Parisi's hypercube, double-scaled SYK, Fock-space fluxes and $NAdS_2/NCFT_1$ duality

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Based on:
Berkooz-YJ-Silberstein 2023 [arxiv:2303.18182]
YJ-Verbaarschot 2020 [arxiv:2005.13017, JHEP11(2020)154]

Outline

- Double-scaled Sachdev-Ye-Kitaev model and near-AdS_2/near-CFT_1
- Parisi's hypercube
- Chord diagrams
- Characterization of the $NAdS_2/NCFT_1$ microscopics

(Double-scaled) SYK and $NAdS_2/NCFT_1$

- Sachdev-Ye-Kitaev:

$$H_{SYK} = i^{p/2} \sum_{i_1,\dots,i_p}^{N} J_{i_1,\dots,i_p} \underbrace{\psi_{i_1} \cdots \psi_{i_p}}_{\text{p << N (p-locality)}}, \qquad \{\psi_i, \psi_j\} = 2\delta_{ij}$$

- Two limits: 1. $p = fixed, N \to \infty$.

  2. $\lambda = \frac{p^2}{N} = fixed, N \to \infty$ (double scaled SYK),

  then $\lambda \to 0$ ("triple scaling").

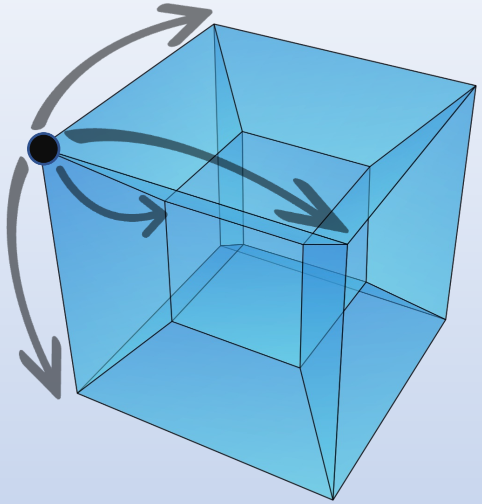
Both limits give nearly-conformal QM ($NCFT_1$) at low temperatures.

(Double-scaled) SYK and $NAdS_2/NCFT_1$

- $NAdS_2$: Jackiw-Teitelboim + matter (dim reduction from higher-dim black holes). Some characteristic behaviours:
  1. $\sim \sinh\sqrt{E}$ density of states
  2. Conformal correlation functions
  3. OTOC $\langle O(t)O(0)O(t)O(0)\rangle \sim \exp(\frac{2\pi}{\beta}t)$ (maximal chaos/fast scrambling)
- $NCFT_1$ from SYK reproduces all the above: $NAdS_2/NCFT_1$ duality
- Puzzle: $NAdS_2$ is a very ubiquitous solution in GR, but microscopic constructions of $NCFT_1$ are comparably rare. Essentially all SYK-like models so far.

  What's the general characterization of $NCFT_1$ microscopics?

Parisi's hypercube



- A useful stepping stone: Parisi's hypercube (Parisi 1994):

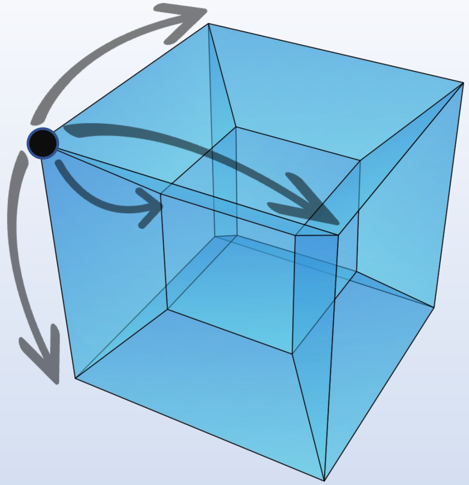
1) d-dim hypercube, $d \to \infty$
2) Single particle hopping, random uniform background fluxes $F_{\mu\nu}$, **i.i.d** with $\langle \sin F_{\mu\nu} \rangle = 0,\ \langle \cos F_{\mu\nu} \rangle \equiv q$

- It's a (continuous-time) quantum random walk model.
- Lattice gauge Hamiltonian (gauge-covariant Laplacian):

$$H_{\vec{x},\vec{y}} = -\frac{1}{\sqrt{d}} \sum_{\mu=1}^{d} U_\mu(\vec{x}) \delta_{\vec{x},\vec{y}+\hat{e}_\mu} + h.c.$$

Holonomy on a plaquette: $U_\nu^{-1}(\vec{x}) U_\mu^{-1}(\vec{x} + \hat{e}_\nu) U_\nu(\vec{x} + \hat{e}_\mu) U_\mu(\vec{x}) = e^{-i F_{\mu\nu}}$

Parisi's hypercube



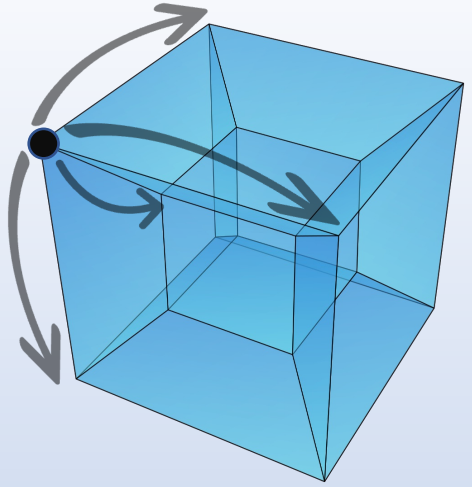
- Hamiltonian in qubit form:

$$H = -\frac{1}{\sqrt{d}}\sum_{\mu=1}^{d}(T_\mu^+ + T_\mu^-), \quad T_\mu^+ \equiv \left(\prod_{\nu,\nu\neq\mu} e^{\frac{i}{4}F_{\mu\nu}\sigma_\nu^3}\right)\sigma_\mu^+ . \quad \textbf{NOT p-local!}$$

Holonomy $\quad T_\nu^- T_\mu^- \, T_\nu^+ T_\mu^+ \propto e^{-i\,F_{\mu\nu}}$

- Superficially looks nothing like an SYK, but will give identical phenomenology.
- Goal: pinpoint what is actually in common, use it as the more general characterization for $NCFT_1$ microscopics.

Parisi’s hypercube



Alternative interpretation: hypercube as a Fock-space graph

1. Take the qubit Hamitonian as the starting point, many-body (but NOT p-local!).

2. Represent each basis vector as a point, connect two points if there is a nonzero transition amplitude.

3. Gives back the hypercube picture. Hypercube as a graphical representation of Fock space evolution (instead of a real-space hypercube)

4. Fluxes are defined in the Fock space.

Holonomy and moments

- A more convenient expression for holonomies:

$$D_\mu \equiv T_\mu^+ + T_\mu^- , \qquad W_{\mu\nu} \equiv D_\nu D_\mu D_\nu D_\mu = \cos F_{\mu\nu} - i \sin F_{\mu\nu}\, \sigma_\mu^3 \sigma_\nu^3,$$
$$\langle W_{\mu\nu} \rangle = \langle \cos F_{\mu\nu} \rangle = q.$$

- Moments

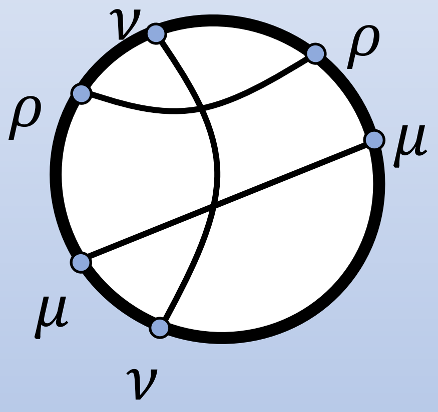
$$\langle Tr\, H^{2k} \rangle = \frac{1}{d^k} \sum_{\mu_1,\ldots,\mu_{2k}} \langle Tr\, D_{\mu_1} \cdots D_{\mu_{2k}} \rangle$$

- Trace → Loops in the Fock space → a forward hopping must be matched with a backward hopping in the same direction ➔ $\{\mu_1, \ldots \mu_{2k}\}$ form k pairs of indices
- Further coincidence among the k pairs ➔ 1/d suppressed

Chord diagrams

- Represent trace as a circle, draw subscripts on the circle, paired indices as chords.

Example:



$$= \frac{1}{d^3} \sum_{\mu \neq \nu \neq \rho} \langle Tr D_\rho D_\nu D_\rho D_\mu D_\nu D_\mu \rangle$$

- Apply $W_{\mu\nu}$ formula repeatedly (and that $D_\mu^2 = 1$), each nontrivial holonomy (interlaced ordering) appears as a chord intersection.

example = $\frac{1}{d^3} \sum_{\mu \neq \nu \neq \rho} \langle \cos F_{\mu\nu} \rangle \langle \cos F_{\nu\rho} \rangle = q^2$

Moments and chords

- In general [Parisi 1994, Marinari-Parisi-Ritort 1995, Cappelli-Colomo 1998, in a different language]

$$\langle Tr\ H^{2k} \rangle = \sum_{\substack{chord \\ diagrams}} q^{number\ of\ chord\ intersections}$$

Same as DSSYK [Erdos-Schroeder 2014, Cotler et al 2016,

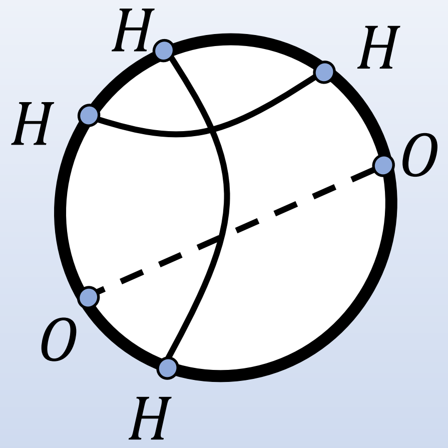
Garcia-Garcia-Verbaarschot 2017, Garcia-Garcia-**YJ**-Verbaarschot 2018,

Berkooz-Narayan-Simon 2018, Berkooz-Isachenkov-Narovlansky-Torrents 2018,

YJ-Verbaarschot 2019, **YJ**-Verbaarschot 2020 …]

$\sim \sinh \sqrt{E}$ density of states

Correlation functions



- Probes:

$$O = -\frac{1}{\sqrt{d}}\sum_\mu\left(\tilde{T}_\mu^+ + \tilde{T}_\mu^-\right), \quad \tilde{T}_\mu^+ \equiv \left(\prod_{\nu,\nu\neq\mu} e^{\frac{i}{4}\tilde{F}_{\mu\nu}\sigma_\nu^3}\right)\sigma_\mu^+ \quad ,$$

recall $$T_\mu^+ \equiv \left(\prod_{\nu,\nu\neq\mu} e^{\frac{i}{4}F_{\mu\nu}\sigma_\nu^3}\right)\sigma_\mu^+$$

- Two-point:

$$\langle \mathrm{Tr}\, H^{k_2}\, O\, H^{k_1} O\rangle = \sum q^{\#\, H-H\ inters.}\, \tilde{q}^{\#\, O-H\ inters.}, \quad \tilde{q} \equiv \langle \cos\frac{F+\tilde{F}}{2}\rangle.$$

➡ Same as DSSYK [Berkooz-Narayan-Simon 2018, Berkooz-Isachenkov-Narovlansky-Torrents 2018]

- True for arbitrary n-point function, all identical to DSSYK.

Correlation functions

- Consquences of such chord rules [Berkooz-Narayan-Simon 2018, Berkooz-Isachenkov-Narovlansky-Torrents 2018] in triple scaling limit:

    1. Conformal correlation functions

    2. OTOC $\langle O(t)O(0)O(t)O(0)\rangle$ ~ $\exp(\frac{2\pi}{\beta}t)$ (maximal chaos).

- Parisi model is at least as good a $NCFT_1$ microscopic construction as DSSYK

Comparison with the (DS)SYK

- Recall

$$H_{SYK} = \sum_I J_I \, \Psi_I, \quad I = \{i_1, i_2, \dots, i_p\}, \quad \Psi_I = i^{p/2} \psi_{i_1} \psi_{i_2} \cdots \psi_{i_p}$$

- Ψ_I is a hopping operator in the Fock space, I specifies the hopping direction (like the μ in the hypercube model).
- Fock-space holonomy $W_{IK} \equiv \Psi_K \Psi_I \Psi_K \Psi_I = (-1)^{|I \cap K|}$.
- Compare with the hypercube $D_\nu D_\mu D_\nu D_\mu = \cos F_{\mu\nu} - i \sin F_{\mu\nu} \, \sigma_\mu^3 \sigma_\nu^3$, we see the SYK holonomies are generated by uniform random fluxes of 0 or π on all plaquettes.

Comparison with the (DS)SYK

- From the Fock-space flux picture SYK is very similar to Parisi, however to achieve a complete analogy we still need

  1)fluxes on different plaquettes to be independent,

  2)the average holonomy to be a tunable parameter.

- These are achieved by going to the double scaled SYK (DSSYK) limit:

$$\frac{p^2}{N} = fixed, \qquad N \to \infty$$

- In the DS limit, set intersections $|I \cap K|$ becomes i.i.d., and average holonomy is

$$\langle (-1)^{|I \cap K|} \rangle = \exp\left(-\frac{2p^2}{N}\right) \equiv q.$$

- This analogy also extends to probes.
- This is essentially how you also obtain DSSYK chord diagrams.

Characterization of $NCFT_1$

- We now have a set of sufficient (not necessary) conditions for the microscopics that give rise to chord combinatorics and hence $NAdS_2/NCFT_1$ physics. All we need is a Fock-space flux that is

  1) uniform and quench-disordered, and

  2) i.i.d on different plaquettes, with a tunable average holonomy.

- In operator language, if $H = \sum_I J_I \widehat{M}_I$ ($\widehat{M}_I$ needs not be p-local):

  1) $\left[\widehat{M}_I, \widehat{M}_K \widehat{M}_L \widehat{M}_K \widehat{M}_L\right] = 0$ almost always.

  2) $Tr \widehat{M}_K \widehat{M}_L \widehat{M}_K \widehat{M}_L = i.i.d, \quad \left\langle Tr \widehat{M}_K \widehat{M}_L \widehat{M}_K \widehat{M}_L \right\rangle = q$.

Characterization of $NCFT_1$

- (double-scaled) p-local approach is an effective way to generate such fluxes, but it’s not the only way.
- Larger tool box for model building.
- One may wonder where the fluxes come from, I speculate they could arise as Berry curvatures.

Thank you!