Boundary induced dynamical phase transition via inhomogeneous quenches

Dongsheng Ge

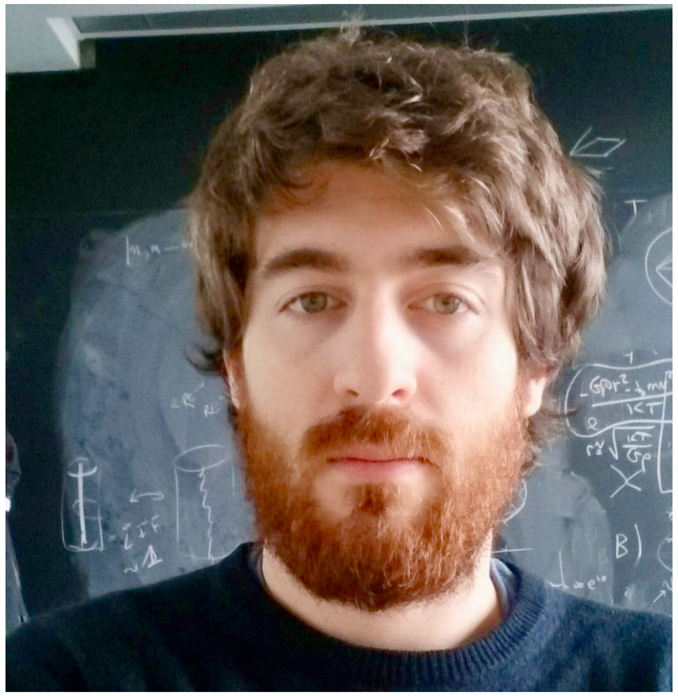
(Osaka University)

16.09.2023 @ QIMG2023

In collaboration with

Alice Bernamonti

(Università degli Studi di Firenze)

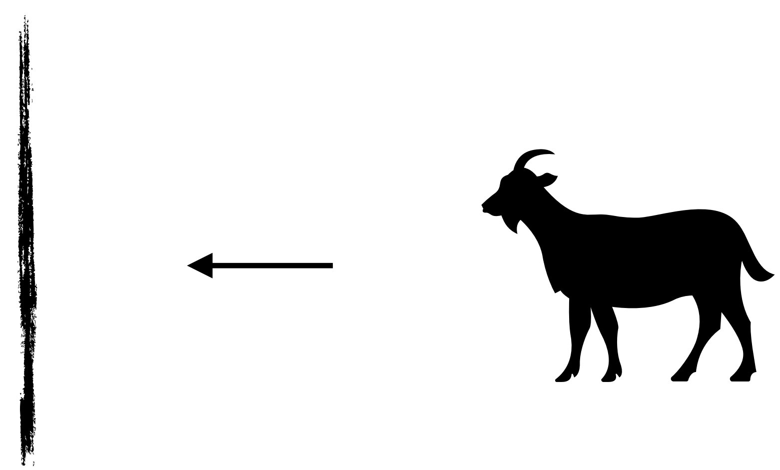


Federico Galli

(INFN Firenze)

The question we are asking

A physical system usually has boundaries, subject to certain boundary conditions, they can give rise to boundary effects. For instance,



The situation becomes more interesting when some dynamics is injected into the system. A natural question we are asking is whether and how

the boundary effects influence the bulk dynamics ?

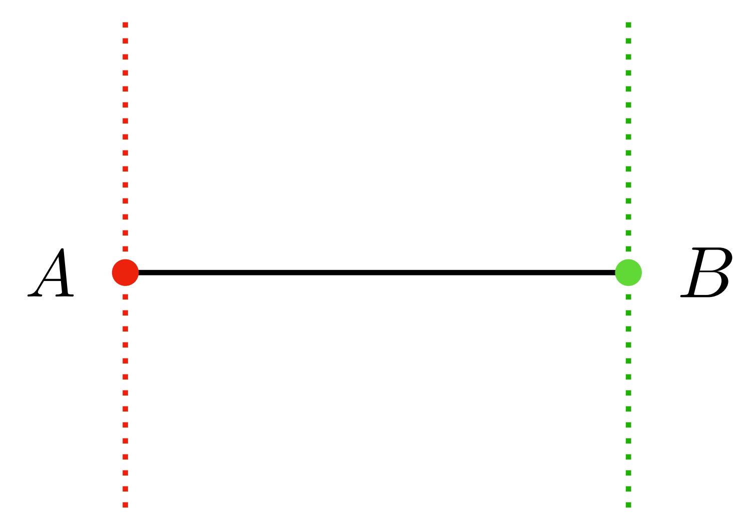
The way we are answering

We choose to answer this question in the "1+1" critical systems with conformal boundaries. We have in mind that they have holographic duals, in the sense that they have the following properties

[Hartman, Keller, Stoica '14]

1. Gapped system
2. Large central charge c
3. Sparse light (h<< c) spectrum

The system is simply illustrated as the figure. It has a finite size L, with generically distinct boundary conditions A & B on its two ends.



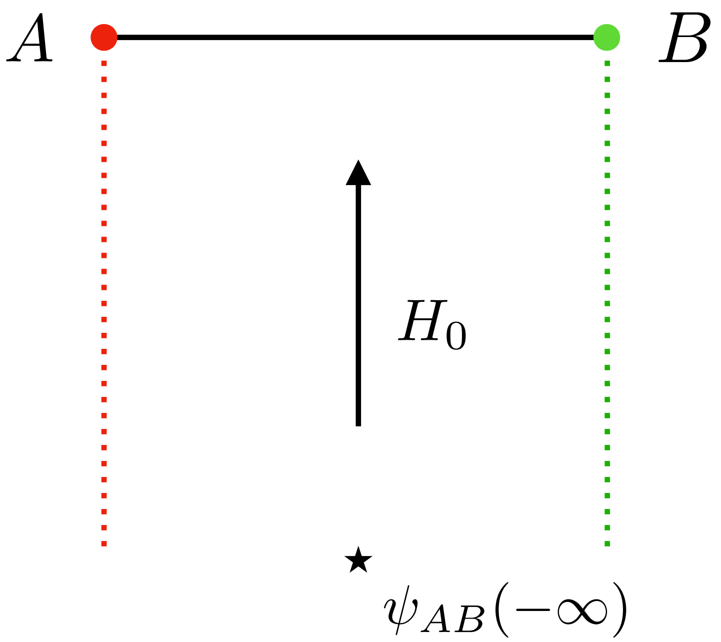
The procedures:

- Preparing the initial state with bcc operators
- Injecting dynamics through inhomogeneous quenches
- Probing the real-time evolution via entanglement entropy

Prepare the initial state

We want to have distinct boundary conditions on the two edges, this can be done through an insertion of the boundary condition changing (bcc) operator at the past infinity of the Euclidean strip. [Cardy 1989]

$$|\psi_{AB}\rangle = \psi_{AB}(-\infty)\,|0\rangle_{AA}$$



- An eigenstate of the BCFT Hamiltonian

$$H_0\,|\psi_{AB}\rangle \propto h_{AB}\,|\psi_{AB}\rangle$$

- The state is normalized

Inject dynamics through deformed Hamiltonians

We want to quench the system using the inhomogeneous Hamiltonians, they are deformed from the usual BCFT Hamiltonian,

$$H = H_0 - \frac{\tanh(2\theta)}{2}(H^- + H^+)$$

The components are written in terms of an integration over the stress tensors

$$H_0 = \frac{1}{2\pi}\int_0^L d\sigma\left(T(w) + \bar{T}(\bar{w})\right) = \frac{\pi}{L}\left(L_0 - \frac{c}{24}\right)$$
$$H^{\pm} = \frac{1}{2\pi}\int_0^L d\sigma\left(e^{\mp\frac{2\pi w}{L}}T(w) + e^{\mp\frac{2\pi \bar{w}}{L}}\bar{T}(\bar{w})\right) = \frac{\pi}{L}L_{\pm 2}$$

Sine-square-deformed (SSD) Hamiltonian is obtained once the deformation parameter is taken to be infinity $\theta \to \infty$, as

[Katsura '11]

[Okunishi '16]

$$1 - \frac{1}{2}\left(e^{\frac{2\pi i\sigma}{L}} + e^{-\frac{2\pi i\sigma}{L}}\right) = 2\sin^2\left(\frac{\pi\sigma}{L}\right)$$

Probe with Entanglement entropy

[Cardy, Calabrese '04 & '09]

We can calculate the Entanglement entropy using the replica trick. It is equivalent to insert a twist operator of conformal dimension $\frac{c}{24}\left(n-\frac{1}{n}\right)$ in the bulk. Then it is just a calculation of the Bulk-boundary-boundary three-point function for the reduced density matrix. The time-dependence can be absorbed into the coordinate transformation. So far in Euclidean signature, later the Wick rotation will be performed.

$$\langle\psi_{AB}|\, e^{H\tau}\Phi_n(w,\bar{w})e^{-H\tau}\,|\psi_{AB}\rangle = \left|\frac{\partial z_\tau}{\partial w}\right|^{2h_n} \langle\psi_{AB}|\,\Phi_n(z_\tau,\bar{z}_\tau)\,|\psi_{AB}\rangle_{UHP}$$

With

$$z_\tau^2 = \frac{(\cosh(2\theta)+\coth(\lambda)e^{2\pi w/L}-\sinh(2\theta)}{\sinh(2\theta)e^{2\pi w/L}+\coth(\lambda)-\cosh(2\theta)}\,, \quad \lambda = \frac{\pi\tau}{L\cosh(2\theta)}$$

Bulk-boundary-boundary correlation

[Cardy, Lewellen, 1991]

The bulk-boundary-boundary correlator on the UHP can be evaluated using the mirror image method, where the anti-chiral part of the bulk operator becomes a chiral operator inserted at its complex conjugate position. This actually results in a four-pt function on the full complex plane

$$\langle\psi_{AB}|\,\Phi_n(z_\tau,\bar{z}_\tau)\,|\psi_{AB}\rangle_{UHP} = \langle\psi_{AB}|\,\Phi_n(z_\tau)\Phi_n(\bar{z}_\tau)\,|\psi_{AB}\rangle$$

[Fitzpatrick, Kaplan, Walters '14]

In the heavy-heavy-light-light (HHLL) regime, such a four-pt correlator is dominated by the identity block in the t-channel as the leading contribution

$$\langle\psi_{AB}|\,\Phi_n(z_\tau)\Phi_n(\bar{z}_\tau)\,|\psi_{AB}\rangle \approx C_n^{A,B}\alpha^{2h_n}(z_\tau\bar{z}_\tau)^{-h_n}\left(2\sinh\left(-\frac{\alpha}{2}\ln\eta\right)\right)^{-2h_n}$$

Entanglement entropy

The boundary entropies contribute to the entanglement entropy, in the holographic context, they can be tuned arbitrarily

$$s_i = \frac{1}{4G} \tanh^{-1}(T_i)$$

Collecting all the pieces and taking the replica index to 1, the entanglement entropy has to be minimized over the two "phases"

$$\boxed{S(t) = \frac{c}{6} \ln \left(\frac{2L}{\pi \alpha \epsilon} \sqrt{f_\theta^2(t) + \sin^2\left(\frac{2\pi\sigma}{L}\right)} \right) + \min\{S^A(t), S^B(t)\}}$$

with

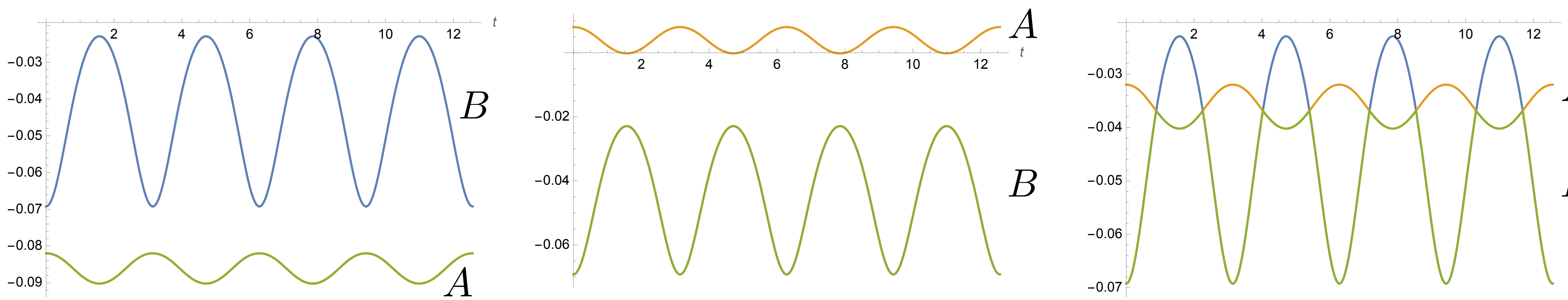
$$S^i(t) = \frac{c}{6} \ln \left| \sin\left(\frac{\alpha}{2}\delta^i\right) \right| + s_i$$

$$f_\theta(T) = -\sin^2(T)\sinh(4\theta) + \left(\cos^2(T) + \cosh(4\theta)\sin^2(T)\right)\cos\left(\frac{2\pi\sigma}{L}\right)$$

For a general deformation, they are oscillating with a periodicity of $L\cosh(2\theta)$

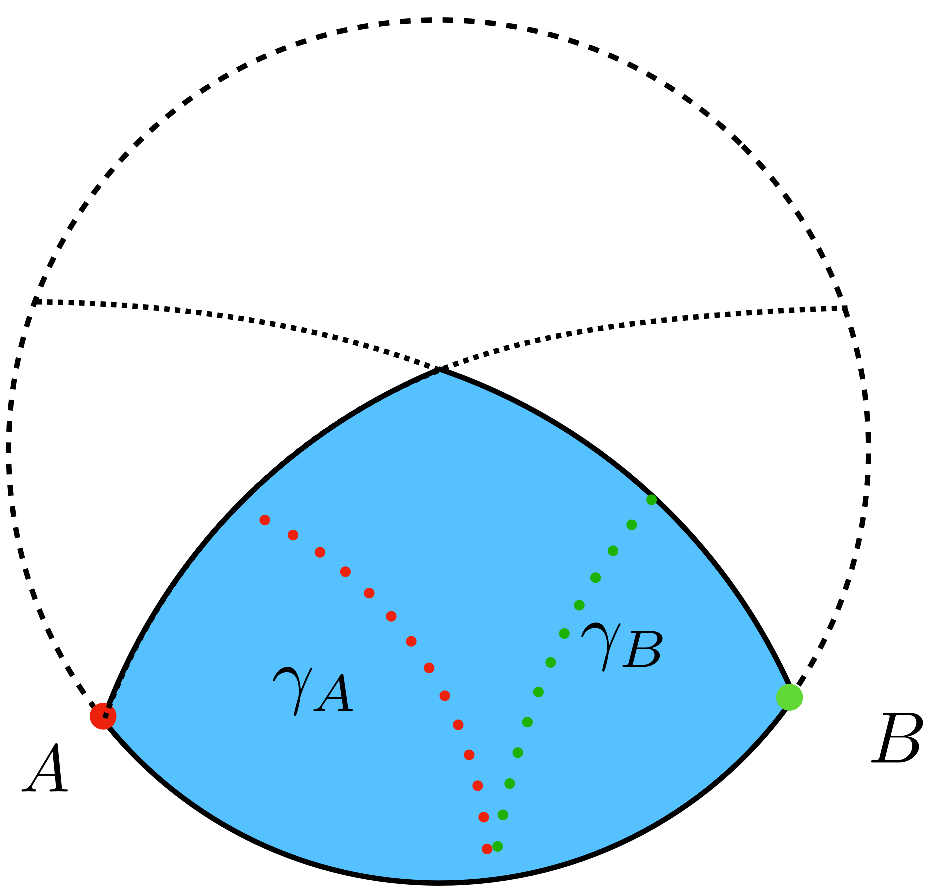
Boundary entropies determine the dynamical patterns

1. Phase A dominant $s_A \ll s_B$
2. Phase B dominant $s_A \gg s_B$
3. Transition from B to A $|s_A - s_B| < \#$



Holographic interpretation

Competition of the two minimal geodesics gives rise to the dynamical behavior.



Future questions:

- Rational models exhibit similar phase transition features ?

  Or it is the large c behavior, due to its chaotic nature ?

- Thermal case, strip -> rectangle ? [Cardy '14]

ありがとございます