Unconventional states of matter in the quantum-wire network of moiré systems

Chen-Hsuan Hsu

Institute of Physics, Academia Sinica (IoP, AS), Taiwan

CHH et al., Phys. Rev. B 108, L121409 (2023)
(partially done @YITP)

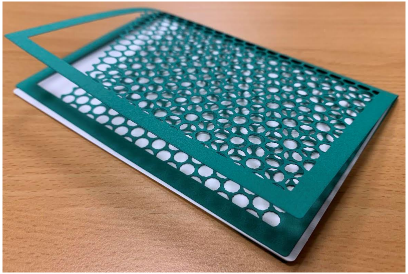
QIMG Workshop, YITP, Kyoto U
(Oct. 5th, 2023)

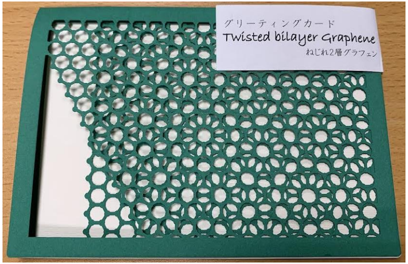
Jelena Klinovaja & Daniel Loss (University of Basel, Switzerland)

$$ NSTC, AS & IoP (Taiwan), Kakenhi & YITP (Japan), NSF & NCCR QSIT (Switzerland)

Fabrication of moiré systems

- twisted bilayer graphene in Kyoto University (gift shop)



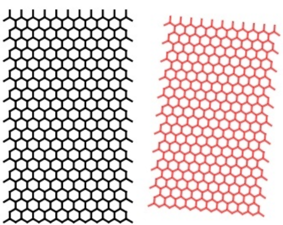


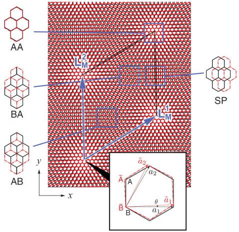
デザイン・製作・解説：きゐ（ペンネーム／京都大学大学院理学研究科所属）
解説協力：高三 和晃（京都大学大学院理学研究科修了、現在カリフォルニア大学バークレー校所属）
このグリーティングカードは京都大学アントレプレナープラットフォームにて開発されました。

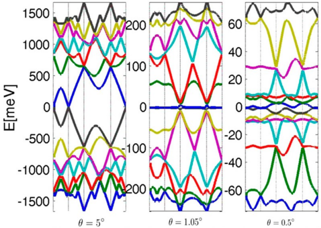
design: きゐ (graduate student @Kyoto U)
commentary: Kazuaki Takasan (postdoc @UC Berkeley)

2D twisted nanostructures forming moiré systems

- Twist angle between 2D monolayers:
  a tunable parameter allowing for continuously varying the band structure
  ⇒ band-structure engineering





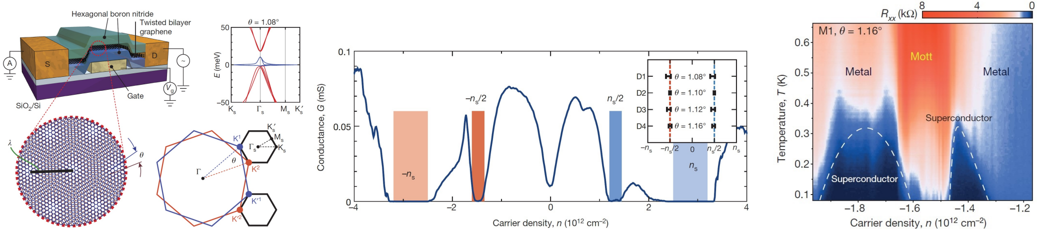


Nam and Koshino, PRB 2017 Bistritzer and MacDonald, PNAS 2011

- Moiré pattern with wavelength $\lambda = a_0/[2\sin(\theta/2)]$
  - θ: twist angle between layers; a_0: lattice constant of graphene monolayer
- (Quasi-)flat bands close to the magic angle (e-e interaction $>$ bandwidth $\approx$ kinetic energy)
  ⇒ a platform for strongly correlated electron systems

Strongly correlated systems in twisted bilayer graphene

- Magic-angle twisted bilayer graphene (TBG)



Cao et al., Nature 556, 43 (2018); Cao et al., Nature 556, 80 (2018)

- Carrier density electrically tuned by voltage gate
- Band insulator for 4e (or 4h) per moiré unit cell and semimetal at charge neutrality point
- Unconventional states of matter when the Fermi energy lies within the (quasi-)flat bands
- Phase diagram: resembling high-T_c materials
  - (Mott-like) correlated insulating phase at half filling
  - dome-like superconductivity regions in e- and h-doped sides of Mott phase

Anomalous Hall effect in TBG

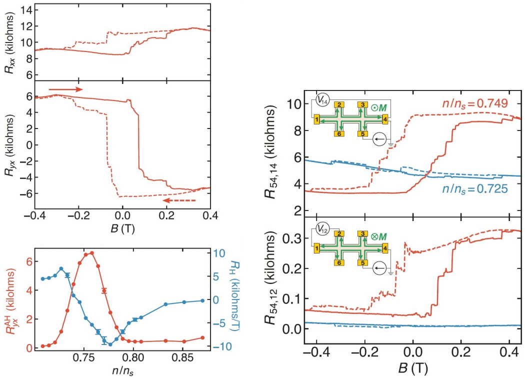
RESEARCH

GRAPHENE

Emergent ferromagnetism near three-quarters filling in twisted bilayer graphene

Aaron L. Sharpe[1,2]*, Eli J. Fox[2,3]*, Arthur W. Barnard[3], Joe Finney[3], Kenji Watanabe[4], Takashi Taniguchi[4], M. A. Kastner[2,3,5,6], David Goldhaber-Gordon[2,3]†

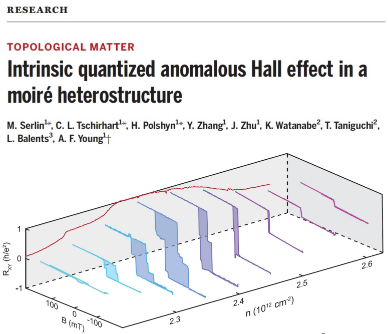
When two sheets of graphene are stacked at a small twist angle, the resulting flat superlattice minibands are expected to strongly enhance electron-electron interactions. Here, we present evidence that near three-quarters ($^3/_4$) filling of the conduction miniband, these enhanced interactions drive the twisted bilayer graphene into a ferromagnetic state. In a narrow density range around an apparent insulating state at $^3/_4$, we observe emergent ferromagnetic hysteresis, with a giant anomalous Hall (AH) effect as large as 10.4 kilohms and indications of chiral edge states. Notably, the magnetization of the sample can be reversed by applying a small direct current. Although the AH resistance is not quantized, and dissipation is present, our measurements suggest that the system may be an incipient Chern insulator.



Sharpe et al., Science 2019

- TBG nearly aligned to the top hBN layer
- Ferromagnetic hysteresis with a coercive field $B \sim O(0.1$ T$)$ at 3/4 filling
- Large Hall resistance and chiral edge modes at $B = 0$ (upper flat band)
- Indication of topological phases

Experimental indication of topological matter in TBG

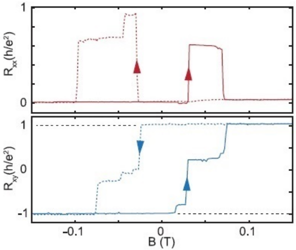


RESEARCH

TOPOLOGICAL MATTER

Intrinsic quantized anomalous Hall effect in a moiré heterostructure

M. Serlin[1]*, C. L. Tschirhart[1]*, H. Polshyn[1]*, Y. Zhang[1], J. Zhu[1], K. Watanabe[2], T. Taniguchi[2], L. Balents[3], A. F. Young[1]†

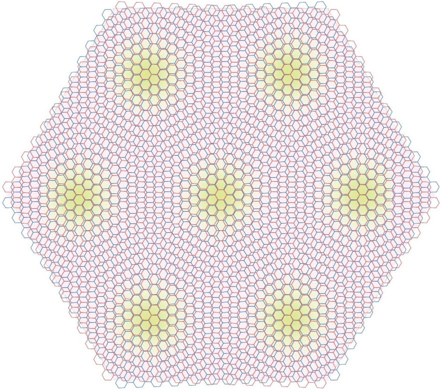


Serlin et al., Science 2020

- Quantized Hall resistance $R_{xy} = h/e^2$ at 3/4 filling at $B = 0$ in TBG aligned to hBN
  $\Rightarrow$ quantum anomalous Hall insulator (QAHI) or Chern insulator with Chern number $C = 1$
- A sequence of Chern insulator states with Chern number $C = \pm 1, \pm 2$ and ± 3 observed at the filling factor $\nu = \pm 3/4, \pm 2/4$ and $\pm 1/4$, respectively
  - complete sequence: Nuckolls et al., Nature 2020; Choi et al., Nature 2021; Das et al., Nat. Phys. 2021
  - partial sequence: Park et al., Nature 2021; Saito et al., Nat. Phys. 2021; Stepanov et al., PRL 2021; Lin et al., Science 2022; Tseng et al., Nat. Phys. 2022

$\Rightarrow$ topologically nontrivial phases as a common feature across samples and setups

Challenge for theoretical analysis

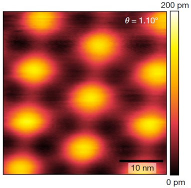


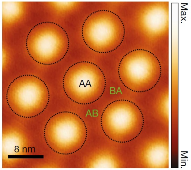
Cao et al., Science 2021

- Experimental observations of unconventional electronic states in TBG motivated numerous theoretical works
- Challenge for theoretical analysis:
  - a large number of atoms $\sim O(10^4)$ due to large moiré unit cells
  - correlation: beyond single-particle picture
- To develop tractable analytic tools, a theoretical framework identifying relevant degrees of freedom is highly desirable!

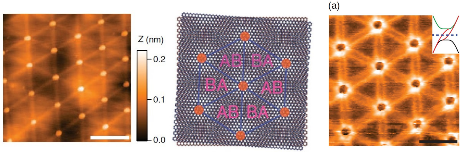
2D network or array of 1D channels in TBG and similar nanostructures

- STM images of domain walls between AB- and BA-stacking areas



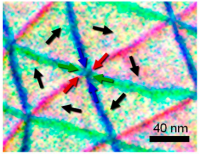


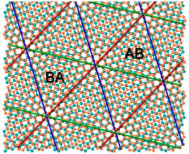
Kerelsky et al., Nature 2019; Jiang et al., Nature 2019

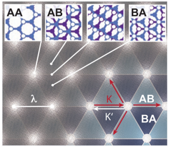


Huang al., PRL 2018

- TEM and transport features of domain walls

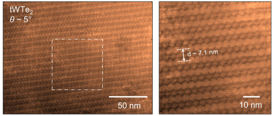


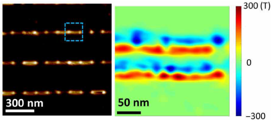




Alden et al., PNAS 2013; Rickhaus et al., Nano Lett. 2018

- 1D channels in twisted bilayer WTe_2 and strain-engineered graphene device

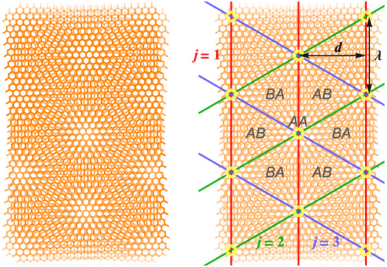




Wang et al., Nature 2022; Hsu et al., Sci. Adv. 2020

Incorporating e-e interactions in 2D network of moiré bilayer systems

- 2D network of interacting quantum wires at nanoscales:



- Unconventional states of matter in 1D or quasi-1D systems:
  - interacting electrons in 1D: (Tomonaga-)Luttinger liquid (TLL)
  - coupled parallel interacting wires: sliding TLL

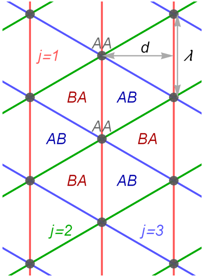
    $\Rightarrow$ intrawire and interwire forward scattering of e-e interactions on equal footing
  - triangular network of 1D wires: 3 sets of sliding TLL

    Wu et al., PRB 2019; Chen et al., PRB 2020; Chou et al., PRB 2021

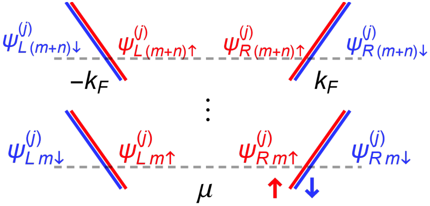
    *related work on square network: Chou et al., PRB 2019

2D network formed by gapless domain wall modes

- Electrons in 2D network consisting of interacting quantum wires
- Fermion field operator $\psi^{(j)}_{\ell m \sigma}(x)$:
  - array index $j \in \{1, 2, 3\}$
  - wire index $m \in [1, N_\perp]$ within each array
  - moving direction $\ell \in \{R, L\}$
  - spin index $\sigma \in \{\uparrow, \downarrow\}$
  - local coordinate x



- Parallel wires within an array:
  - chemical potential μ and Fermi wave vector k_F (identical for all wires)



Bosonization

- Bosonization of the field operator:

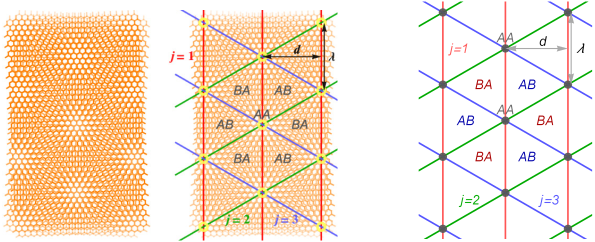
$$\psi^{(j)}_{\ell m\sigma}(x) = \frac{U^j_{\ell m\sigma}}{\sqrt{2\pi a}} e^{i\ell k_F x} e^{\frac{-i}{\sqrt{2}}[\ell\phi^j_{cm}(x)-\theta^j_{cm}(x)+\ell\sigma\phi^j_{sm}(x)-\sigma\theta^j_{sm}(x)]}$$

  - $U^j_{\ell m\sigma}$: Klein factor; a: short-distance cutoff
- Commutation relation between the boson fields:

$$\left[\phi^j_{\xi m}(x), \theta^{j'}_{\xi' m'}(x')\right] = i\frac{\pi}{2}\mathrm{sign}(x'-x)\delta_{jj'}\delta_{\xi\xi'}\delta_{mm'}$$

  - index ξ, ξ' for charge (c) or spin (s) sector
  - charge density operator $\propto \partial_x\phi^j_{cm}$; spin density operator $\propto \partial_x\phi^j_{sm}$
  - charge current operator $\propto \partial_x\theta^j_{cm}$; spin current operator $\propto \partial_x\theta^j_{s,m}$
- Intrawire or interwire Coulomb (density-density) interaction $\propto \partial_x\phi^j_{cm}\partial_x\phi^j_{cn}$
  $\Rightarrow$ forward-scattering terms ($R \leftrightarrow R$ & $L \leftrightarrow L$) in the quadratic form
  $\Rightarrow$ diagonalizable

Bosonized model for the quantum-wire network



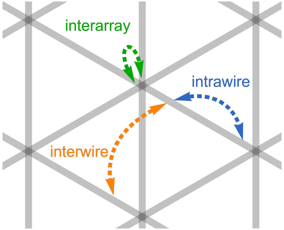
- Quantum-wire network with the quadratic interaction terms:

$$H_{0,c}^{(j)} = \sum_{mn} \int \frac{\hbar dx}{2\pi} \left[V_{\phi,mn}^{j} \partial_x \phi_{cm}^{j} \partial_x \phi_{cn}^{j} + V_{\theta,mn}^{j} \partial_x \theta_{cm}^{j} \partial_x \theta_{cn}^{j} \right]$$

$$H_{0,s}^{(j)} = \sum_{n} \int \frac{\hbar dx}{2\pi} \left[\frac{u_s}{K_s} (\partial_x \phi_{sn}^{j})^2 + u_s K_s (\partial_x \theta_{sn}^{j})^2 \right]$$

  - $V_{\phi,mn}^{j}$, $V_{\theta,mn}^{j}$, K_s: forward-scattering terms ($R_m \leftrightarrow R_n$ & $L_m \leftrightarrow L_n$)
  - ϕ_{cn}^{j}, θ_{cn}^{j}, ϕ_{sn}^{j}, θ_{sn}^{j}: boson fields

General scattering operator



- Backscatterings ($R \leftrightarrow L$): non-quadratic (sine-Gordon) form
  - analyzed by perturbative renormalization-group (RG) technique
  - potential for various electronic states

- General operator describing various scattering processes:

$$O_{\{s^j_{\ell p\sigma}\}}(x) \quad = \quad \sum_{m=1} \prod_p \prod_j \left[\psi^{(j)}_{R(m+p)\uparrow}(x)\right]^{s^j_{Rp\uparrow}} \left[\psi^{(j)}_{L(m+p)\uparrow}(x)\right]^{s^j_{Lp\uparrow}} \left[\psi^{(j)}_{R(m+p)\downarrow}(x)\right]^{s^j_{Rp\downarrow}} \left[\psi^{(j)}_{L(m+p)\downarrow}(x)\right]^{s^j_{Lp\downarrow}}$$

  - specific scattering process characterized by the integer set $\{s^j_{\ell p\sigma}\}$
  - constraints on $s^j_{\ell p\sigma}$ due to conservation laws

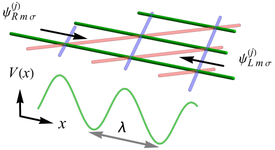
- Scatterings involving different arrays at intersections:
  - generically allowed but typically less RG relevant
  - we focus on the (intrawire/interwire) scatterings within an array (j suppressed)

Constraints on $s_{\ell p\sigma}$ from conservation laws

- Energy conservation:
  scatterings taking place at Fermi level
- Global particle number or charge conservation (without "external" pairing):

$$\sum_{p,\sigma}(s_{Rp\sigma} + s_{Lp\sigma}) = 0$$

- Momentum conservation:
  more general condition than non-moiré systems
- In moiré systems, electrons experience a moiré potential with a spatial period of λ



$\Rightarrow$ moiré periodic potential provides "crystal momentum" $\propto$ reciprocal lattice vector $2\pi/\lambda$

Unconventional scatterings allowed by moiré periodic potential

- Moiré periodic potential: partially relaxing the constraint from the momentum conservation
- Generalized condition from momentum conservation (for clean systems):

$$k_F \sum_{p,\sigma} (s_{Rp\sigma} - s_{Lp\sigma}) = \frac{2\pi}{\lambda} \times \text{ integer}$$

  $\Rightarrow$ momentum difference compensated by crystal momentum of the moiré potential
  $\Rightarrow$ additional processes at certain fillings
- Resonance condition for the filling factor $\nu = k_F \lambda / \pi$:

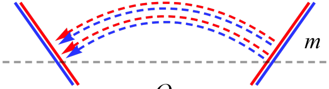
$$\nu \quad = \quad \frac{P}{\sum_{p,\sigma} s_{Rp\sigma}}, \quad P \in \text{nonzero integer}$$

  - $\nu = 1$ corresponds to 4 electrons per moiré unit cell in TBG
- We refer to this type of processes as *moiré umklapp scatterings*
  $\Rightarrow$ destabilizing the network: *moiré correlated states*

Examples for moiré umklapp scatterings (O_i and O_ii)

- Further categorized into 4 subtypes: O_i–O_iv
- Moiré umklapp scatterings allowed at fractional fillings ($\nu = P/4$ for illustration)
- O_i: processes involving only intrawire scatterings in individual wires

$$(s_{R0\sigma}, s_{L0\sigma}) \to (N_\sigma, -N_\sigma)$$
$$N_\sigma \in \mathbb{N}$$
$$O_\mathrm{i} = \sum_m \left(\psi^\dagger_{Lm\uparrow}\psi_{Rm\uparrow}\right)^{N_\uparrow}\left(\psi^\dagger_{Lm\downarrow}\psi_{Rm\downarrow}\right)^{N_\downarrow}$$

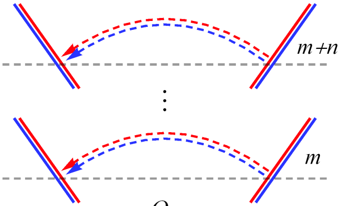


O_i

with $N_\sigma = 2$

- O_ii: processes involving correlated intrawire scatterings in multiple wires

$$(s_{R0\sigma}, s_{L0\sigma}, s_{Rn\sigma}, s_{Ln\sigma}) \to (N_{0\sigma}, -N_{0\sigma}, N_{n\sigma}, -N_{n\sigma})$$
$$N_{0\sigma}, N_{n\sigma} \in \mathbb{N}$$
$$O_\mathrm{ii} = \sum_m \left(\psi^\dagger_{Lm\uparrow}\psi_{Rm\uparrow}\right)^{N_{0\uparrow}}\left(\psi^\dagger_{Lm\downarrow}\psi_{Rm\downarrow}\right)^{N_{0\downarrow}} \times \left[\psi^\dagger_{L(m+n)\uparrow}\psi_{R(m+n)\uparrow}\right]^{N_{n\uparrow}}\left[\psi^\dagger_{L(m+n)\downarrow}\psi_{R(m+n)\downarrow}\right]^{N_{n\downarrow}}$$



O_ii

with $N_{0\sigma} = N_{n\sigma} = 1$

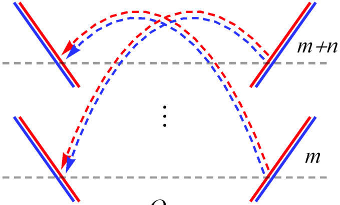
Examples for moiré umklapp scatterings ($O_{\rm iii}$ and $O_{\rm iv}$)

- $O_{\rm iii}$: processes involving interwire scatterings but still conserving the particle number for each wire

$$(s_{R0\sigma}, s_{L0\sigma}, s_{Rn\sigma}, s_{Ln\sigma}) \to (N_\sigma, -N_\sigma, N_\sigma, -N_\sigma)$$

$$N_\sigma \in \mathbb{N}$$

$$O_{\rm iii} = \sum_m \left[\psi^\dagger_{L(m+n)\uparrow}\psi_{Rm\uparrow}\right]^{N_\uparrow}\left[\psi^\dagger_{L(m+n)\downarrow}\psi_{Rm\downarrow}\right]^{N_\downarrow} \times \left[\psi^\dagger_{Lm\uparrow}\psi_{R(m+n)\uparrow}\right]^{N_\uparrow}\left[\psi^\dagger_{Lm\downarrow}\psi_{R(m+n)\downarrow}\right]^{N_\downarrow}$$



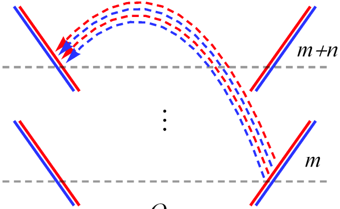
$O_{\rm iii}$
with $N_\sigma = 1$

- $O_{\rm iv}$: scattering processes that do not conserve particle numbers for individual wires

$$(s_{R0\sigma}, s_{L0\sigma}, s_{Rn\sigma}, s_{Ln\sigma}) \to (N_\sigma, -M_\sigma, M_\sigma, -N_\sigma)$$

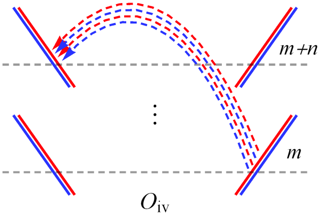
$$N_\sigma, M_\sigma \in \mathbb{N}, \quad N_\sigma \neq M_\sigma$$

$$O_{\rm iv} = \sum_m \left[\psi^\dagger_{L(m+n)\uparrow}\psi_{Rm\uparrow}\right]^{N_\uparrow}\left[\psi^\dagger_{L(m+n)\downarrow}\psi_{Rm\downarrow}\right]^{N_\downarrow} \times \left[\psi^\dagger_{Lm\uparrow}\psi_{R(m+n)\uparrow}\right]^{M_\uparrow}\left[\psi^\dagger_{Lm\downarrow}\psi_{R(m+n)\downarrow}\right]^{M_\downarrow}$$



$O_{\rm iv}$
with $(N_\sigma, M_\sigma) = (2, 0)$

Gapless chiral edge modes from O_{iv} process



- $\bar{S}_{p,c} \neq 0$ for O_{iv}: particle number not conserved for individual wires
- Simplest case involving the n-th nearest neighbor wires:
  $S_{n,c} = S_{0,c}$, $\bar{S}_{n,c} = -\bar{S}_{0,c}$, and $S_{p,c}, \bar{S}_{p,c} = 0$ otherwise
- Introducing chiral fields $\Phi_{\ell m} = -\ell\phi_{cm} + f\theta_{cm}$ for each wire:

$$\left[\Phi_{\ell m}(x), \Phi_{\ell' m'}(x')\right] = i\ell\pi\delta_{\ell\ell'}\delta_{mm'} f \,\text{sign}(x - x'),$$
$$f = -\bar{S}_{0,c}/S_{0,c}$$

- The perturbation from O_{iv} process:

$$\delta H_{\text{iv}} = g_{\text{iv}} \int dx \left(O_{\text{iv}} + O_{\text{iv}}^{\dagger}\right) \propto g_{\text{iv}} \sum_{m=1} \int dx \;\cos\left\{\frac{S_{0,c}}{\sqrt{2}}\left[\Phi_{L(m+n)} - \Phi_{Rm}\right]\right\}$$

  $\Rightarrow$ involving right- and left-moving modes in the interior of the system

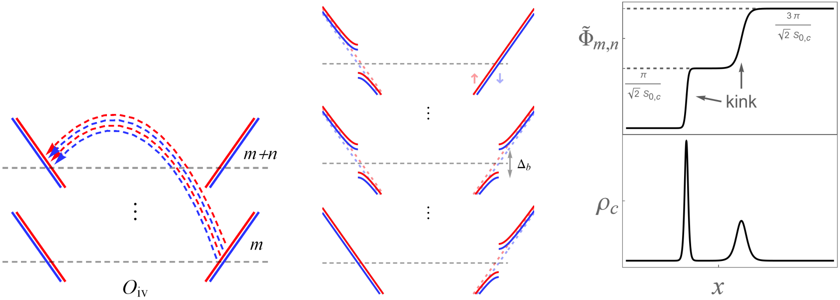
- There remain gapless chiral modes:
  $\Phi_{L,1}, \cdots, \Phi_{L,n}$ at one edge and $\Phi_{R,N_\perp}, \cdots, \Phi_{R,(N_\perp - n+1)}$ at the opposite edge
  (similarly for the other arrays)

Fractional excitations

- Defining $\tilde{\Phi}_{m,n} = [\Phi_{L(m+n)} - \Phi_{Rm}]/2$:

$$\delta H_{\rm iv} \quad \propto \quad g_{\rm iv} \sum_{m=1} \int dx \; \cos\left(\sqrt{2} S_{0,c} \tilde{\Phi}_{m,n}\right)$$

  - gapping out bulk modes in the interior of the system
    $\Rightarrow$ moiré correlated state with an insulating bulk and gapless edge modes



- $\tilde{\Phi}_{m,n}$ pinned to minima: $\tilde{\Phi}_{m,n} \to$ odd integer $\times\, \pi/(\sqrt{2} S_{0,c})$
- Fractional excitations with charge $2e/S_{0,c}$ associated with the kink

Exploring moiré correlated states through gapless edge modes

- At certain fractional fillings, $O_{\rm iv}$ leads to an insulating bulk with gapless chiral edge modes
  $\Rightarrow$ resembling quantum anomalous Hall effect in TBG

- It would be challenging to directly detect the fractional charge
  $\Rightarrow$ probing the moiré correlated state through the edge modes
- Assuming a single mode $\Phi_{R,N_\perp} \to \phi$ at an edge for simplicity, where the chiral field ϕ satisfies

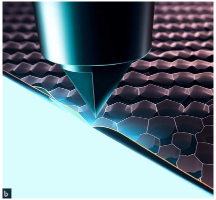
$$\left[\phi(x), \phi(x')\right] = i\pi f \,\mathrm{sign}(x - x')$$

- Effective edge theory from the commutator:

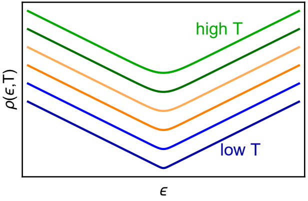
$$\frac{S_{\rm edge}}{\hbar} = \int \frac{dx d\tau}{4\pi f} \left[- i\partial_x\phi\partial_\tau\phi + v_{\rm e}\left(\partial_x\phi\right)^2\right]$$

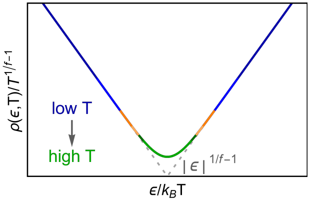
$\Rightarrow$ experimental setups to detect and characterize the edge modes

Scanning tunneling spectroscopy (STS)



created by Microsoft Image Creator





- Local density of states at the edge:

$$\rho(\epsilon) \quad = \quad \frac{1}{\pi}\mathrm{Re}\left[\int_0^\infty dt\, e^{i\epsilon t/\hbar}\left\langle \psi_\mathrm{e}(t)\psi_\mathrm{e}^\dagger(0)\right\rangle\right]$$

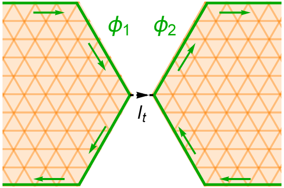
- Universal scaling curve for temperature T and energy ϵ (measured from Fermi level):

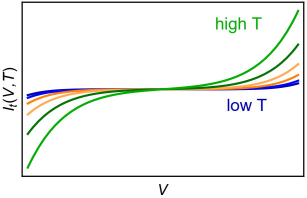
$$\rho(\epsilon, T) \quad \propto \quad T^{\frac{1}{f}-1}\cosh\left(\frac{\epsilon}{2k_\mathrm{B}T}\right)\left|\Gamma\left(\frac{1}{2f}+i\frac{\epsilon}{2\pi k_\mathrm{B}T}\right)\right|^2$$

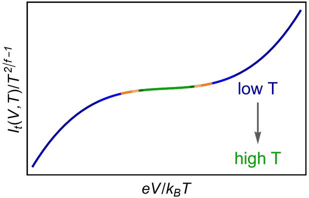
  - power law $|\epsilon|^{1/f-1}$ at very low T
  - scaling parameter determined by universal fraction f, independent of system details

Current-bias curve of interedge tunneling

- Proposed edge transport measurement:







- Interedge tunneling process:

$$S_t = t_0 \int d\tau \, e^{i(\phi_1 - \phi_2)/f}$$

  - t_0: non-universal tunnel amplitude
  - ϕ_1, ϕ_2: chiral fields in two separate edges

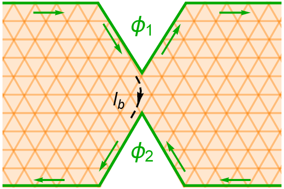
- Current-bias ($I_t - V$) curve at temperature T:

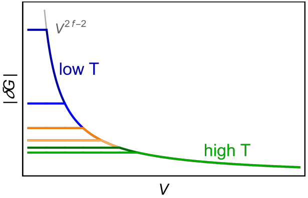
$$I_t \propto T^{\frac{2}{f}-1} \sinh\left(\frac{eV}{2k_B T}\right) \left|\Gamma\left(\frac{1}{f} + i\frac{eV}{2\pi k_B T}\right)\right|^2$$

$\Rightarrow$ another universal scaling formula with a scaling parameter set by f

Conductance correction induced by interedge backscattering

- Proposed edge transport measurement:





- Interedge backscattering process:

$$S_b = v_b \int d\tau \, e^{i(\phi_1 - \phi_2)}$$

  - v_b: non-universal backscattering strength
  - ϕ_1, ϕ_2: chiral fields in two separate edges

- Conductance correction depending on the bias (V) and temperature (T):

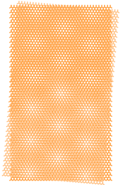
$$|\delta G| \propto \begin{cases} V^{2f-2}, & \text{for } eV \gg k_B T \\ T^{2f-2}, & \text{for } eV \ll k_B T \end{cases}$$

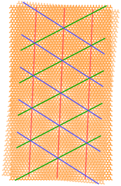
$\Rightarrow$ power-law behavior with a scaling parameter set by f

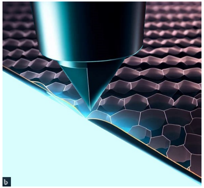
Summary

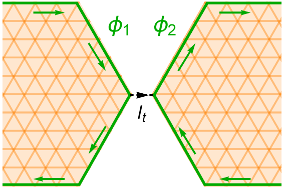
- Moiré correlated states and fractional excitations from moiré umklapp scatterings
- Correlated states hosting a gapped bulk and gapless edge modes at fractional fillings (resembling quantum anomalous Hall effect observed in experiments)
- Proposed spectroscopic and transport setups for experimental verification

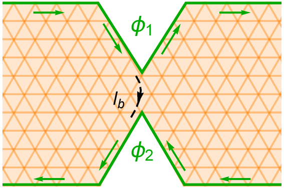
  CHH et al., Phys. Rev. B 108, L121409 (2023)











- Recruitment information (IoP, AS, Taiwan):
  - faculty positions (all subfields in Physics):
    https://tinyurl.com/2h2tj69s
  - postdoc and student positions (condensed matter):
    https://sites.google.com/view/qmtheory