New faces of holographic entanglement

Matthew Headrick

based on

arXiv:2208.10507 w/ Veronika Hubeny

work in progress w/ Brianna Grado-White, Guglielmo Grimaldi, Veronika Hubeny

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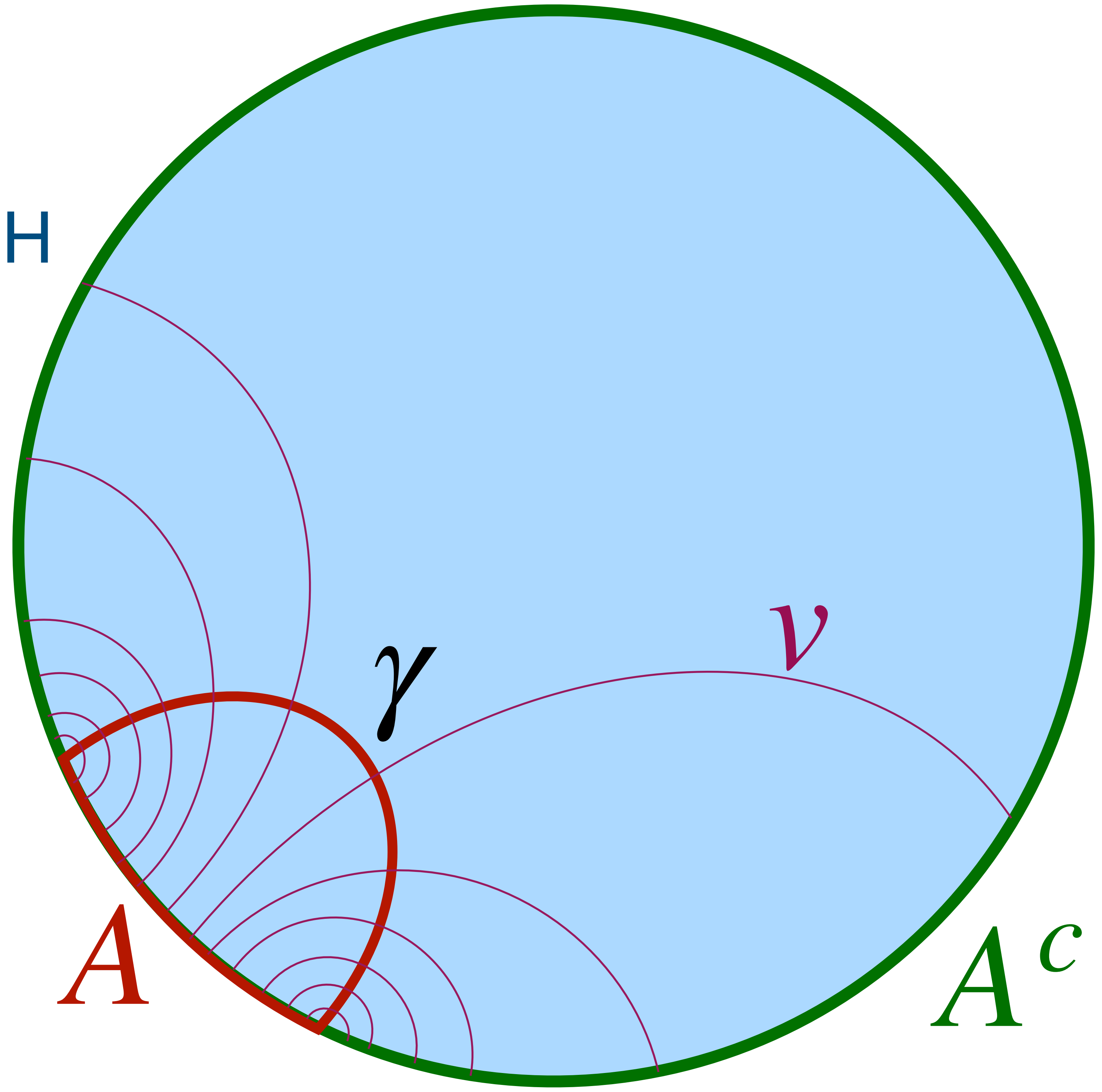
The Ryu-Takayanagi formula & its generalizations have revolutionized our understanding of holography & quantum gravity

$$S(A) = \frac{1}{4G} \min_{\gamma \sim A} \mathrm{area}(\gamma)$$

$$= \max_{v} \int_A v \qquad (\nabla \cdot v = 0, |v| \leq 1)$$

Freedman-MH

convex program; field lines of v are "bit threads"



time slice of static spacetime

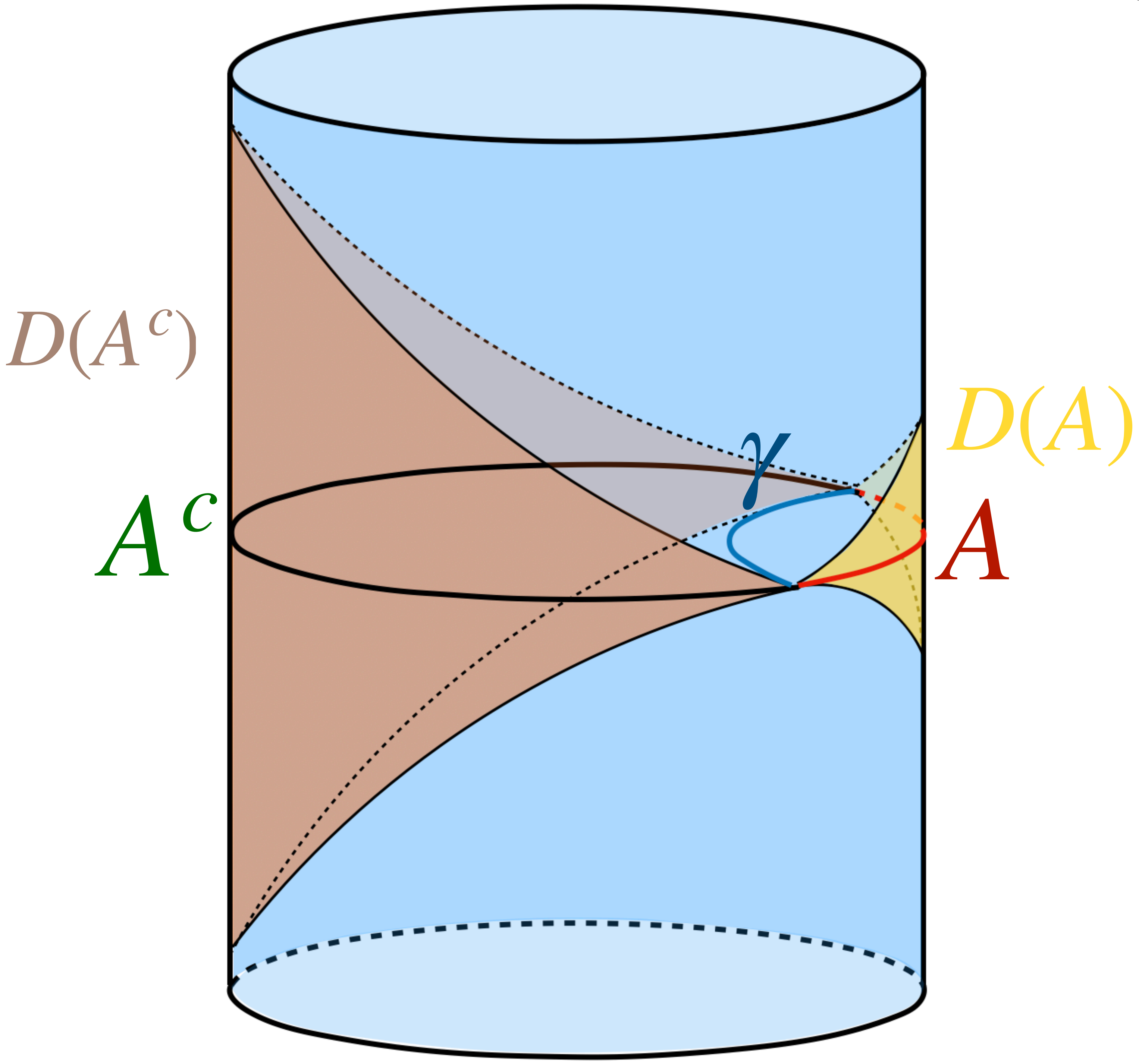
Generalizations in many directions:

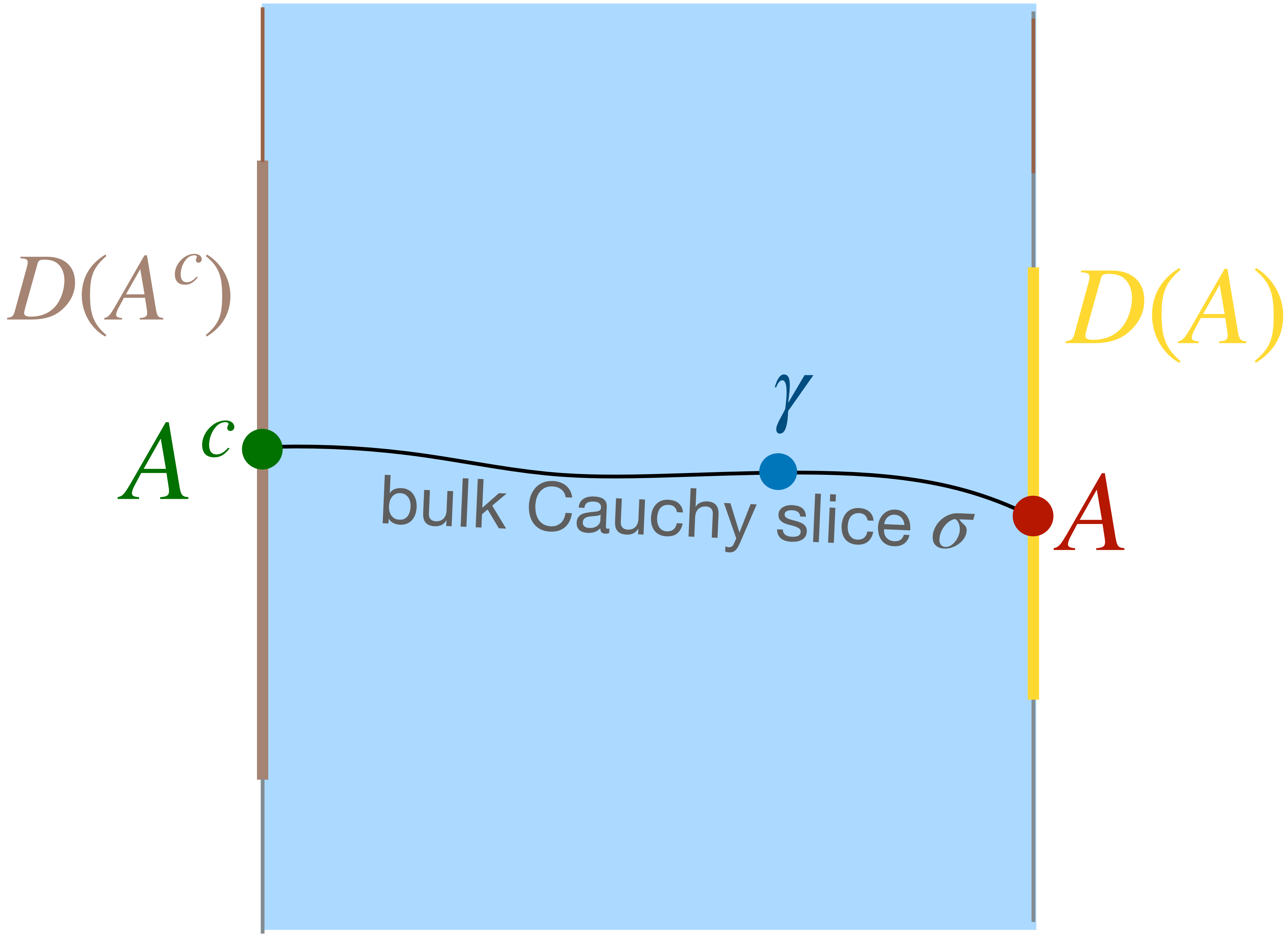
- time dependence
- quantum corrections
- higher-derivative corrections
- Rényi entropies
- reflected entropies
- Python's lunch
- flat space, de Sitter, cosmology, …
- …

Hubeny-Rangamani-Takayanagi *covariant* entanglement entropy formula:

$$S(A) = \min_{\substack{\gamma \sim A \\ \text{extremal}}} \text{area}(\gamma) = \max_{\sigma} \min_{\sigma \supset \gamma \sim A} \text{area}(\gamma)$$

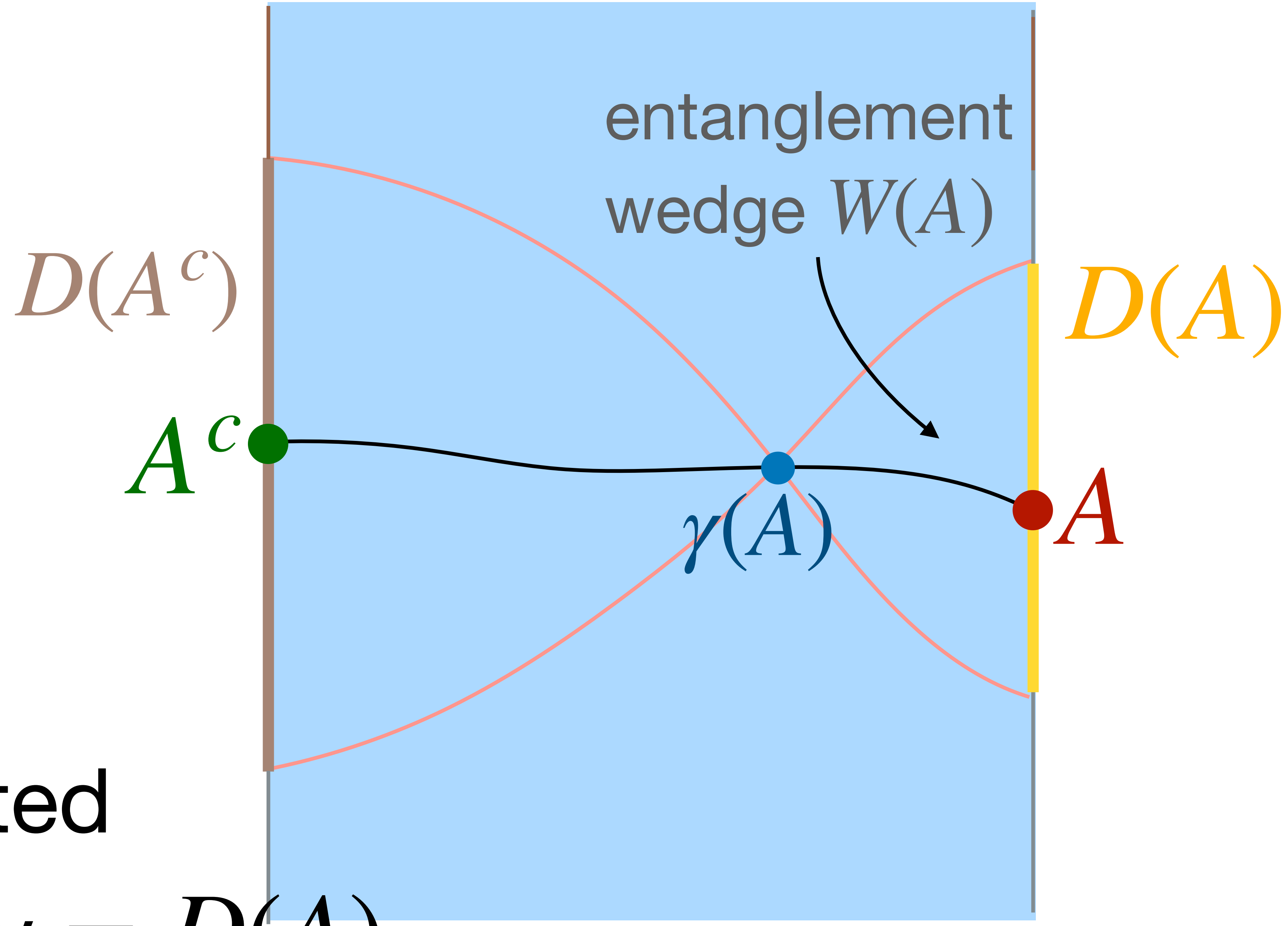
(maximin) Wall





HRT maximin

$$S(A) = \min_{\substack{\gamma \sim A \\ \text{extremal}}} \text{area}(\gamma) = \max_{\sigma} \min_{\sigma \supset \gamma \sim A} \text{area}(\gamma)$$



Can be used to prove:

- existence of HRT surface
- reduces to RT w/time-reflection symmetry
- competing HRT surfaces are spacelike-separated
- consistency w/boundary causality: $W(A) \cap \text{bdy} = D(A)$
- entanglement wedge nesting & complementarity:

$$W(A) \subset W(AB), \;\; W(A) \cap W(A^c) = \gamma(A) = \gamma(A^c)$$

- entropy inequalities:
  - subadditivity: $S(AB) \leq S(A) + S(B)$
  - strong subadditivity: $S(B) + S(ABC) \leq S(AB) + S(BC)$

Crucial consistency checks on HRT formula & subregion duality

Use full dynamics: Einstein eq, null energy condition, AdS boundary conditions

For RT, an infinite set of further inequalities, not general properties of quantum states, have been proven:

- MMI: $S(A) + S(B) + S(C) + S(ABC) \leq S(AB) + S(BC) + S(AC)$
- 5-party dihedral:

Hayden-MH-Maloney

$$S(AB) + S(BC) + S(CD) + S(DE) + S(EA) + S(ABCDE) \leq S(ABC) + S(BCD) + S(CDE) + S(DEA) + S(EAB)$$

Bao-Nezami-Ooguri-Stoica-Sully-Walter

- …

These inequalities define the *RT entropy cone*

Full set of inequalities (or other characterization of allowed entropies) is unknown

Constrain entanglement structure of static holographic states, but meaning and implications remain unclear

Bao, Czech, Fadel, Hayden, He, MH, Hernández-Cuenca, Hubeny, Mezei, Rangamani, Rota, Shuai, Stoica, Walter, Wang,…

Valid for time-dependent states?

Using maximin one can prove *only* MMI Wall, Rota-Weinberg

Proved for topologically trivial 3d bulk Czech-Dong

In the belief that it’s useful to have more perspectives on this crucial entry in the holographic dictionary, I’ll describe 3 more equivalent formulations of the HRT formula:

1) V-threads
2) U-threads
3) Minimax

(Equivalence to HRT requires NEC, AdS bounday conditions)

I’ll then give two applications of minimax:

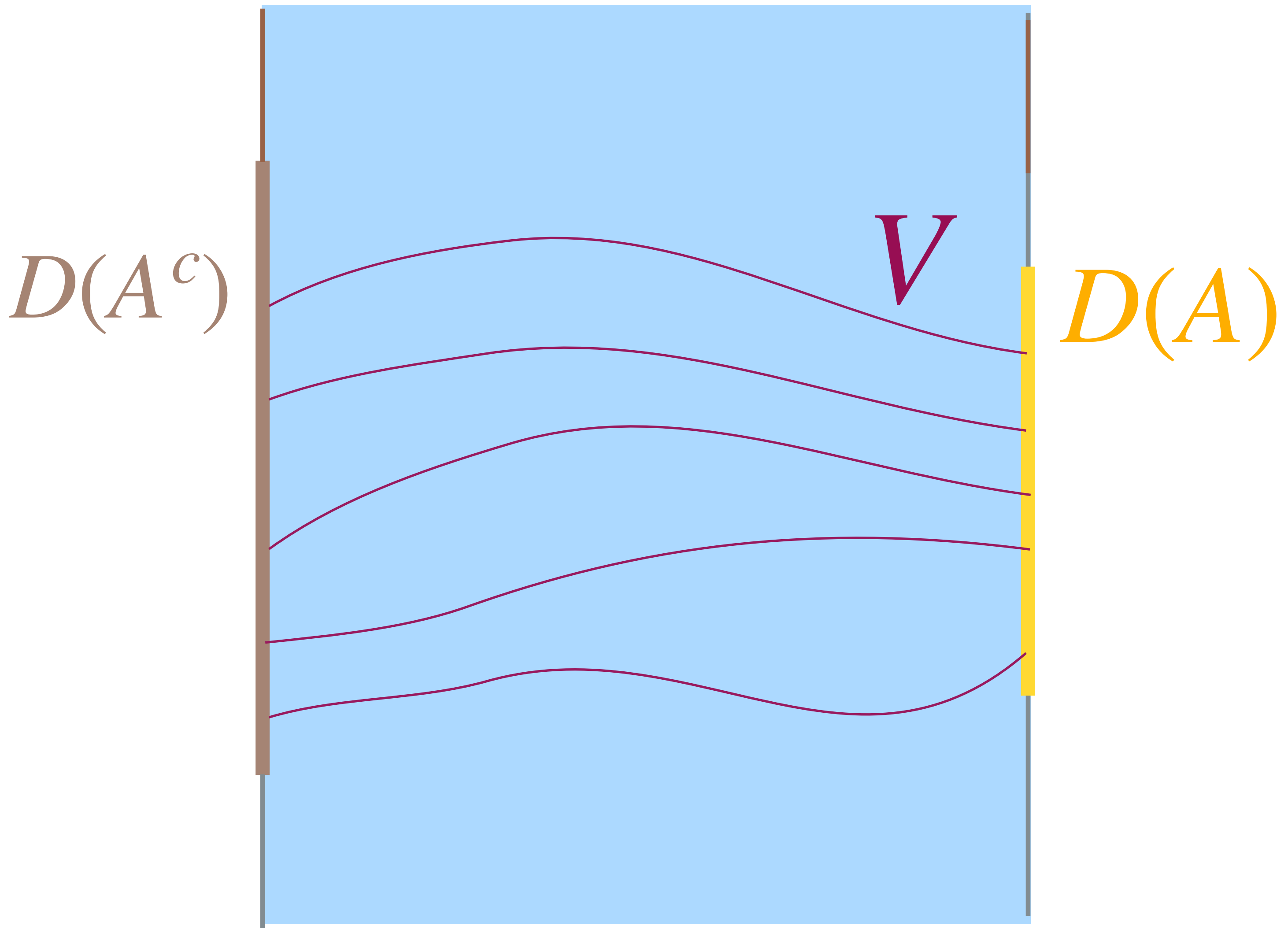
a) Graph model & entropy inequalities
b) Entangled universes

1) *V-threads:* $S(A) = \max_V \int_{D(A)} V$

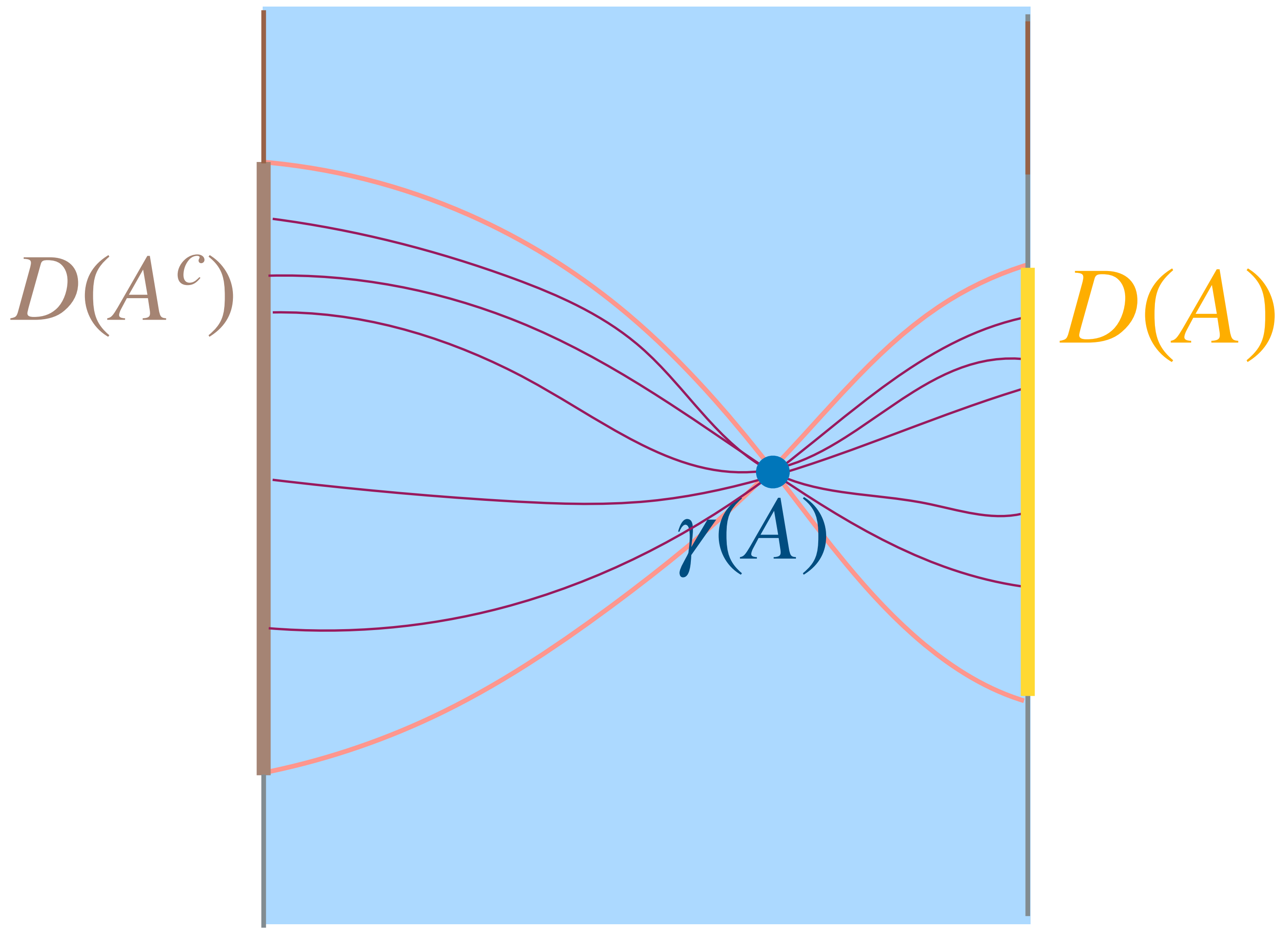
$$\nabla \cdot V = 0\,, \quad V|_{(D(A)\cup D(A^c))^c} = 0\,, \quad \int_{\mathscr{C}} d\tau\, |V_\perp| \leq 1 \quad (\mathscr{C} = \text{any timelike curve})$$

(convex program)

(can be written as local constraint by adding clock function)



allowed V



optimal V: finds $\gamma(A)$, entanglement wedges

Morally, V-threads are *Bell pairs*

MH-Hubeny

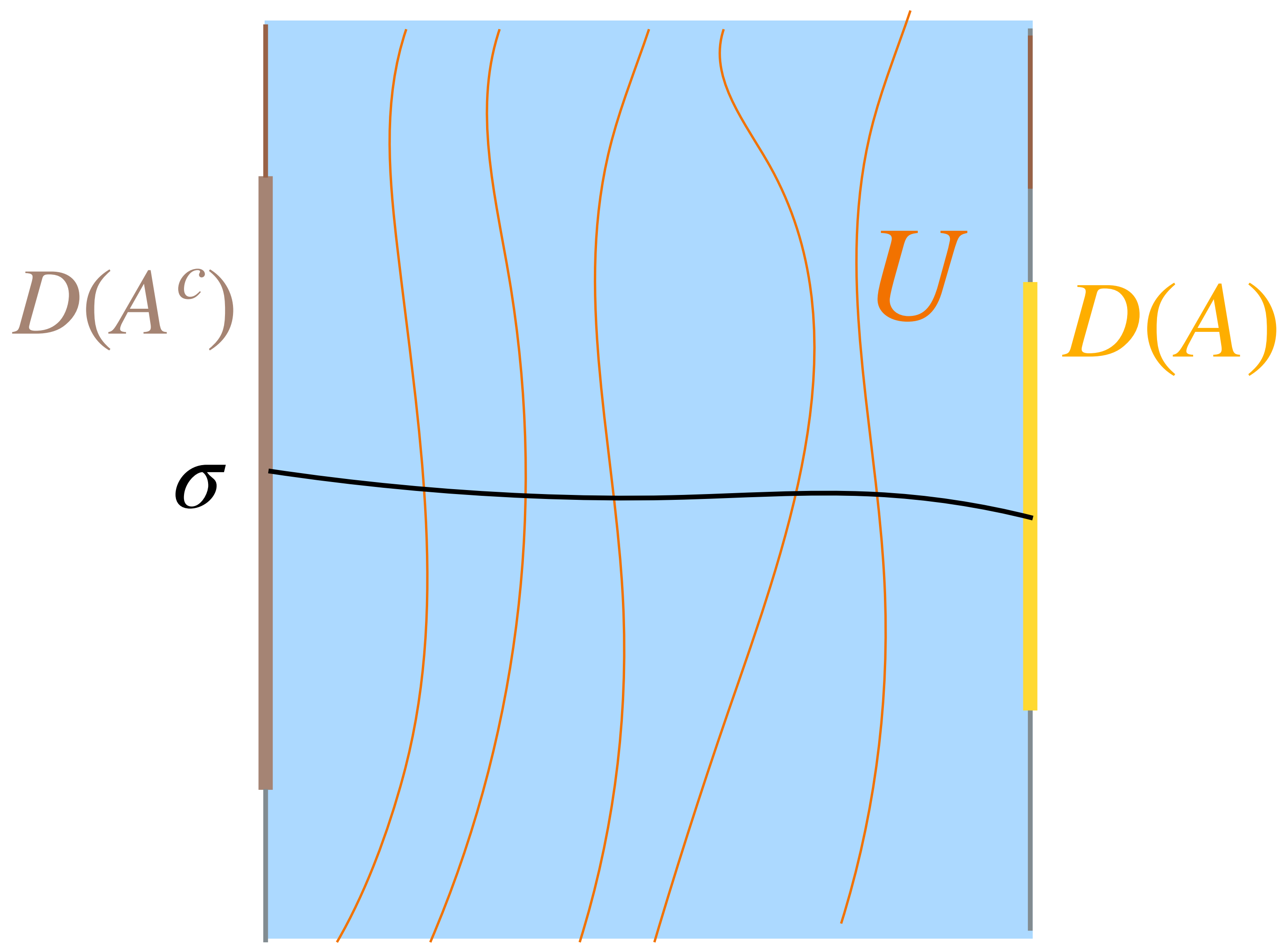
2) *U-threads*: $S(A) = \min_U \int_\sigma U$ (any bulk Cauchy slice containing A, A^c)

U timelike, $\nabla \cdot U = 0\,,\ U|_{D(A) \cup D(A^c)} = 0$

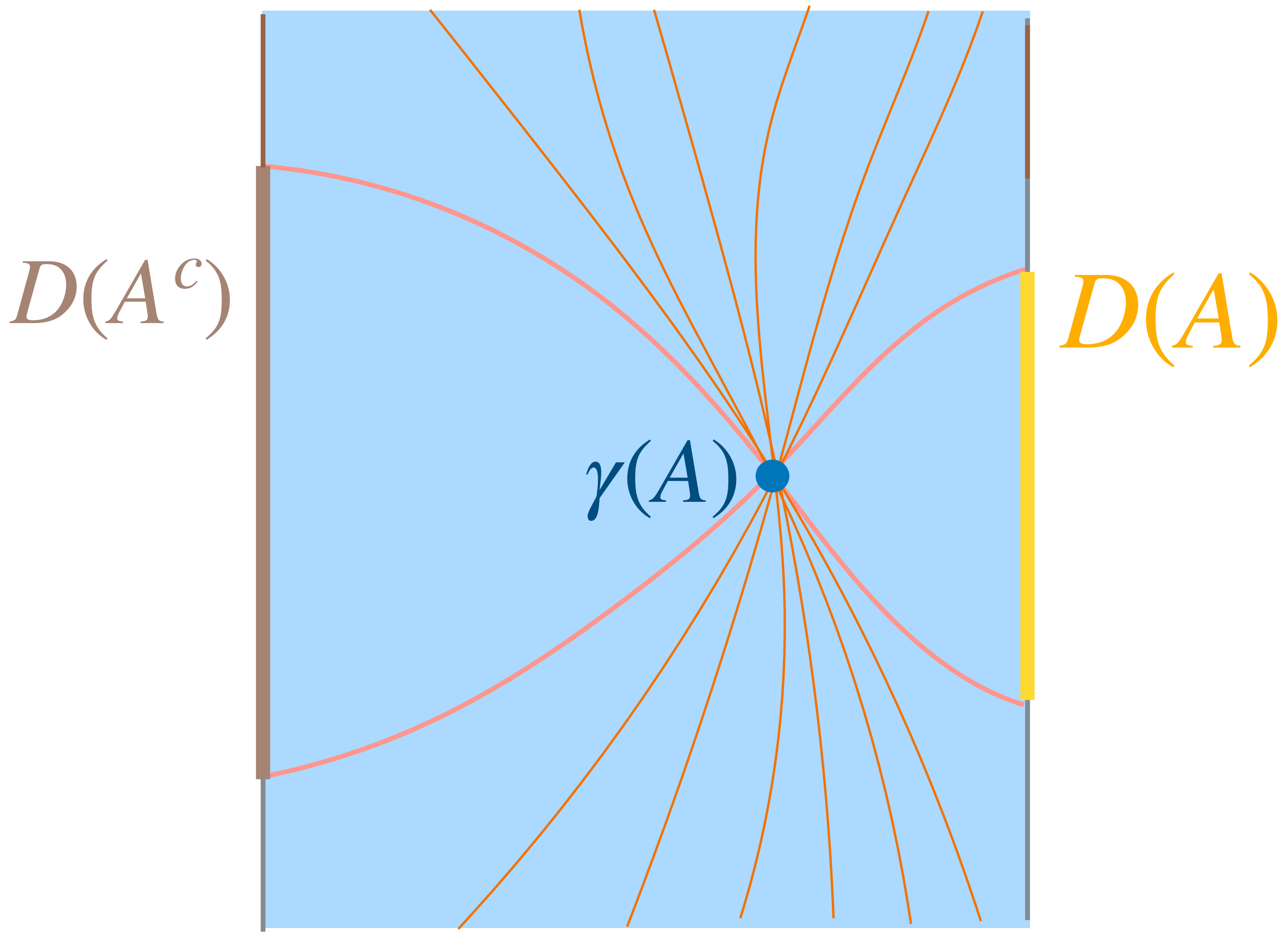
(convex program, dual of V-threads)

$\int_{\mathscr{C}} ds\, |U_\perp| \geq 1$ ($\mathscr{C}$ = any curve from $D(A)$ to $D(A^c)$)

(can be rewritten as local constraint by adding function interpolating between $D(A)$ & $D(A^c)$)



allowed U



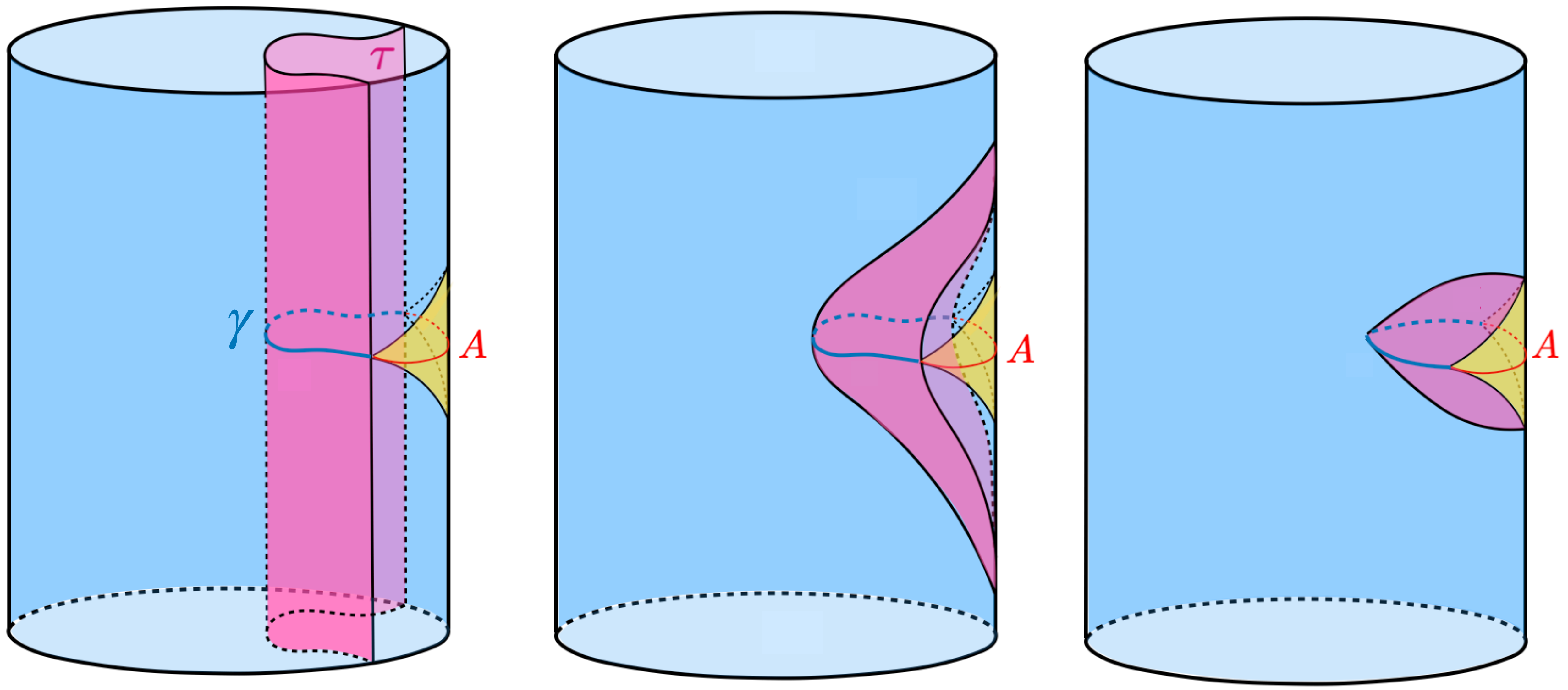
optimal U: finds $\gamma(A)$, entanglement wedges

Morally, U-threads are *disentanglers*

3) Minimax: $S(A) = \min_{\tau \sim D(A)} \max_{\gamma \subset \tau \text{ achronal}} \text{area}(\gamma)$ (τ is a timelike hypersurface: "timesheet")

MH-Hubeny

spacetime homology, rel. $(D(A) \cup D(A^c))^c$

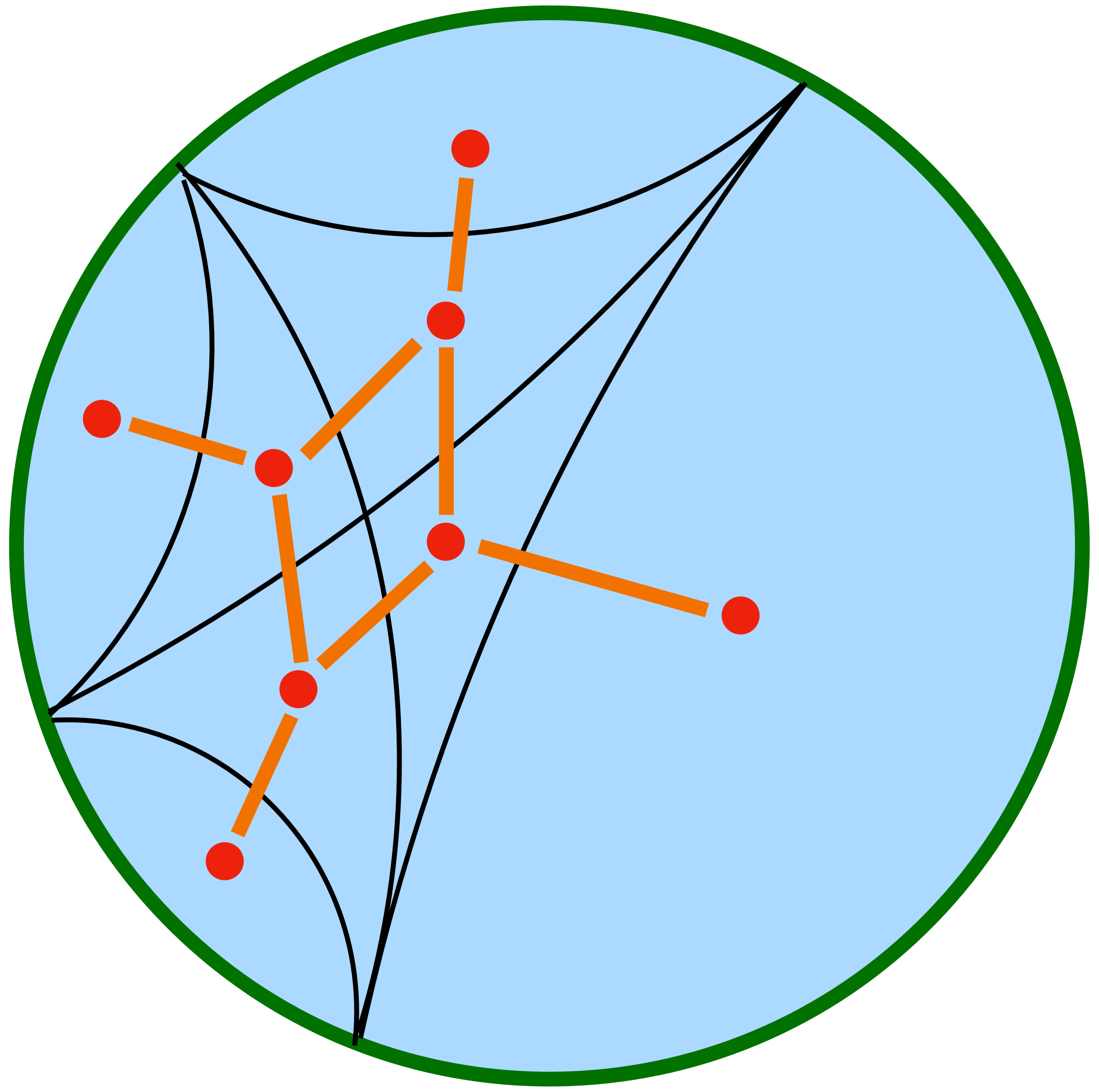


Minimax timesheet is highly non-unique (floppy) away from HRT

Using minimax, can *define* $W(A)$ as the smallest spacetime homology region, prove its properties

Grado-White-Grimaldi-MH-Hubeny

Graph model & entropy inequalities



Back to RT:

- Fix a set of boundary regions $A, B, \ldots$
- Decompose bulk along all RT surfaces $\gamma(A), \gamma(AB), \ldots$
- Vertex = bulk cell
- Edge = partial surface, weight = area
- On resulting weighted graph ("dessication"), entropies are min cuts

Bao-Nezami-Ooguri-Stoica-Sully-Walter

Is there a *covariant dessication* for HRT:

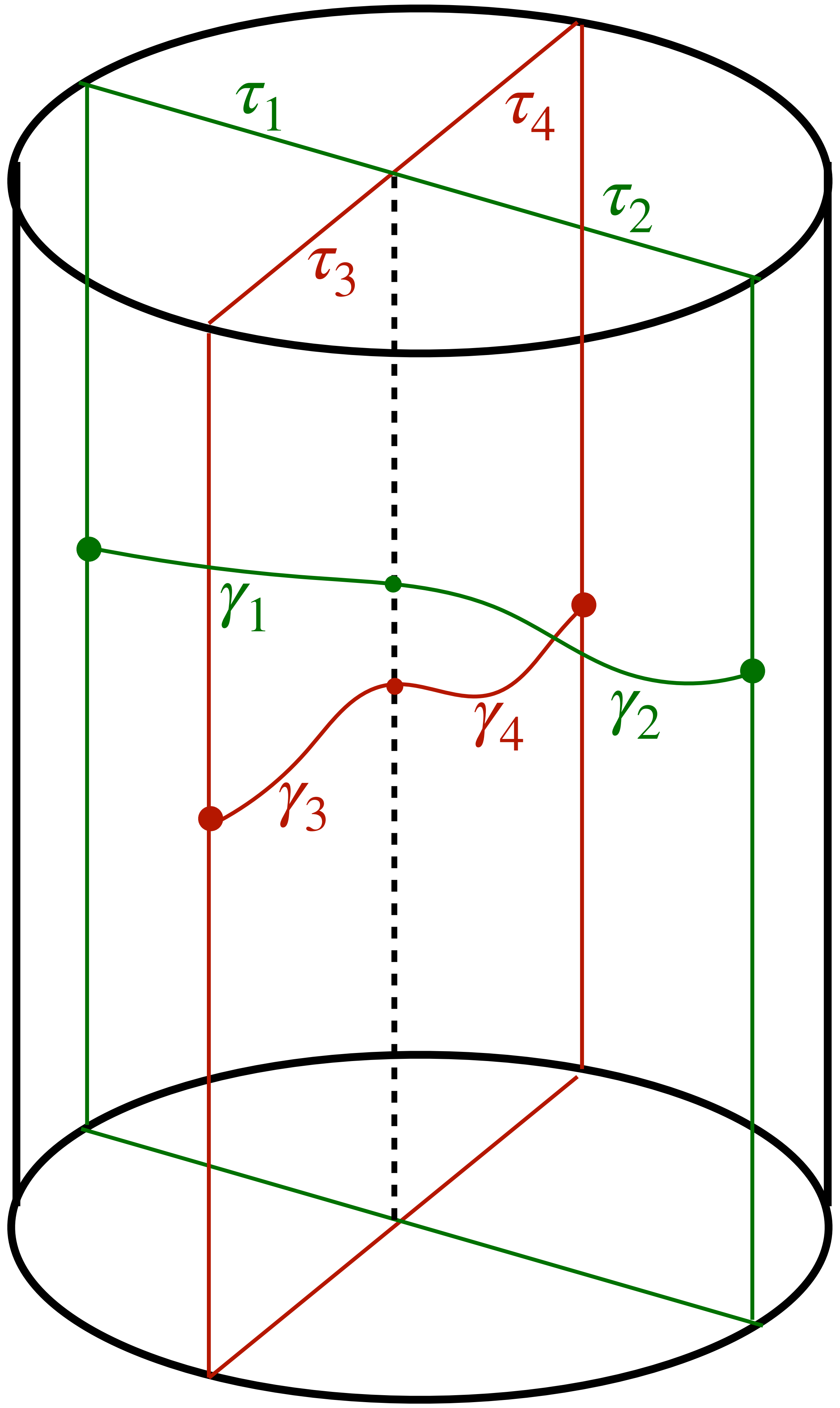
weighted graph that encodes all entropies by *min cuts*?

Intersecting timesheets cut HRT surfaces

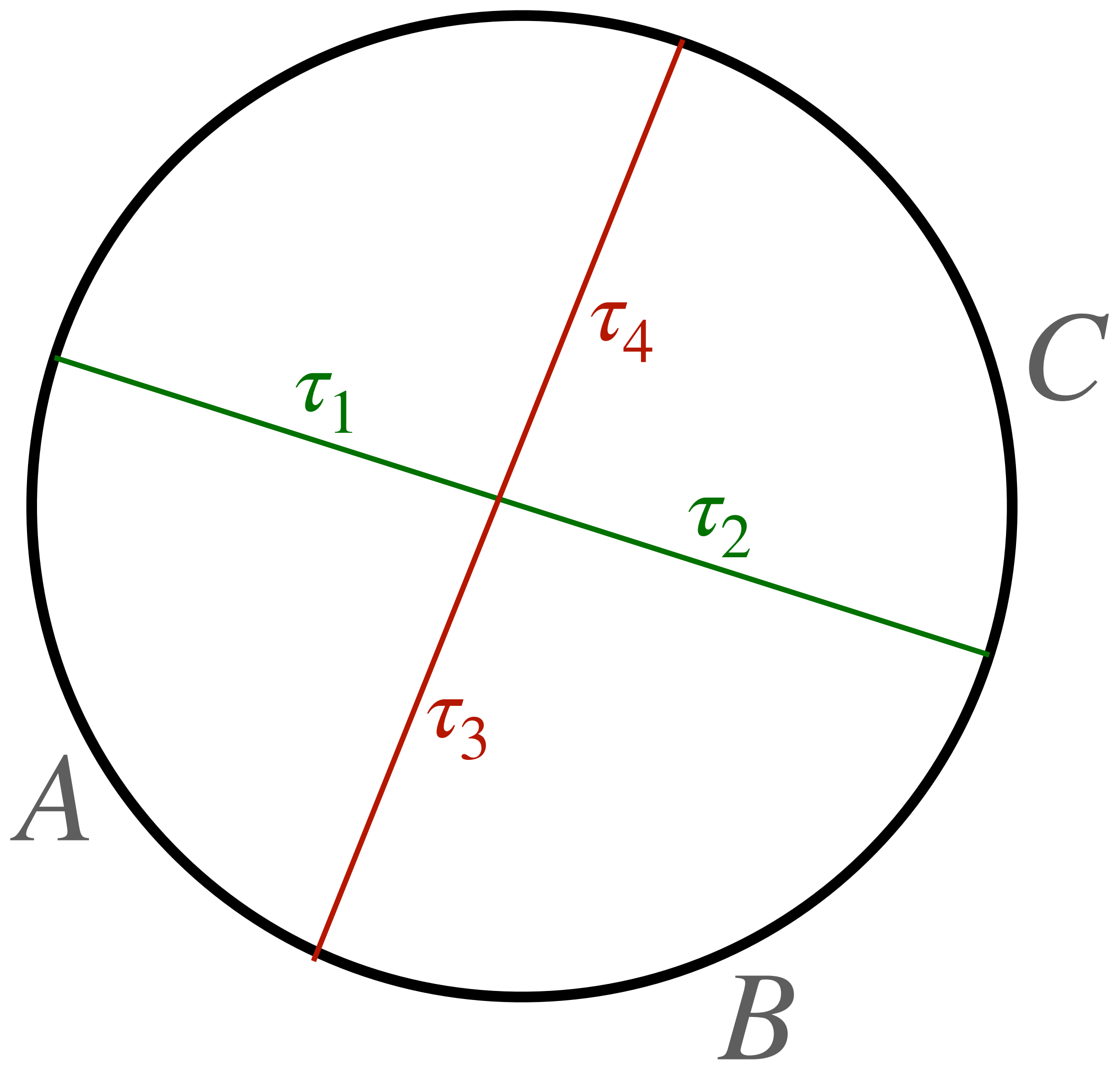
Grado-White-Grimaldi-MH-Hubeny

Timesheets **cooperate** if every partial HRT surface γ_i is maximal on partial timesheet τ_i

Conjecture: For any set of boundary regions (on common Cauchy slice), cooperating timesheets exist



top view



Graph model (dessication):

- vertex = spacetime cell (of WdW patch)
- edge weight = area(γ_i)

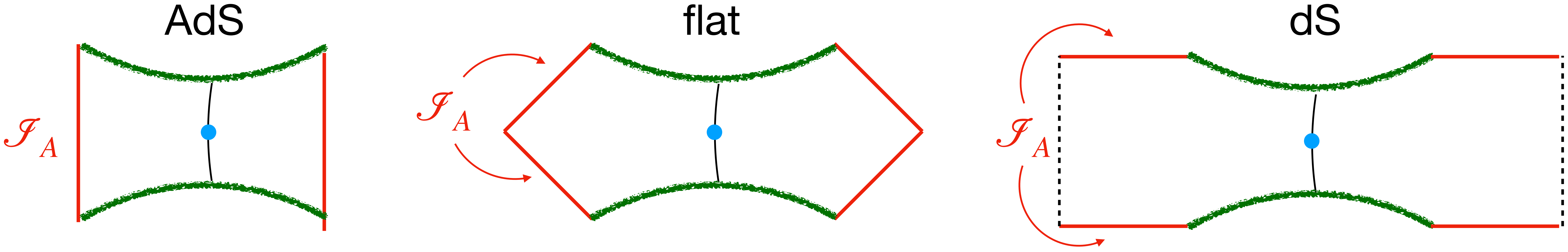
Theorem: Entropies = min cuts on graph

Corollary: All RT entropy inequalities are valid for HRT

HRT cone = RT cone

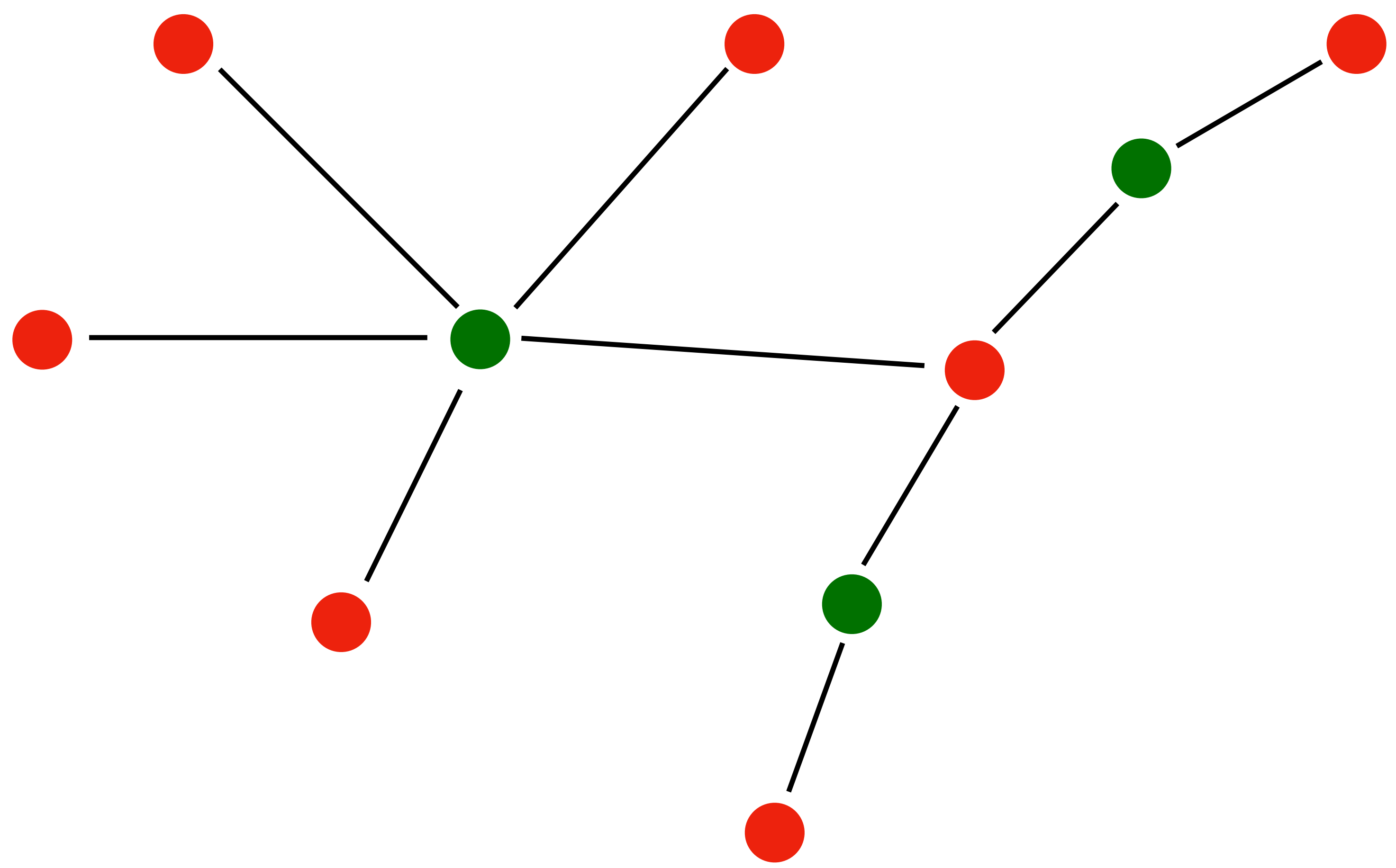
Entangled universes

Asymptotically…



Conjectures:

- Each universe has a Hilbert space
- Universes are entangled
- $S(A) = \frac{1}{4G_N}\text{area}(\gamma) + \cdots$
- γ defined via minimax, spacetime homology: $\tau \sim \mathscr{I}_A$ (rel. singularities)



In general, universes & wormholes define bipartite graph

Entropies computed via minimax, spacetime homology w/asymptotic boundaries

どうもありがとうございます!