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Stark effect and dissociation of mesons in holographic conductor

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D3/D7 brane system

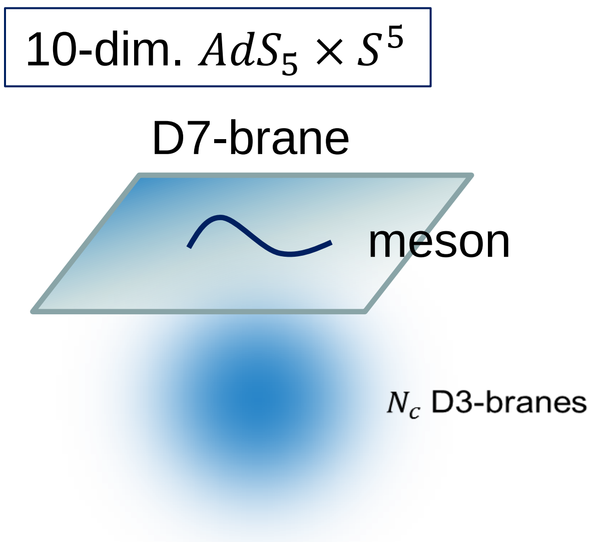
Karch, Katz (2002), Grana, Polchinski (2002), Bertolini *et al*. (2002)

- The AdS/CFT states that a strongly correlated quantum system can be analyzed by classical gravity
  - Nonequilibrium steady state, nonlinear conductivity
- The D3/D7 models can describe quark-like charged particles in addition to gluon sector
  - A stack of N_c D3-branes and a few D7-branes
- In the large N_c limit, we should study classical dynamics of a probe D7-brane in the $AdS_5 \times S^5$ spacetime
  - Embedding functions of the brane and worldvolume gauge fields are determined by EoM derived from the DBI action
  - Fluctuations of D7-brane correspond to "mesonic" excitations, which are bound states of "quark/anti-quark" pair in gauge theory side

- Dynamics and configuration of a probe D7-brane can be determined by the DBI action

DBI action:

$$S_{\rm D7} = -\mu_7 \int \mathrm{d}^8\xi \sqrt{-\det\left(g_{ab}+F_{ab}\right)}$$
$$= -\int d\rho\, d^4x \mathcal{L}[w, a_x]$$



Embedding functions: $w = W(\rho), \quad \psi = 0$

Worldvolume gauge field: $A_a d\xi^a = A_x(t,\rho)dx$

10-dim. spacetime: $ds^2 = r^2[-dt^2 + d\vec{x}_3^2] + \frac{1}{r^2}[d\rho^2 + \rho^2 d\Omega_3^2 + dw^2 + w^2 d\psi^2]$

From classical solutions, we can read obsevables in the field theory

$$W(\rho) = m + \frac{\langle \bar{q}q \rangle}{\rho^2} + \cdots, \quad A_x(t,\rho) = -E_x t + \frac{\langle \bar{q}\gamma_x q \rangle}{2\rho^2} + \cdots$$

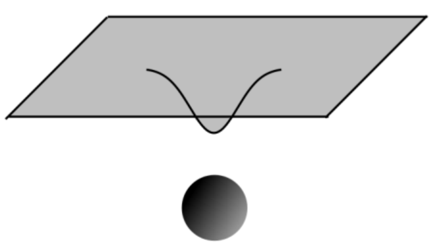
Phase transition in the D3/D7 system

- If we introduce a worldvolume gauge field living on the D7 brane or a black hole in bulk spacetime, topology of the brane configuration will change

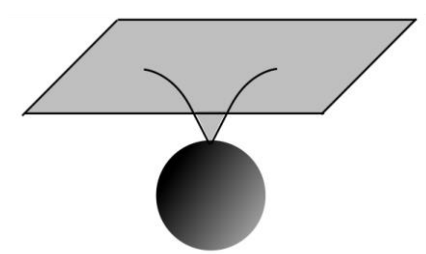
Frolov (2006), Mateos, Myers, Thomson (2006, 2007)

Electric field or black hole become larger

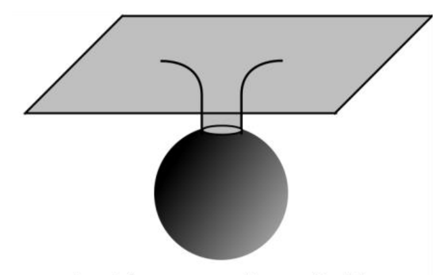
Minkowski embedding



Critical embedding



Black hole embedding



Fluctuations are confined
normal modes
with real frequencies

Fluctuations are absorbed into horizon
quasi-normal modes
with complex frequencies

Brief dictionary of the correspondence

Gravity side

Black hole in the bulk
or
Gauge field on the brane

The brane is bending

$$W(\rho) = m + \frac{c}{\rho^2} + \cdots$$

$$A_x(t,\rho) = -E_x t + \frac{J_x}{2\rho^2} + \cdots$$

The brane intersects a horizon or not

The fluctuations dissipate or are confined (quasi-normal modes or normal modes)

Gauge theory side

Finite temperature in the gluon sector
or
Finite electric field

Expectation values change

$\langle \bar{q} q \rangle \propto c$: quark condensate

$\langle \bar{q} \gamma_\mu q \rangle \propto J_\mu$: electric current

m:quark mass, E_x:electric field

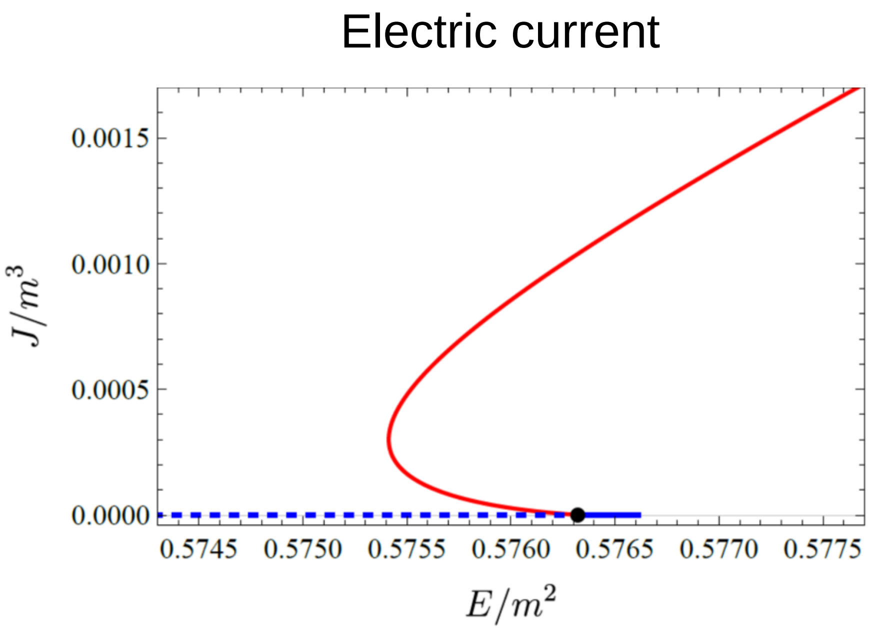
conductor or insulator

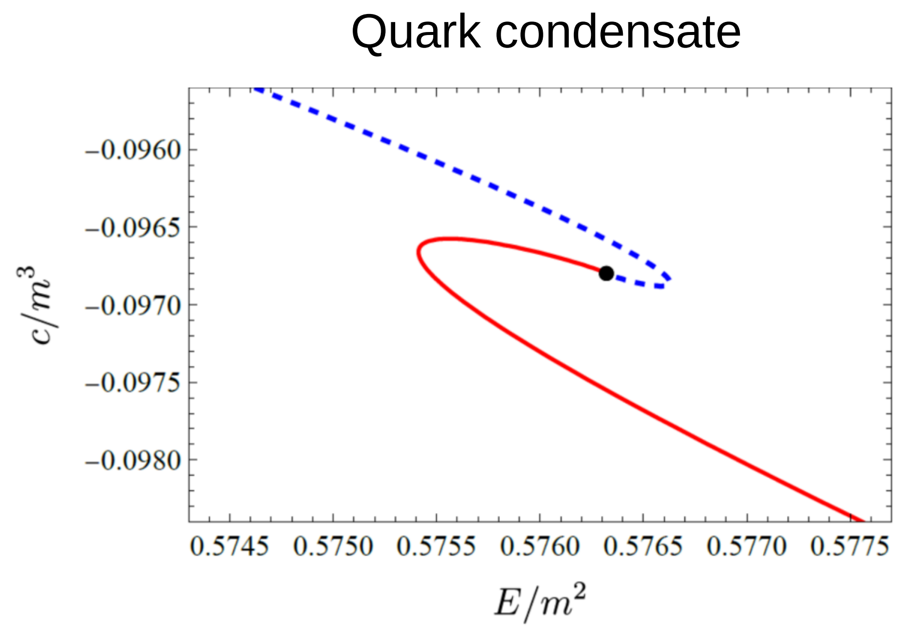
Mesons (bound state) are dissociated or stable

Mateos, Myers, Thomson (2006, 2007)
Karch, O'Bannon (2007)

Dielectric breakdown at the zero temperature

- E-J curve and E-c curve





- At the critical electric field, the D7-brane configuration transit from the Minkowski embeddings to the BH embeddings
- In the vicinity of the critical value, multiple solutions appear for a given parameter

Our work

- We explore how an external electric field affects the spectrum of mesons over all the values of the electric field
  - Solving eigenvalue problem for linear perturbations on the D7-brane solutions
  - Vacuum state with no electric field
  - Strong electric field limit (zero quark-mass limit)
- Some modes are coupled in the presence of finite electric fields
- Normal modes/quasinormal modes

Stark (decoupled mode) Erdmenger, Meyer, Shock (2007)

Zeeman Filev, Johnson, Rashkov, Viswanathan (2008)

Albash, Filev, Johnson, Kundu (2008)

Linear perturbations

- We consider linear perturbations for background solutions of the probe D7-brane
  - Brane embedding functions

$$W = \bar{W}(\rho) \to \bar{W}(\rho) + \epsilon w(t,\rho), \qquad \psi = 0 \to 0 + \epsilon \psi(t,\rho),$$

scalar   pseudo-scalar

  - Worldvolume gauge field

$$A_a d\xi^a \to (-Et + h(\rho))dx + \epsilon a_{\|}(t,\rho)dx + \epsilon \vec{a}_{\perp}(t,\rho)\cdot d\vec{x}_{\perp}$$

vector (parallel)   vector (transverse)

$$S^{(2)} = \frac{1}{2}\int d^4x d\rho \gamma^{\alpha\beta}\partial_\alpha \vec{a}_\perp \partial_\beta \vec{a}_\perp + \frac{1}{2}\int d^4x d\rho \tilde{\gamma}^{\alpha\beta}\partial_\alpha\psi\partial_\beta\psi$$

Decoupled sector

$$+ \int d^4x d\rho \Big[\partial_\alpha \Phi^T A^{\alpha\beta}\partial_\beta\Phi + \Phi^T B^\alpha \partial_\alpha \Phi + \Phi^T C \Phi\Big]$$

Coupled sector

$$\Phi^T \equiv (w, a_{\|})$$

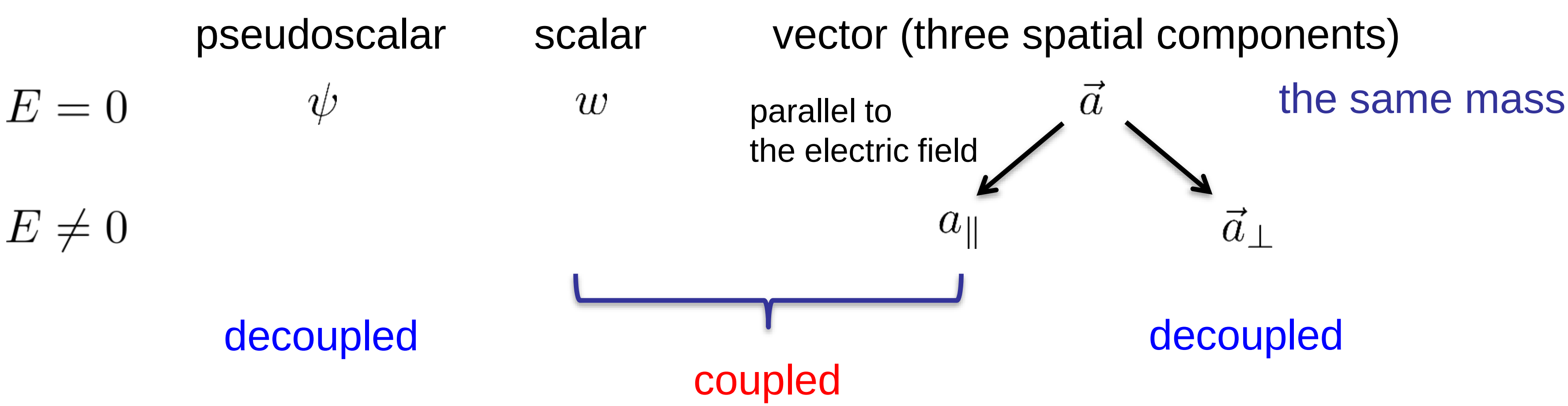
To focus on the lightest modes, we consider homogeneous perturbations

Meson spectrum in the vacuum

- In the supersymmetric vacuum, spectra of the mesons are analytically obtained
  - Every mode is decoupled
  - Mass spectrum of the lightest modes is degenerate

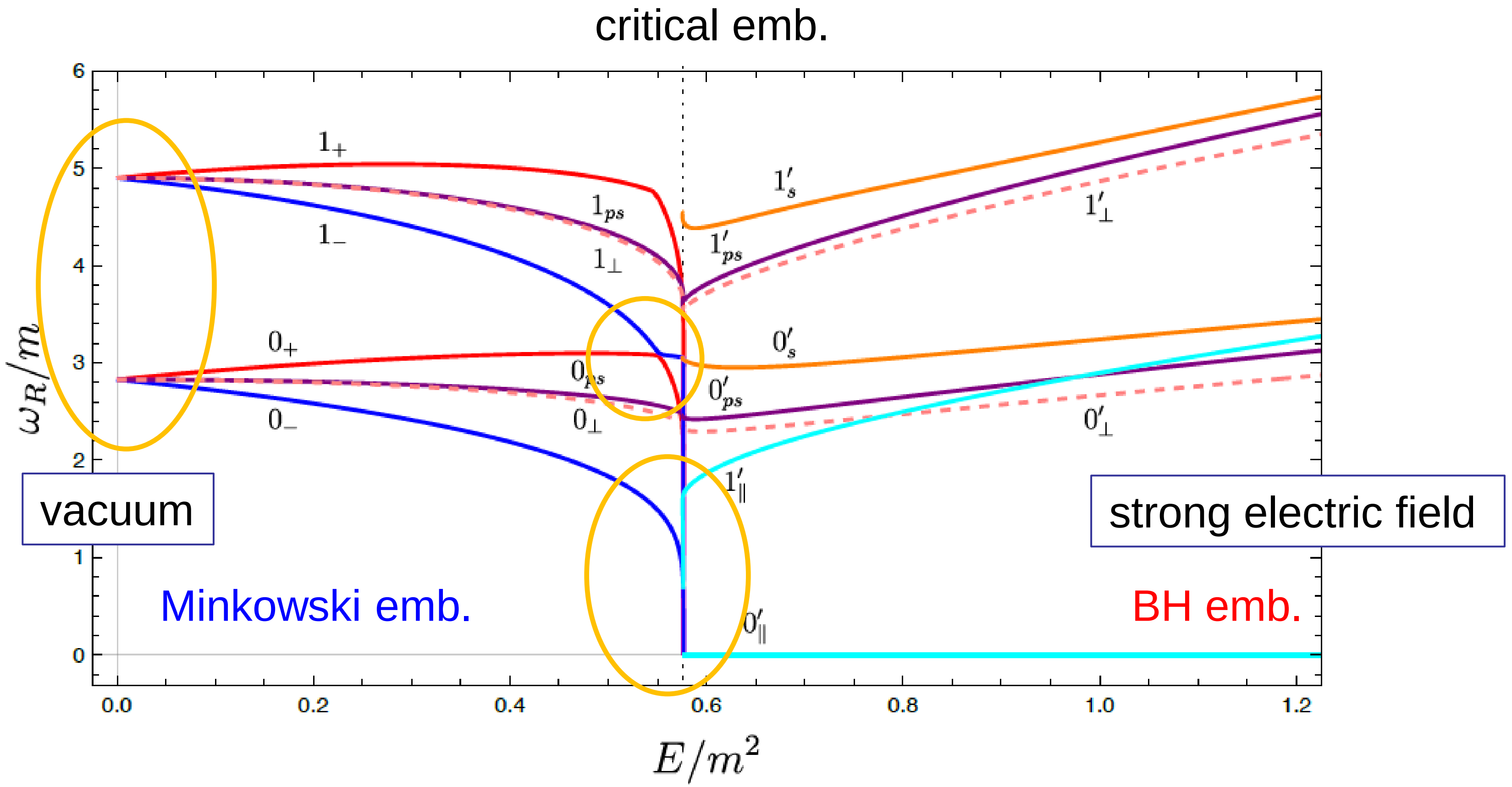
$$\frac{\omega_n}{m} = 2\sqrt{(n+1)(n+2)} \qquad n = 0, 1, 2, \ldots$$

Kruczenski, Mateos, Myers, Winters (2003)



Meson spectrum in the presence of the electric field

- Real part of eigenfrequencies



- The spectra of each decoupled mode are continuously connected
- The coupled modes exhibit three characteristic features

Stark effect for small E

- In a finite electric field, we cannot distinguish the coupled modes
- If the electric field is small, we can analytically treat mass shift in a perturbative method

Hashimoto, SK, Murata, Oka (2014)

$$S^{(2)}_{w,a_\parallel} = \frac{1}{2}\int dt \int_0^{1/m} du \frac{1-m^2u^2}{u}\left[\dot{\chi}_+\dot{\chi}_- - (1-m^2u^2)\chi'_+\chi'_- \right.$$
$$\left. -2iEmu^2\left(\dot{\chi}_+\chi_- - \chi_+\dot{\chi}_-\right)\right] + O(E^2)$$

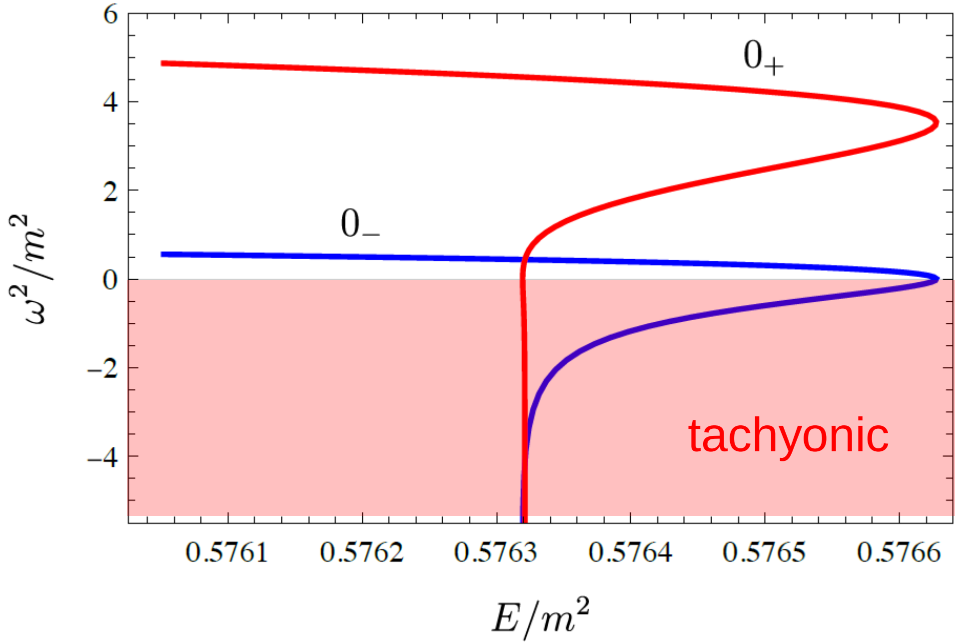
The quadratic action can be diagonalized in terms of new variables: $\chi_\pm \equiv w \pm ia_\parallel$

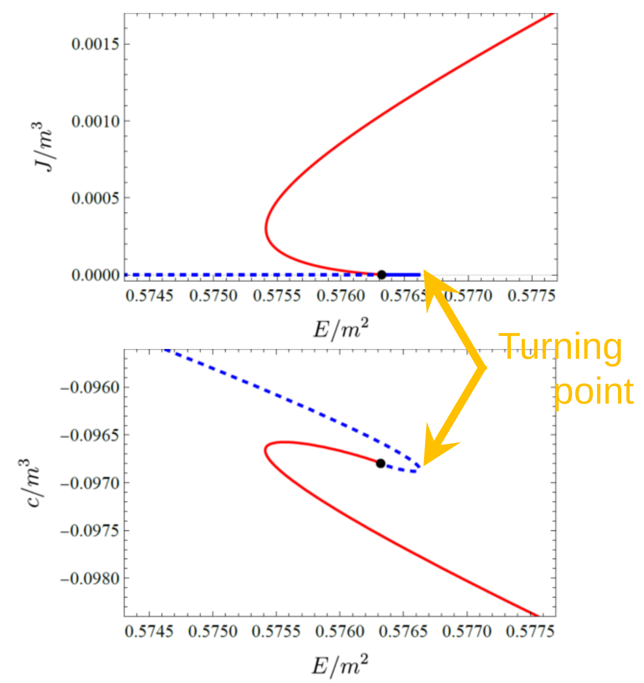
$$\frac{\omega_n^\pm}{m} = 2\sqrt{(n+1)(n+2)} \pm \frac{E}{m^2}$$

Degenerate mass spectra split into the upper and lower masses

Near-critical region (1)

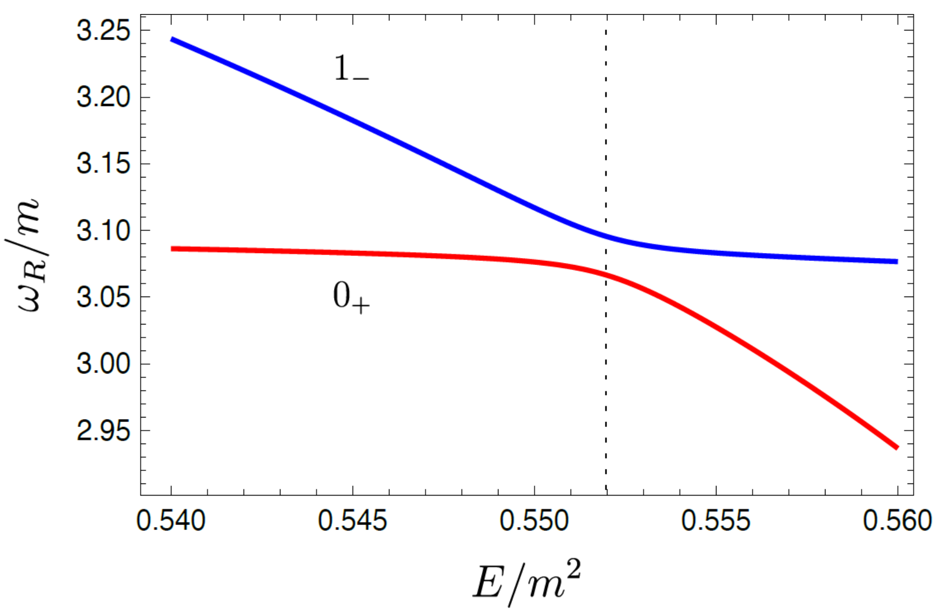
- Tachyonic instability: $\omega^2 < 0$
  - In the vicinity of the critical electric field, multiple background solutions coexist for a given electric filed
  - This causes turnovers in the E-J/E-c curves, and the spectrum; at the turning points linear stability changes





Near-critical region (2)

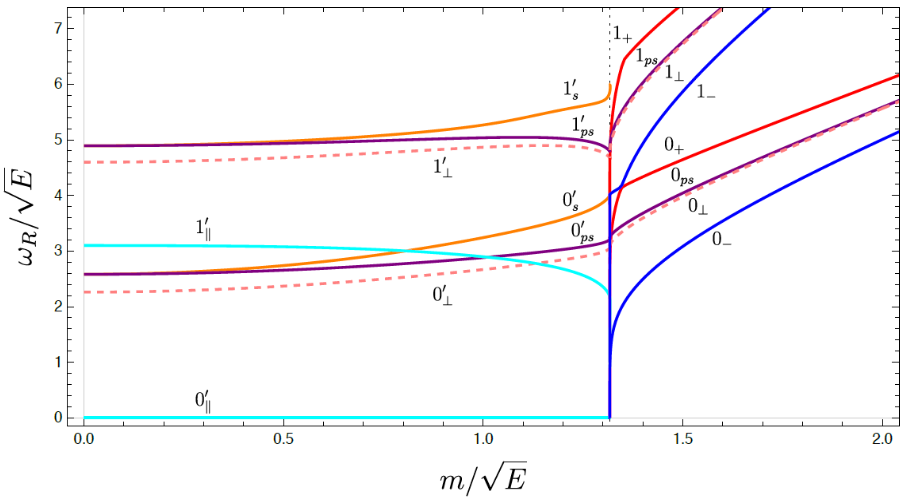
- Avoided crossing
  - We find that the coupled modes are repulsive in the spectrum
  - Each level of the mass spectrum shifts but never collides



Quasinormal modes

- Complex frequencies

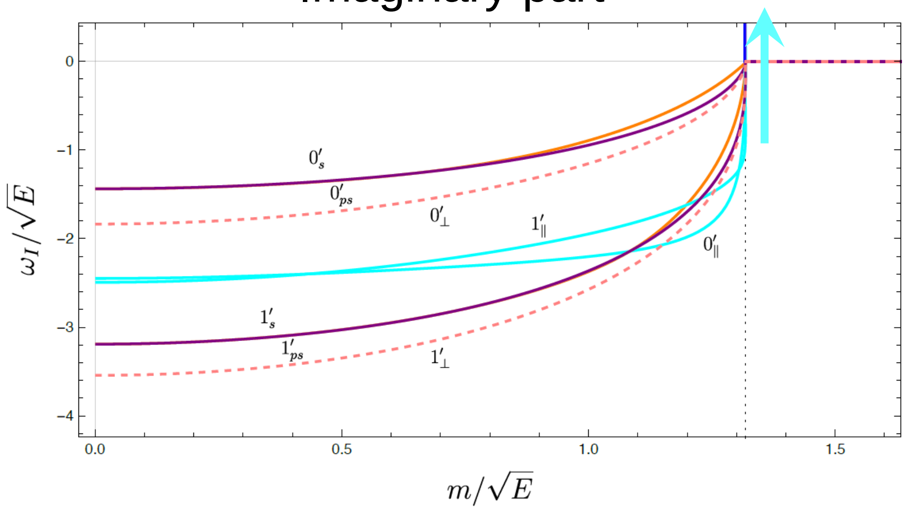
Real part



Imaginary part

$\mathrm{Im}\,\omega > 0$

instability



(rescaled by the electric field instead of m)

- The decoupled modes are always stable and are connected to the normal modes
- One of the coupled modes becomes pure-imaginary mode and eventually unstable at the turning point of the E-J curve
- The others do not cause instability

Our previous work
Ishigaki, SK, Matsumoto (2022)

Summary

- By using the D3/D7 model, we have studied mass spectrum of mesons in the whole range of electric fields at zero temperature
  - When an electric field is applied, one scalar meson and one component of the vector meson are coupled
  - For small electric fields the spectra are described by the normal modes on fluctuations of the D7-brane, while for large electric fields they are described by the quasinormal modes

- We can know which modes originate from the spectra in the decoupling limit
- One of the coupled modes becomes tachyonic near the critical electric field; the other never becomes tachyonic even at the critical electric field
  - Dynamical stability of the background D7-brane is governed by this mode
- The spectra of the other decoupled modes continuously connect between the normal modes and the quasinormal modes