De Sitter tensor networks with overlapping qubits

[arXiv:2304.02673]

ChunJun Cao[1,2,3], Wissam Chemissany[4],
Alexander Jahn[2,5], and Zoltán Zimborás[6]

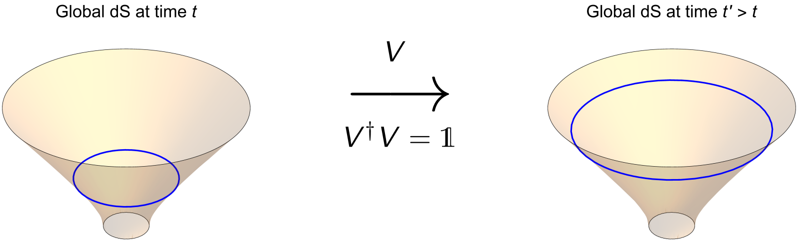
[1] University of Maryland, USA, [2] Caltech, USA, [3] Virginia Tech, USA,
[4] University of Pennsylvania, USA, [5] Freie Universität Berlin, Germany,
[6] Wigner Research Centre, Hungary

Kyoto, Sep 4, 2023

Hilbert space maps

Applying quantum information theory to (quantum) gravity leads to interesting Hilbert space maps.

Hilbert space $\mathcal{H}_m$ | Hilbert space $\mathcal{H}_n$



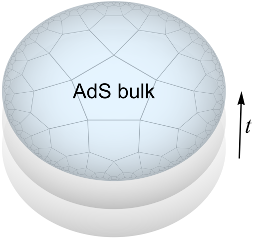
Evolution between de Sitter time-slices by *isometries*.
[Cotler/Strominger '22]

Hilbert space maps

Applying quantum information theory to (quantum) gravity leads to interesting Hilbert space maps.

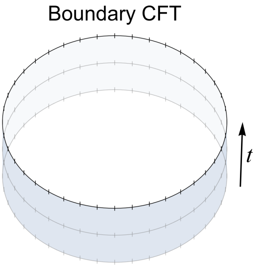
Hilbert space $\mathcal{H}_m$

Hilbert space $\mathcal{H}_n$



$$\xrightarrow{V}$$

$$V^\dagger V \approx \mathbb{1}$$



Approximate bulk-to-boundary isometry in AdS/CFT.
[Almheiri/Dong/Harlow '15]

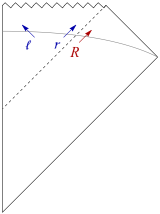
Hilbert space maps

Applying quantum information theory to (quantum) gravity leads to interesting Hilbert space maps.

Hilbert space $\mathcal{H}_m$

Hilbert space $\mathcal{H}_n$

Black hole with Hawking radiation



$$\xrightarrow[V^\dagger V \neq \mathbb{1}]{V \otimes \mathbb{1}}$$

Holographic description

Black hole evaporation as a *non-isometric code*.

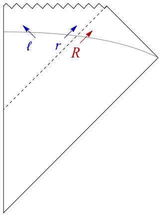
[Akers/Engelhardt/Harlow/Penington/Vardhan '22]

Hilbert space maps

Applying quantum information theory to (quantum) gravity leads to interesting Hilbert space maps.

Hilbert space $\mathcal{H}_m$

Black hole with Hawking radiation



$$\xrightarrow[V^\dagger V \neq \mathbb{1}]{V \otimes \mathbb{1}}$$

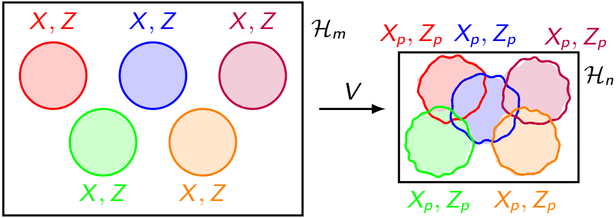
Hilbert space $\mathcal{H}_n$

Holographic description

Our question: Can non-isometric maps also describe the **Hilbert space truncation** that occurs in dS quantum gravity?

Overlapping qubits

What happens to qubits under a non-isometric map V?



Pauli operators in $\mathcal{H}_m$ are compressed to

$$X \to X_p = VXV^\dagger \ , \quad Z \to Z_p = VZV^\dagger \ .$$

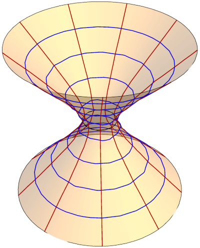
The n qubit algebra of the $X_p^{(j)}$ and $Z_p^{(k)}$ is *overlapping*, e.g.

$$||[X_p^{(j)}, X_p^{(k)}]|| = O(\epsilon) \quad \text{for } j \neq k \ .$$

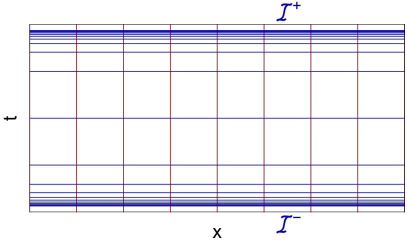
Surprisingly, small overlaps are possible for compression into **exponentially** fewer qubits: $\epsilon^2 \sim \frac{\log m}{n}$
[Chao/Reichardt/Sutherland/Vidick '17]

De Sitter spacetime

A physical setting with apparent Hilbert space growth:
dS spacetime, GR solution to pure $\Lambda > 0$ universe



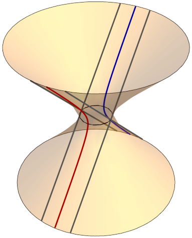
(Higher-dim. embedding)



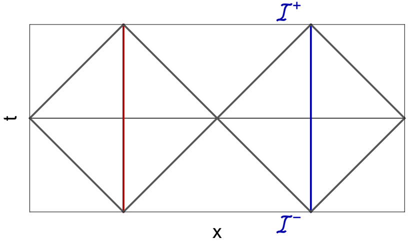
(Conformal coordinates)

De Sitter spacetime

A physical setting with apparent Hilbert space growth:
dS spacetime, GR solution to pure $\Lambda > 0$ universe



(Higher-dim. embedding)



(Conformal coordinates)

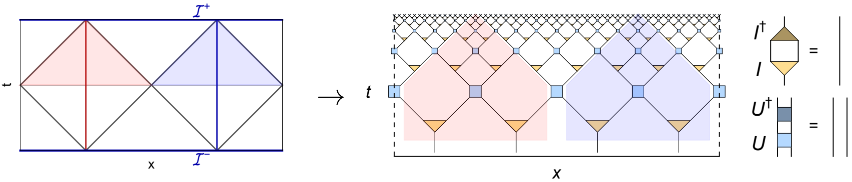
dS can be divided into *static patches* of stationary observers and rapidly expanding *exterior regions*.

De Sitter MERA

The *multi-scale entanglement renormalization ansatz* (MERA) [Vidal '06] can be used as a **toy model of dS spacetime**.

[Bény '11] [Czech/Lamprou/McCandlish/Sully '15]

[Bao/Cao/Carroll/Chatwin-Davis '17]

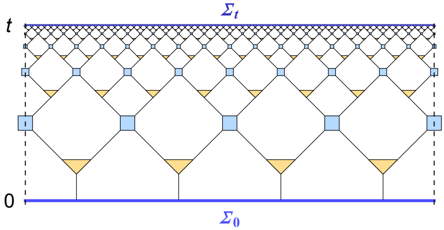


The **causal structure** of dS is reflected in the MERA tensors, built from isometries I and unitaries (disentanglers) U.

De Sitter MERA: non-isometric maps

From the MERA we can build two types of non-isometric maps for dS time-slices: A **global** and a **local map**.

The map $V_{\text{global}} : \mathcal{H}_{\Sigma_t} \to \mathcal{H}_{\Sigma_0}$ relates the global Hilbert space on a time-slice Σ_t to the "initial" one at $t = 0$.

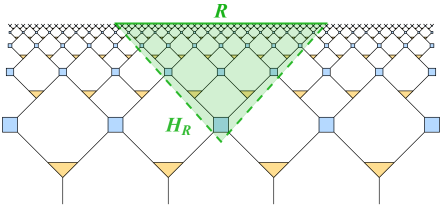


In a universe with such overlapping qubits in a fixed Hilbert space, non-local commutators become $O(1)$ at $\frac{t}{t_H} = O(S_{\text{dS}}) \approx 10^{120}$.

Curiously, this is consistent with models of slow-roll inflation. [Dubovsky/Senatore/Villadoro '08]

De Sitter MERA: non-isometric maps

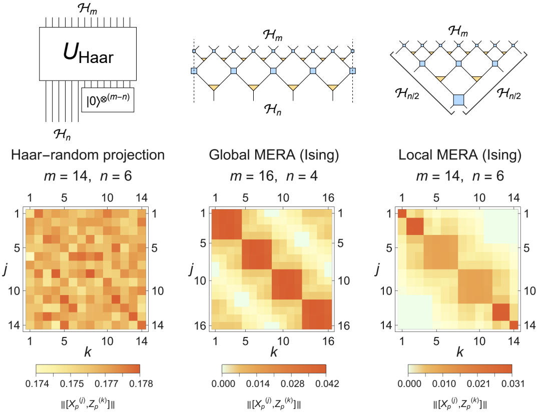
The alternative map $V_{\text{local}} : \mathcal{H}_R \to \mathcal{H}_{H_R}$ relates the Hilbert space on a time-slice *subregion* R to the horizon H_R of its past domain of dependence.



In this model, $\dim \mathcal{H}_R \sim e^{\dim \mathcal{H}_{H_R}}$. H_R is the "RT surface" of R, bounding its entanglement.

De Sitter MERA: Commutators

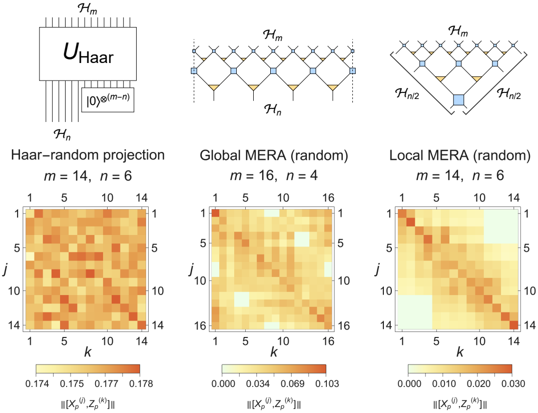
To preserve **approximate locality**, commutators $\|[X_p^{(j)}, Z_p^{(k)}]\|$ should decay with distance $|j-k|$.



The causal structure of MERA/dS preserves approximate commutator locality!

De Sitter MERA: Commutators

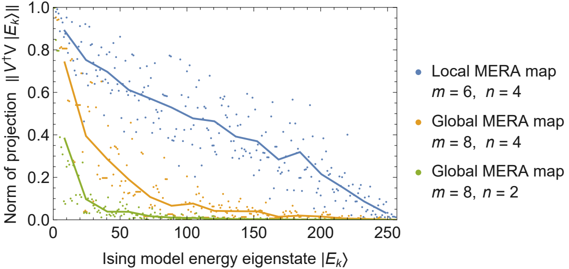
To preserve **approximate locality**, commutators $\|[X_p^{(j)}, Z_p^{(k)}]\|$ should decay with distance $|j-k|$.



The causal structure of MERA/dS preserves approximate commutator locality!

De Sitter MERA: Low-energy physics

Global and local MERA models have the same effect:
Low-energy eigenstates are preserved,
high-energy states are represented with low fidelity.

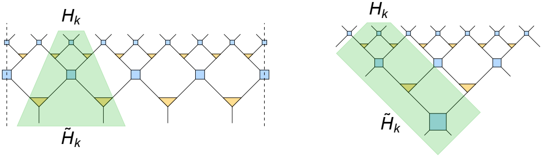


The global model sharply truncates after the first 2^n energy eigenstates, whereas the local model has a smooth behavior.

De Sitter MERA: Effective theories

Both the models describe an *effective theory* H of (subregions of) dS timeslices by a corresponding *fundamental theory* $\tilde{H}$.

- In the global MERA, $\tilde{H}$ is a truncation of H up to fixed scale/energy, **preserving locality**.
- In the local MERA, $\tilde{H}$ describes a very different theory: A local H leads to a **non-local** $\tilde{H}$.



Relationship to large-q SYK holographic proposal [Susskind '21]?

Discussion

Describing quantum gravity with overlapping qubits relates two old problems:

1. The problem of **degree of freedom counting** of Hilbert spaces when gravity is included
2. The problem of **Hilbert space dimension verification** for qubit systems

But many questions remain:

- Black hole evaporation with overlapping qubits?
- Is quantum gravity generally quantum mechanics in a different (smaller) Hilbert space?
- What about flat spacetime?
- Relation to "static patch holography"?
- Low-energy CFT simulations with local MERA model?

Thank you for your attention!

"Quantum gravity with overlapping qubits in Kyoto"
(Midjourney v5)