

2023/10/3 “Quantum Information, Quantum Matter and Quantum Gravity” @YITP

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# **Blackhole shadow of a spatially superposed Schwarzschild spacetime**

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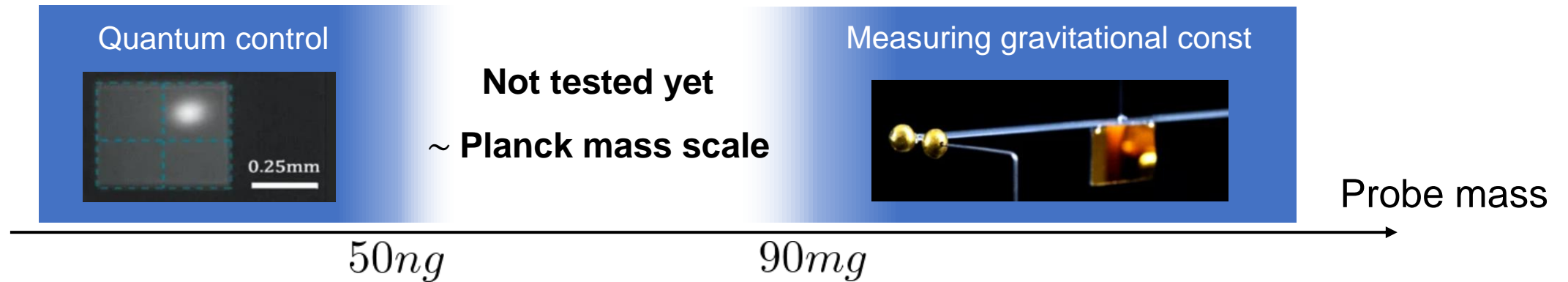
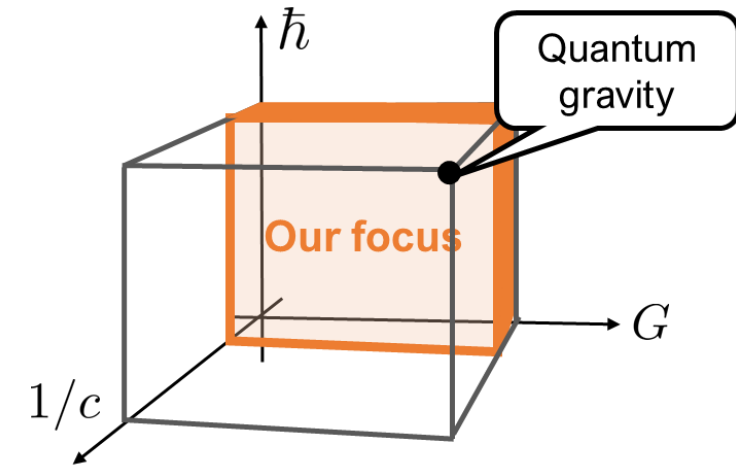
Supervisor: Yasusada Nambu

(On-going)

# Introduction

# Introduction

- We need experimental evidence to explore quantum gravity
- Quantum gravity theory  $\ni$  non-relativistic gravity in QM regime  
**Our focus**
- Regarding the recent progress of quantum experiment in mesoscopic scale, it is expected to see gravitational effect of quantum object in near future.



**How to test the quantum nature of non-relativistic gravity targeting on mesoscopic object?**

# Quantum gravity witness

Feynman (1957)

Bose et al. (2017), Marletto, Vedral (2017)

Carney, Muller, Taylor (2021)

- **Idea** “Is the gravitational field superposed when the mass source is in quantum superposition?”

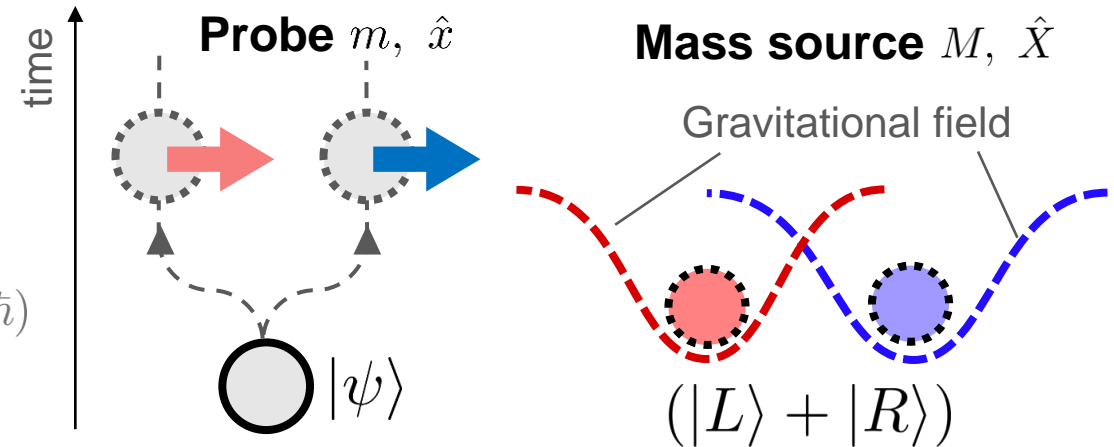
If Yes... Gravitational potential is

$$\Phi(\hat{x}, \hat{X}) = -\frac{GM}{|\hat{x} - \hat{X}|} : \text{depends on src operator}$$

Time evolution of probe and mass source is

$$e^{-im\Phi t/\hbar}|\psi\rangle \otimes (|L\rangle + |R\rangle) \quad (\phi_j = -im\Phi(\hat{x}, j)t/\hbar)$$
$$= e^{\phi_L}|\psi\rangle \otimes |L\rangle + e^{\phi_R}|\psi\rangle \otimes |R\rangle : \text{entangled state}$$

Probe feels superposed gravity and evolves to superposed state.



- **Issue** Other quantum interaction can also let the probe superposed and create entanglement.

How can we pick out quantum feature unique to gravity?

We need to deal with relativistic phenomena to distinguish gravity and other interactions!

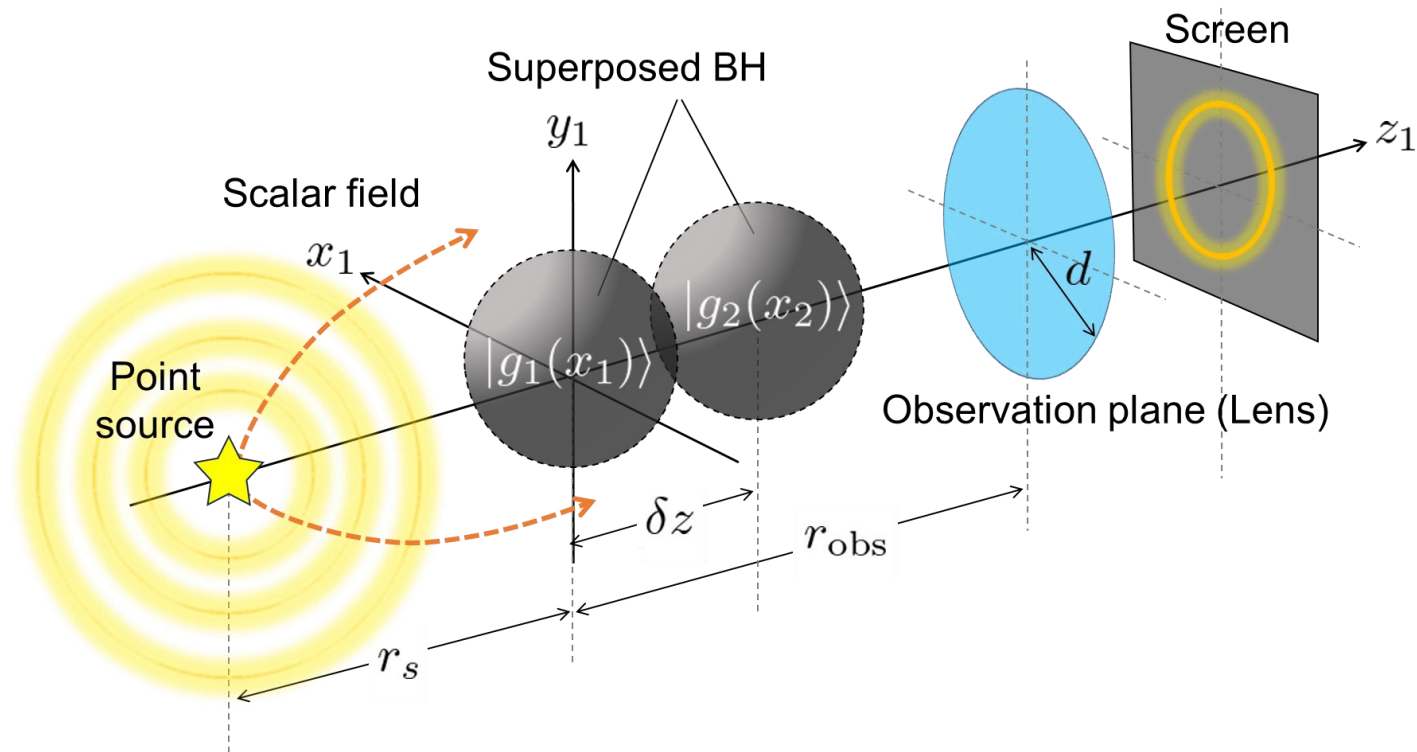
– Gravitational redshift using quantum clock: YK, Maeda, Nambu, Osawa (2022)

– **Gravitational lensing** ← Today's talk!! As a first step, we consider BH as toy model.

**BH shadow of a spatially superposed  
Schwarzschild spacetime  
(on-going work)**

# Setup

- Suppose a quantized scalar field propagating from the point source on a **spatially superposed Schwarzschild spacetime**
- Quantized scalar field propagate through a lens, and we observe the intensity of the point source (BH image) on the screen.



## BG spacetime in superposition

$$\frac{1}{\sqrt{2}} (|g_1(x_1)\rangle + |g_2(x_2)\rangle)$$

Assume that BH states are orthogonal

$$\langle g_1(x_1) | g_2(x_2) \rangle = 0$$

Each BH state gives Sch. metric:

$$\left\{ \begin{aligned} ds^2 &= (g_i)_{\mu\nu} d(x_i)^\mu d(x_i)^\nu \\ &= -F(r_i) dt^2 + \frac{dr_i^2}{F(r_i)} + r_i^2 d\Omega^2(\theta_i, \phi_i) \\ F(r_i) &= 1 - 2M/r_i \\ (x_2, y_2, z_2) &= (x_1, y_1, z_1 - \delta z) \end{aligned} \right.$$

# How to construct the BH image from quantum state?

Noda, Nambu (2022)

- Suppose 2 Unruh-de Witt detectors A and B on the observation plane and detect two point function of the scalar field.

Detector has two internal energy levels  $|0\rangle, |1\rangle$ .

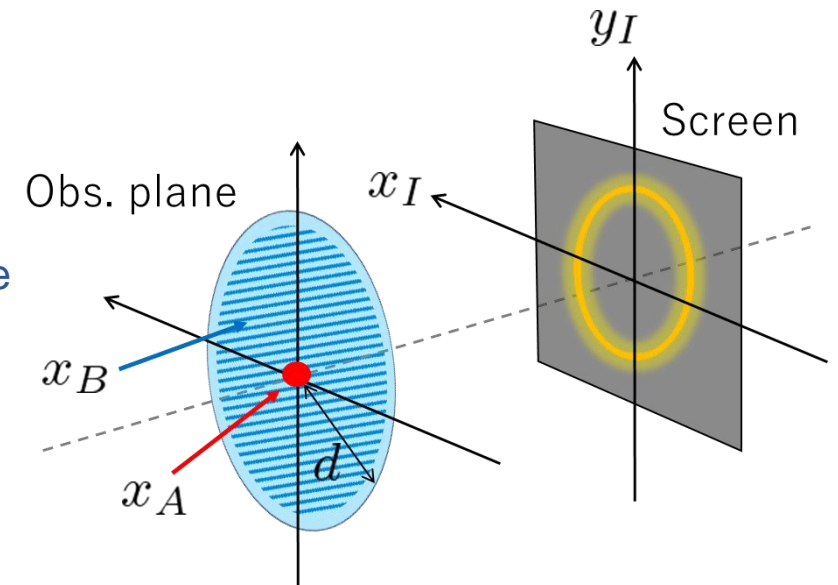
Detector position:  $\mathbf{x}_A = (r_{\text{obs}}, 0, 0)$ ,  $\mathbf{x}_B = (r_{\text{obs}}, \theta, \phi)$

Fixed at the center of obs. plane    Move freely on the plane

Interaction between detectors and scalar field:

$$H_I(t) = \lambda \sum_{D=A,B} (\sigma_D^+(t) + \sigma_D^-(t)) \Phi(t, \mathbf{x}_D)$$

$$\sigma_D^+(t) = e^{i\omega_0 t} |1, \mathbf{x}_D\rangle \langle 0, \mathbf{x}_D|, \quad \sigma_D^-(t) = e^{-i\omega_0 t} |0, \mathbf{x}_D\rangle \langle 1, \mathbf{x}_D|$$



Two point function is given by  $\langle \tilde{\Phi}(\omega_0, \mathbf{x}_A) \tilde{\Phi}(\omega_0, \mathbf{x}_B) \rangle \propto \text{Tr}_{\text{BH, scalar}} [{}_A \langle 0 | {}_B \langle 1 | \rho(t) | 1 \rangle_A | 0 \rangle_B]^*$

$$\tilde{\Phi}(\omega, \mathbf{x}) = (2\pi)^{-1} \int dt e^{i\omega_0 t} \Phi(t, \mathbf{x})$$

Time evolution of BH + scalar + 2 detectors

- Fourier transform the two point function to obtain the BH image on the screen.

$$\mathcal{F}(x_I, y_I) \propto \int_{\text{obs. plane}} d^2 \mathbf{x}_{AB} \text{Tr}_{\text{BH, scalar}} [{}_A \langle 0 | {}_B \langle 1 | \rho(t) | 1 \rangle_A | 0 \rangle_B]^* \exp \left[ \frac{i\omega_0}{r_{\text{obs}}} \mathbf{x}_I \cdot \mathbf{x}_{AB} \right]$$

: Explains optical deflection on lens

$$\mathbf{x}_{AB} := \mathbf{x}_A - \mathbf{x}_B$$

# Time evolution

$$\left( \begin{array}{l} \text{Lensing formula using UdW detectors} \\ \mathcal{F}(x_I, y_I) \propto \int d^2 x_{AB} \text{Tr} [{}_A\langle 0|_B\langle 1|\rho(t)|1\rangle_A|0\rangle_B]^* \exp \left[ \frac{i\omega_0}{r_{\text{obs}}} \mathbf{x}_I \cdot \mathbf{x}_{AB} \right] \end{array} \right)$$

- Let's calculate time evolution of “superposed BH + scalar + 2 detectors” systems to obtain BH image:

$$\rho(t) = e^{-i\mathcal{T}[\int dt H_I(t)]/\hbar} |\psi_{\text{ini}}\rangle \langle \psi_{\text{ini}}| e^{i\mathcal{T}[\int dt H_I(t)]/\hbar}$$

– Initial state:  $|\psi_{\text{ini}}\rangle = \frac{1}{\sqrt{2}} (|g_1(x_1)\rangle + |g_2(x_2)\rangle) \otimes \underbrace{|\Psi_{\text{vac}}\rangle}_{\text{Scalar}} \otimes \underbrace{|0, x_{A1}\rangle \otimes |0, x_{B1}\rangle}_{\text{2 detectors in ground state}}$

– Hamiltonian:  $H_I(t) = \sum_{i=1,2} h_i(t) |g_i\rangle \langle g_i|$  Time evolution depends on the BH state  
⇒ superposition of probe scalar, BH-others entanglement

Here,  $h_i(t) = \lambda \sum_{D=A,B} (\sigma_D^+(t) + \sigma_D^-(t)) \Phi_{g_i}(t, x_{Di})$

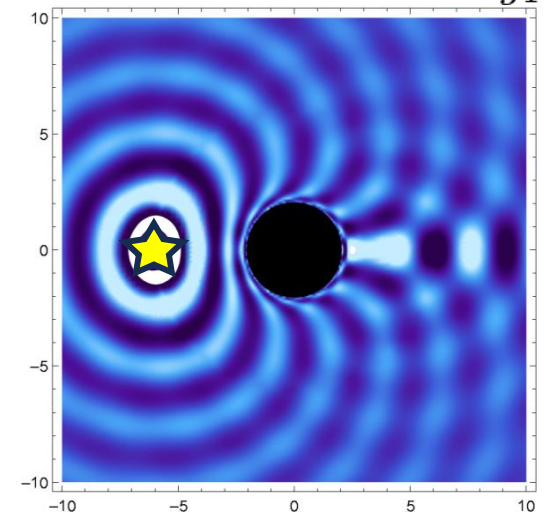
is an interaction btw scalar and detectors for  $|g_i\rangle$ , and

$$\sigma_D^+(t) = e^{i\omega_0 t} |1\rangle_D \langle 0|, \quad \sigma_D^-(t) = e^{-i\omega_0 t} |0\rangle_D \langle 1|$$

$$\square \Phi_{g_i}(t, x_i) = (g_i)^{\mu\nu} \nabla_\mu \nabla_\nu \Phi_{g_i}(t, x_i) = S \delta(\mathbf{x} - \mathbf{x}_s)$$

KG eq. on BH spacetime  $g_i(x_i)$  with the point source term  
 (Boundary conditions: In-going at the horizon, Out-going at infinity)

Numerical sols of  $\langle \Phi_{g_1}^2 \rangle$





# Image of superposed BH

- BH image

After we substitute the time evolved state into the lensing formula, we obtain

$$\mathcal{F}(x_I, y_I) = \frac{1}{2} \{ \mathcal{F}_1(x_I, y_I) + \mathcal{F}_2(x_I, y_I) \}$$

Here,  $\mathcal{F}_i(x_I, y_I)$  is the image of the classical BH spacetime  $g_i(x_i)$

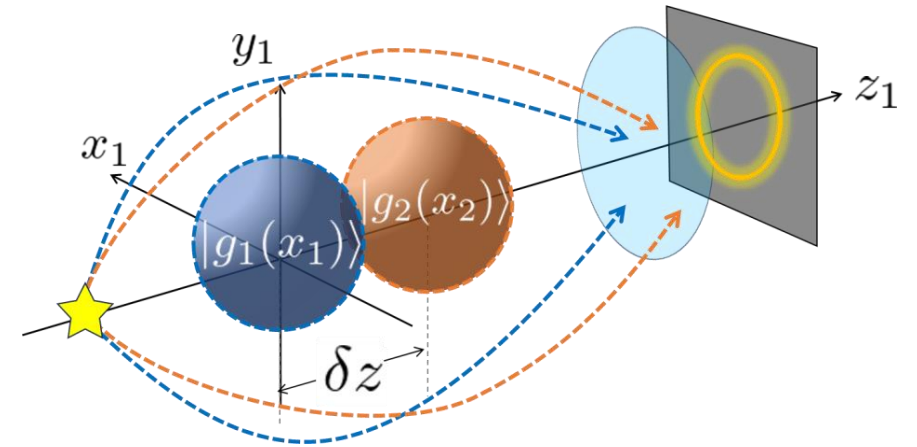
$$\mathcal{F}(x_I, y_I) \propto \int_{\text{obs. plane}} d^2 \mathbf{x}_{ABi} \left\langle \tilde{\Phi}_{g_i}(\omega_0, \mathbf{x}_{Ai}) \tilde{\Phi}_{g_i}(\omega_0, \mathbf{x}_{Bi}) \right\rangle \exp \left[ \frac{i \omega_0}{r_{\text{obs}}} \mathbf{x}_I \cdot \mathbf{x}_{ABi} \right]$$

2 point function of the scalar field on the classical Sch. Spacetime.  
We can calculate this by solving KG equation numerically!

If we assume that the BH states are orthogonal  $\langle g_1(x_1) | g_2(x_2) \rangle = 0$ ,

**BH image of superposed Sch. spacetime is simply given by**

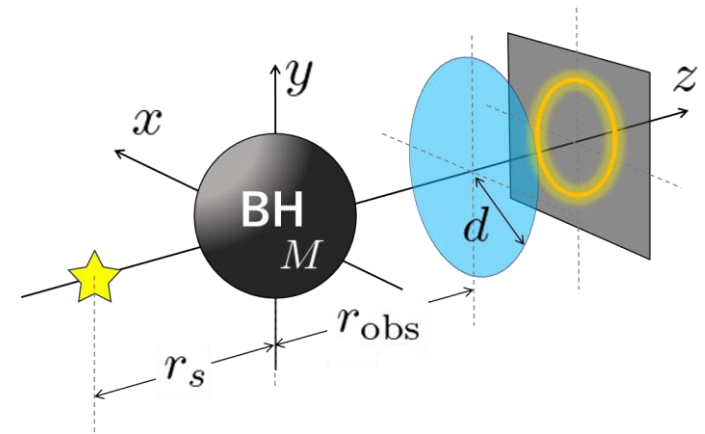
**the summation of BH images of each spacetime.**



# Result 1: Classical BH case

- BH image of a classical Schwarzschild spacetime

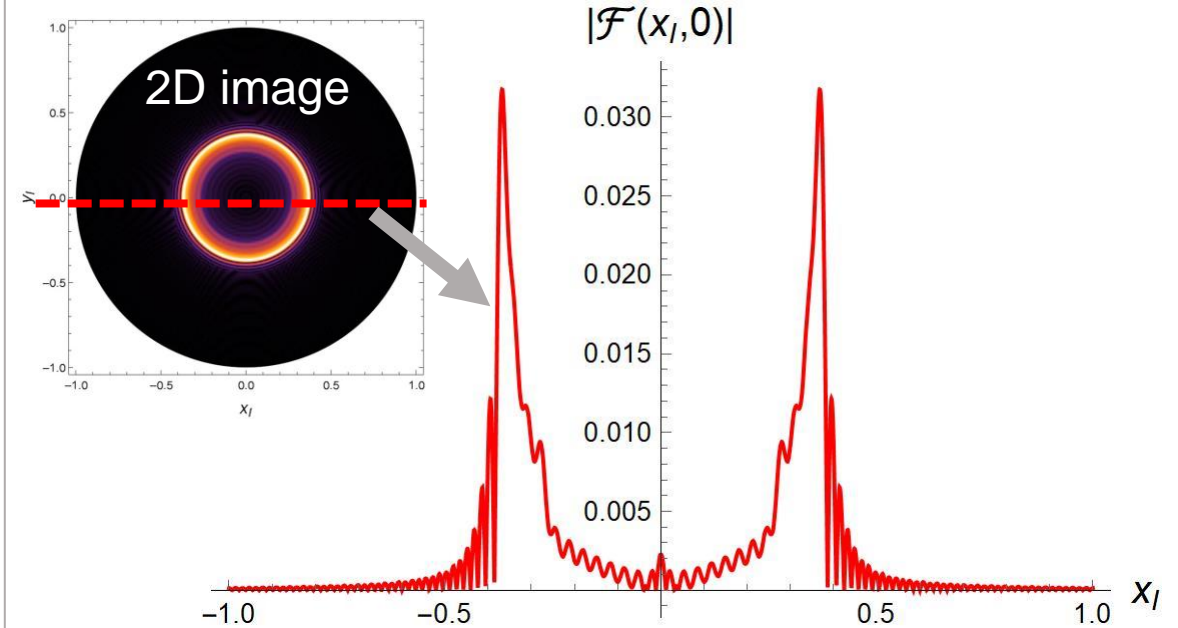
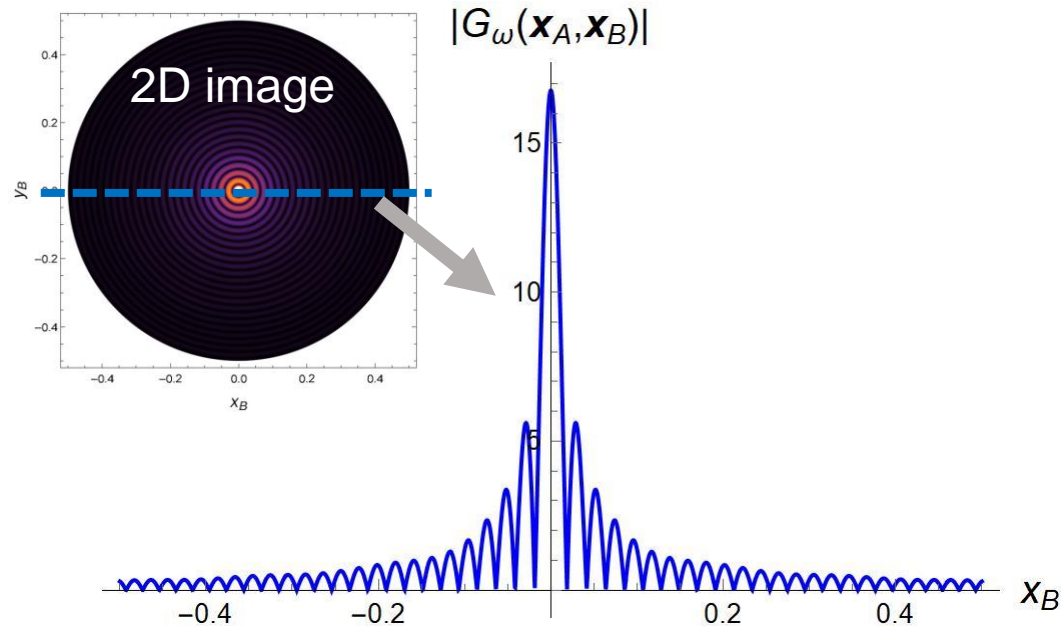
$$M = 1, \omega = 20, r_{\text{obs}} = 20, r_s = 6, d = 0.5$$



2 point func. on the obs. plane

Fourier trf.

BH image on the screen

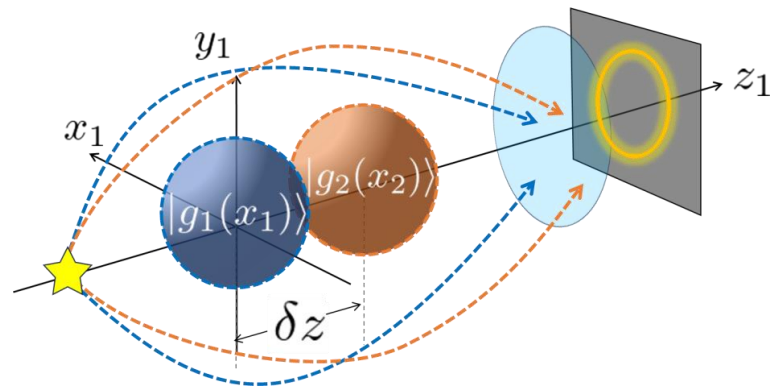


Einstein ring appears!

# Result2: Superposed BH case

- BH image of a superposed Schwarzschild spacetime

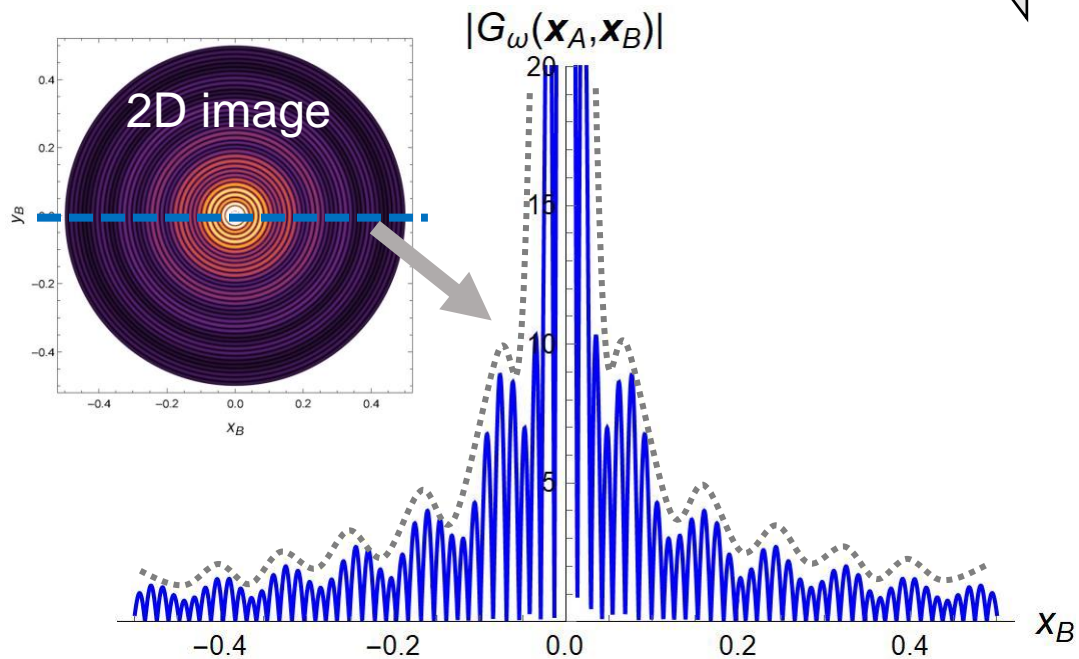
$$M = 1, \omega = 20, r_{\text{obs}} = 20, r_s = 6, d = 0.5, \delta z = 4$$



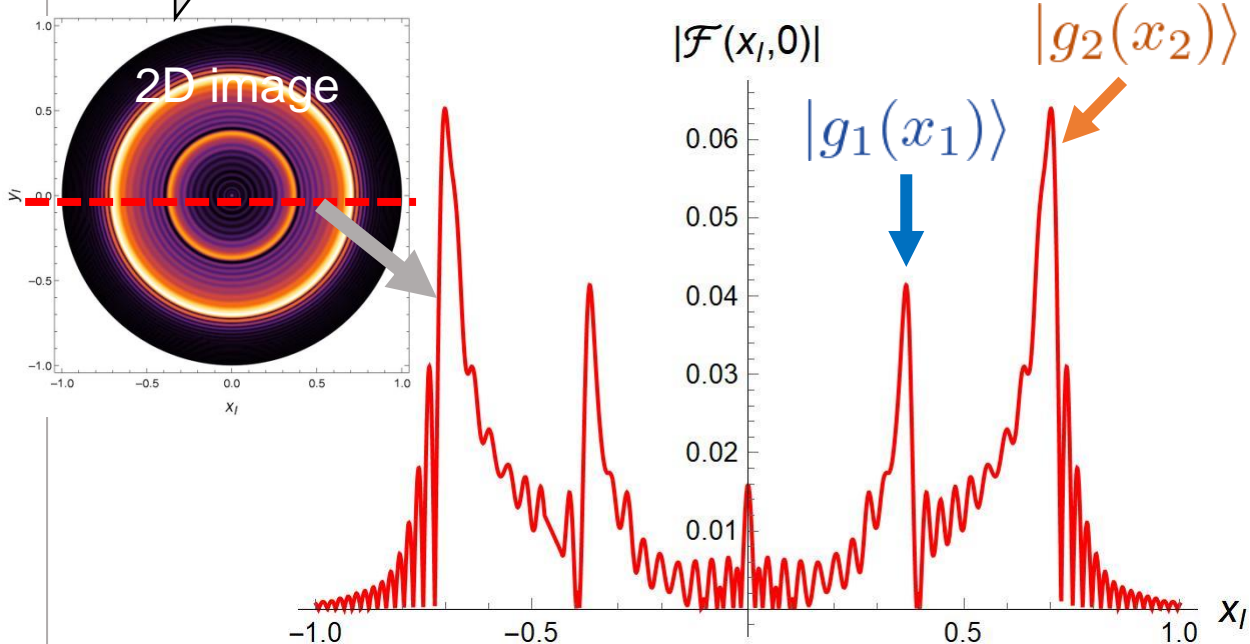
2 point func. on the obs. plane

Fourier trf.

BH image on the screen



Interference pattern of two wave modes scattered by  $|g_1(x_1)\rangle$  and  $|g_2(x_2)\rangle$

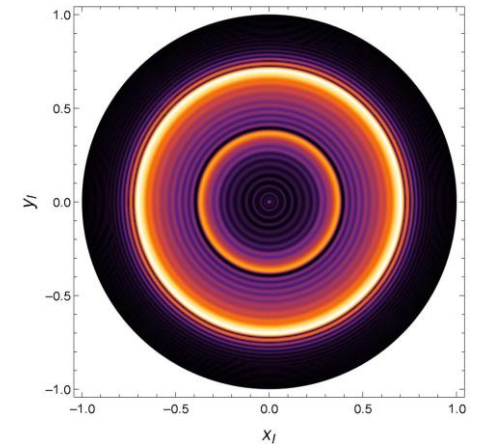


Double Einstein rings appear which peak corresponds to  $|g_1(x_1)\rangle$  and  $|g_2(x_2)\rangle$

# Conclusion

# Conclusion

- Issue of the BMV proposal : We cannot distinguish the quantum feature of gravity and other quantum interactions.
  - **We stepped into a relativistic phenomena to capture quantum feature unique to gravity.**
- BH shadow of a spatially superposed Schwarzschild spacetime
  - Superposition of Sch. BH  $\frac{1}{\sqrt{2}} (|g_1(x_1)\rangle + |g_2(x_2)\rangle)$   $\left[ \langle g_1(x_1) | g_2(x_2) \rangle = 0 \right]$
  - **Image of superposed BH = Summation of images of each classical BH**
- Future work
  - What if BH states are not orthogonal?  $\langle g_1(x_1) | g_2(x_2) \rangle \neq 0$
  - What is the difference between scattering by superposed Coulomb potential?
    - Did we really see the quantum feature unique to gravity?
  - Extension to tabletop experiment (weak gravity case)



**Thank you!**

**Backup**

# 導入

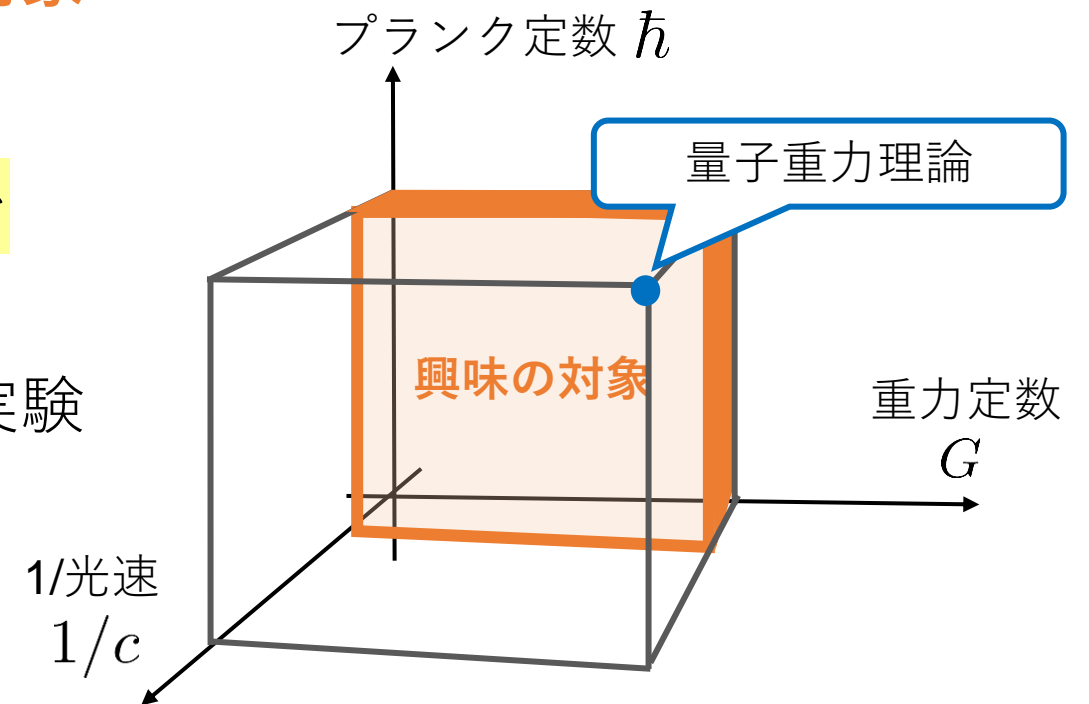
# 重力の量子性の検証

- 重力の量子論はよく分かっていない
- 量子重力を探るためには、実験事実が不可欠！
- 量子重力理論  $\Rightarrow$  量子力学の枠組みにおける非相対論的な重力理論

## 興味の対象

非相対論的な重力の量子的性質を  
検証実験で明らかにしよう

Bose et al., Marletto, Vedral による提案: BMV実験



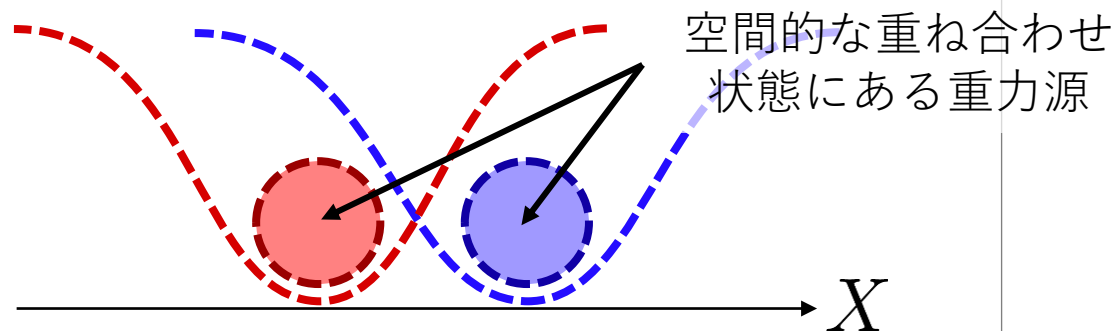


# 重力の量子性の検証

Feynman (1957), Zeh (2008)

「重力源が重ね合わせ状態のとき、重力場も重ね合わさるか？」を明らかにしたい

Yes → 量子化された重力 (QG)



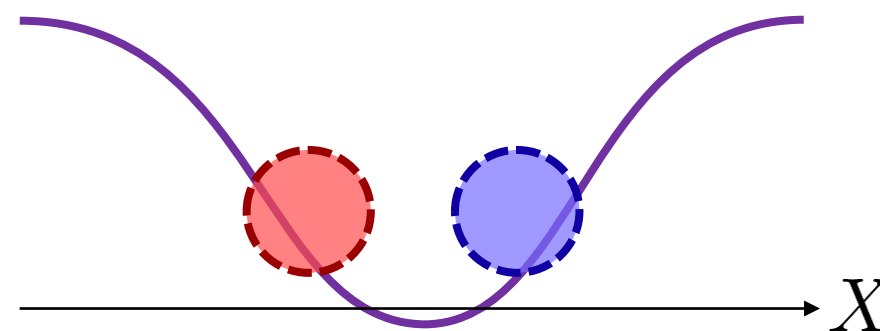
$x$  におけるニュートンポテンシャルが重力源系に対する**演算子**として作用

$$\Phi(\hat{x}, \hat{X}) = -\frac{GM}{|\hat{x} - \hat{X}|}$$

言い換えれば、

「重力ポテンシャルが重力源の演算子を含む/ 含まないか？」を明らかにしたい

No → 準古典的な重力 (CG)



$$\Phi(\hat{x}) = \left\langle -\frac{GM}{|\hat{x} - \hat{X}|} \right\rangle_{\text{重力源}}$$

# BMV実験

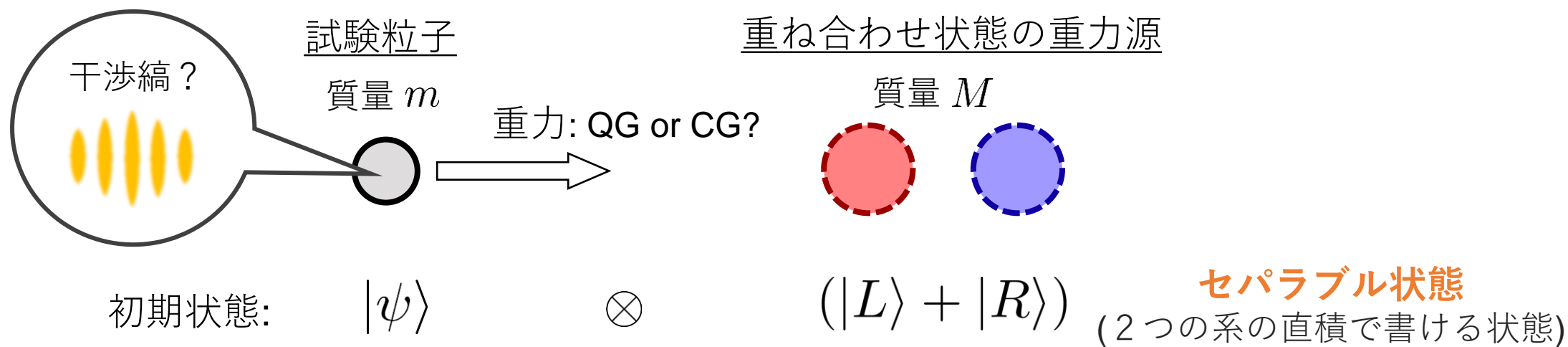
Bose et al. (2017), Marletto, Vedral (2017)  
Carney, Muller, Taylor (2021)

## ● 主旨

重力下の粒子の干渉縞の観測から、ニュートン重力の重ね合わせが実現するか否かを検証する

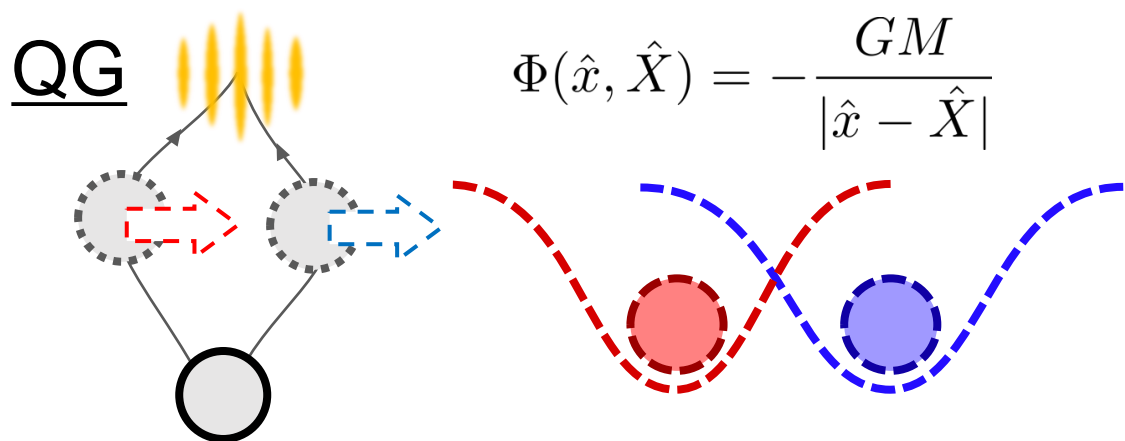
## ● セットアップ

空間的な重ね合わせ状態にある重力源と相互作用する粒子を用意し、時間発展後の試験粒子の位置の測定を行う。



# BMV実験

Bose et al. (2017), Marletto, Vedral (2017)  
Carney, Muller, Taylor (2021)

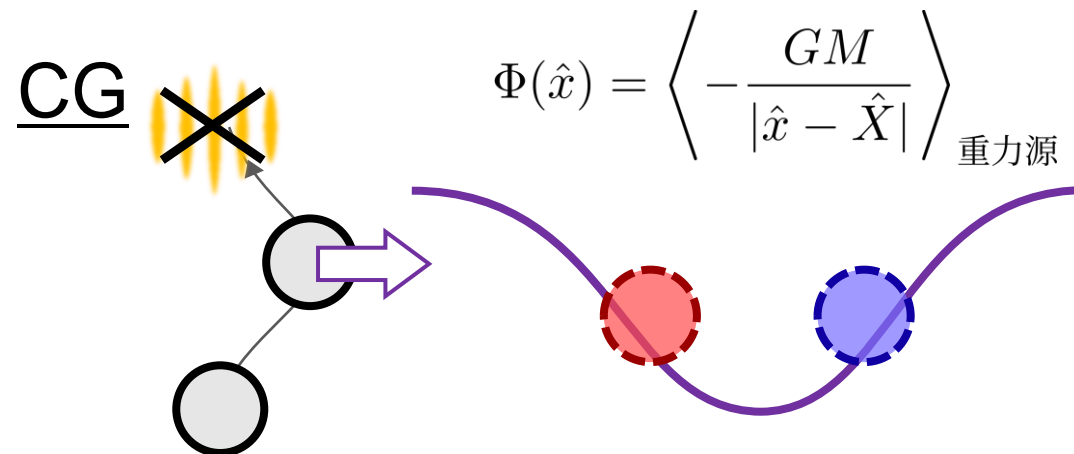


$$e^{-im\Phi t/\hbar}|\psi\rangle \otimes (|L\rangle + |R\rangle)$$
$$= e^{\phi_L}|\psi\rangle \otimes |L\rangle + e^{\phi_R}|\psi\rangle \otimes |R\rangle$$

量子もつれ状態へ発展

$$(\phi_j = -im\Phi(\hat{x}, j)t/\hbar)$$

⇒ QGによって干渉縞ができる



$$e^{-im\Phi t/\hbar}|\psi\rangle \otimes (|L\rangle + |R\rangle)$$
$$= e^{\phi}|\psi\rangle \otimes (|L\rangle + |R\rangle)$$

セパラル状態のまま

$$(\phi = -im\Phi(\hat{x})t/\hbar)$$

⇒ 干渉縞は見えない

干渉縞の有無を見れば、重力場の重ね合わせ状態が作られるか否かを検証できる

# モチベーション

## ● BMV実験の課題

- 重力以外の他の量子相互作用でも、量子もつれは生成され干渉縞が見える。

$$\text{QG: } \Phi(\hat{x}, \hat{X}) = -\frac{GM}{|\hat{x} - \hat{X}|} \quad \text{クーロン力: } A_t(\hat{x}, \hat{X}) = \frac{Q}{4\pi\epsilon_0} \frac{1}{|\hat{x} - \hat{X}|}$$

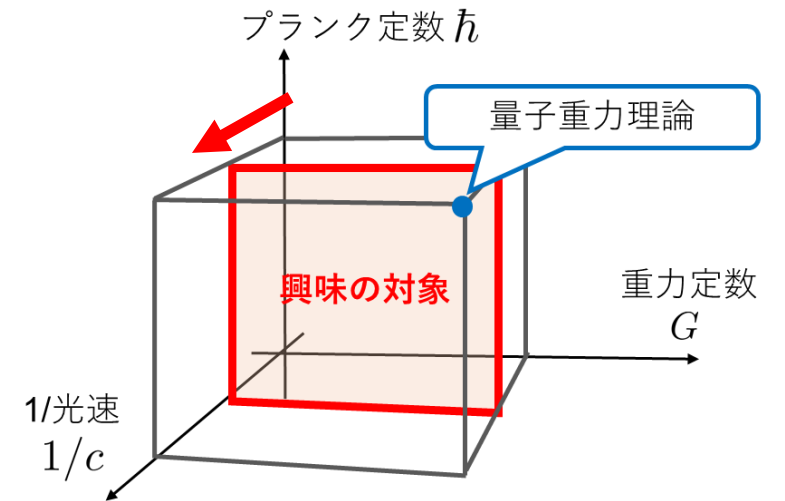
- クーロン力が混在すると、BMV実験は機能しない！  
**重力に固有の量子性を検証する方法はないか？**

## ● 本研究でやりたいこと

- 重力とそれ以外の力を区別するには、  
**相対論的な現象**に踏み込む必要がある

例：重力赤方偏移 ⇒ YK, Maeda, Nambu, Osawa (2022)

**重力レンズ効果** ⇒ 今回の話 (on-going work)



# 古典的なSchwarzschild時空における 量子スカラー場の重力レンズ効果

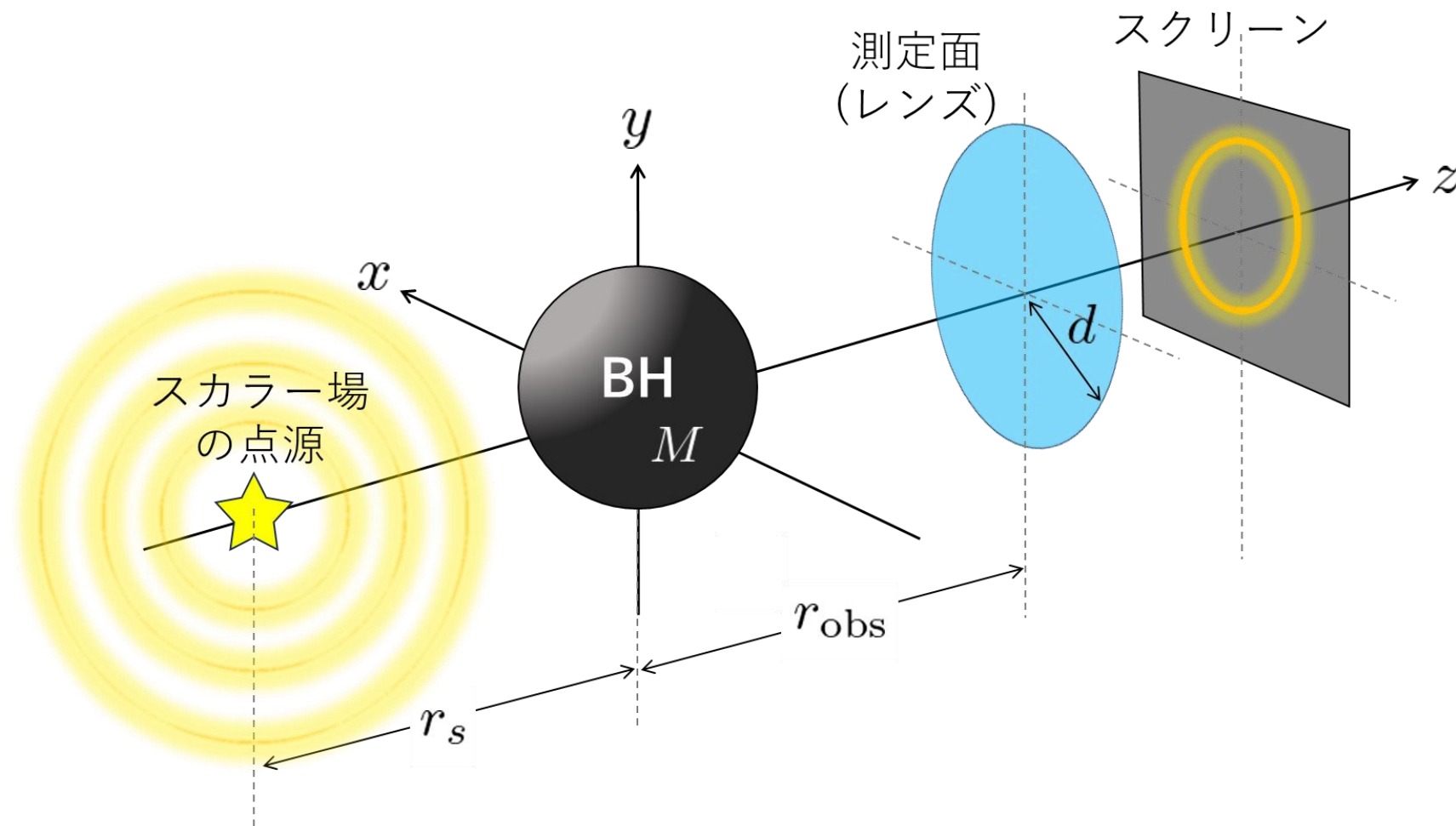
[先行研究 Noda, Nambu 2022, Caribe Jonsson, Casals 2023]

※ Hawking radiationの重力レンズ効果の研究

今回は、horizon外の光源から伝播する量子スカラー場のlensingを考える

# セットアップ

- 古典的なSchwarzschild時空において、点源から伝播する量子スカラー場
- 量子スカラー場の測定から、スクリーン上での点源のintensity (BH像)を求める



# スカラー場の時間発展

Noda, Nambu (2022)  
Caribe Jonsson, Casals (2023)

- 背景時空: 古典的なSchwarzschild時空 ※ Eternal blackhole

$$ds^2 = -F(r)dt^2 + \frac{1}{F(r)}dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad F(r) = 1 - \frac{2M}{r}$$

- Massless, real 量子スカラー場の時間発展

モード展開  $\hat{\Phi}(x) = \int_0^\infty d\omega \hat{a}_{\ell\omega} e^{-i\omega t} \Phi_\omega(\mathbf{x}) + \hat{a}_{\ell\omega}^\dagger e^{i\omega t} \Phi_\omega^*(\mathbf{x})$

$$\Phi_\omega(\mathbf{x}) = \sum_\ell \frac{1}{r} R_{\ell\omega}(r) P_\ell(\cos \theta)$$

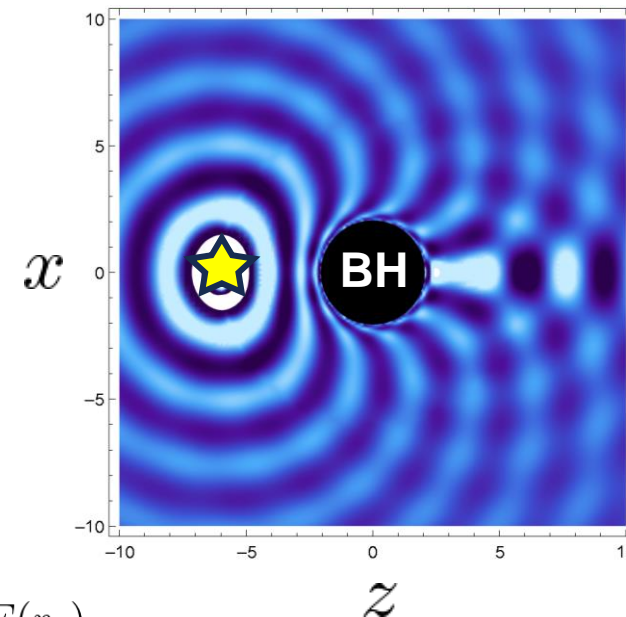
Klein-Gordon eq.  $\square \hat{\Phi}(x) = \hat{\rho}(x), \quad \hat{\rho}(x) = S \delta(x - x_s)$  より、

$r$ 方向のEoM  $\frac{d^2}{dr_*^2} R_{\ell\omega}(r) + (\omega^2 - V_{\text{eff}}(r)) R_{\ell\omega}(r) = S_\ell \delta(r - r_s)$

$$V_{\text{eff}} = \omega^2 - F(r) \left( \frac{F'(r)}{r} + \frac{\ell(\ell+1)}{r^2} \right), \quad S_\ell = (-)^\ell \left( \ell + \frac{1}{2} \right) S r_s F(r_s)$$

境界条件  $R_{\ell\omega}(r) \sim \begin{cases} e^{-i\omega r^*} & (r \rightarrow r_h) \\ e^{i\omega r^*} & (r \rightarrow \infty) \end{cases}$

スカラー場のamplitude



# Unruh-deWitt測定器を用いた撮像

Noda, Nambu (2022)

## ● Van Cittert-Zernike theorem

スクリーン上の点源のintensityは、測定面上のスカラー場の2点関数のフーリエ変換

$$\mathcal{F}(x_I, y_I) = \int_{|\mathbf{x}_{AB}| \leq d} d^2 \mathbf{x}_{AB} \frac{G_\omega(\mathbf{x}_A, \mathbf{x}_B)}{r_{\text{obs}}} \exp \left[ i \frac{\omega}{r_{\text{obs}}} \mathbf{x}_I \cdot \mathbf{x}_{AB} \right]$$

測定面上におけるスカラー場の2点相関関数  $G_\omega(\mathbf{x}, \mathbf{y}) = \langle \Phi_\omega(\mathbf{x}) \Phi_\omega(\mathbf{y}) \rangle$

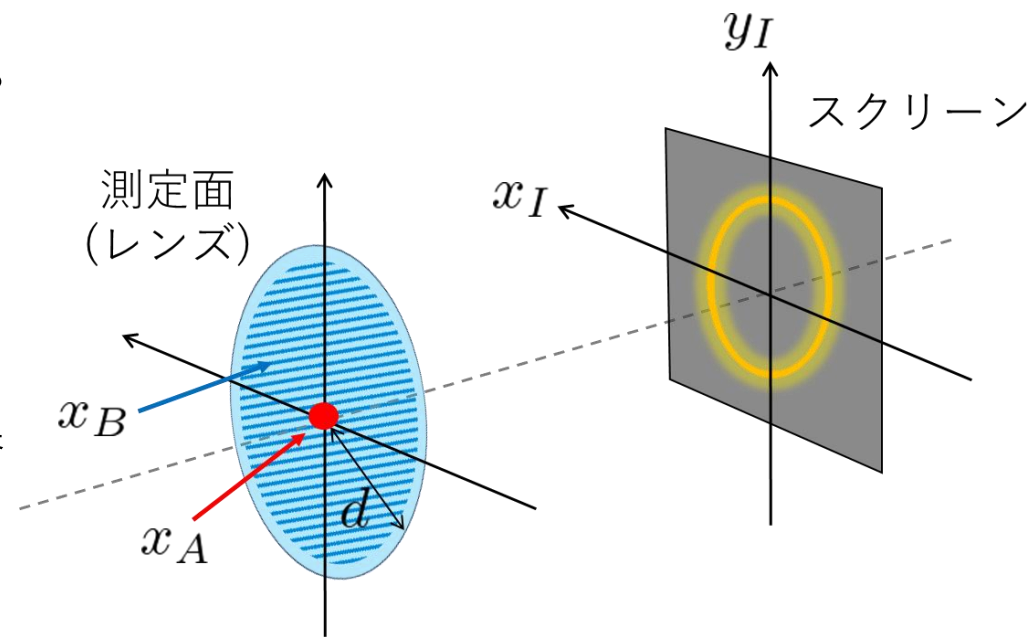
ただし、 $\mathbf{x}_I$ はスクリーン上の座標、 $\mathbf{x}_{AB} := \mathbf{x}_A - \mathbf{x}_B$ は測定面上の2点間の距離

今回は  $\mathbf{x}_A = (r_{\text{obs}}, 0, 0)$ ,  $\mathbf{x}_B = (r_{\text{obs}}, \theta, \phi)$  とする。

1点はz軸上(点源の対極)に固定 積分変数

後のために、任意の量子状態にどのような操作をすれば点源のintensityが得られるかを考える。

⇒ 測定面上の位置  $\mathbf{x}_A$  と  $\mathbf{x}_B$  にUnruh-deWitt 測定器を置いて2点関数を測定





# Unruh-deWitt測定器を用いた撮像

Noda, Nambu (2022)

- 任意の量子状態から点源のintensityを得る公式

- 測定面上の2つのUnruh-deWitt測定器とスカラー場の相互作用ハミルトニアン

$$H_I(t) = \lambda \sum_{D=A,B} (\sigma_D^+(t) + \sigma_D^-(t)) \Phi(t, x_D)$$

ただし  $\sigma_D^+(t) = e^{i\omega_0 t} |1, x_D\rangle \langle 0, x_D|$ ,  $\sigma_D^-(t) = e^{-i\omega_0 t} |0, x_D\rangle \langle 1, x_D|$

エネルギー準位 (0 or 1)      測定器の位置

測定器の2準位間のエネルギー差

- 初期状態  $|\psi\rangle = |\Psi_{\text{vac}}\rangle \otimes |0, x_A\rangle \otimes |0, x_B\rangle$

Unruh vacuum  $\hat{a}_{\ell\omega} |\Psi_{\text{vac}}\rangle = 0$

2つの測定器の干渉から、スカラー場の2点相関関数が得られる。

$$\text{Tr}_F [{}_A\langle 0|{}_B\langle 1|\rho(t)|1\rangle_A|0\rangle_B] = 2\pi\lambda^2 \left( \int dt_+ \right) G_{\omega_0}^*(x_A, x_B)$$

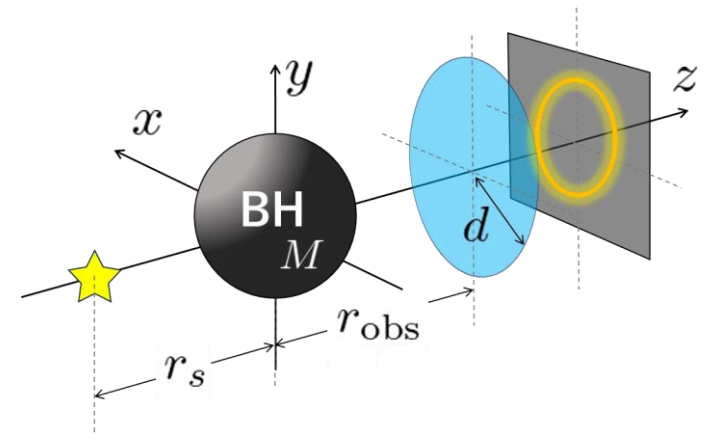
これより、量子状態  $\rho(t)$  から点源のintensityを計算する公式が以下で与えられる

$$\mathcal{F}(x_I, y_I) = \left( 2\pi\lambda^2 \int dt_+ \right)^{-1} \int d^2x_{AB} \text{Tr}_F [{}_A\langle 0|{}_B\langle 1|\rho(t)|1\rangle_A|0\rangle_B]^* \exp \left[ i \frac{\omega}{r \mathbf{x}_I \cdot \mathbf{x}_{AB}} \right]$$

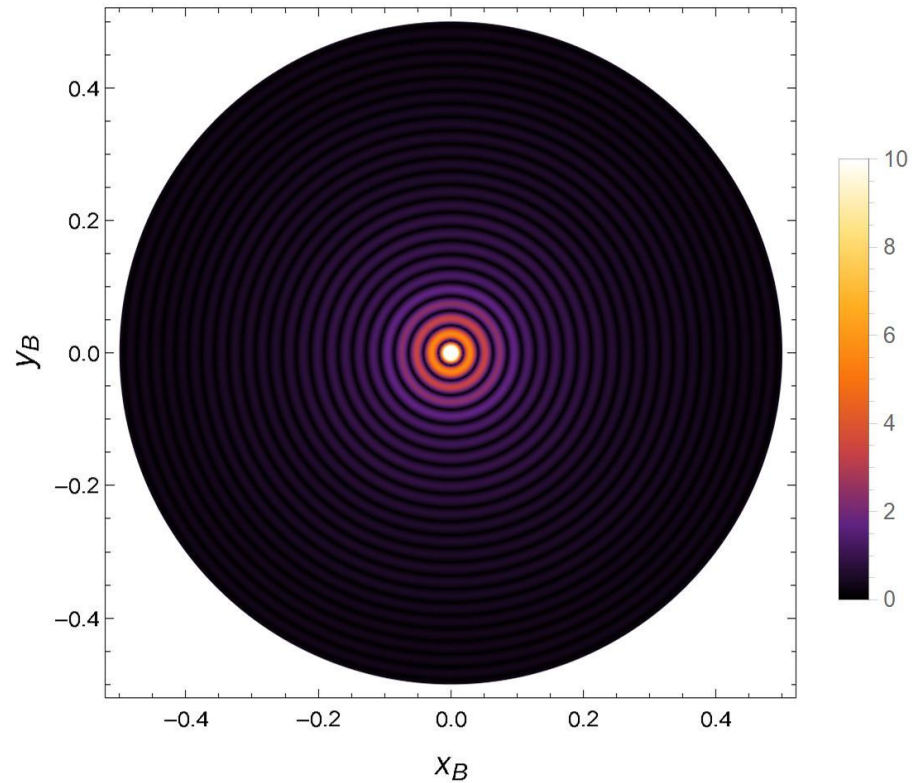
# 結果

- Schwarzschild時空におけるBH像（2次元面上）

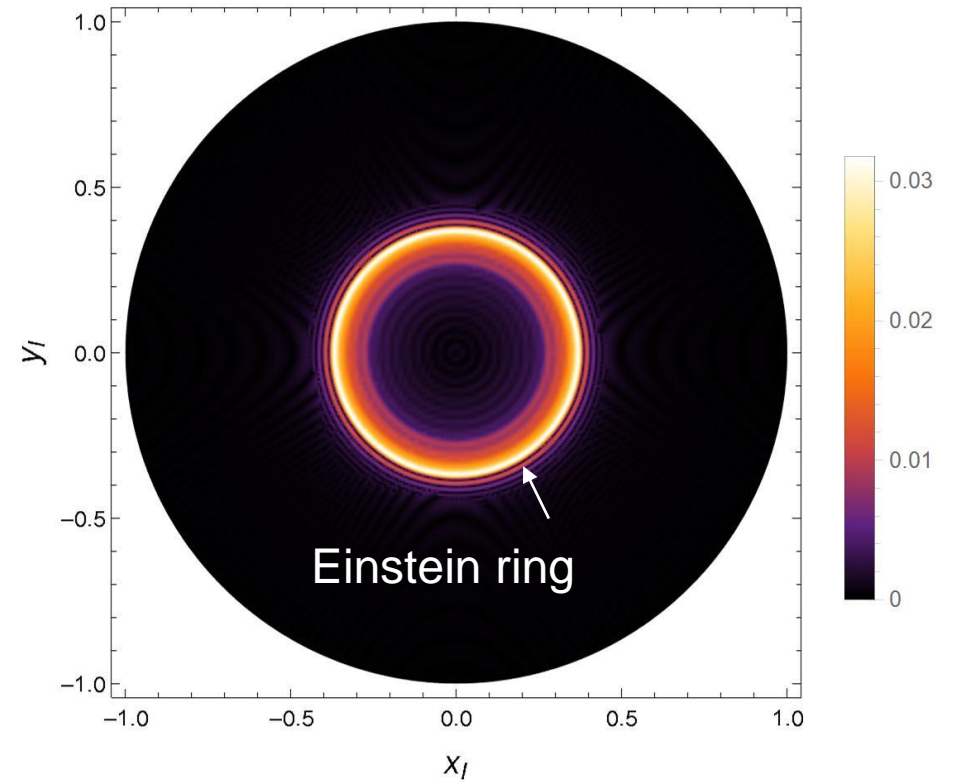
$$M = 1, \omega = 20, r_{\text{obs}} = 20, r_s = 6, d = 0.5$$



測定面上の2点関数  $|G_\omega(\mathbf{x}_A, \mathbf{x}_B)|$



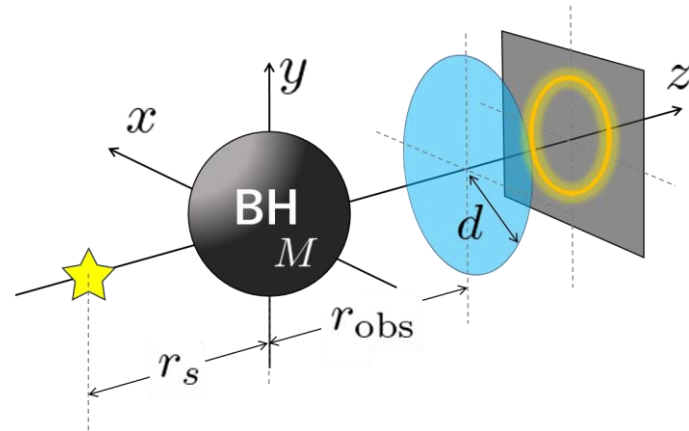
スクリーン上のintensity  $|\mathcal{F}(x_I, y_I)|$



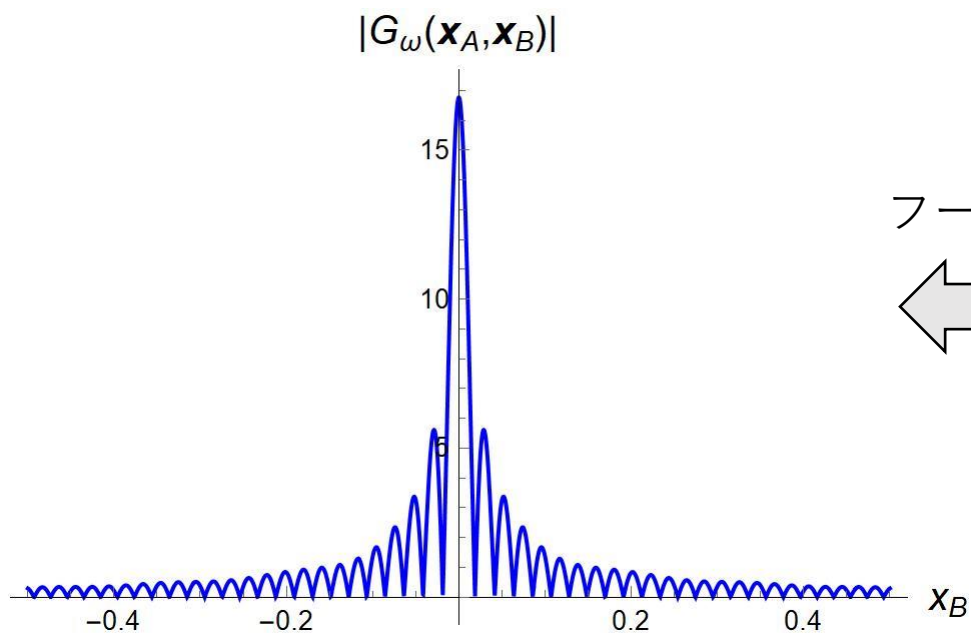
# 結果

- Schwarzschild時空におけるBH像 (1次元)

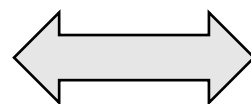
$$M = 1, \omega = 20, r_{\text{obs}} = 20, r_s = 6, d = 0.5$$



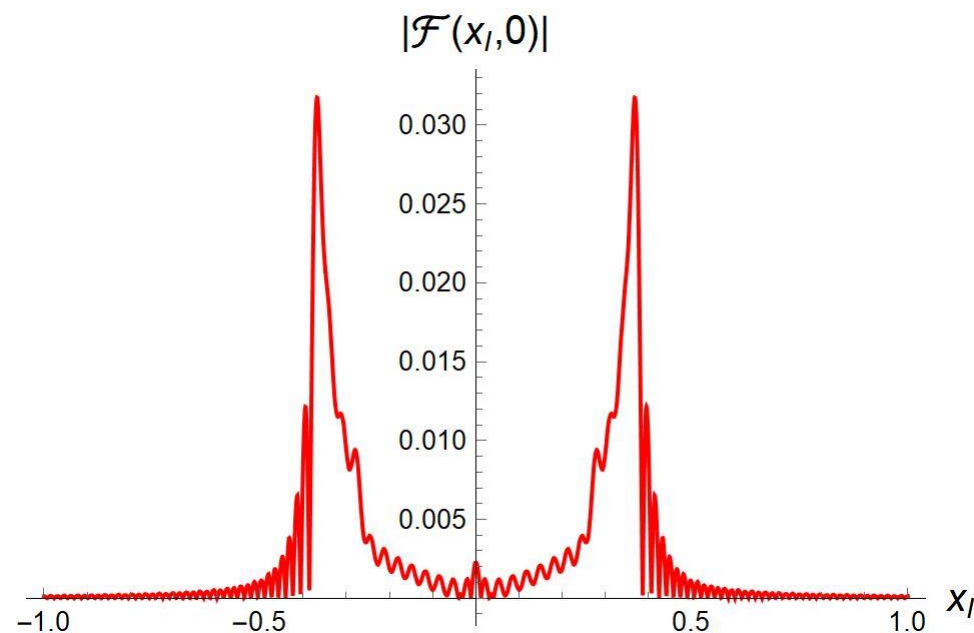
測定面上の2点関数  $|G_\omega(\mathbf{x}_A, \mathbf{x}_B)|$



フーリエ変換



スクリーン上のintensity  $|\mathcal{F}(x_I, y_I)|$



**おまけ**

# Unruh-deWitt 測定器の量子状態

- 古典的な Schwarzschild 時空における 2 つの測定器の量子状態

$$\rho_D = \text{Tr}_F \left[ U_I |\psi\rangle \langle \psi| U_I^\dagger \right] = \sum_{\alpha\beta=1}^4 S_{\alpha\beta} |e_\alpha\rangle \langle e_\beta|$$

$$|e\rangle = \{|0\rangle_A |0\rangle_B, |0\rangle_A |1\rangle_B, |1\rangle_A |0\rangle_B, |1\rangle_A |1\rangle_B\}^T$$

$$S = \begin{pmatrix} 1 - E_{\omega_0}(x_A, x_A) - E_{\omega_0}(x_B, x_B) & 0 & 0 & X(x_A, x_B)^* \\ 0 & E_{\omega_0}(x_B, x_B) & E_{\omega_0}(x_A, x_B) & 0 \\ 0 & E_{\omega_0}(x_B, x_A) & E_{\omega_0}(x_A, x_A) & 0 \\ X(x_A, x_B) & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

ただし

$$E_{\omega_0}(x, y) = \lambda^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' e^{-i\omega_0(t-t')} W(t, x; t', y),$$

$$X(x, y) = -\lambda^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \theta(t-t') e^{-i\omega_0(t+t')} (W(t, x; t', y) + W(t, y; t', x))$$

$$W(t, x; t', y) = \langle \Phi(t, x) \Phi(t', y) \rangle \text{ は Wightman function}$$

# もつれ構造

- 古典BH時空中の2つの測定器+スカラー場の系
  - 測定器A, Bの間の量子もつれ

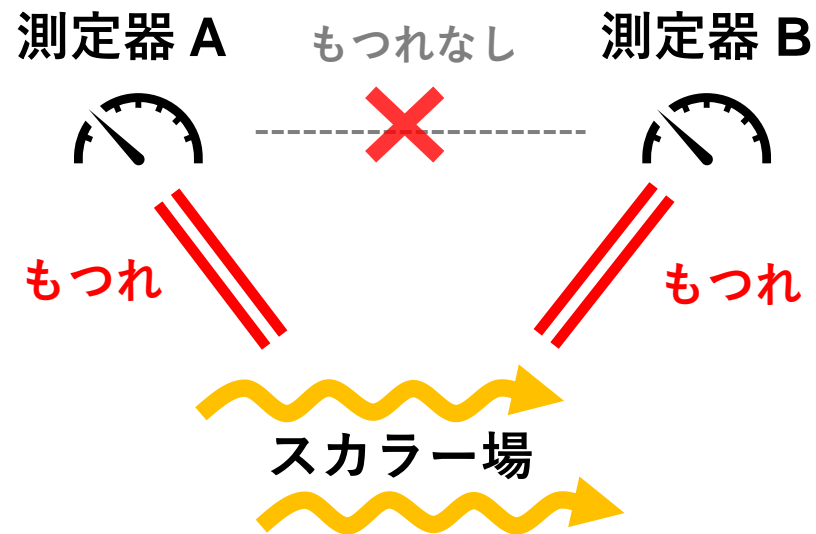
$$\frac{|X(x_A, x_B)|^2}{= 0} - \frac{E(x_A, x_A)E(x_B, x_B)}{\geq 0} > 0 \quad \text{ならば測定器A, Bはもつれている}$$

: 測定器のswitching functionが一定のため  $\lambda = (\text{const.})$

⇒ 測定器A, Bはもつれていない

- もつれ構造

BH  
古典



測定器とスカラー場が絶えず相互作用して強くもつれているので、測定器間にもつれない。

# Unruh-deWitt 測定器の量子状態

- 重ね合わせ状態のSchwarzschild時空における2つの測定器の量子状態

$$\rho_D = \text{Tr}_F \left[ U_I |\psi\rangle\langle\psi| U_I^\dagger \right] = \sum_{i=1,2} \sum_{\alpha\beta=1}^4 (S_i)_{\alpha,\beta} |e_\alpha\rangle\langle e_\beta|$$

$$|e\rangle = \{|0\rangle_A |0\rangle_B, |0\rangle_A |1\rangle_B, |1\rangle_A |0\rangle_B, |1\rangle_A |1\rangle_B\}^T$$

$$S_i = \begin{pmatrix} 1 - \sum_{D=A,B} E_{\omega_0}(x_{Di}, x_{Di}) & 0 & 0 & X(x_{Ai}, x_{Bi})^* \\ 0 & E_{\omega_0}(x_{Bi}, x_{Bi}) & E_{\omega_0}(x_{Ai}, x_{Bi}) & 0 \\ 0 & E_{\omega_0}(x_{Bi}, x_{Ai}) & E_{\omega_0}(x_{Ai}, x_{Ai}) & 0 \\ X(x_{Ai}, x_{Bi}) & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

ただし

$$E_{\omega_0}(x, y) = \lambda^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' e^{-i\omega_0(t-t')} W(t, x; t', y),$$

$$X(x, y) = -\lambda^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \theta(t-t') e^{-i\omega_0(t+t')} (W(t, x; t', y) + W(t, y; t', x))$$

$$W(t, x; t', y) = \langle \Phi(t, x) \Phi(t', y) \rangle \text{ は Wightman function}$$

# もつれ構造

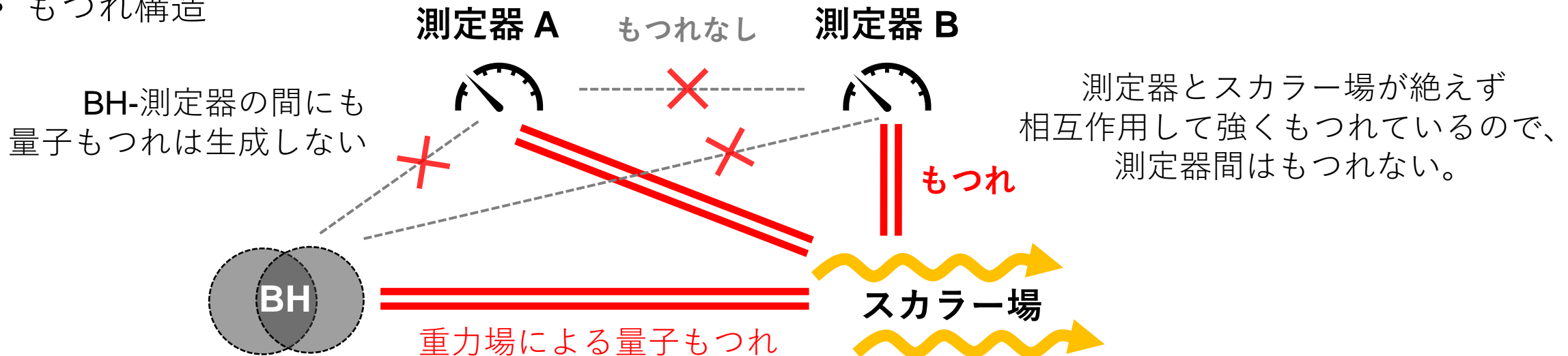
- 重ね合わせ状態のBH+ 2つの測定器+スカラー場の系
  - 測定器A, Bの間の量子もつれ

$$\sum_{i=1,2} \frac{|X(x_{Ai}, x_{Bi})|^2}{= 0} - \frac{E(x_{Ai}, x_{Ai})E(x_{Bi}, x_{Bi})}{\geq 0} > 0 \text{ ならば測定器A, Bはもつれている}$$

: 測定器のswitching functionが一定のため  $\lambda = (\text{const.})$

⇒ 測定器A, Bはもつれていない

- もつれ構造



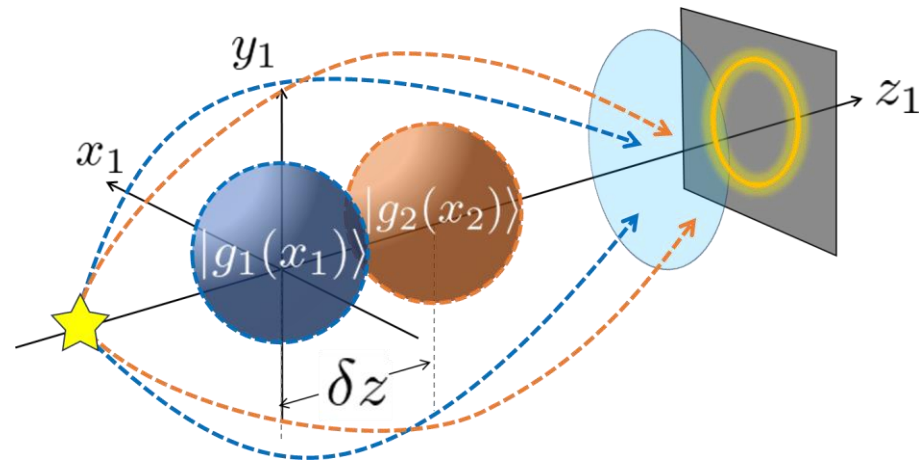


Backup

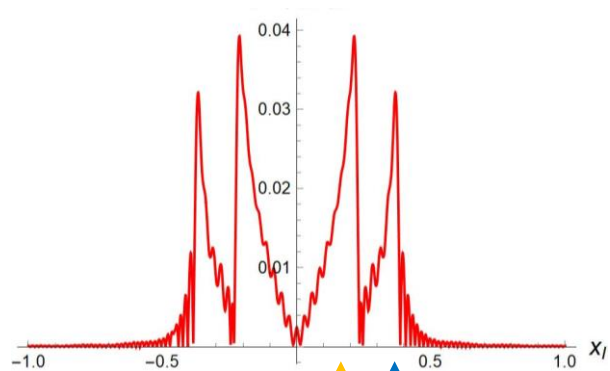
# 結果2

- $\delta z$  の依存性

$$M = 1, \omega = 20, r_{\text{obs}} = 20, r_s = 6, d = 0.5$$

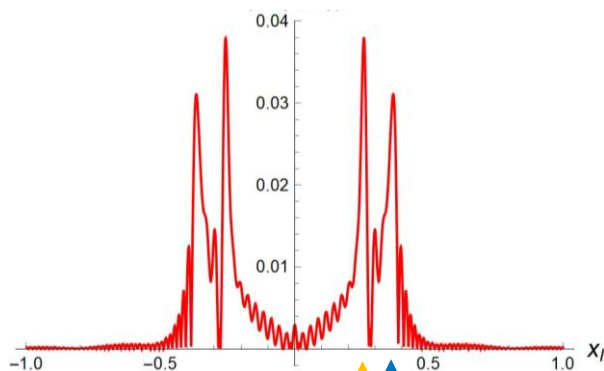


$$\delta z = -3.5$$



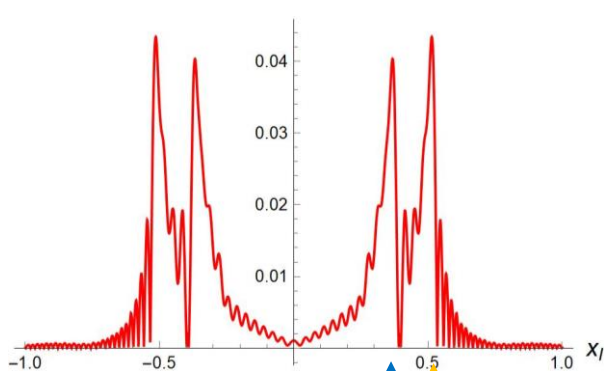
$$|g_2(x_2)\rangle |g_1(x_1)\rangle$$

$$\delta z = -2$$



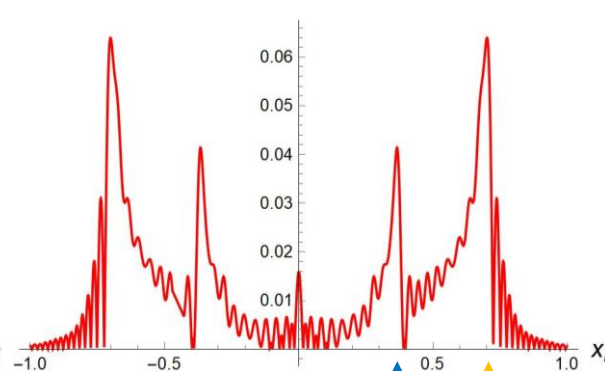
$$|g_2(x_2)\rangle |g_1(x_1)\rangle$$

$$\delta z = 2$$



$$|g_1(x_1)\rangle |g_2(x_2)\rangle$$

$$\delta z = 4$$



$$|g_1(x_1)\rangle |g_2(x_2)\rangle$$