Holographic torus correlators of stress tensor in AdS3/CFT2

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ExU-YITP workshop on Holography, Gravity and Quatum information

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Motivations

AdS/CFT correspondence, Maldacena 1997

Dictionary: GKPW

S. S. Gubser, I. R. Klebanov and A. M. Polyakov, 9802109
E. Witten, 9802150

$$Z_{\mathrm{CFT}}[g_{ij}, J] = \int_{G_{\mu\nu}|_{\mathrm{bdy}}=g_{ij}, \Phi|_{\mathrm{bdy}}=J} [dG_{\mu\nu}][d\Phi] e^{-S_{\mathrm{grav}}[G_{\mu\nu}, \Phi]}$$

To check ("prove") the AdS3/CFT2 correspondence:

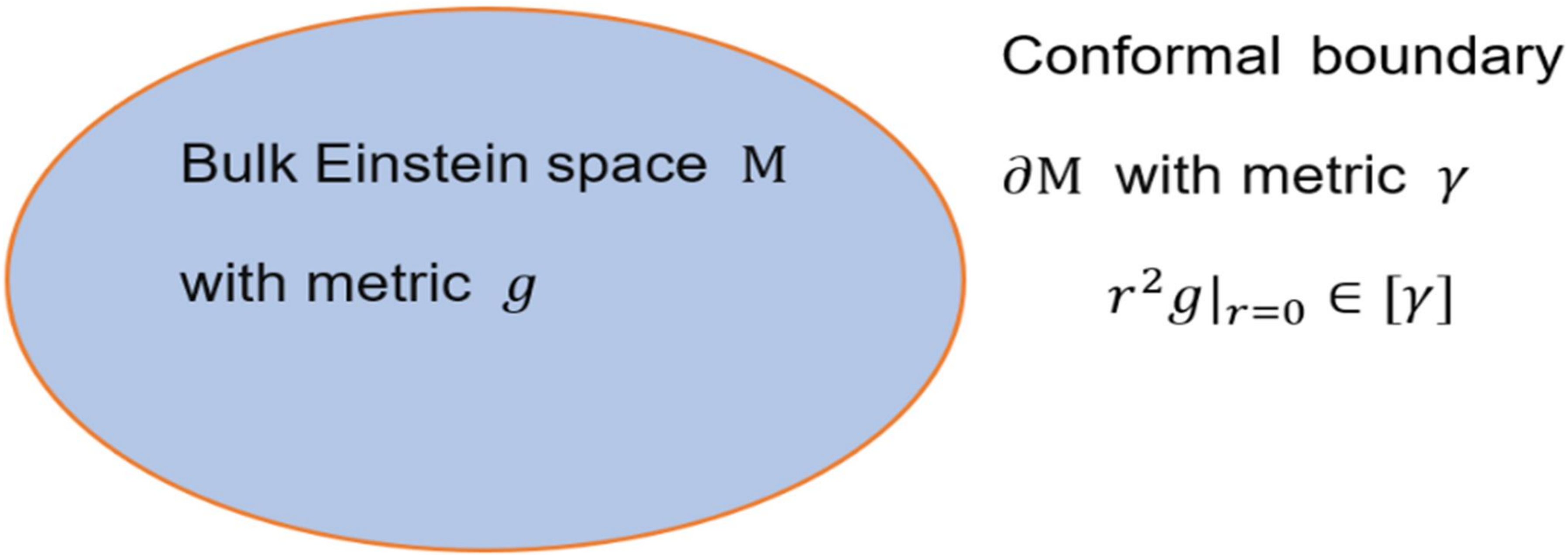
$$\langle O \rangle = -i \frac{\delta Z[\phi_0]}{\delta \phi_0} = \frac{\delta S[\phi_0]}{\delta \phi_0}$$

Partition functions, generic correlation functions, etc.

$$\langle O(x_1) \ldots O(x_n) \rangle_{CFT} \sim \frac{\delta^n I_{grav}}{\delta \psi_0(x_1) \ldots \delta \psi_0(x_n)}$$

Stress tensor Correlators: Distribution of energy, **M**omentum in a given system.

The boundary value problem

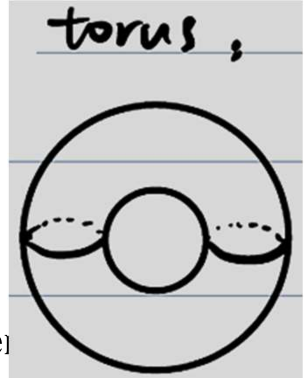


- **In general, for the given conformal boundary torus, we need to consider all gravity saddles with different topology and metric.**
- **Near boundary geometry is well-understood** **[Charles Fefferman, C. Robin Graham, arXiv: 0710.0919, Commun. Math. Phys. 217 (2001) 595-622)]**
- **The global boundary value problem is much more difficult.**

AdS3/CFT2

In AdS3/CFT2, the partition function Alexander Maloney, Edward Witten, 0712.0155

$$\sum_{\alpha} e^{-I^{(\alpha)}_{on-shell}} = Z_{CFT}$$



α labels the saddle points.

- **Thermal AdS3** for low temperature
- S transformation of moduli parameter
- **BTZ black hole** for high temperature

$$S_E = -\frac{1}{16\pi G}\int\sqrt{g}(R-2\Lambda) - \frac{1}{8\pi G}\int\sqrt{\gamma}K + \frac{1}{8\pi G l}\int\sqrt{\gamma}.$$

$$Z = \sum_{g_{\mu\nu}\in\Gamma} e^{-S[g_{\mu\nu}]} Z[g_{\mu\nu}]^{(\text{quantum})}$$

saddle points $\frac{l}{4G} \gg 1$

Holographic stress tensor correlators:

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2}\left[dzd\bar{z} - r^2\pi^2(dz^2 + d\bar{z}^2) + r^4\pi^4 dzd\bar{z}\right]$$

Variation of boundary metric

$$\delta\gamma_{ij}dx^i dx^j = \epsilon f_{ij}(z,\bar{z})dx^i dx^j .$$

Variation of bulk metric

How to find the global well-behaved solution of varied Einstein equations

$$\langle T_{i_1 j_1}(z_1)\ldots T_{i_n j_n}(z_n)\rangle$$

$$= -\frac{(-2)^n \delta^n I[\gamma]}{\sqrt{\det(\gamma(z_1))}\ldots\sqrt{\det(\gamma(z_n))}\delta\gamma^{i_1 j_1}(z_1)\ldots\delta\gamma^{i_n j_n}(z_n)}$$

Holographic prescriptions:

1. **Top-down: Regularity boundary conditions**

2. **Bottom-up: Metric variation= Modular variation** **up to boundary coordinate transformation**

Top-down approach

Thermal AdS3 gravity $S_E = -\frac{1}{16\pi G}\int\sqrt{g}(R-2\Lambda) - \frac{1}{8\pi G}\int\sqrt{\gamma}K + \frac{1}{8\pi G l}\int\sqrt{\gamma}.$

(z, zbar) ~ (z + 1, zbar + 1) ~ (z + τ, zbar + τbar)

Conformal boundary at rho = ∞ or r = 0

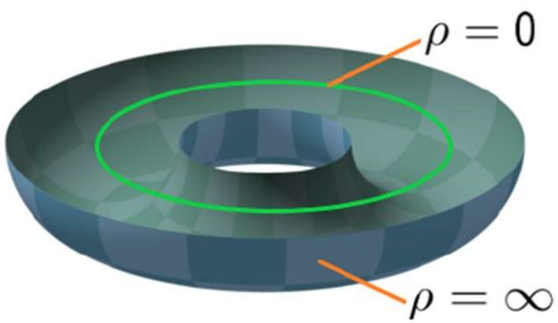
$$ds^2 = d\rho^2 + \cosh^2\rho dt^2 + \sinh^2\rho d\phi^2$$

$$r = \frac{1}{\pi e^\rho},\ z = \frac{\phi + it}{2\pi},\ \bar{z} = \frac{\phi - it}{2\pi}.$$

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2}\left[dzd\bar{z} - r^2\pi^2(dz^2 + d\bar{z}^2) + r^4\pi^4 dzd\bar{z}\right]$$

M. T. Anderson, math/0402198, hep-th/0403087, …

To the first order $ds^2 = (1 + \epsilon\mathcal{L}_{V^{[1]}})(d\rho^2 + \cosh^2\rho dt^2 + \sinh^2\rho d\phi^2) + \epsilon g_{ij}^{FG[1]} dx^i dx^j.$



Regularity boundary conditions: bulk metric at rho=0 be regular.

$$\int_{T^2} d^2z\, g_{0t\phi}^{FG[1]} = 0. \qquad \int_{T^2} d^2z\, g_{2\phi\phi}^{FG[1]} = 0.$$

Two-point function:

$$T_{zz}^{[1]}(z) = \int d^2w \mathcal{G}(z-w)(2T_{zz}^{[0]}\partial_w - \frac{1}{16\pi G}\partial_w^3)f_{\bar{z}\bar{z}}(w) + \frac{1}{16\pi G}(-\partial_z\partial_{\bar{z}}f_{zz} + 2\partial_z^2 f_{z\bar{z}})(z) + C^{[1]}(z),$$

Regularity at rho=0

vary $T_{zz}^{[1]}$ with respect to $f_{\bar{z}\bar{z}}$

$$C^{[1]} = \frac{\pi}{4G\text{Im}\tau}\int_{T^2} d^2z f_{\bar{z}\bar{z}}$$

$$\langle T(z)T(w)\rangle = \frac{c}{12}[\wp_\tau''(z-w) + 4\pi^2\wp_\tau(z-w) + 8\pi^2\zeta_\tau(\frac{1}{2})]$$

Consistent with:
T. Eguchi and H. Ooguri, Nucl. Phys. B 282, 308 (1987)
SH and Y. Sun，arXiv:2004.07486

Hard to obtain the higher point correlation function by using top-down approach !!!

Bottom-up approach

1. Global Variation

   $\bar{\alpha} dz^2 + \alpha d\bar{z}^2$

2. Wely Transformation

   $(1 - \alpha - \bar{\alpha})$

3. Diffeomorphism

   $z + \alpha(\bar{z} - z)$

$\simeq$

- Modular Parameter Variation

  $\Delta\tau = \alpha(\bar{\tau} - \tau)$

$$\int d^2z\left(\frac{\delta}{\delta\gamma_{\bar{z}\bar{z}}(z)} - \frac{\delta}{\delta\gamma_{z\bar{z}}(z)}\right) + \mathcal{L}_{(\bar{z}-z)\partial_z} = (\bar{\tau} - \tau)\frac{\partial}{\partial\tau},$$

$$\int d^2z\left(\frac{\delta}{\delta\gamma_{zz}(z)} - \frac{\delta}{\delta\gamma_{z\bar{z}}(z)}\right) + \mathcal{L}_{(z-\bar{z})\partial_{\bar{z}}} = (\tau - \bar{\tau})\frac{\partial}{\partial\bar{\tau}}.$$

Key ingredient to obtain the higher point correlation function !!!

Acting on lower-point functional

$$(\bar{\tau}-\tau)\partial_\tau\langle O\rangle = \mathcal{L}_{(z-\bar{z})\partial_z}\langle O\rangle + \int_{T^2} d^2z\left(\frac{\delta\langle O\rangle}{\delta\gamma_{\bar{z}\bar{z}}(z)} - \frac{\delta\langle O\rangle}{\delta\gamma_{z\bar{z}}(z)}\right)$$

$$T^{[1]}_{zz}(z) = \int d^2w\,\mathcal{G}(z-w)\left(2T^{[0]}_{zz}\partial_w - \frac{1}{16\pi G}\partial_w^3\right)f_{\bar{z}\bar{z}}(w) + \frac{1}{16\pi G}\left(-\partial_z\partial_{\bar{z}}f_{zz} + 2\partial_z^2 f_{z\bar{z}}\right)(z) + C^{[1]}(z),$$

Then the integral constant is determined as

$$\frac{\delta C^{[1]}}{\delta f_{\bar{z}\bar{z}}(z)} = -\frac{2}{\mathrm{Im}\tau}T^{[0]}_{zz} - 2i\frac{\partial}{\partial\tau}T^{[0]}_{zz}$$

Consistent with top-down approach!

The recurrence relation

- Holographic Virasoro Ward identity for $\gamma_{\bar{z}\bar{z}} = F$

$$\partial_{\bar{z}}\langle T_{zz}\rangle - 2\partial_z F\langle T_{zz}\rangle - F\partial_z\langle T_{zz}\rangle + \frac{1}{16\pi G}\partial_z^3 F = 0.$$

- We obtain a differential equation relating higher and lower point correlators by taking the functional derivative with respect to F, we solve the differential equation with integration constants to obtain the recurrence relation

$$\begin{aligned}\langle T_{zz}(z)T_{zz}(z_1)\dots T_{zz}(z_n)\rangle &= -i\partial_\tau\langle T_{zz}(z_1)\dots T_{zz}(z_n)\rangle + \frac{1}{32\pi^2 G}\delta_{n,1}\wp''_\tau(z-z_1)\\ &- \frac{1}{2\pi}\sum_{i=1}^{n}\Big[2(\wp_\tau(z-z_i)+2\zeta_\tau(\tfrac{1}{2}))\langle T_{zz}(z_1)\dots T_{zz}(z_n)\rangle\\ &+ (\zeta_\tau(z-z_i) - 2\zeta_\tau(\tfrac{1}{2})(z-z_i))\partial_{z_i}\langle T_{zz}(z_1)\dots T_{zz}(z_n)\rangle\Big]\end{aligned}$$

SH and Y. Sun，arXiv:2004.07486

Holographic torus correlators for TTbar deformed CFT

- $T\bar{T}$ deformation

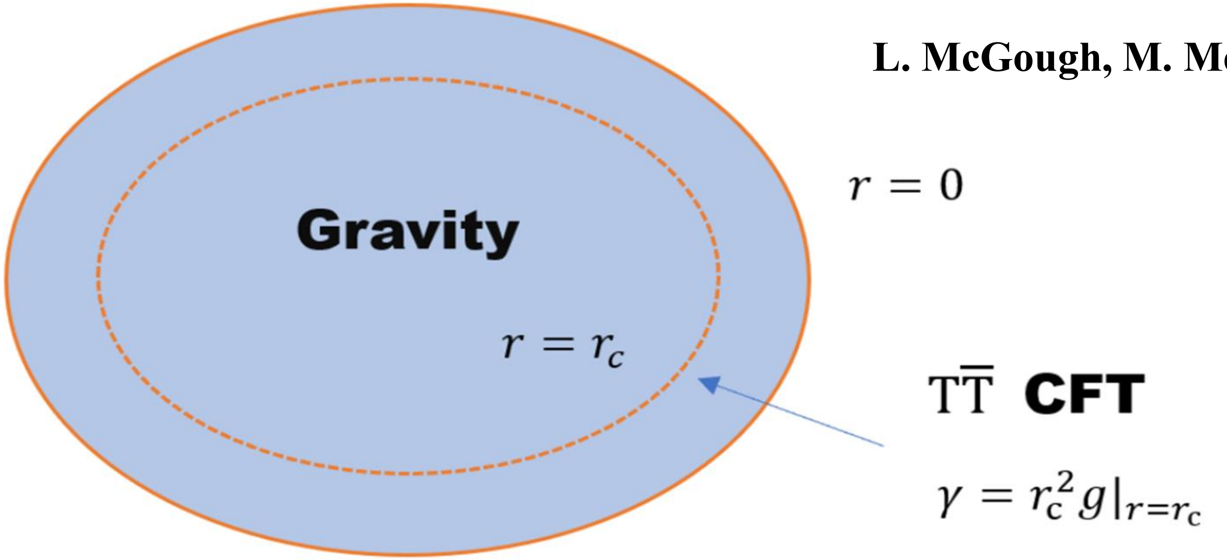
$$\frac{dS^{[\mu]}}{d\mu} = \frac{1}{8}\int d^2x(T^{ij}T_{ij} - T_i^{i\,2})^{[\mu]}$$

Refer to Wei Song's talk

- Cutoff AdS holography

$$\mu = 16\pi G r_c^2$$

L. McGough, M. Mezei, and H. Verlinde, JHEP 04, 010



Correlation function for TTbar CFT

Torus embedded as a cutoff surface $\rho = \rho_c$ in thermal AdS_3

$$ds^2 = d\rho^2 + \pi^2 e^{2\rho}[dZd\bar{Z} - e^{-2\rho}(dZ^2 + d\bar{Z}^2) + e^{-4\rho}dZd\bar{Z}]$$

Taking periods in Z to be 1 and $\Omega = \frac{\tau + e^{-2\rho_c}\bar{\tau}}{1+e^{-2\rho_c}}$ and defining the coordinates for torus $z = \frac{Z - e^{-2\rho_c}\bar{Z}}{1-e^{-2\rho_c}}$, $\bar{z} = \frac{\bar{Z} - e^{-2\rho_c}Z}{1-e^{-2\rho_c}}$

$$r = \frac{1}{\pi e^{\rho}(1 - e^{-2\rho_c})}$$

$$\langle T_{zz}\rangle = -\frac{\pi}{8G}\frac{1-e^{-2\rho_c}}{1+e^{-2\rho_c}}, \quad \langle T_{\bar{z}\bar{z}}\rangle = -\frac{\pi}{8G}\frac{1-e^{-2\rho_c}}{1+e^{-2\rho_c}}$$

$$\langle T_{z\bar{z}}\rangle = \frac{\pi}{8G}\frac{e^{-2\rho_c} - e^{-4\rho_c}}{1+e^{-2\rho_c}}.$$

$$i\partial_\tau I = \langle T_{zz}\rangle - \langle T_{z\bar{z}}\rangle$$

$$-i\partial_{\bar{\tau}} I = \langle T_{\bar{z}\bar{z}}\rangle - \langle T_{z\bar{z}}\rangle$$

J. Cardy, JHEP 10, 186,
S. Datta and Y. Jiang, JHEP 08, 106,
SH, Y. Sun, and Y.-X. Zhang, JHEP 09, 061

Higher point correlators… refer to our paper.

Summary

- **Proposed prescription to study Holographic torus stress tensor correlator which are consistent with CFTs data.**
- **Offer a precise a check AdS3/CFT2.**
- **The correlators of stress tensor in holographic TTbar deformed CFT.**
- **Average ensemble, other topologies, generic operators, higher dimension[SH, Yi Li, 2308.13518[hep-th]], etc.**

Thanks for your attention