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YITP Program on QI, QM and QG

Black Hole Interior

September 8, 2023

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Outline

Motivation

Quantum fields

The charged quantum scalar field

Going to Kerr-de Sitter

MOTIVATION

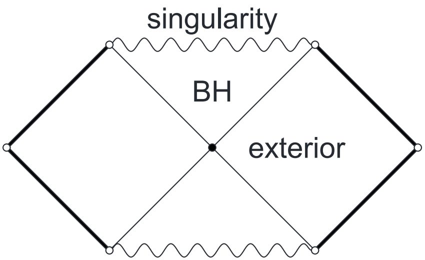
The limits of the world are guarded by MONSTERS

[Map by Abraham Ortelius]

... by SINGULARITIES

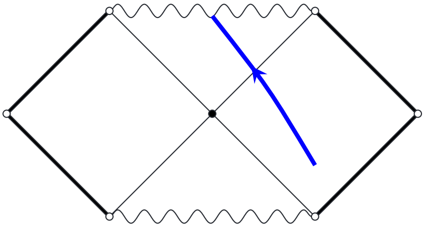
[Map by Abraham Ortelius]

Schwarzschild black hole



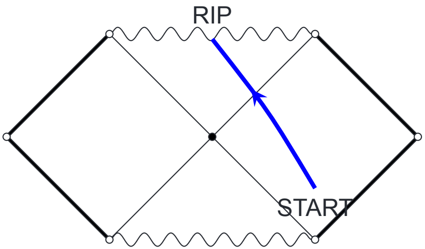
Singularity is spacelike

Schwarzschild black hole



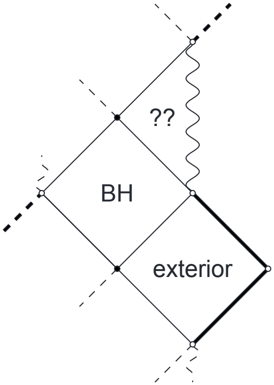
Observers diving into BH end their existence

Schwarzschild black hole

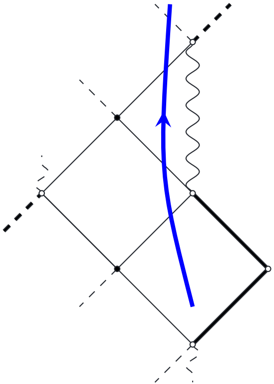


... in finite proper time

Charged black hole

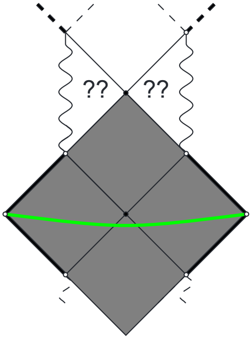


Charged black hole



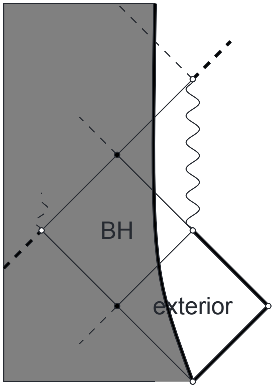
Observers diving into BH may stay clear of singularity

Determinism violation

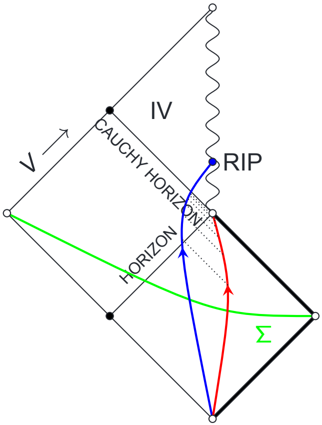


Domain of dependence of Cauchy slice

Collapsing shell [Boulware]

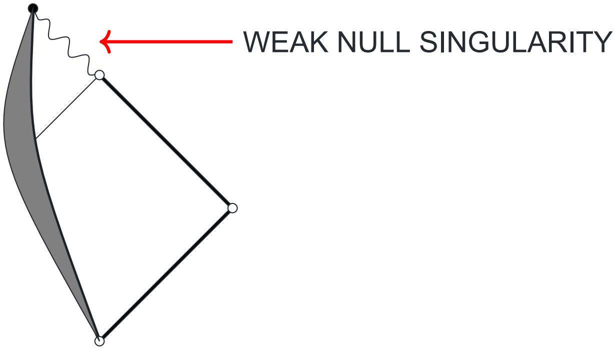


Strong cosmic censorship (sCC) conjecture

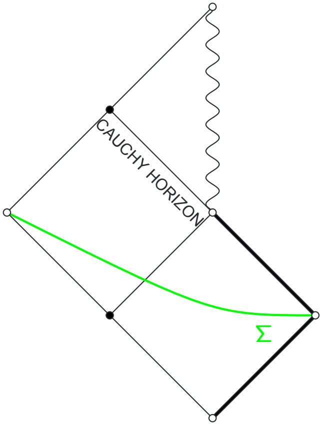


- Fields not determined by initial data on Σ beyond CAUCHY HORIZON
- Signals reaching CAUCHY HORIZON infinitely blueshifted
- sCC idea: Transverse derivatives ∂_V of any field ϕ blow up near CAUCHY HORIZON, making the spacetime inextendible.
- $\Longrightarrow$ Region *IV* does not exist! [physics: Penrose, Chandrasekhar-Hartle, Israel, Poisson, Ori, Brady, Dias-Reall-Santos, ... maths: Christodoulu, Dafermos, Luk, Franzen, Costa, Bony, Häffner, Dyatlov, Zworski, ...]

Upshot: Null singularity

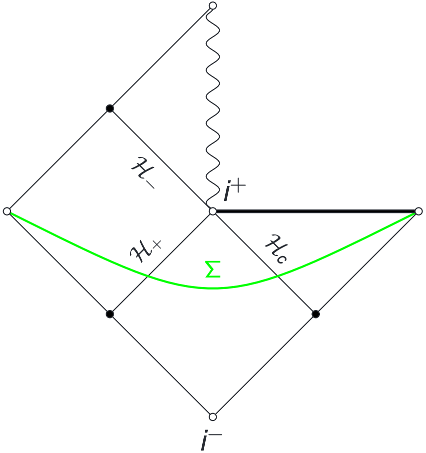


Mathematical formulation of sCC



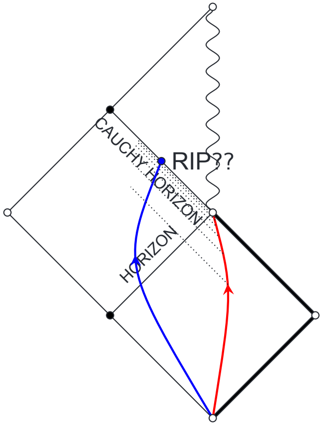
- Strong cosmic censorship conjecture (sCC): For generic initial data, non-linear metric perturbation is inextendible as H^1_{loc}-function across Cauchy horizon as a weak solution to the Einstein equations [Christodoulou]
- For test KG field $(\Box_g - \mu^2)\phi = 0$ this means $T_{VV} = (\partial_V \phi)^2 \notin L^1_{loc}$
- T_{VV} represents energy-momentum flux across Cauchy horizon.

The RNdS spacetime



- $g = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2$
- $f(r) = -\frac{\Lambda}{3}r^2 + 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$
- Horizons: $\mathcal{H}_-, \mathcal{H}_+, \mathcal{H}_c$ at r_-, r_+, r_c
- Null coordinates: $U \sim \nwarrow$, $V \sim \nearrow$
- Temperatures: $\kappa_-, \kappa_+, \kappa_c$

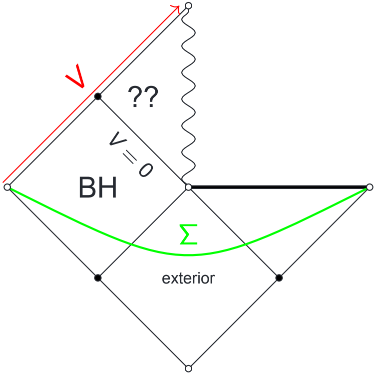
sCC in the RNdS spacetime



- Cosmological redshift $\to$ competition with blueshift
- This competition is mathematically expressed by

$$\beta = \frac{\alpha}{\kappa_-} = \frac{\text{spectral gap of QNMs}}{\text{temperature of CH}}$$

sCC in the RNdS spacetime



- Cosmological redshift $\rightarrow$ competition with blueshift
- sCC violated if $\beta = \frac{\alpha}{\kappa_-} > \frac{1}{2}$ because $\phi \in H^{1/2+\beta}_{loc}$ [Hintz, Vasy] $\Longrightarrow T_{VV} \in L^1_{loc}$ for generic smooth (!) initial data
- $\alpha > 0$ is the spectral gap of $\Box_g - \mu^2$ (numerics)
- $\beta > 1/2$ for large Q by [Cardoso et al.]
- sCC fails for near extremal RNdS black holes!

QUANTUM FIELDS

Why quantum fields?

- When sCC is violated, then $T_{VV} \in L^1_{loc}$ at the Cauchy horizon at classical level.
- At quantum level: could $\langle T_{VV} \rangle_\Psi$ blow up at Cauchy horizon for generic "smooth" initial states Ψ?
- Mathematically: Can we show that $\langle T_{VV} \rangle_\Psi \notin L^1_{loc}$ for generic initial Hadamard state Ψ?
- Physical idea: A quantum state contains virtual particles of arbitrarily high frequency initially. A high frequency mode may give rise to singular behavior at Cauchy horizon.

Main technical result

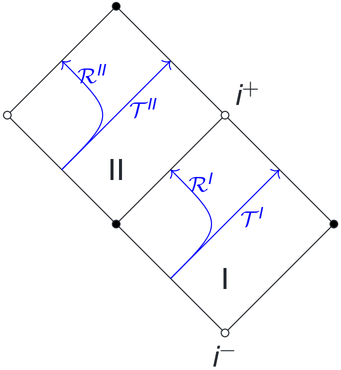
Theorem: [SH, Wald, Zahn] Let Ψ be a Hadamard state near Σ (initial surface) and $\beta > 1/2$ ($\Longrightarrow$ violation of classical sCC!). Then near $\mathcal{CH}$:

$$\langle T_{VV} \rangle_\Psi \sim C|V|^{-2} + t_{VV} \, .$$

Here:

- $C \sim \sum_\ell (2\ell + 1) \int_0^\infty \mathrm{d}\omega \, \omega n_\ell(\omega) \sim \hbar$
- $n_\ell(\omega)$ determined via scattering problem in RNdS, no dependence on Ψ!
- t_{VV} has $(\beta - 1/2-)$ fractional derivatives in L^1_{loc} near $\mathcal{CH}$.
- Quantum singularity **stronger** than classical!
- Different in BTZ BH [Dias, Reall, Santos]

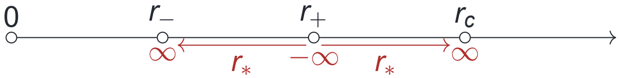
The scattering problem



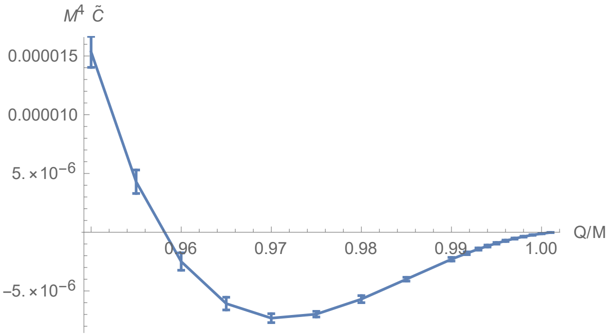
$\mathcal{T}^{I,II}_{\omega\ell}, \mathcal{R}^{I,II}_{\omega\ell} \Longrightarrow n_\ell(\omega) \Longrightarrow C$

- Klein-Gordon-equation: $\left[\Box_g - \mu^2\right]\psi = 0$
- Mode ansatz:
  $\psi_{\omega\ell m} = (4\pi|\omega|)^{-1} r^{-1} Y_{\ell m}(\theta,\phi) e^{-i\omega t} R_{\omega\ell}(r)$
- Equation for $R_{\omega\ell}(r)$:
  $\left[\partial^2_{r_*} - W(r) + \omega^2\right] R_{\omega\ell}(r) = 0$

$\Rightarrow$ 1d scattering problem



Results: charge-dependence of the energy flux



Energy flux $\langle T_{VV} \rangle_\Psi \sim C V^{-2}$ at $\mathcal{CH}$ as a function of Q/M [SH, Klein, Zahn], both signs of C appear [Ori, Zilberman, ...]! Qualitative difference in 3d [Dias, Reall, Santos], similar for Kerr [Casals, Ori, Ottewil, Zilberman]

Sign of C

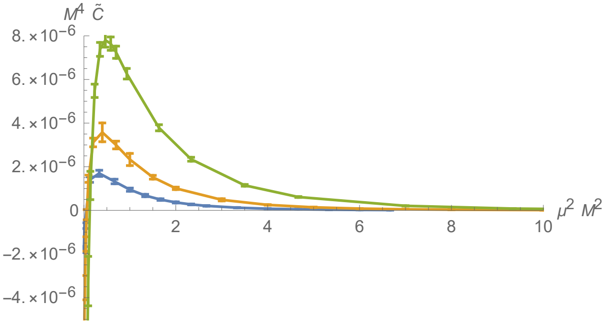
If we could impose semi-classical Einstein equation

$$G_{\mu\nu} = T_{\mu\nu}^{\text{class}} + T_{\mu\nu}^{\text{quant}}$$

- $C > 0$: Observers crossing CH get crushed to death
- $C < 0$: Observers crossing CH get stretched to death

This should be analyzed more fully on evaporating BH spacetime

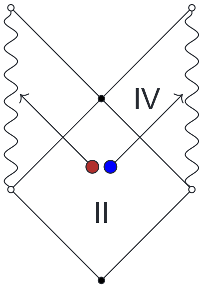
Results: mass-dependence of the energy flux



Energy flux $\langle T_{VV} \rangle_\Psi \sim CV^{-2}$ at $\mathcal{CH}$ as a function of $\mu^2 M^2$ [SH, Klein, Zahn], both signs of C appear! $\Longrightarrow$ infinite streching/squeezing of observers possible

THE CHARGED QUANTUM SCALAR FIELD

Charged scalar fields



- Charged black hole $\Rightarrow$charged scalar field ϕ
- Classically: sCC still violated for large Q

  [Cardoso et.al., Dias et.al.]
- Charge current $J_\mu = iq(\phi^* D_\mu \phi - \phi D_\mu \phi^*)$
- Can this current "discharge" the black hole interior (i.e. $\mathcal{CH}$)?
- Does this drive the black hole away from extremality?
- Is the "Schwinger pair creation picture" valid [Herman, Hiscock]?

The main technical result

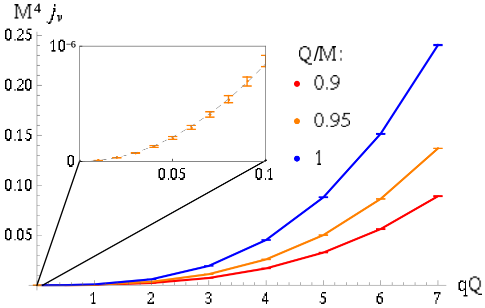
Theorem: [SH, Klein, Zahn] Let Ψ be a Hadamard state near Σ (initial surface) and $\beta > 1/2$ ($\Longrightarrow$ violation of classical sCC!). Then near $\mathcal{CH}$:

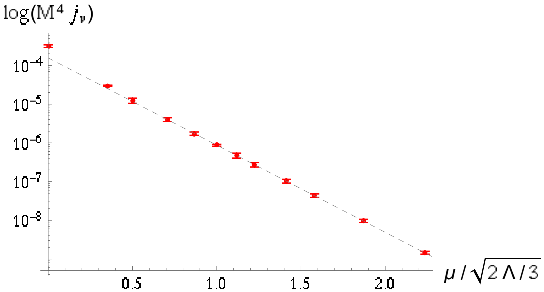
$$\langle J_V \rangle_\Psi \sim C|V|^{-1} + j_V \,.$$

Here:

- $C \sim q \sum_\ell (2\ell + 1) \int_0^\infty \mathrm{d}\omega \, n_\ell(\omega)$
- $n_\ell(\omega)$ determined via scattering problem in RNdS, no dependence on Ψ!
- j_V has $(\beta + 1/2-)$ fractional derivatives in L^1_{loc} near $\mathcal{CH}$.

Results: Discharging of the event horizon

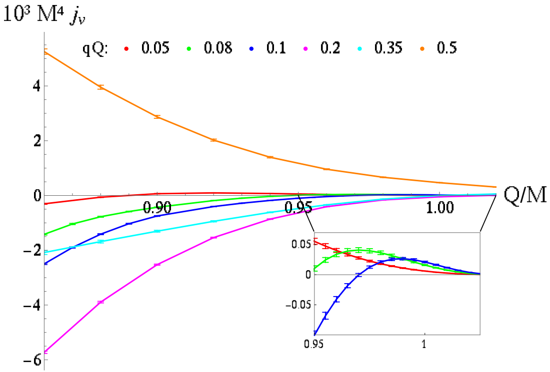




The event horizon always gets discharged by expected charge-current (in Unruh state).
Effect stronger for larger test-charge

Discharging effect gets weaker for larger test mass.

Results: (Dis)charging the Cauchy-horizon



Plot of $C \sim \langle J_v \rangle_U$. The Cauchy horizon can be charged or discharged. Near extremality, it is always discharged!

Summary

Event horizon	Cauchy horizon
current/energy flux finite	infinte
always discharged	charged or discharged
current increases with Q	decreases with Q near extremality
current increases with q	– "– for large enough Q
current decreases with μ	– "– for large enough Q
expected steady state behavior	unexpected

GOING TO KERR-DE SITTER

From RNdS to KdS

RNdS	Kerr-de Sitter
charge Q Extremal limit: $Q = M + \mathcal{O}(\Lambda)$	specific angular momentum a Extremal limit: $a = M + \mathcal{O}(\Lambda)$
spherically symmetric	axisymmetric
exterior region static	exterior region stationary
charge current j_μ	angular momentum density $T_{\mu\varphi}$
sCC classically **violated** [Hintz, Vasy]	sCC **holds** classically [Dias, Santos, Reall]

Why quantum fields?

- Classical: $T_{VV} \sim V^{-(2-2\beta)}$ in Kerr-deSitter at the Cauchy horizon, where $0 < \beta < 1/2$ [Dias, Santos, Reall].
- Quantum: $\langle T_{VV} \rangle_{\mathrm{U}} \sim V^{-2}$ at Cauchy horizon in Kerr [Casals, Ottewil, Ori, Zilberman].
- Expect quantum singularity to be **stronger** than classical singularity in **generic state**!
- Can we find similar effect as for (dis-)charge of RNdS interior for up-/down-spinning of KdS interiors?

Roadmap

- Construct the Unruh vacuum on Kerr- de Sitter and show Hadamard rigorously [Klein]
- Check whether we get a state-independent leading divergence near $\mathcal{CH}$, i.e. that $\langle T_{VV}\rangle_\Psi = \langle T_{VV}\rangle_\text{U} + t_{VV}$ where $t_{VV} \sim V^{-(2-2\beta+))}$ [Hintz, Klein, in progress]
- Similarly for $T_{V\varphi}$.
- Find a computable expressions for $\langle T_{V\phi}\rangle_\text{U}, \langle T_{VV}\rangle_\text{U}$ [Klein, Soltani, Casals, SH, to appear]
- Check sign of quantum fluxes, e.g. whether quantum effects spin down the hole near extremality [Klein, Soltani, Casals, SH, to appear]

Renormalization

By well-known procedures [SH, Wald] we can compute $\langle T_{\mu\nu} \rangle_\Psi$ once we have the 2-point correlation function $w_\Psi(x, x') = \langle \phi(x)\phi(x') \rangle_\Psi$.

To give finite $\langle T_{\mu\nu} \rangle_\Psi$, the singularities of the 2-point function must have special form.

Hadamard condition

Microlocal spectrum condition [Radzikowski,Fredenhagen,SH,...]

$$
\begin{aligned}
WF(w_\Psi) = \{ & (x,k,x',-k') \in T^*(M \times M) \setminus 0 : \\
& (x,k),(x',k') \text{ lie on same bicharacteristic strip}, k^0 > 0 \}
\end{aligned}
$$

This condition is stating that Ψ contains no extractable negative energy in the UV limit. It is a generalization of the spectrum condition to curved spacetime.

Renormalization

By well-known theorem [Radzikowski] : Ψ Hadamard $\Longrightarrow$

$$\langle\phi(x)\phi(x')\rangle_\Psi \sim \frac{(\text{geometric})}{\sigma_\varepsilon(x,x')} + (\text{geometric})\log\sigma_\varepsilon(x,x') + (\text{smooth}_\Psi(x,x')) \qquad (1)$$

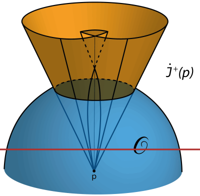
Then ϕ^2 is defined by point-split renormalization (=OPE for free fields in curved space):

$$\langle\phi^2(x)\rangle_\Psi = \lim_{x'\to x}\left(\langle\phi(x)\phi(x')\rangle_\Psi - (\text{geometric singular part})\right) \qquad (2)$$

- ϕ^2 is “defined in same way on all spacetimes” [SH & Wald, Brunetti et al.]
- similar for $T_{\mu\nu}, J^\mu$.

Hadamard states from characteristic surfaces [SH thesis]

- Project onto "positive frequency" conjugate to affine parameter on null generators of characteristic surface $\mathcal{N}$ [Hawking, Unruh,...]
- Propagate "positive frequency" condition [SH thesis] using propagation of singularities theory [Duistermaat, Hörmander]
- In BH spacetimes $\mathcal{N}$ is a union of horizons/scri [Dappiaggi, Moretti, Pinamonti], [SH, Klein, Wald]



Defining the Unruh state in KdS [Klein]

Theorem

Let $M = \mathrm{I} \cup \mathrm{II} \cup \mathrm{III}$ the union of the exterior and interior (up to the Cauchy horizon) of a slowly rotating Kerr-de Sitter black hole with a small cosmological constant. Then

$$\begin{aligned} w_{\mathrm{U}}(f,h) = &- \lim_{\epsilon \to 0^+} \frac{r_+^2 + a^2}{\chi\pi} \int \frac{E(f)|_{\mathcal{H}}(U_+, \Omega_+) E(h)|_{\mathcal{H}}(U'_+, \Omega_+)}{(U_+ - U'_+ - i\epsilon)^2} dU_+ dU'_+ d^2\Omega_+ \\ &- \lim_{\epsilon \to 0^+} \frac{r_c^2 + a^2}{\chi\pi} \int \frac{E(f)|_{\mathcal{H}_c}(V_c, \Omega_c) E(h)|_{\mathcal{H}_c}(V'_c, \Omega_c)}{(V_c - V'_c - i\epsilon)^2} dV_c dV'_c d^2\Omega_c \end{aligned}$$

is the two-point function of a quasi-free Hadamard state for the massive real scalar field on M.

Strategy of Proof [Klein]

- Use $|\partial^N E(f)(r, t_*, \theta, \varphi)| \lesssim e^{-\alpha|t_*|}||f||_{C^m}$ in region I [Hintz, Vasy]
- To prove: positive spectral gap $\alpha > 0$, which is hard. Partial results [Casals, Teixeiro-daCosta] , improved results [SH, Zahn, in progress] !

$\Rightarrow$ w_{U} well-defined distribution and, by a limit deformation of Cauchy surfaces, commutator property holds

- For geodesics ending at one of the horizons, use propagation theorems for wavefront sets to show Hadamard property
- Assume: Geodesics not ending at one of the horizons must pass through a region where both $v_+ = \partial_t + \omega_+ \partial_\varphi$ and $v_c = \partial_t + \omega_c \partial_\varphi$ are timelike

$\Rightarrow$ Adapt the arguments from the RNdS case [SH, Wald, Zahn, Dappiaggi, Moretti, Pienamonti, Gerard, Wrochna, SH PhD thesis]

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THANK YOU FOR YOUR ATTENTION

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