1} \qquad , \qquad \Sigma = \mathcal{J}^2 G^{q-1}\,.$$

- At large q, they reduce to an ordinary differential equation

$$G(\tau) = \frac{\mathrm{sgn}(\tau)}{2}\left(1 + \frac{g(\tau)}{q} + \cdots\right) \qquad , \qquad \partial_\tau^2 g(\tau) = \mathcal{J}^2 e^{g(\tau)}\,.$$

- At low temperatures, the theory is dominated by a Schwarzian mode with linear-in-temperature entropy (as JT gravity).

Deformed SYK models

- We want to deform the SYK model with **relevant** deformations.

$$H_{total} = H^{(q)}_{\text{SYK}} + s\,\mathcal{O}_{rel}\,.$$

- Problem: simple operators with fermions are irrelevant.

See e.g. [Gross, Rosenhaus, '17]

- Idea: consider disordered operators, like the Hamiltonian itself!

$$\Delta_\psi = 1/q \quad , \quad \sum J_{i_1,\cdots i_{\tilde{q}}} \psi_{i_1}\psi_{i_2}\cdots\psi_{i_{\tilde{q}}} \to \Delta = \tilde{q}/q ?$$

Deformed SYK models

- Deformed Hamiltonian:

$$H_{total} = H^{(q)}_{\text{SYK}} + s\, H^{(\tilde{q})}_{\text{SYK}} \quad \text{with} \quad \tilde{q} < q\,, s \in \mathbb{R}\,.$$

- Deformed Schwinger-Dyson equations:

$$G = (\partial_\tau - \Sigma)^{-1} \qquad , \qquad \Sigma = \mathcal{J}^2 G^{q-1} + \mathcal{J}^2 s^2 G^{\tilde{q}-1}\,.$$

- Large q equation:

$$\begin{cases} q \to \infty\,, \tilde{q} \to \infty\,, \\ q/\tilde{q} = n > 1 (\text{fixed})\,. \end{cases} \qquad \partial_\tau^2 g(\tau) = \mathcal{J}^2 e^{g(\tau)} + \mathcal{J}^2 s^2\, n\, e^{g(\tau)/n}\,.$$

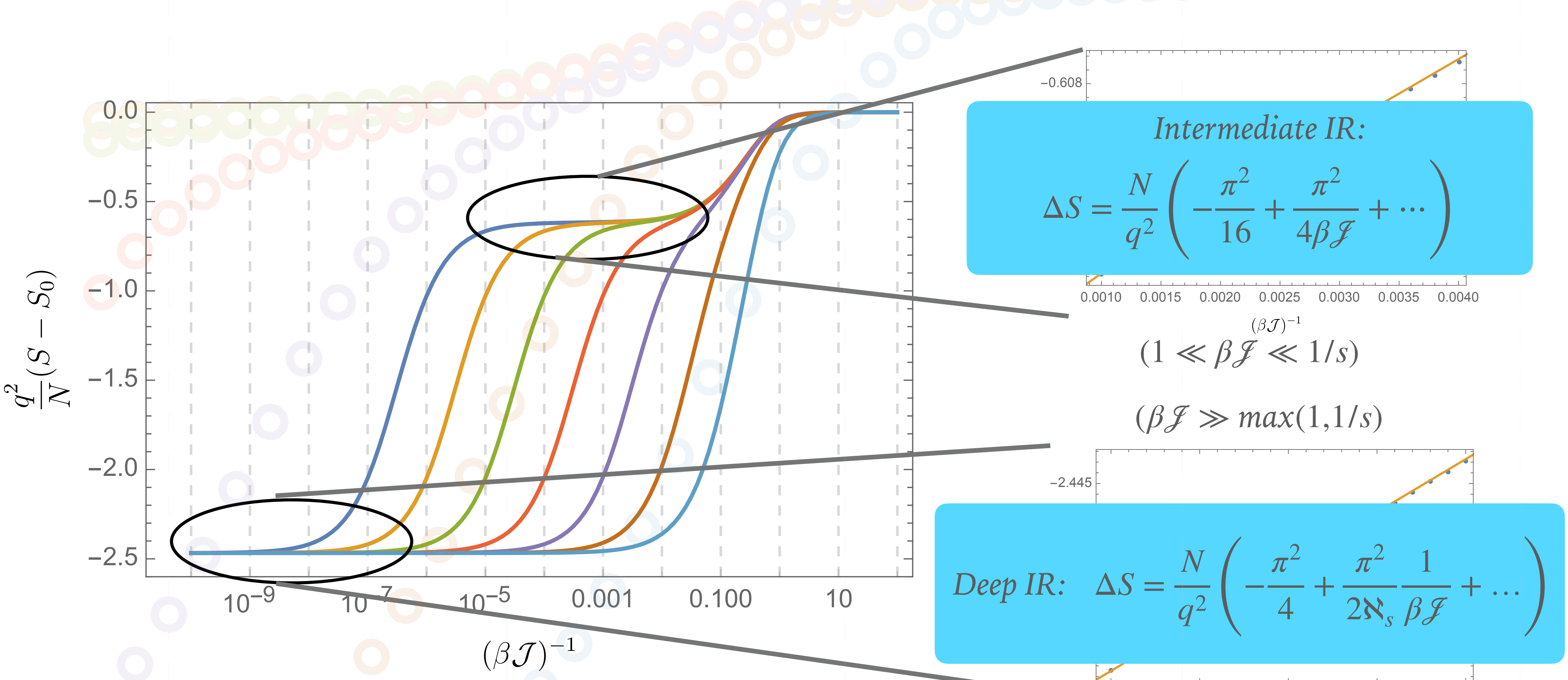
- Remarkably, n=2 has an analytic solution in terms of dimensionless variables $\beta\mathcal{J}$ and s!

$$e^{g(\tau)} = \frac{2\nu^2}{\sqrt{(\beta\mathcal{J})^2\nu^2 + s^4(\beta\mathcal{J})^4}\cos(\nu(\frac{2\tau}{\beta} - 1)) + s^2(\beta\mathcal{J})^2} \qquad \cos\nu = \frac{2\nu^2 - s^2(\beta\mathcal{J})^2}{\sqrt{(\beta\mathcal{J})^2\nu^2 + s^4(\beta\mathcal{J})^4}}$$

[Jiang, Yang, '19; Anninos, DAG, '20]

Thermodynamics of deformed SYK models

[Anninos, DAG '20]



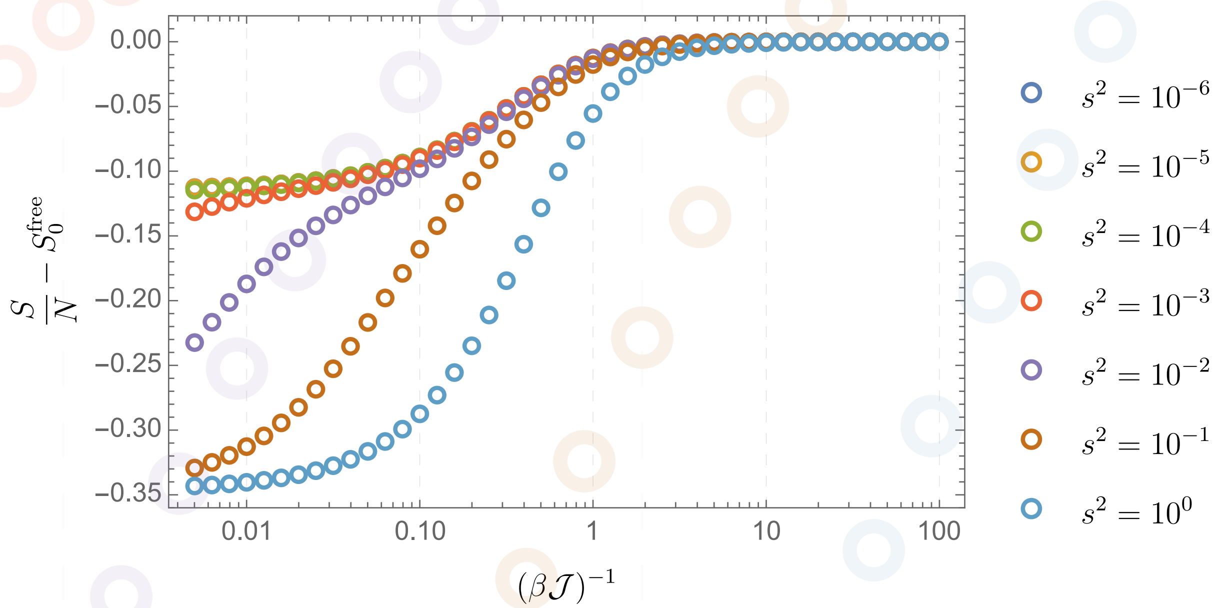
$s^2 = 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 10^{0}$

$$\aleph_s = \frac{s^2}{\sqrt{1 + 4s^2}}$$

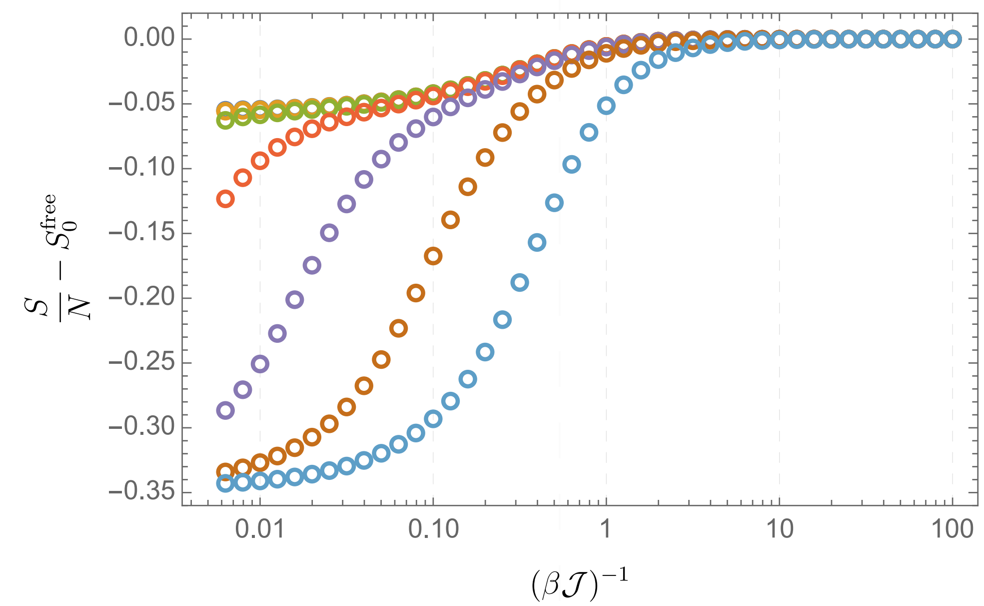
Thermodynamics of deformed SYK models at finite *q* and at finite *N*

[Anninos, DAG, Sheorey '22 + wip]

Finite q entropies:



$q = 4\,, \tilde{q} = 2$



$q = 6\,, \tilde{q} = 2$

Conformal perturbation theory:

$$\beta F = \beta F_{\text{CFT}} - \frac{g_h^2}{2}\int_0^\beta\int_0^\beta d\tau_1 d\tau_2 \left\langle O_h(\tau_1)\, O_h(\tau_2)\right\rangle_\beta + \cdots \,.$$

$$\frac{\delta^2 S_{h=1/n}}{N} \propto \left(1 - \left(2 - \frac{2}{n}\right)\right) \frac{\pi^{\frac{2}{n}-\frac{1}{2}}\Gamma\left(\frac{1}{2} - \frac{1}{n}\right)}{2\Gamma\left(1 - \frac{1}{n}\right)} \left(\beta\mathcal{J}\right)^{2-\frac{2}{n}}$$

[Tikhanovskaya, Guo, Sachdev, Tarnopolsky '20; Cruz, Tarnopolsky '22; Anninos, DAG, Sheorey '22]

Thermodynamics of deformed SYK models

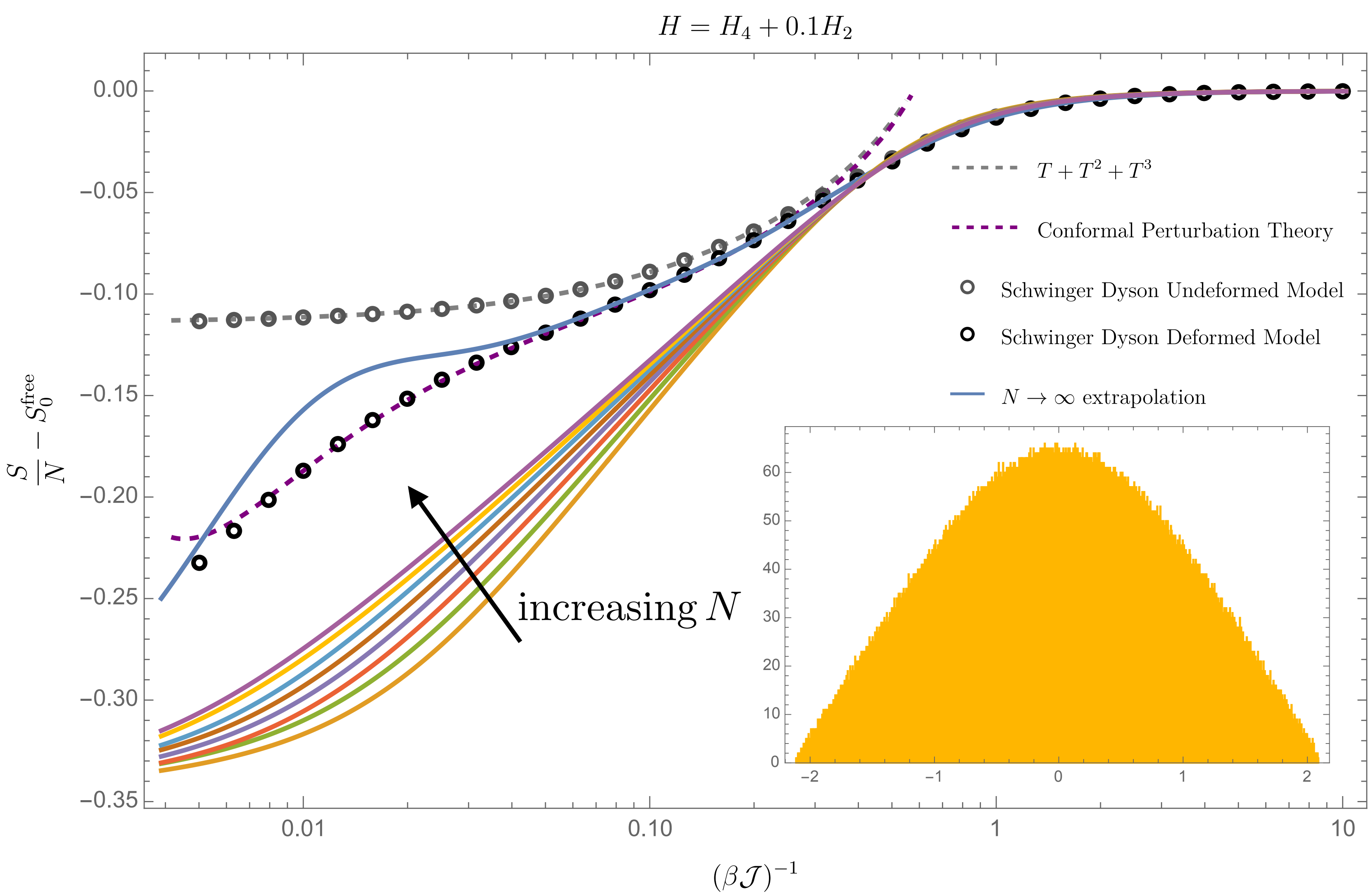
Finite *N* flows

[Anninos, DAG, Sheorey, wip]

A naïve counting will tell that this is impossible to achieve:

$$N \gg \beta\mathcal{J} \gg 1/s \gg 1$$

Nevertheless, it is possible to see the effect of the deformation by extrapolation of exact diagonalisation results!



[Tikhanovskaya, Guo, Sachdev, Tarnopolsky ’20; Cruz, Tarnopolsky ’22; Anninos, DAG, Sheorey ’22]

Remarks on deformed SYK models

- These relevant deformations of the SYK model are robust even at finite *q* and finite *N*, both at zero and finite temperature.
- Consistent with **Conformal Perturbation Theory** predictions.
- New IR (near) fixed points.
- **Landscape** of relevant deformations.

$$H_{total} = H^{(q)} + \sum_i s_i H^{(q_i)}$$

$$\aleph_s = \frac{s^2}{\sqrt{1+4s^2}}$$

- De Sitter thermodynamics reproduced if $s \to is$, which generates a **non-Hermitian** deformation of SYK.

[García-García, Sá, Verbaarschot '21; García-García, Jia, Rosa, Verbaarschot, '22]

- These models have interesting properties: PT symmetry, real spectrum at large *N*, consistency with conformal perturbation theory, etc.

[Anninos, DAG, Sheorey, wip]

- Double scaling? Other probes? Krylov complexity?

Thank you!