Symmetries, Einstein Equations, and Carrollian Hydrodynamics at Null Boundaries

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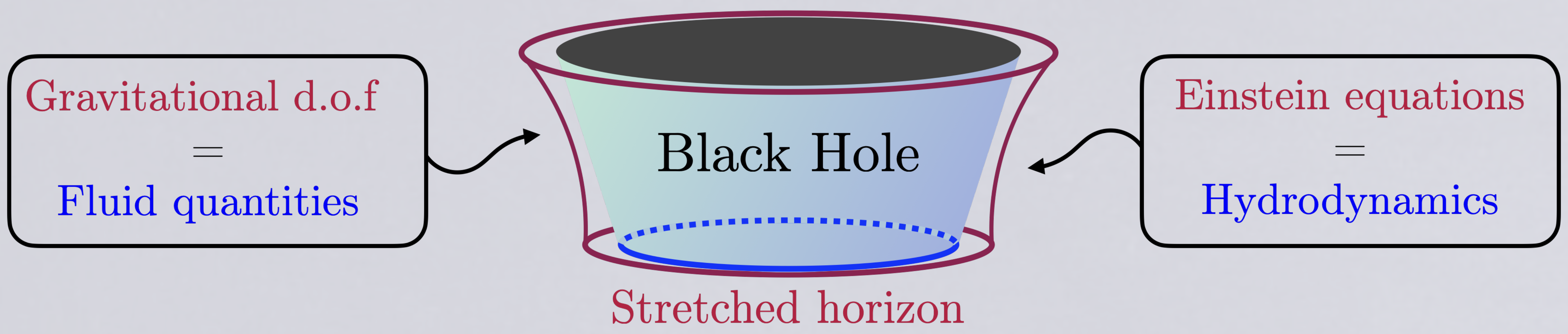
1. INTRODUCTION

The study of **gravitational subsystems** [1] when spacetime is divided into local subregions enclosed by some boundaries have opened an interesting avenue of exploration, offering novel insights into the nature of gravity and shed light on its quantum nature:

- Boundary/corner symmetries and edge modes
- Boundary dynamics and conservation laws
- Quasi-local holography

We study a spacetime geometry near a finite-distance null boundary (e.g., event horizons of black holes) and aim at exploring the relations between symmetries, dynamics of gravity, and hydrodynamics

- We tackle this problem from the perspective of the **membrane paradigm** [2, 3], acknowledging that some information of the null boundary can be accessed once considering small geometrical fluctuations around the boundary.
- We consider a timelike surface (a ***stretched horizon*** or a membrane) that can be used as a probe to study the true (null) boundary. It can be endowed with fluid quantities and the gravitational dynamics, governing by the Einstein equations, can be written as hydrodynamic conservation laws.



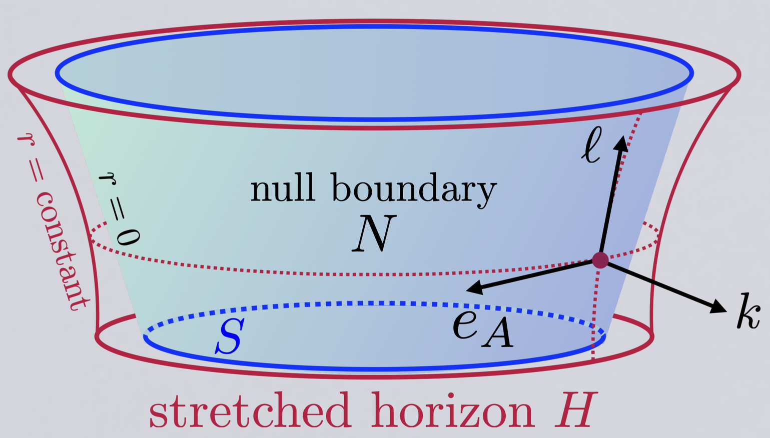
- There are some issues in the conventional membrane paradigm:
  - Some geometrical objects corresponding to fluid quantities *blow up* when taking the limit from the stretched horizon to the null boundary
  - Recent developments have argued that both the intrinsic geometry and the corresponding hydrodynamics at the null boundary should be ***Carrollian*** ($c \to 0$ limit) [4–7] instead of Galilean ($c \to \infty$ limit)

Objectives

- **Provide a unified treatment of both timelike and null hypersurfaces such that**
  - the null limit of horizon fluid quantities and the associated conservation laws are non-singular
  - the intrinsic Carrollian geometry is apparent on the surfaces
- **Draw a correspondence between gravitational d.o.f at the stretched horizon and Carrollian fluid**
- **Study the phase space of gravity, analyze the symmetries and the corresponding conservation laws and Noether charges**

2. GEOMETRY OF A STRETCHED HORIZON

- Both a stretched horizon and a null boundary can be thought of as hypersurfaces foliated in a spacetime and situated at $r =$ constant. A null boundary is at $r = 0$
- To construct the geometry of the stretched horizon, we employ the *rigging technique* for hypersurfaces [8].
- Starting from a ***null rigged structure*** on the stretched horizon, a ***Carroll structure*** can be induced, hence making a Carroll geometry fully manifests



Null rigged structure

- Normal form n_a
- Null rigging vector k^a
- Transversality: $k^a n_a = 1$
- Projector: $\Pi_a{}^b := \delta_a^b - n_a k^b$

$\Downarrow$

Carroll structure

- Vertical vector $\ell^a := n^b \Pi_b{}^a$
- Horizontal projector $q_a{}^b := \Pi_a{}^b - k_a \ell^b$

Spacetime metric
$g_{ab} = q_{ab} + k_a n_b + \ell_a k_b$

- Intrinsic geometry consists of a null-ness parameter ρ, a scale factor α, a Carrollian connection β_A, a velocity field V^A, and a 2-sphere metric q_{AB}
- Extrinsic geometry contains in the **Weingarten tensor**, $\mathcal{W}_a{}^b := \Pi_a{}^c(\nabla_c n^d)\Pi_d{}^b$, whose components serve as canonical conjugate momenta to the intrinsic variables
- **The null limits ($r \to 0$) of all geometrical quantities are regular**

3. CARROLLIAN HYDRODYNAMICS

- ***Brown-York-like Formalism:*** The energy-Momentum tensor of the stretched horizon, $(T_{\text{gravity}})_a{}^b$, is related to its extrinsic geometry $\mathcal{W}_a{}^b$
- It admits the decomposition in the same way as the energy-momentum tensor of Carrollian fluids, allowing us to construct a **gravitational dictionary** between physical d.o.f of gravity and Carrollian fluid quantities

$$\boxed{(T_{\text{gravity}})_a{}^b = \mathcal{W}_a{}^b - \mathcal{W}\Pi_a{}^b} \quad \Longleftrightarrow_{\textbf{Dictionary}} \quad \boxed{(T_{\text{fluid}})_a{}^b = -k_a\left(\mathcal{E}\ell^b + \mathcal{J}^b\right) + \pi_a \ell^b + \left(\mathcal{S}_a{}^b + \mathcal{P} q_a{}^b\right)}$$

$\mathcal{E}$ is energy density, $\mathcal{P}$ is pressure, $\mathcal{J}^a$ is Carrollian heat current, π_a is fluid momentum, and $\mathcal{S}_a{}^b$ is viscous stress tensor

- The vacuum **Einstein equations** $G_{ab} = 0$, governing the dynamics of the stretched horizon, implies the **Carrollian hydrodynamic conservation laws**, and vice versa

$$\boxed{\Pi_a{}^b G_{b\underline{n}} = D_b (T_{\text{fluid}})_a{}^b = 0}$$

- ***Covariant Phase Space:*** The gravity-hydrodynamic correspondence goes beyond the level of equations of motion. In addition, the phase space of gravity coincides with the phase space of Carrollian hydrodynamics

$$\boxed{\underbrace{\Theta_{\text{gravity}}}_{\text{Pre-symplectic potential}} = \underbrace{\delta S_{\text{fluid}}}_{\text{Carrollian hydrodynamic action}} - \underbrace{\int_H \bar{\theta}\delta\rho}_{\text{Null-ness of the surface}}}$$

4. SYMMETRIES, DYNAMICS, & CHARGES

- ***Near-Horizon Symmetries:*** A diffeomorphism ξ^a, characterized by functions (T, W, X^A), that preserves the structures of the null boundary,

$$\xi^a = T\ell^a + rWk^a + X^A e_A{}^a + \mathcal{O}(r)$$

It forms $\mathrm{Diff}(N) \Subset \mathrm{Weyl}(N)$ symmetry algebra

- ***Noether Current:*** Using the covariant phase space technology, we compute the Noether current associated with the near-horizon symmetries

$$\boxed{\underbrace{J_\xi}_{\substack{\text{Noether current}\\ \mathrm{d}J_\xi = 0}} = \underbrace{C_\xi}_{\text{Constraints}} + \underbrace{\mathrm{d}Q_\xi}_{\text{Noether charges}}}$$

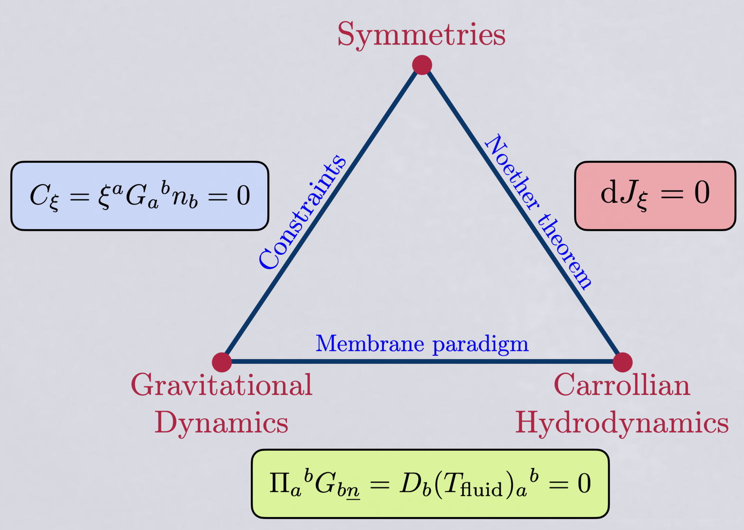
$$\boxed{\begin{aligned} C_\xi &= \xi^a G_a{}^b n_b \\ &= TG_{\underline{\ell\ell}} + X^A G_{\underline{\ell A}} + r\left(WG_{\underline{\ell k}} + D^{(A}X^{B)}G_{\underline{AB}}\right) + \ldots \end{aligned}}$$

The vanishing of the constraints, $C_\xi = 0$, imposes the dynamical Einstein equations on the null boundary

$$\boxed{G_{\underline{\ell\ell}} = 0, \qquad G_{\underline{\ell A}} = 0, \qquad G_{\underline{\ell k}} = 0, \qquad G_{\underline{AB}} = 0}$$

The last two equations only appear at the sub-leading order in r

5. CONCLUSIONS AND FUTURE DIRECTIONS



Future Directions

- Link with *null infinities*
- Flat space holography
- Thermodynamics of the stretched horizons and black holes
- Quantization of gravitational subsystems bounded by a null boundary

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