

**Subregion von Neumann Algebras  
&  
Entanglement Entropies in AdS/CFT**

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**Work in progress with Herman Verlinde**

# Quantum gravity - AdS/CFT -



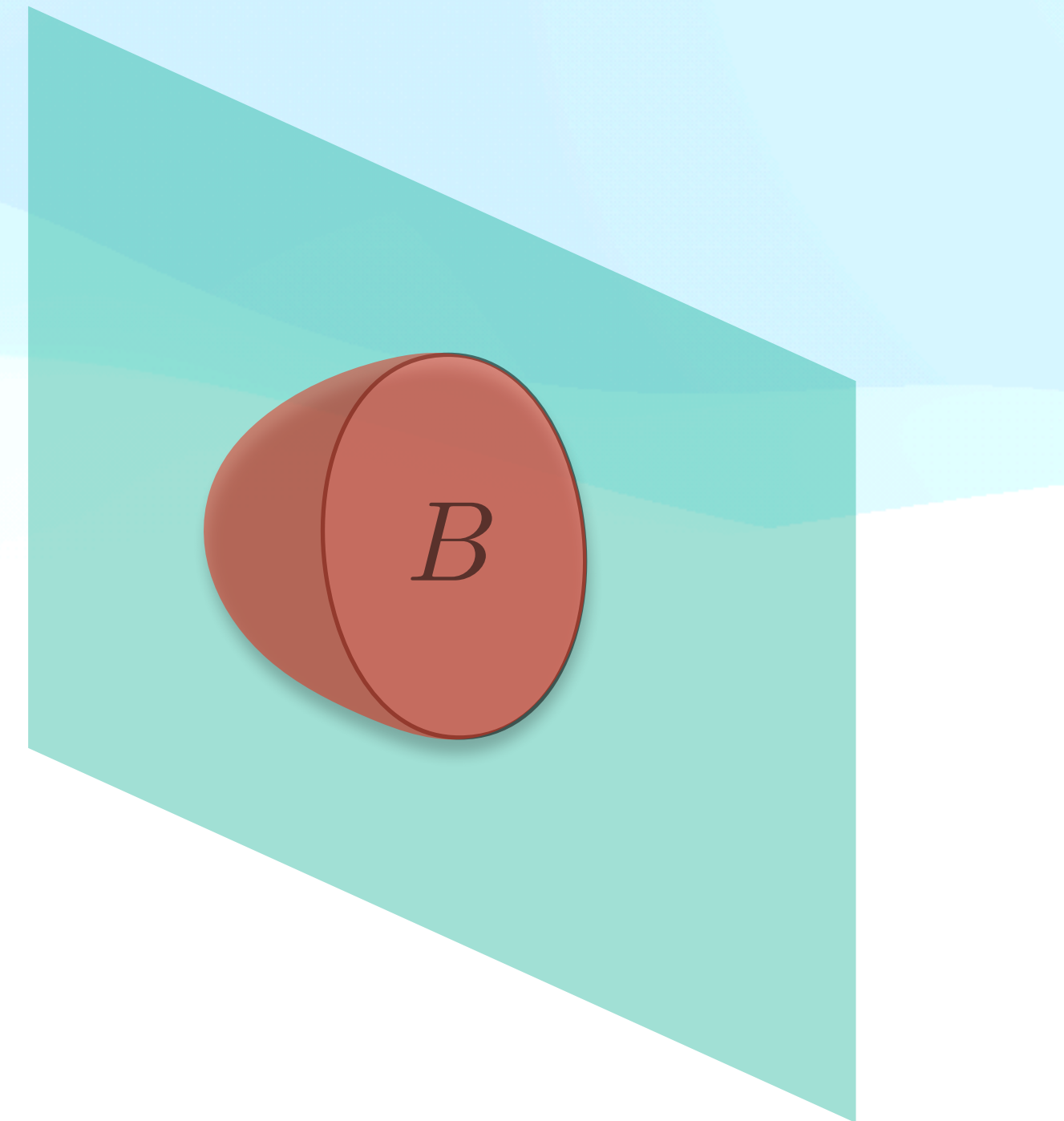
- How does **spacetime geometry** emerge?
- Fate of **information inside a black hole**?

**Quantum Extremal Surface formula (=Ryu-Takayanagi)**

$$S(B) = \frac{\text{Area}(\partial b)}{4G_N} + S_{\text{bulk}}(b)$$

**bdy entanglement**  $\leftrightarrow$  **bulk geometry**

entanglement wedge of  $B$

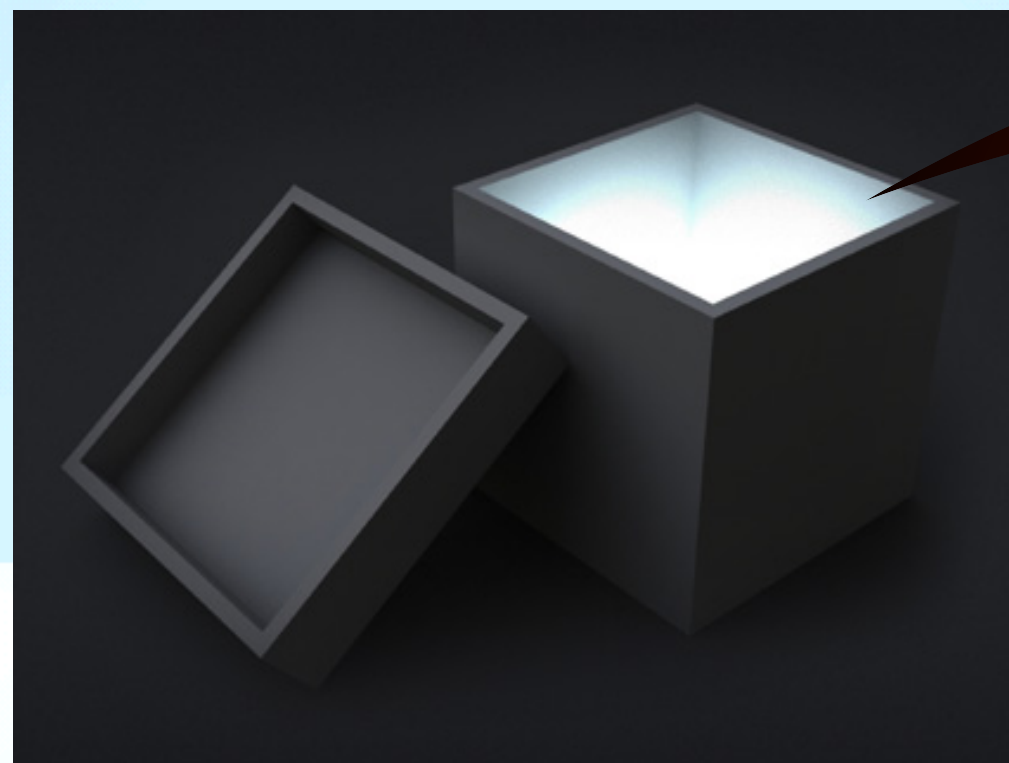


**QES** formula may be “proved” through **Euclidean gravity path-integral**

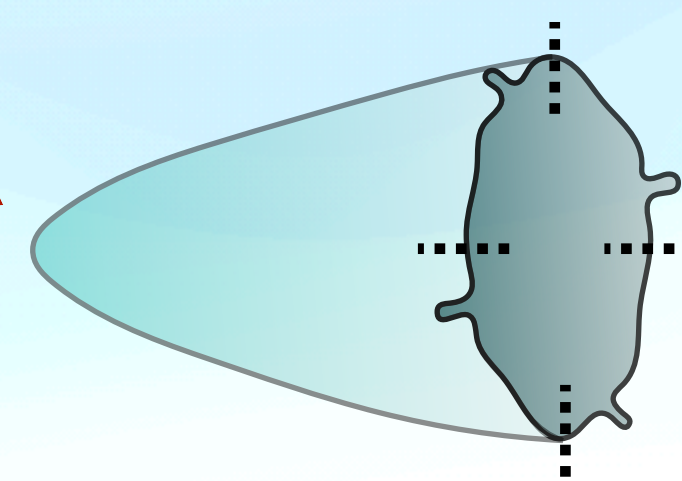
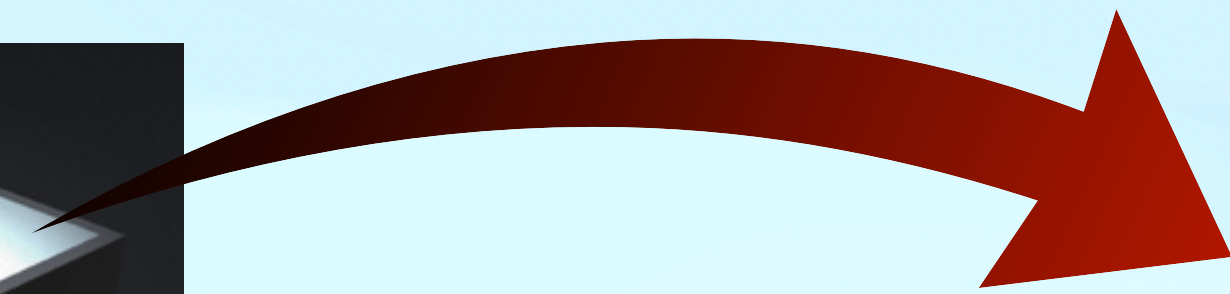
[Lewkowycz-Maldacena, Almheiri-Hartman-Maldacena-Shagoulian-Tajdini,.....]

# Path-Integral v.s. Operator Algebra

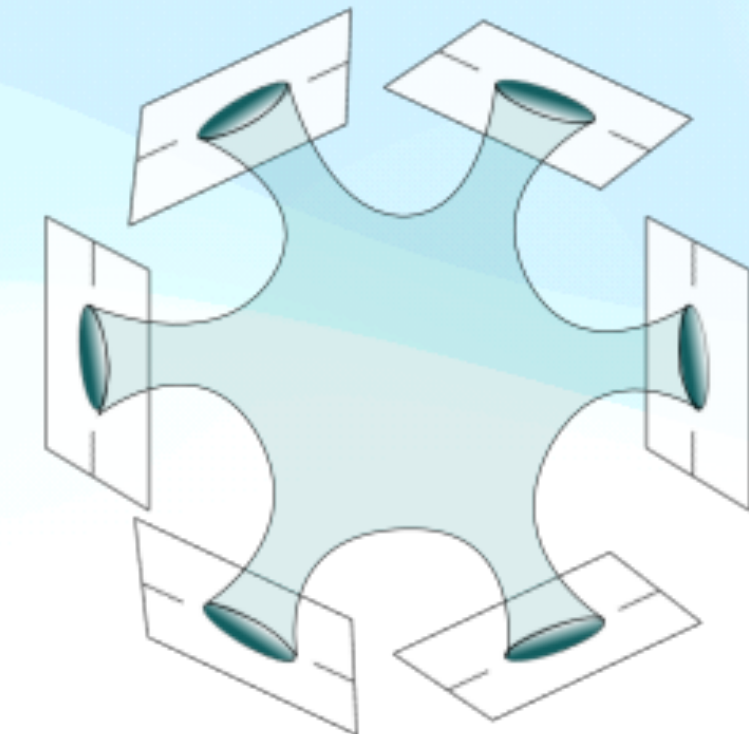
**! NOT CLEAR!**: Why does path-integral know correct answers in quantum gravity??



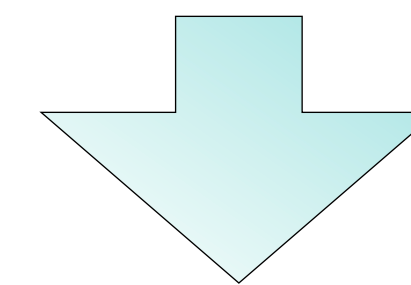
**Gravity path-integral**  
= “**magical black box**”  
that only gives correct answers



[Lewkowycz-Maldacena]



[Almheiri-Hartman-Maldacena-Shagoulian-Tajdini]



**QES formula, Page curve...**

**What's inside the black box ?**

→ **Operator algebra as a complementary approach**

# Operator Algebra for QFT in a Subregion

Entanglement entropy  $\rightarrow$  **Operator algebra** in a local subregion **A**



**von Neumann algebra** : algebra of observables (i.e. self-adjoint operators)

**KNOWN**: vN algebra for a local subregion becomes “**Type III**”

**! NO trace, density matrices or entropies!**

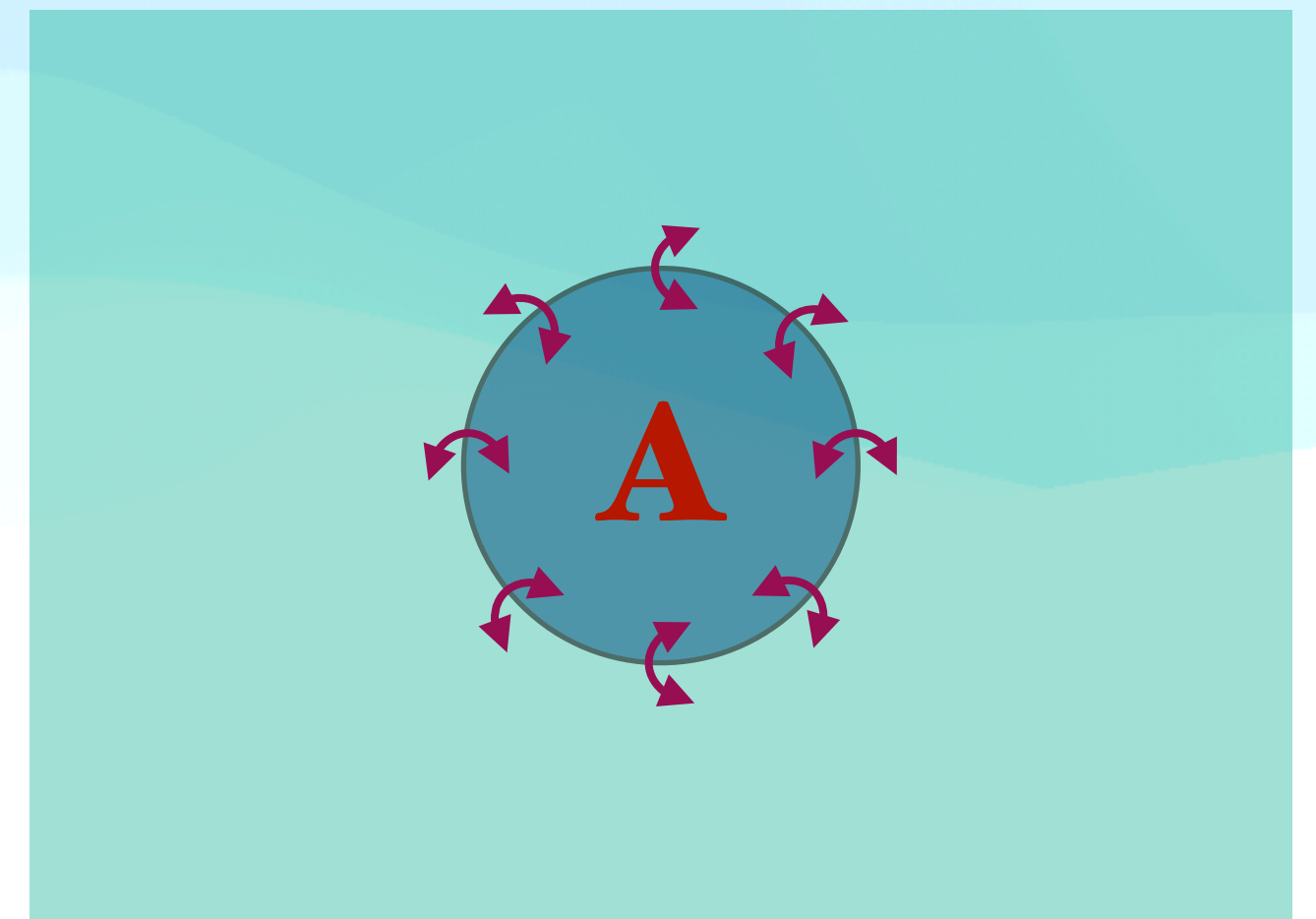
**KNOWN:** vN algebra for a local subregion becomes “**Type III**”

→ Due to **infinite amount of entanglement** near the edge of A

We usually take **a lattice cut-off** by hand

$$S(A) \propto \frac{\text{Area}(\partial A)}{\epsilon^{d-2}} + \dots (d > 2, \text{vacuum})$$

→ violates local properties of QFTs!



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Is there a “natural” way to improve the subregion algebra of a QFT from **Type III** to **Type II** without violating any local properties?

○ trace, density matrices and entropies

# Recent Arguments by Witten

[Leutheusser-Liu]

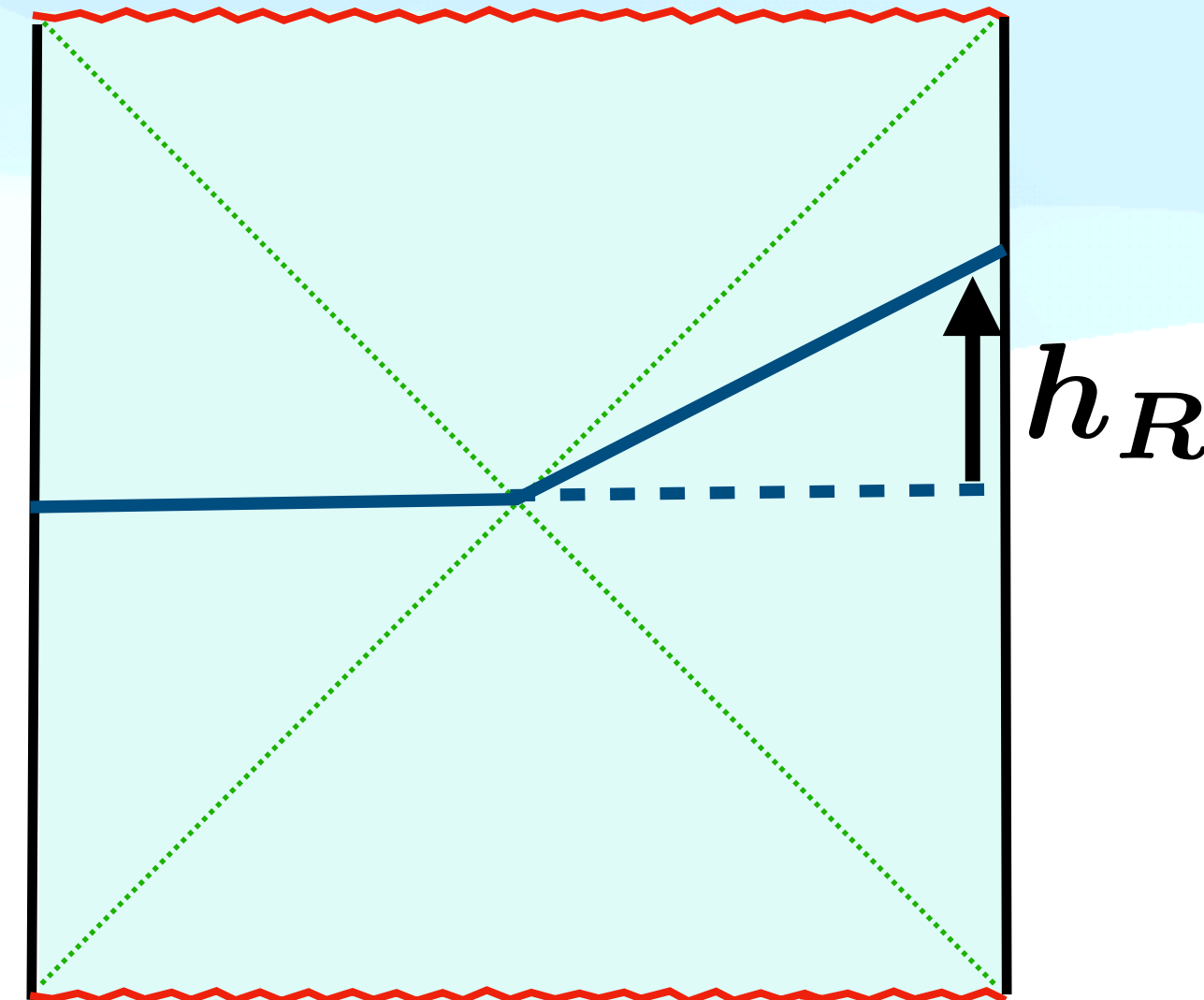
**Type III** vN algebra  $\mathcal{A}_{R,0}$  also appears for a large  $N$  CFT above  $T_{HP}$

$\Leftrightarrow$  dual to an **AdS black hole**

Add (dual to) **ADM Hamiltonian**  $h_R$  to  $\mathcal{A}_{R,0}$ :

$$\mathcal{A}_R = \mathcal{A}_{R,0} \rtimes \mathbb{R} h_R$$

“crossed product”



(Similar to **semi-direct product**, an operation of  $h_R$  generates a group of automorphism of  $\mathcal{A}$ )

**This new algebra becomes Type II $_{\infty}$ !** [proof by Takesaki, Acta Mathematica 131 (73) 249]

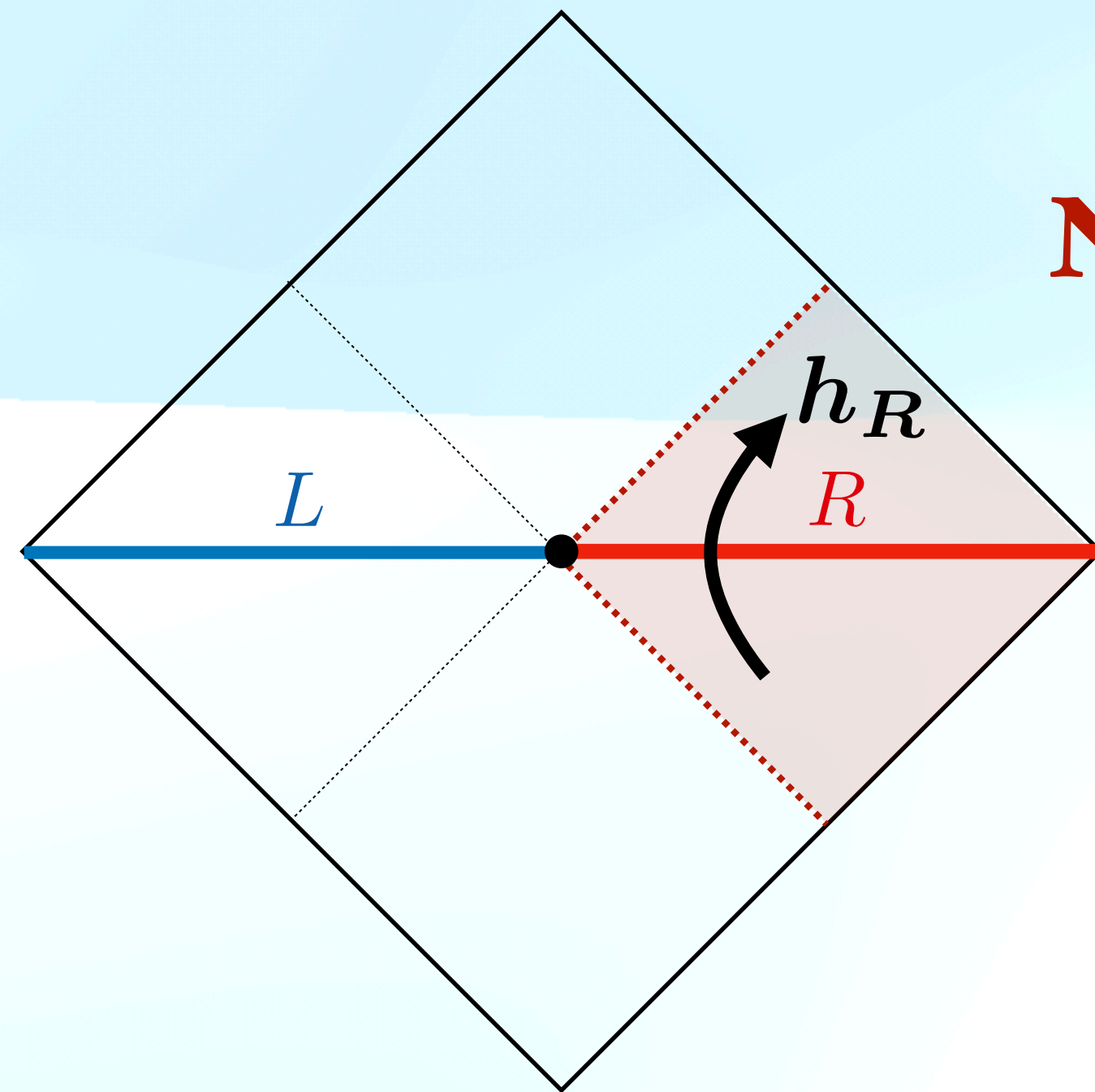
$h_R$ : **gravity** d.o.f, which **does not exist in QFT** on a fixed AdS background

generates “**time-shift**” (physical mode in quantum gravity) on the bdy

# In Our Case of QFT in a Subregion...

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Can we add (one-sided) **modular Hamiltonian** to the subregion algebra??



**NO!:** (one-sided) **modular Hamiltonian**  $h_R$  **does not exist** in a QFT in a subregion  $R$  due to the divergence from the edge of the subregion!

**Need a completely new strategy to get Type II!**

# Main Idea

- Demonstrate our strategy in **CFT<sub>2</sub>** on Minkowski space (& its AdS dual)
- Our strategy can be applied to generic QFTs in any dimensions

Strategy:  **$T\bar{T}$ -deform** the original CFT

$$S_{T\bar{T}} = -\frac{1}{2\Lambda} \int (T_{\alpha\beta} T^{\alpha\beta} - T_{\gamma}^{\gamma 2})$$

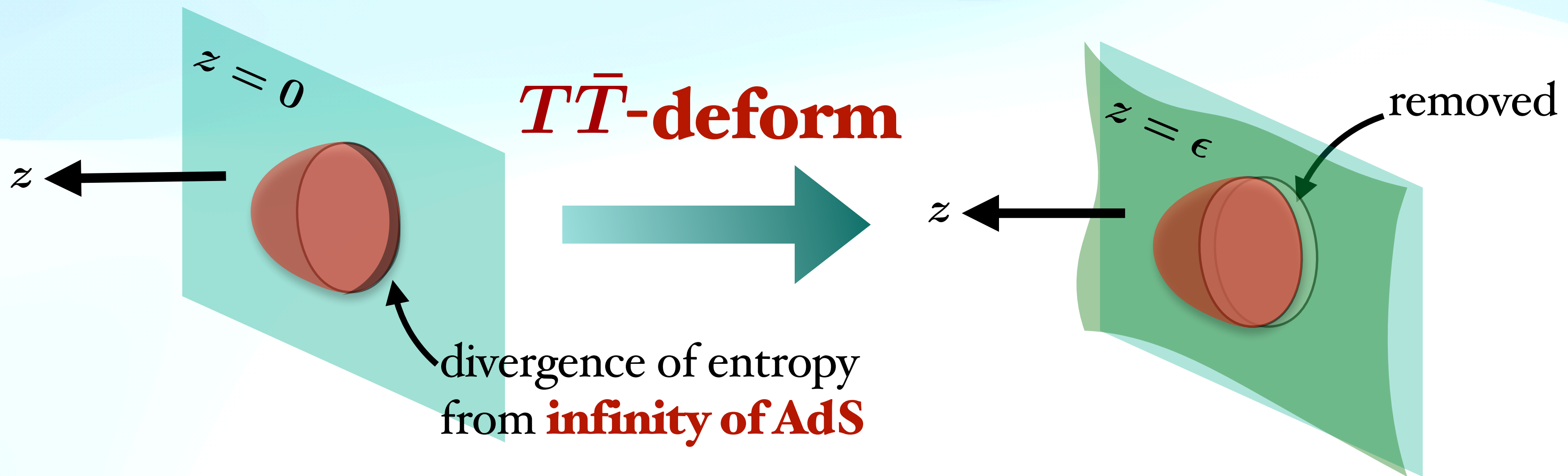
**Why is the subregion algebra uplifted to Type II with entropies?**



# Holographic Interpretation

Strategy:  $T\bar{T}$ -deform the original CFT

$$S_{T\bar{T}} = -\frac{1}{2\Lambda} \int (T_{\alpha\beta} T^{\alpha\beta} - T_{\gamma}^{\gamma 2})$$



$T\bar{T}$ -deformation pushes AdS bdy toward the bulk interior

→ removes divergence coming from infinity of AdS!

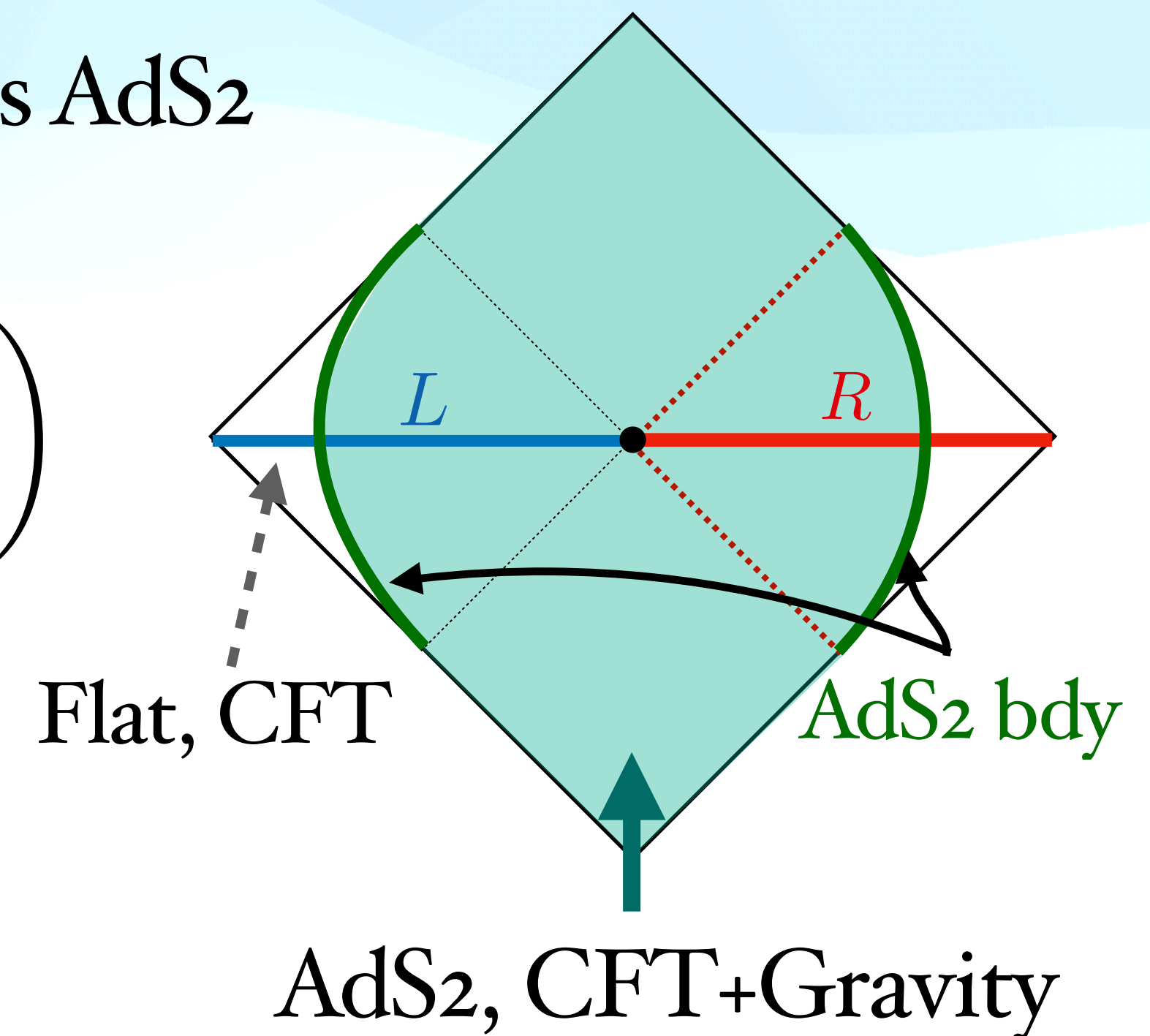
# Coupling Gravity to CFT

**Equivalent** [Dubovsky-Gorbenko-Mirbabayi]
 
 $\longleftrightarrow$ 
 **$T\bar{T}$ -deformation**

 $\longleftrightarrow$ 
**Coupling *flat limit* of JT gravity**

**JT gravity** with finite  $\ell$  : 2d dilaton gravity that describes AdS<sub>2</sub>

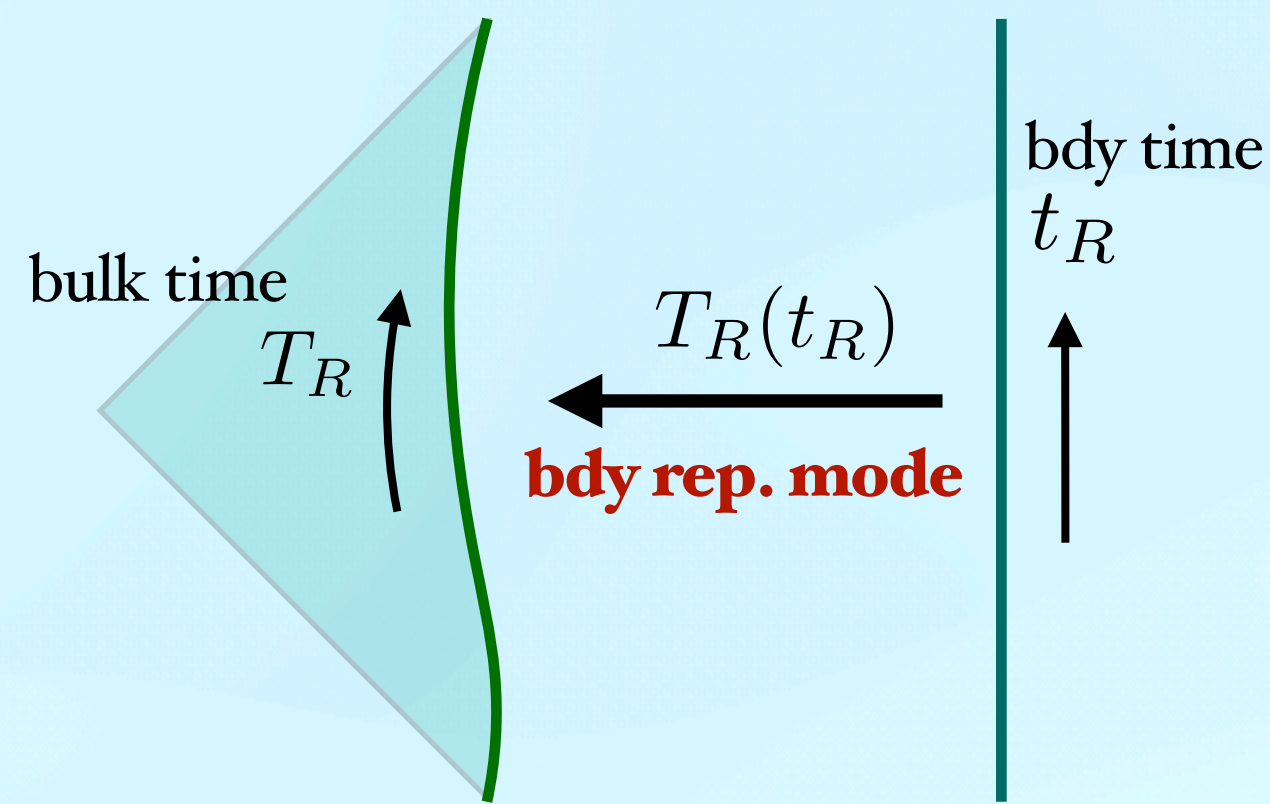
$$S_{\text{JT}} = \int_{\text{bulk}} \left[ \underset{\substack{\uparrow \\ \text{dilaton}}}{\Phi} \left( R + \frac{2}{\underset{\substack{\uparrow \\ \text{AdS radius}}}{\ell^2}} \right) - \Lambda \right] + 2 \int_{\text{bdy}} \Phi \left( K - \frac{1}{\ell} \right)$$



We study **vN algebra with finite  $\ell$** ,  
 and then **take flat limit  $\ell \rightarrow \infty$**

[same setup as “island outside the horizon” by Almheiri-Mahajan-Maldacena’19]

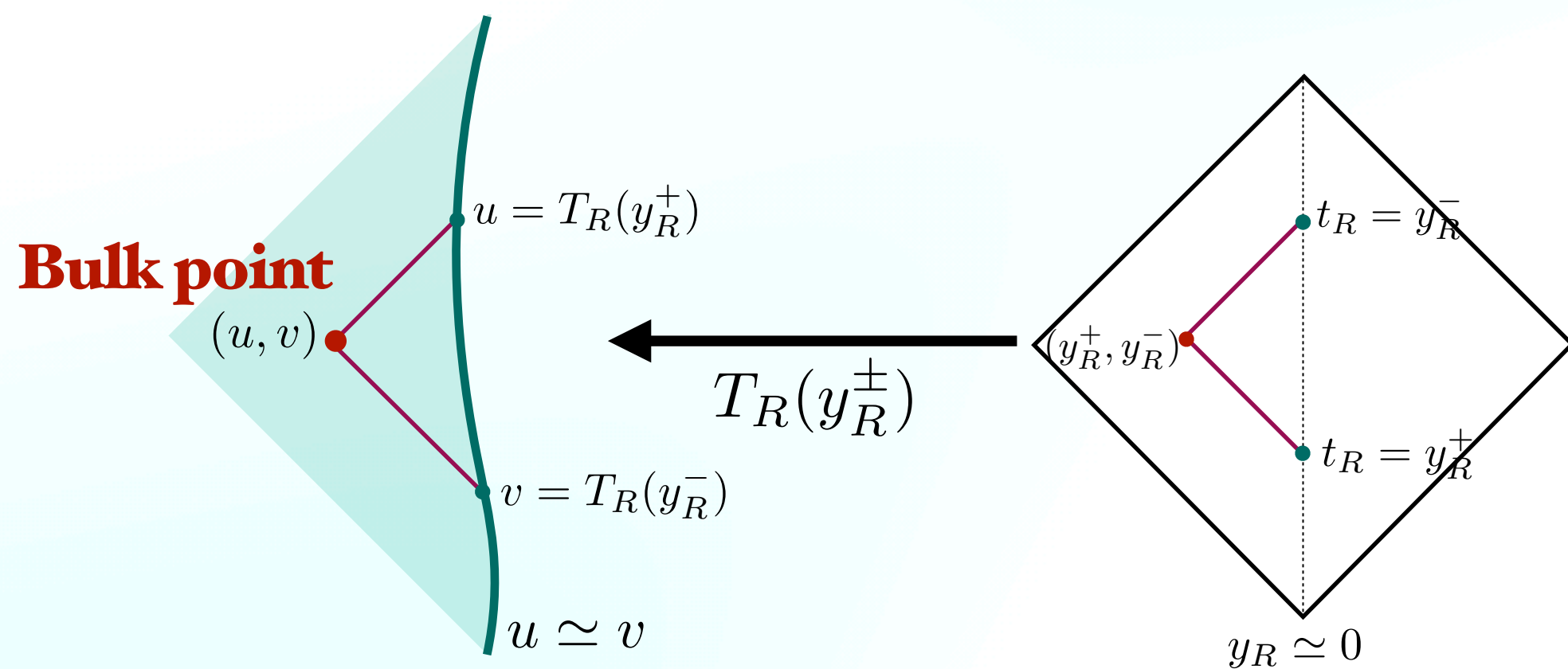
# Gravitationally Dressed Observables



- Dynamics of JT reduces to **boundary time reparametrization**  
**“Schwarzian action”**

$$S_{\partial M} = \frac{\phi_b}{16\pi G_N} \int dt \left( \dot{T}_R^2 + \left( \frac{\ddot{T}_R}{\dot{T}_R} \right)^2 \right)$$

- Gravitationally-dressed observables are defined through **boundary time reparametrization modes**



$$\hat{\mathcal{O}}(u, v) \equiv \mathcal{O} \left( u = T_R \left( y_R^+ \right), v = T_R \left( y_R^- \right) \right)$$

Dynamical frame:  
CFT+Gravity on  $\text{AdS}_2$

Reference frame:  
Rindler wedge in Flat space

Bulk coordinates in  
dynamical frame

Coordinates in reference frame

# Upshot

- **Dilaton at the edge** (=“**area operator**”)  $\Leftrightarrow$  one-sided modular Hamiltonian

$$\begin{array}{ccc} p_{T_R} \sim \frac{\Phi_H}{4G_N} \sim -H_{\text{mod}}^{T_R} \\ \uparrow \qquad \qquad \qquad \qquad \qquad \qquad \uparrow \\ \text{generator of bdy "time-shift"} \qquad \qquad \qquad \text{generator of time-evolution of observables} \end{array}$$

- **Crossed product** arises by **quantizing** reparametrization mode  $T_R$

$$e^{i(p_{T_R} + H_{\text{mod}}^{T_R})t} \hat{\mathcal{O}}(\hat{T}_R) e^{-i(p_{T_R} + H_{\text{mod}}^{T_R})t} = \hat{\mathcal{O}}(\hat{T}_R)$$

**Crossed product**  $\sim$  Hamiltonian constraint in the matter + gravity system

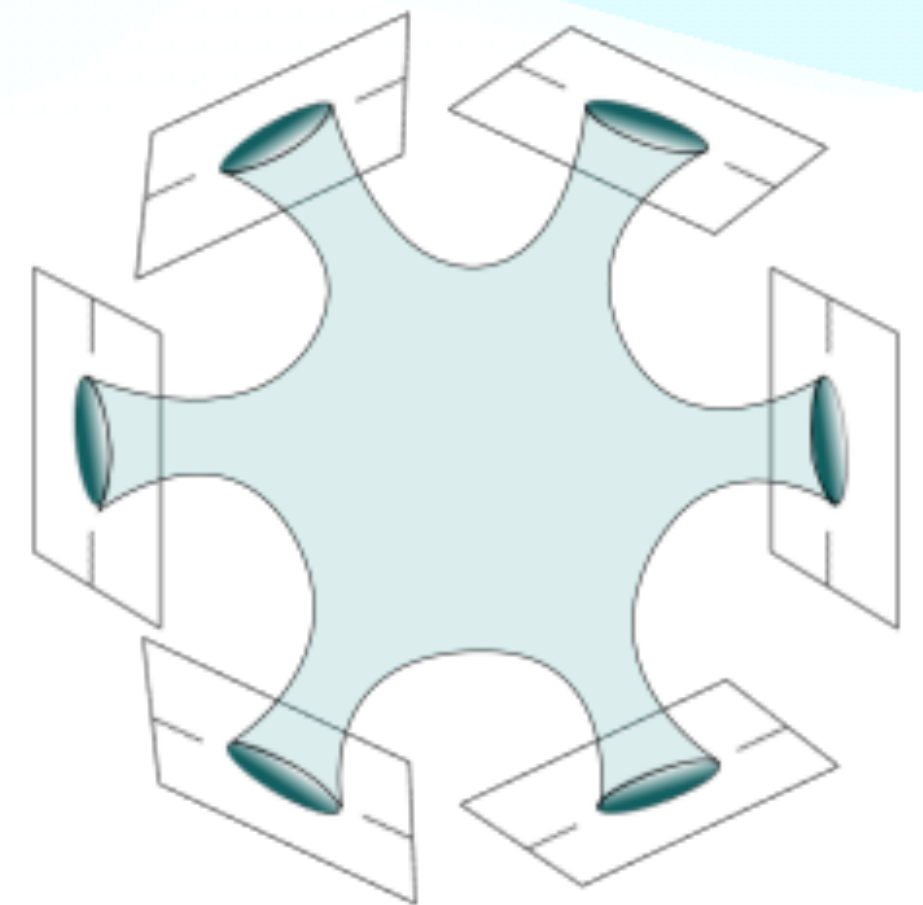
- **QES** for a generic subregion is derived by similar arguments by Witten et al.

# Summary & Future Direction

- Subregion algebra may be uplifted from **Type III** to **Type II** with notion of entropies
- **Full quantum level** of quantum gravity: **Type I**
- To resolve **BH info paradox**  $\Leftrightarrow$  How vN algebra is uplifted to **Type I**

## On the other hand....

- **Gravity path-integral** somehow know its resolution  
(e.g., replica wormhole)



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**Structure of vN algebra & Geometries in gravity path-integral?**

(c.f. Zhencheng's talk yesterday)