Entanglement and confinement in quantum field and lattice gauge theories

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האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

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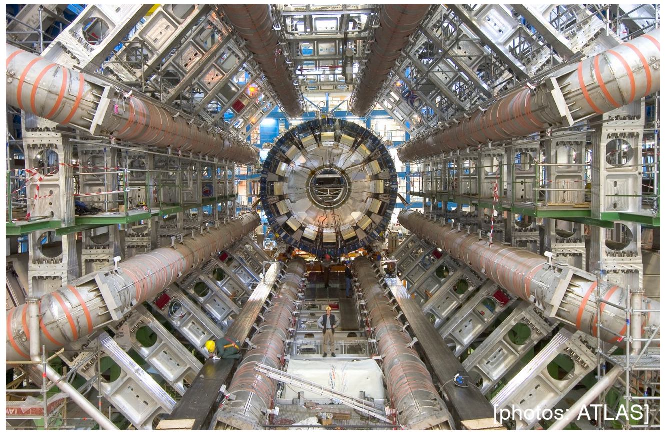
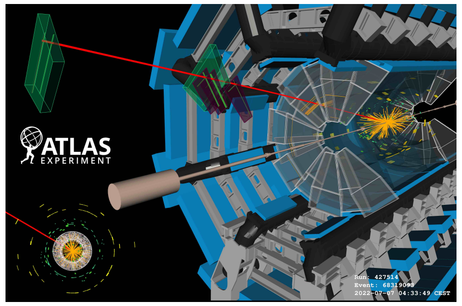
Introduction and Motivation

- How can we understand fundamental matter in extreme places in the universe?





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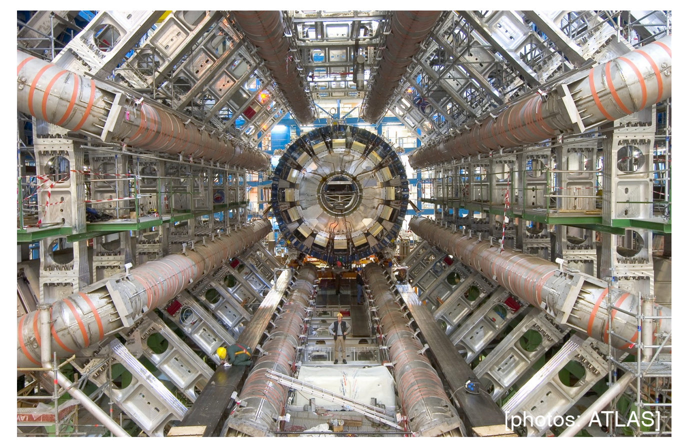
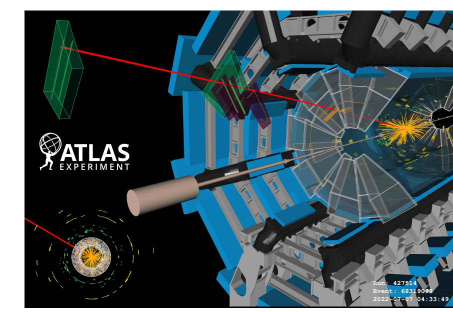
- Quantum Information concepts as a guiding principle and new perspective to describe physical systems


(photos: ATLAS)



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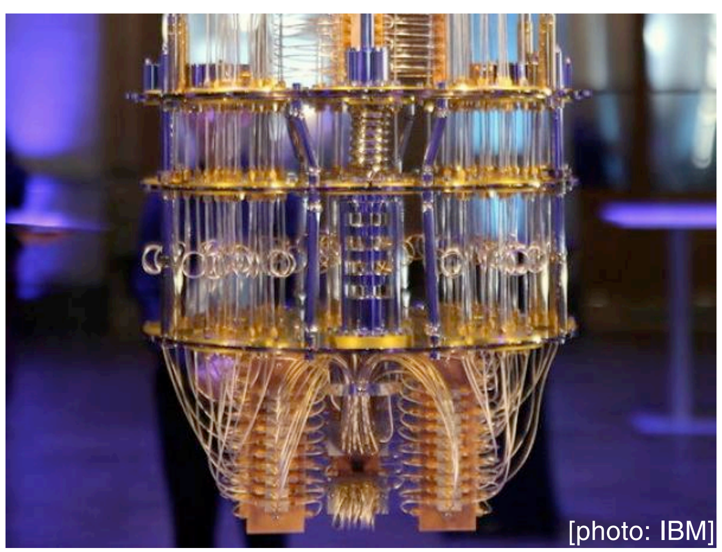

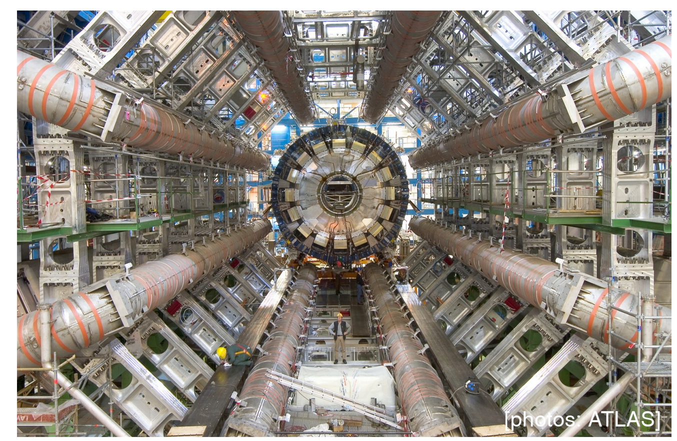

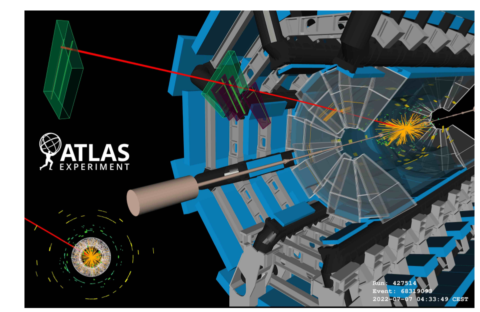

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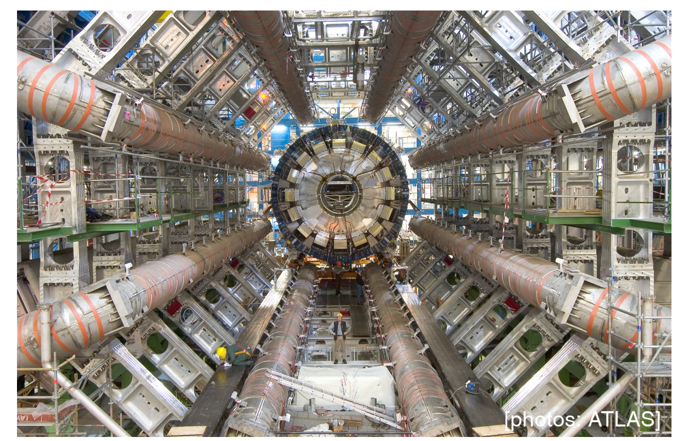
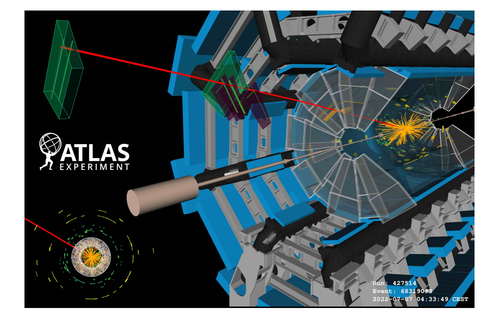
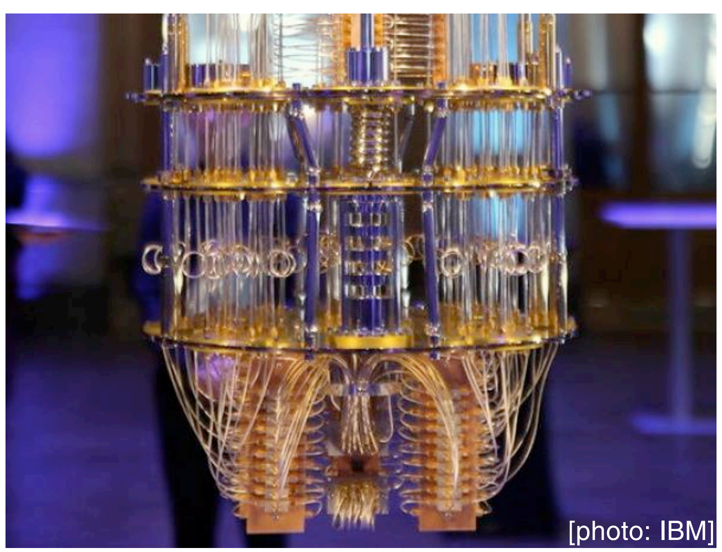






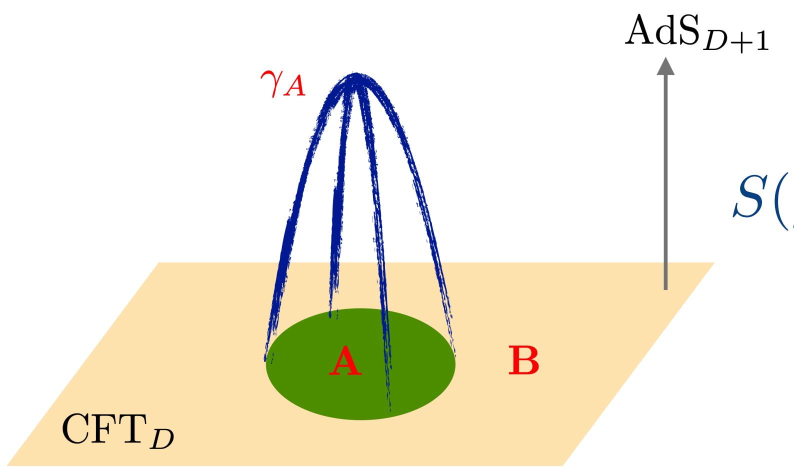
- Explore the interplay of **entanglement** and **confinement** with Tensor Networks (TNs) as representations of quantum states

⇒ use TNs to explore thermal and dynamical entanglement properties of the (1+1)D Ising QFT

⇒ explore connections between Rényi entropies and (de)confinement in 2D LGTs

Entanglement and Confinement

- Holographic models suggest connections between entanglement and confinement properties in QFTs:

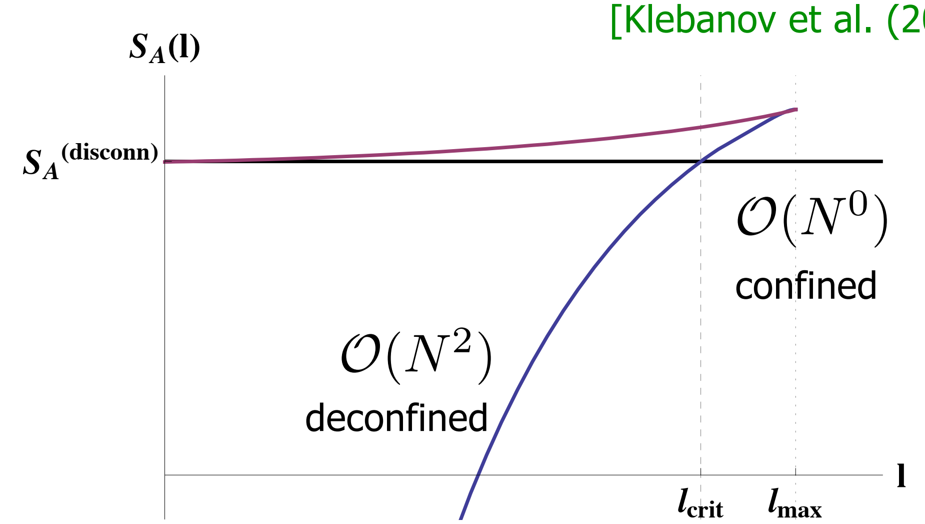


[Ryu, Takayanagi (2006)]

$$S(\rho_A) = -\,\mathrm{Tr}\left[\rho_A \log \rho_A\right] = \frac{\mathrm{Area}(\gamma_A)}{4G_N^{(D+1)}}$$

entanglement as a probe of confinement:

[Klebanov et al. (2007)]



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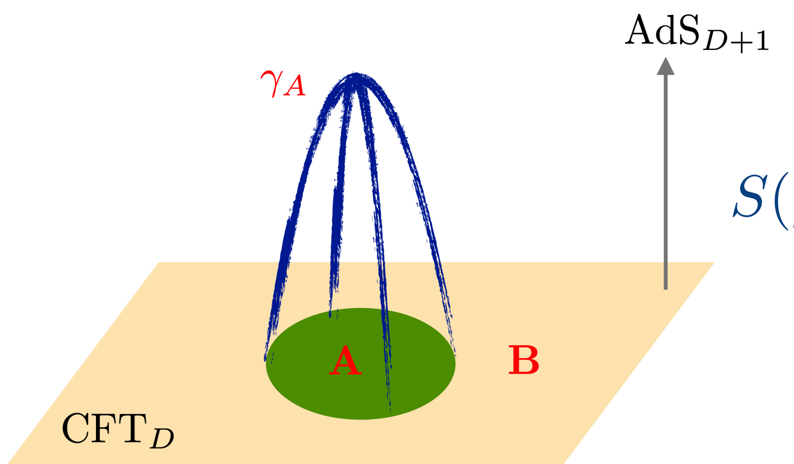

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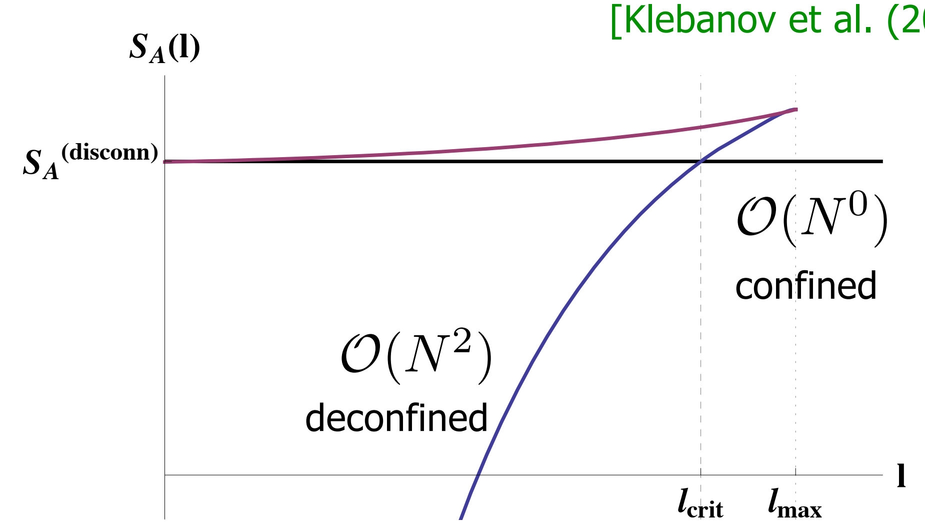
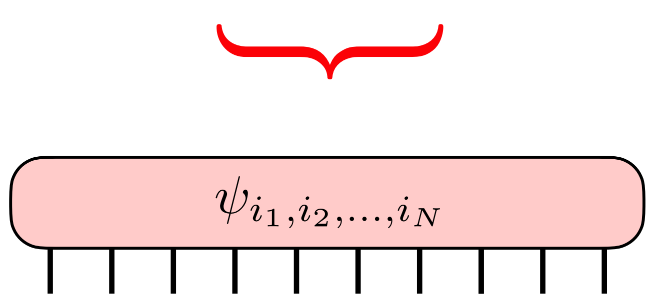
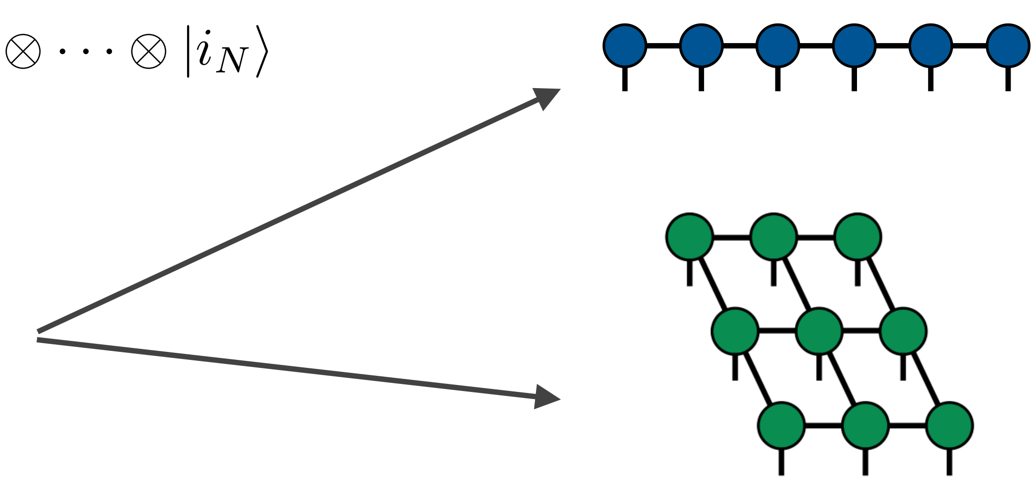


Tensor network ansätze and diagrams

- Tensor networks decompose a QMB wave function into smaller local tensors based on the entanglement structure [Verstraete et al.; Schollwöck; Orus; Eisert; Montangero; Cirac et al.; ...]

$$|\Psi\rangle = \sum_{i_1,i_2,\ldots,i_N=1}^{d} \psi_{i_1,i_2,\ldots,i_N}\, |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$



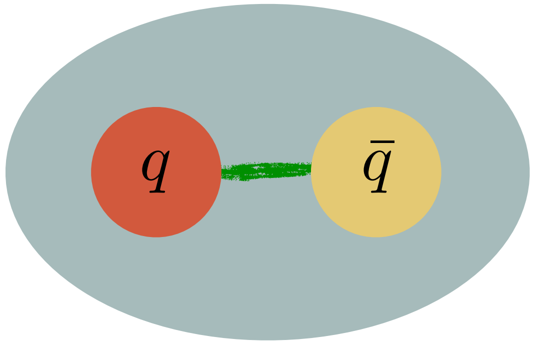


MPS

PEPS

Meson Melting

[Bañuls, Heller, Jansen, JK, Svensson, arXiv:2206.10528 (2022)]



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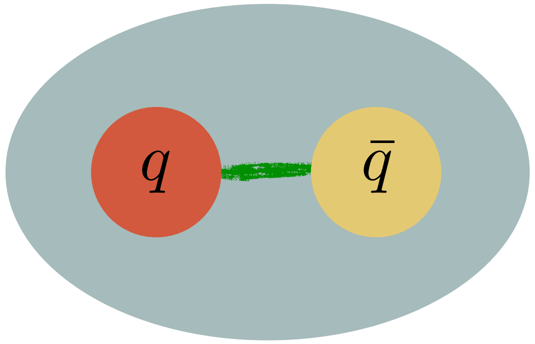

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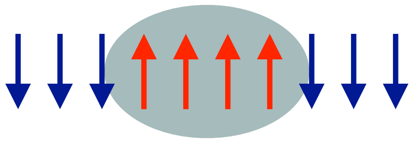
- Mesons exist also as confined fermion pairs (domain walls) in relativistic regimes of the Ising model

$$H = -J \left(\sum_{j=1}^{N-1} \sigma_j^z \sigma_{j+1}^z + h \sum_{j=1}^{N} \sigma_j^x + g \sum_{j=1}^{N} \sigma_j^z \right)$$



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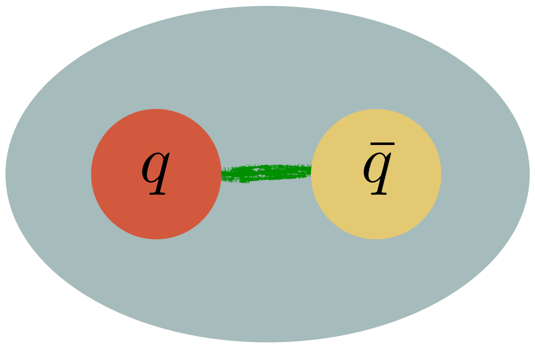

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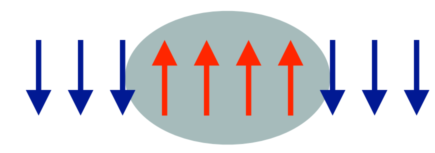

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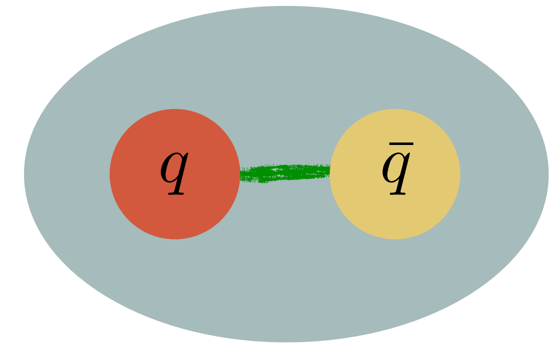

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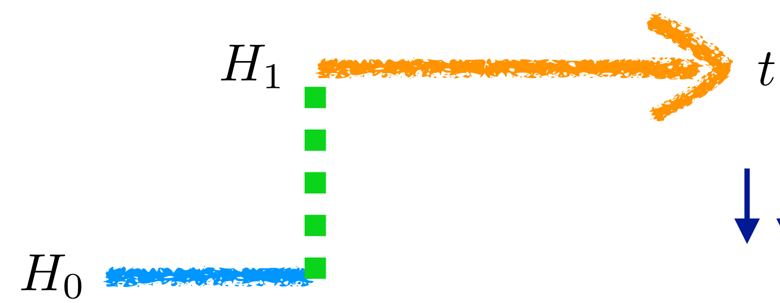
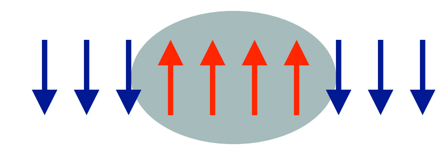
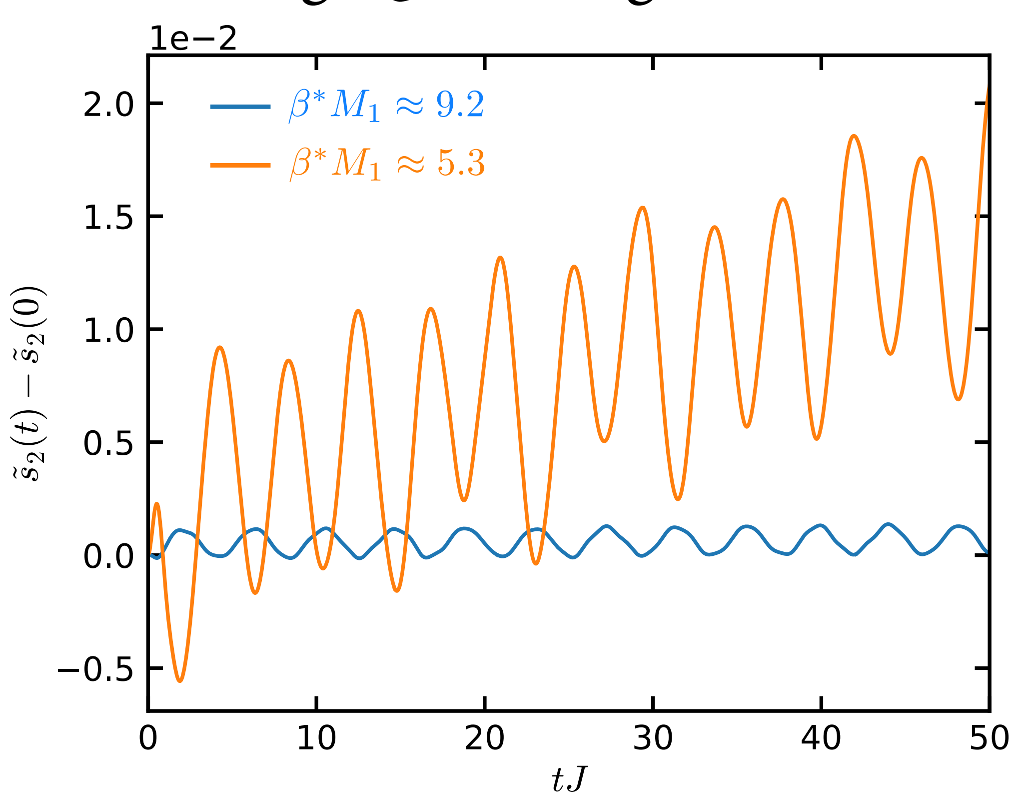
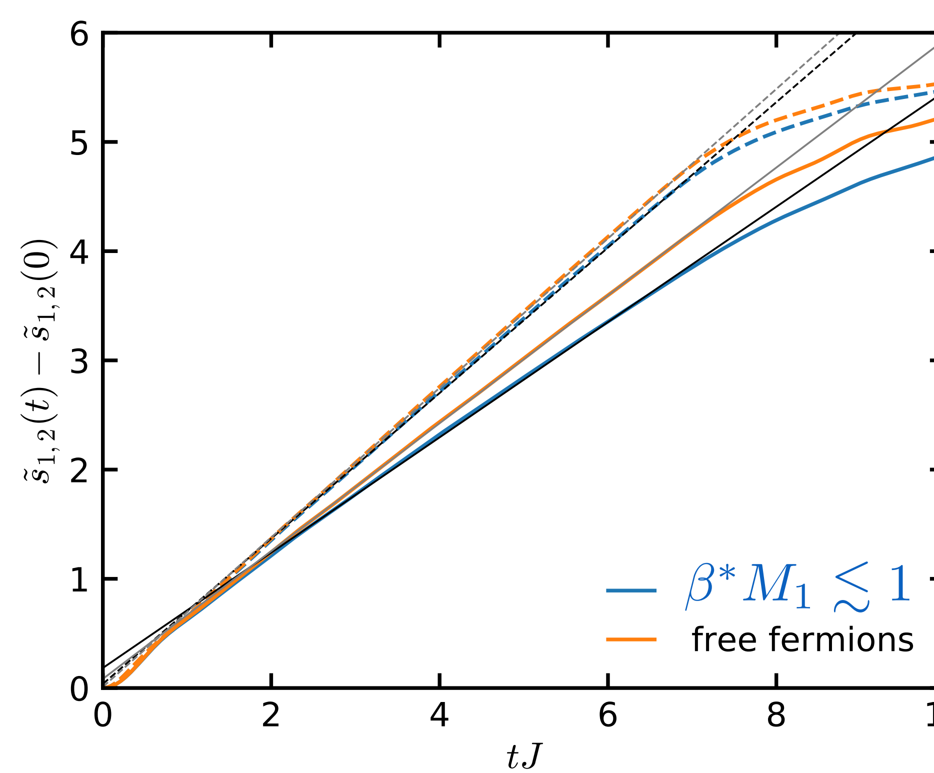




- We simulate a quantum quench to induce a dynamical out-of-equilibrium system
- Entanglement measures witness the melting of mesons in the ferromagnetic phase of the Ising QFT at high effective temperatures:





Meson content of entanglement spectra

[JK, PRB 107, L100303 (2023); arXiv:2210.15682]

- Spectral properties of the entanglement Hamiltonian reveal important information of QMB systems and phases [Li, Haldane (2008)]

  ⇒ entanglement spectrum is available from Schmidt decomposition of MPS via

  $\rho_A = \mathrm{Tr}_B\, \rho_{AB} = \sum_r \lambda_r \, |\Psi_r^L\rangle \langle \Psi_r^R| \equiv \mathrm{e}^{-\mathcal{H}_{\mathrm{mod}}}$

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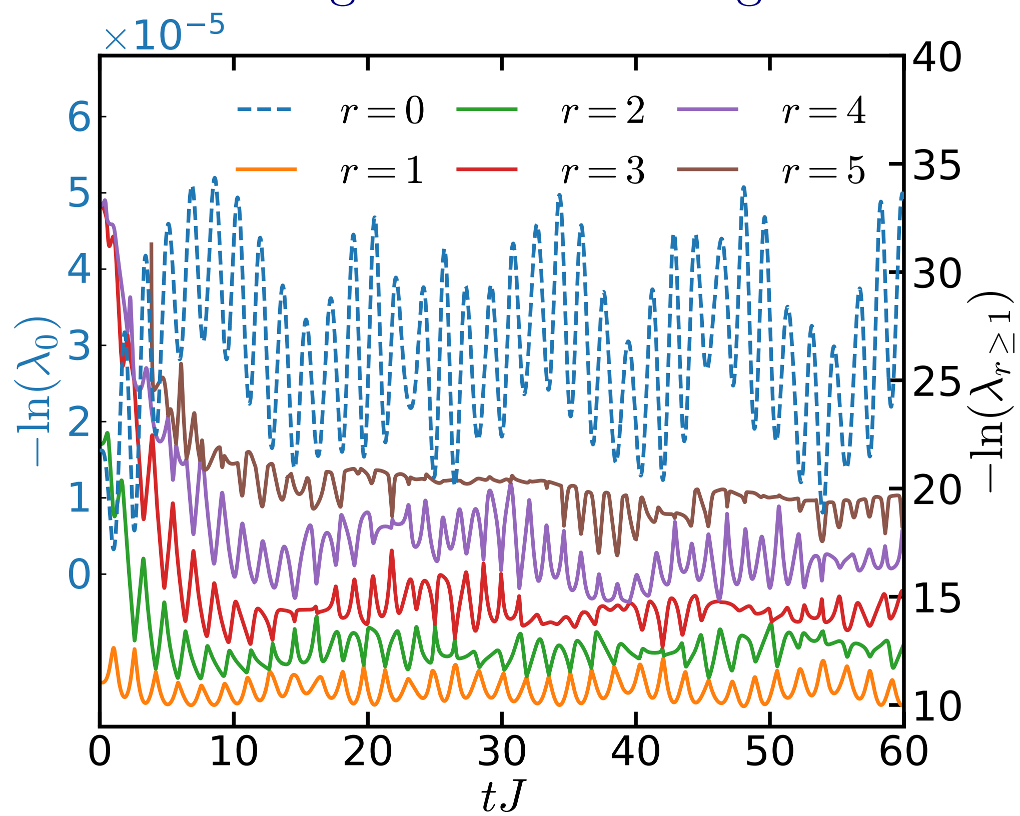
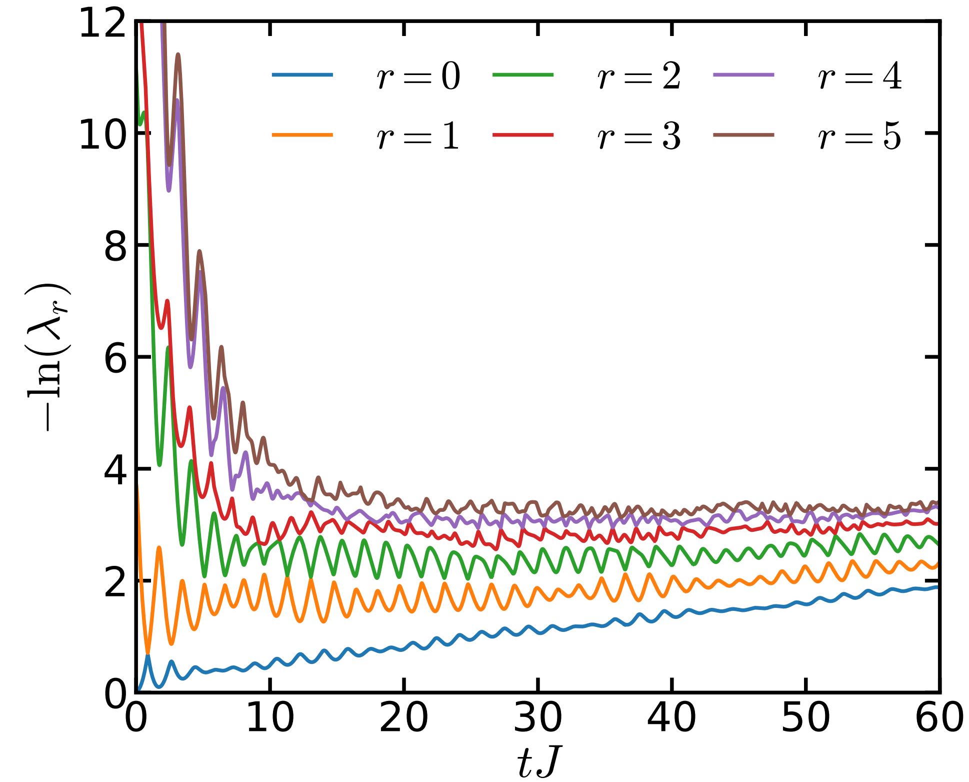
- Results after integrable and nonintegrable quantum quenches in relativistic and semiclassical regimes:

ferromagnetic → ferromagnetic



paramagnetic → ferromagnetic



  - only the (experimentally accessible) dominant eigenvalue fully encodes the meson content of the QMB system or QFT via entanglement oscillations
  - regular DQPTs at irregular times leave the entanglement spectrum gapped

Rényi entropies in 2D LGT TNs

[JK, Zohar, to appear]

- Projected entangled pair states (PEPS) can be used to study gauge-invariant nonlocal Wilson loops as a probe of confinement in (pure) 2D LGTs [Zohar (2021)]

- **Aim:** Transfer operator approach for the calculation of (normalized) Rényi entropies

$$\bar{S}_n = \frac{1}{1-n} \ln \frac{\mathrm{Tr}[\rho_A^n]}{\mathrm{Tr}^n[\rho_A]} \qquad \rho_A = \mathrm{Tr}_B \left[|\Psi\rangle \langle\Psi| \right]$$

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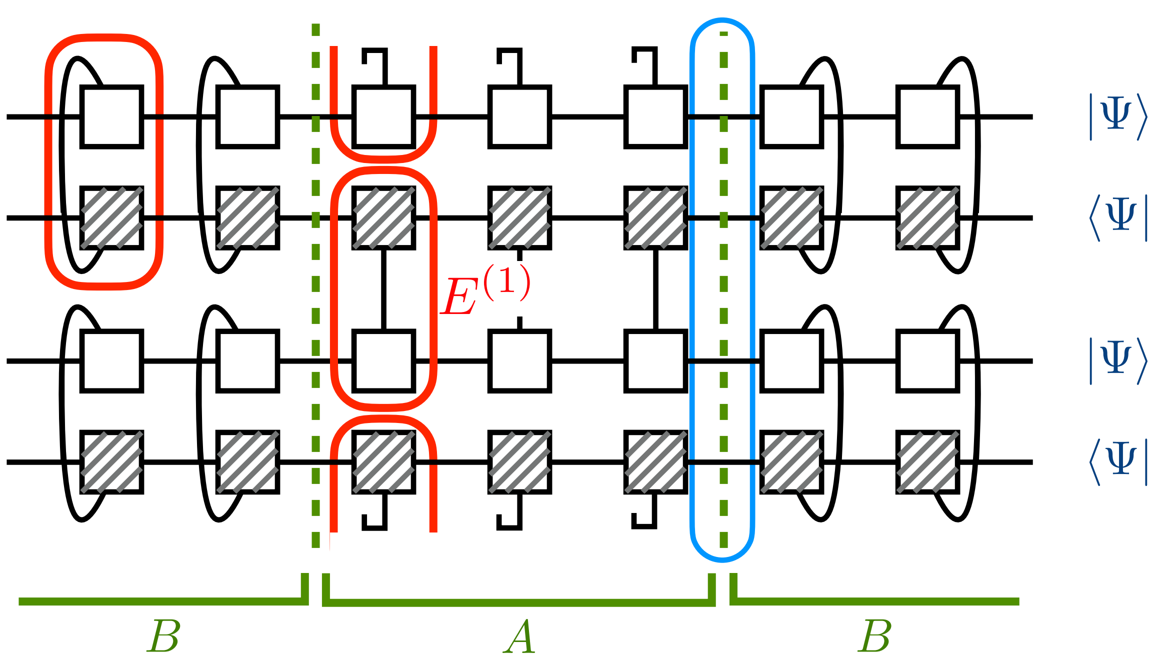
$$p_2 \equiv \mathrm{Tr}[\rho_A^2] =$$



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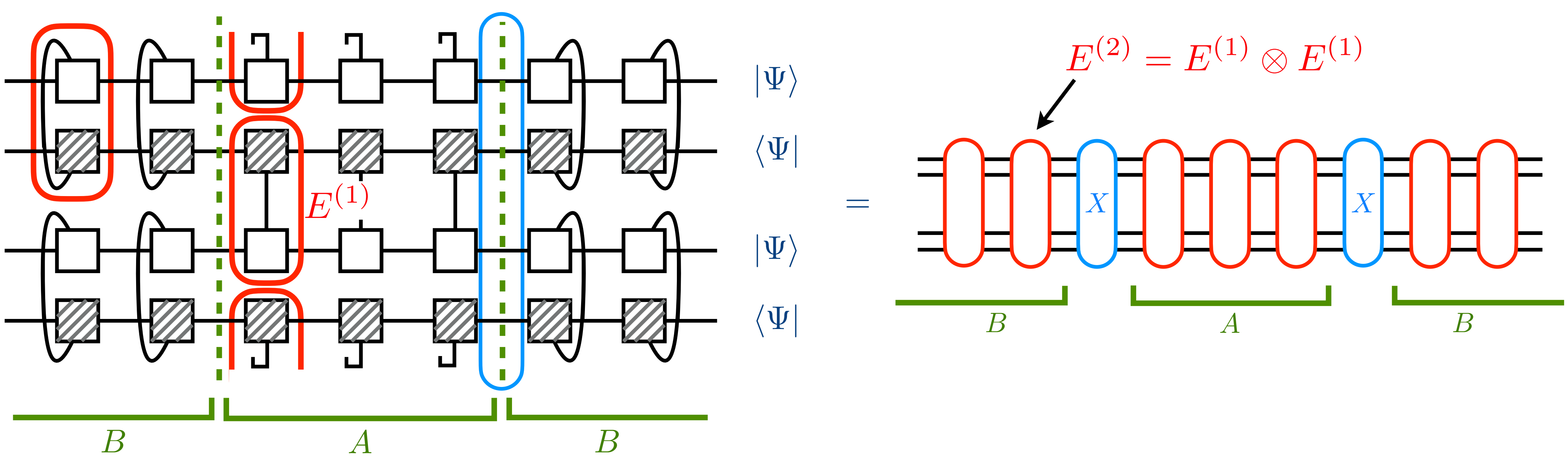

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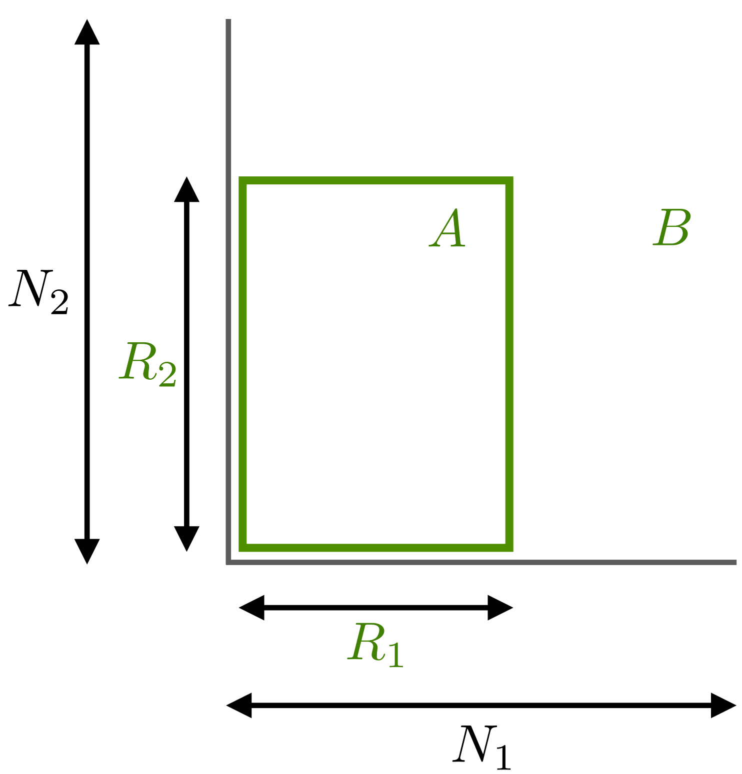
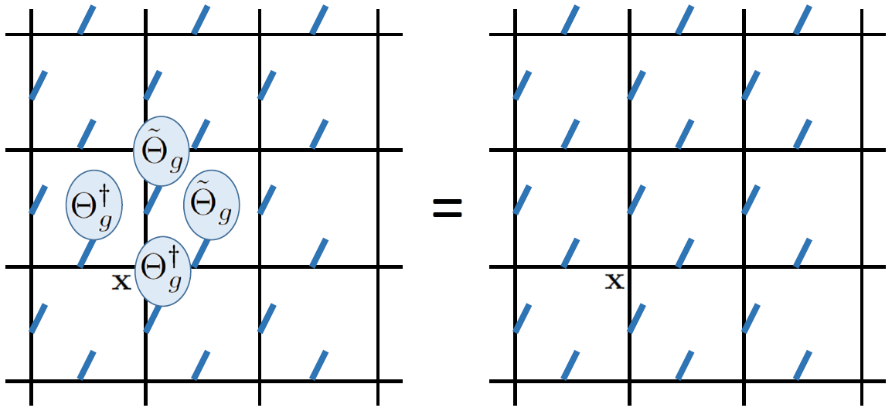
- **Setup in 2D:**

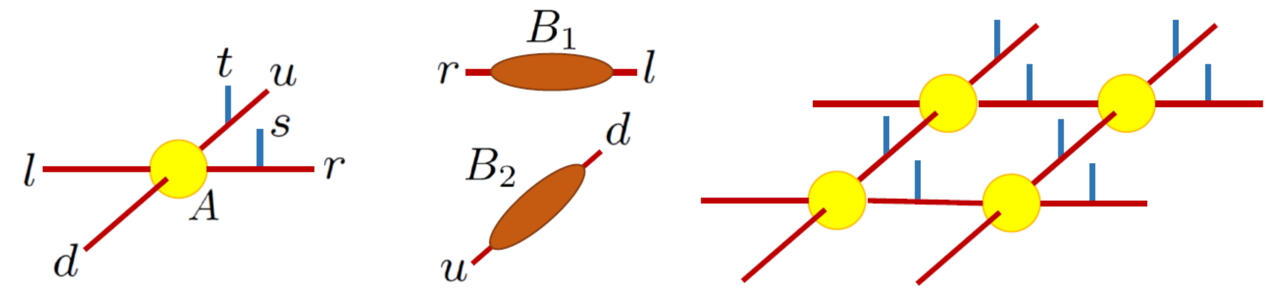


$\Rightarrow$ we are interested in long-range and continuum properties of $\bar{S}_n$ for a large subsystem A: $1 \ll R_1, R_2 \ll N_1, N_2$

- **Gauge-invariant PEPS ansatz:**



⇒ local invariance under arbitrary (Abelian/ non-Abelian) groups can be achieved with a gauged PEPS ansatz [Zohar et al. (2015, 2016, 2018, 2021,...)]



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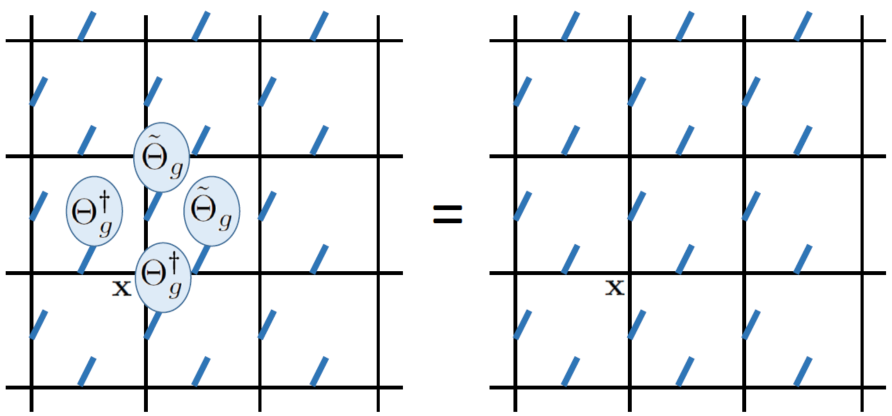

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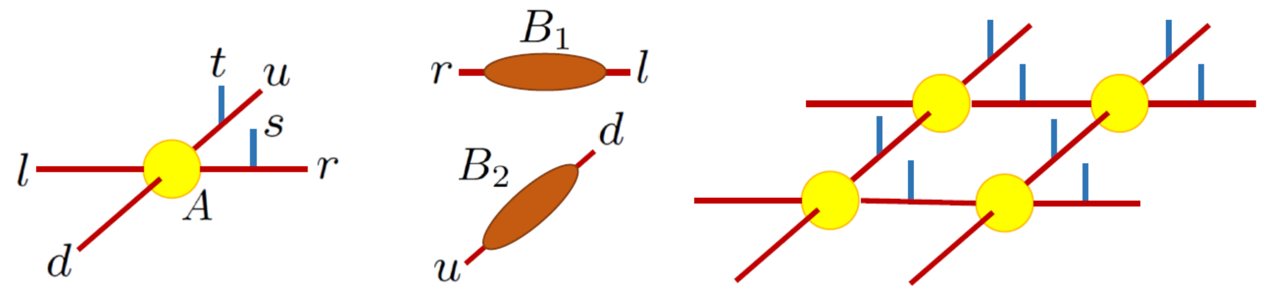

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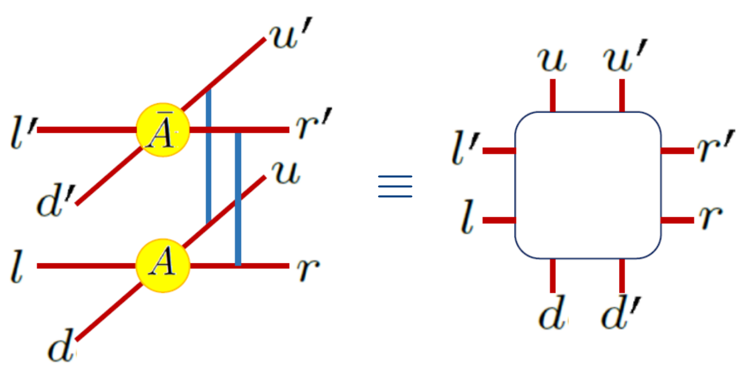
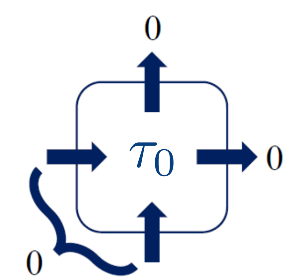
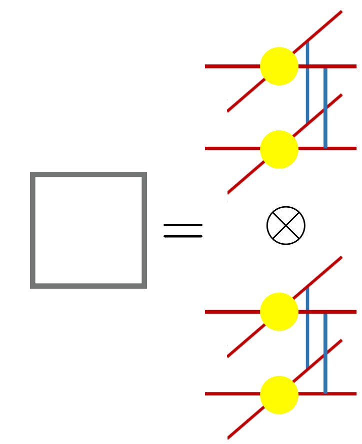


⇒ acts as an index map of singlets when the local gauge symmetry is taken into account:





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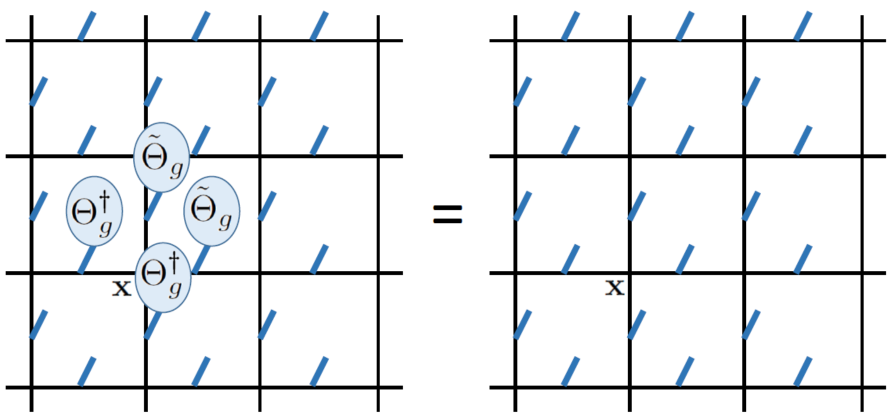

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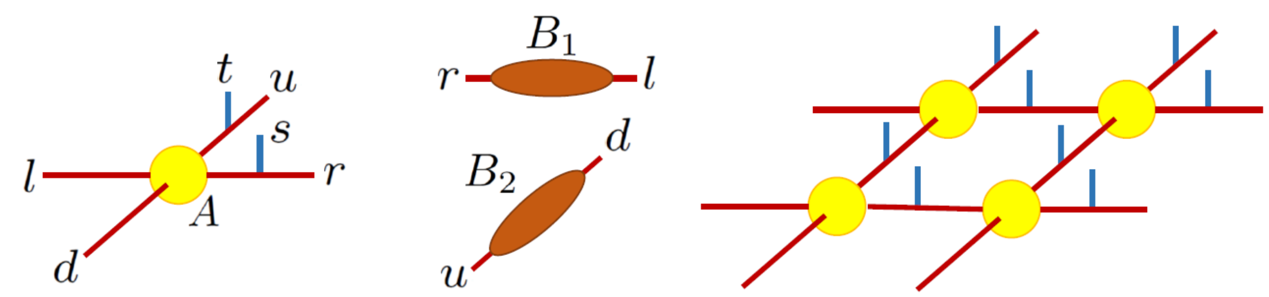

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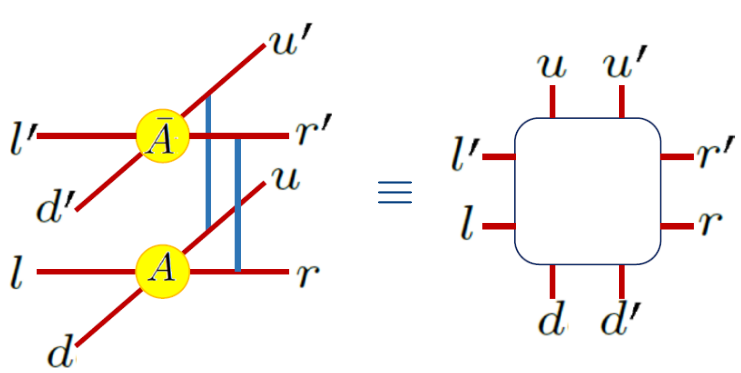

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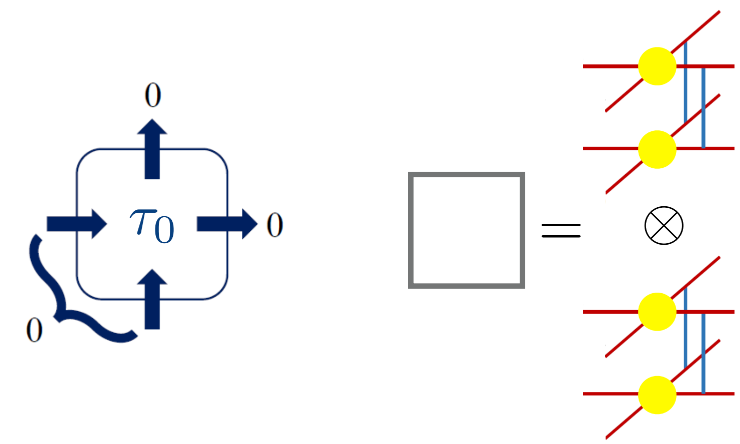

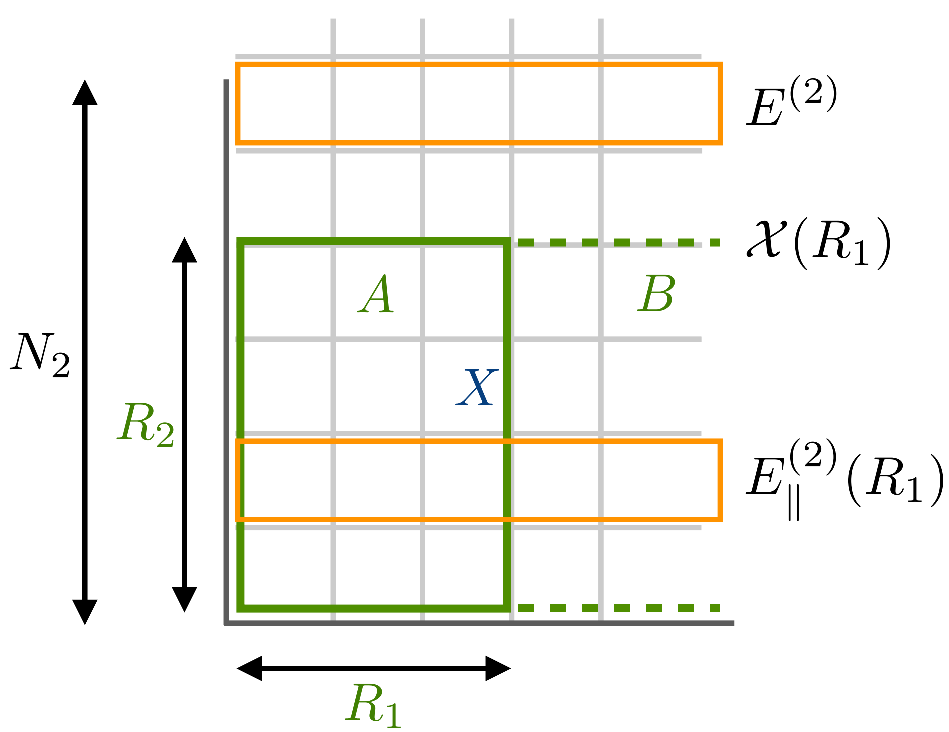

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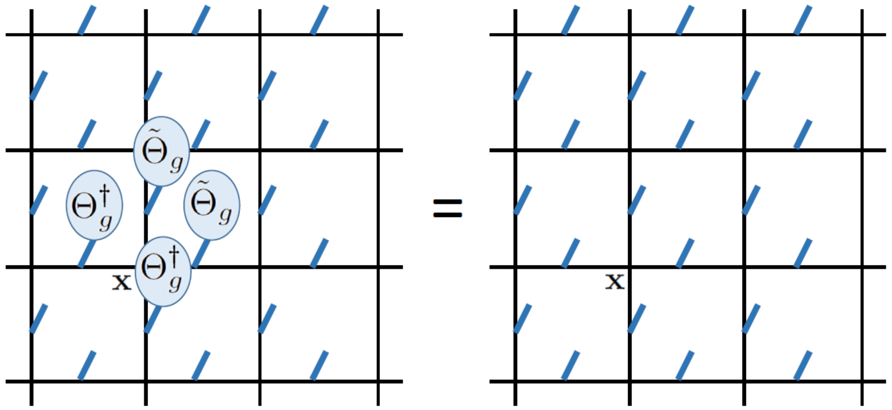

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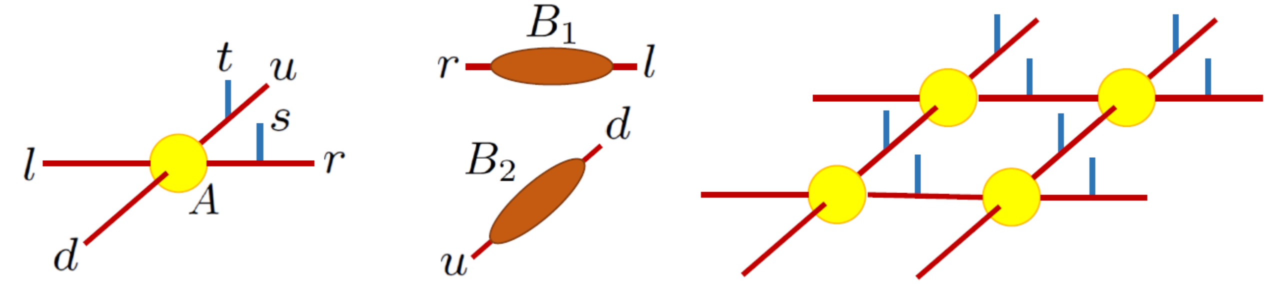

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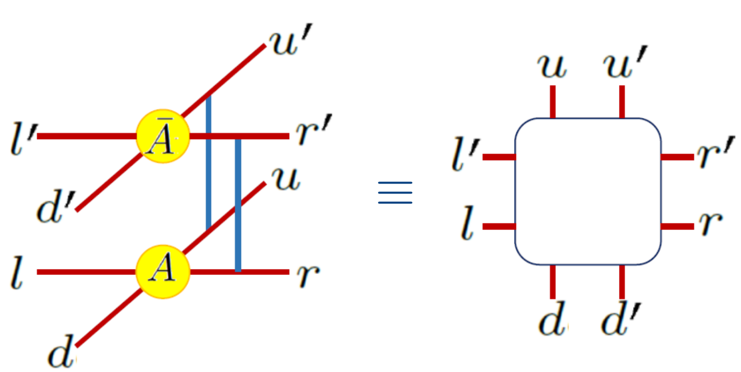

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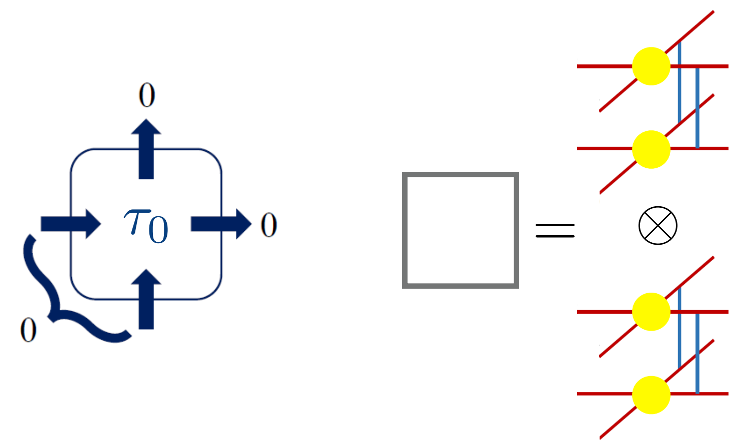
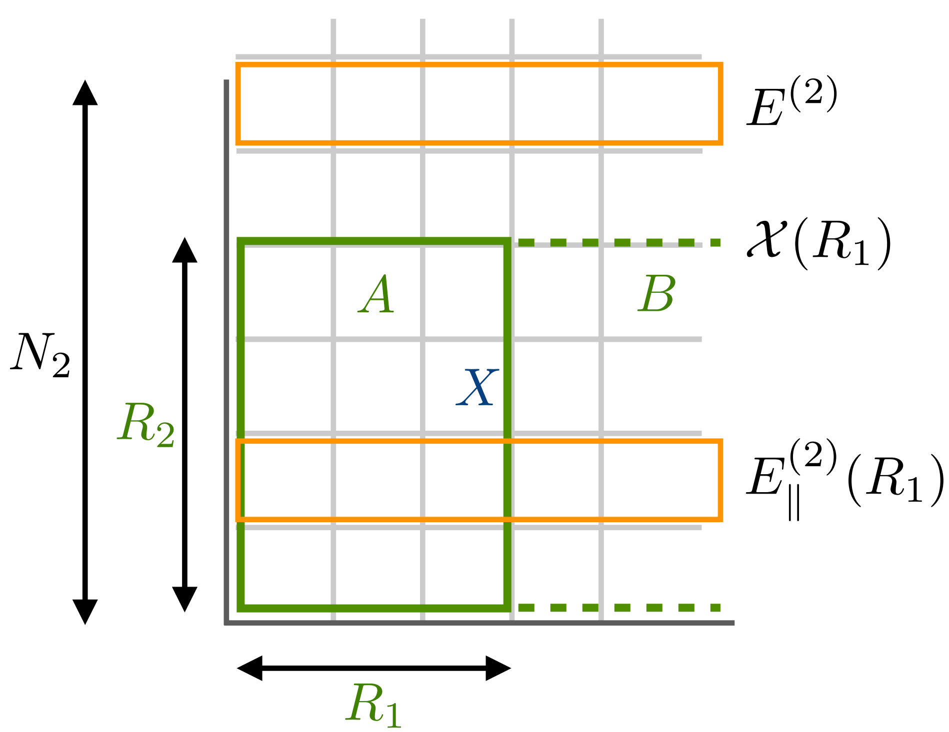


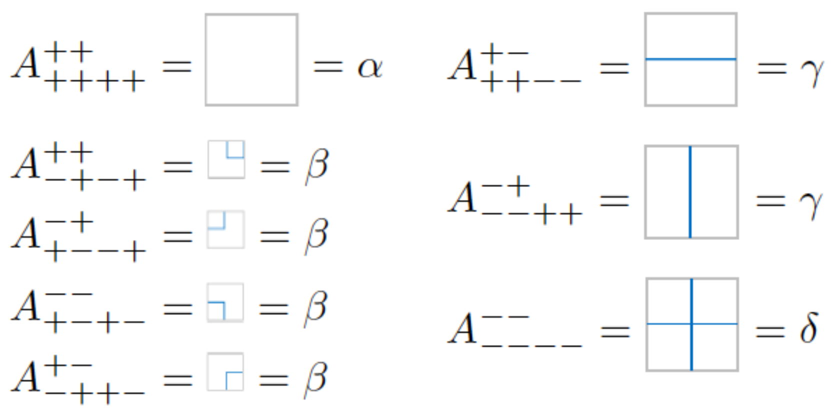


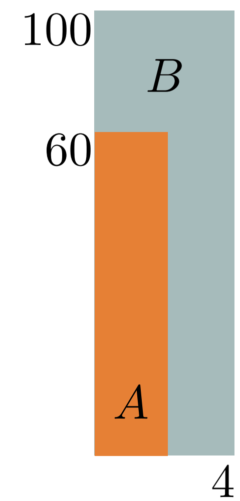
$$p_2 = \mathrm{Tr}\left[\mathcal{X}(R_1) E_{\parallel}^{(2)\,R_2}(R_1)\mathcal{X}(R_1) E^{(2)\,N-R_2}\right]$$

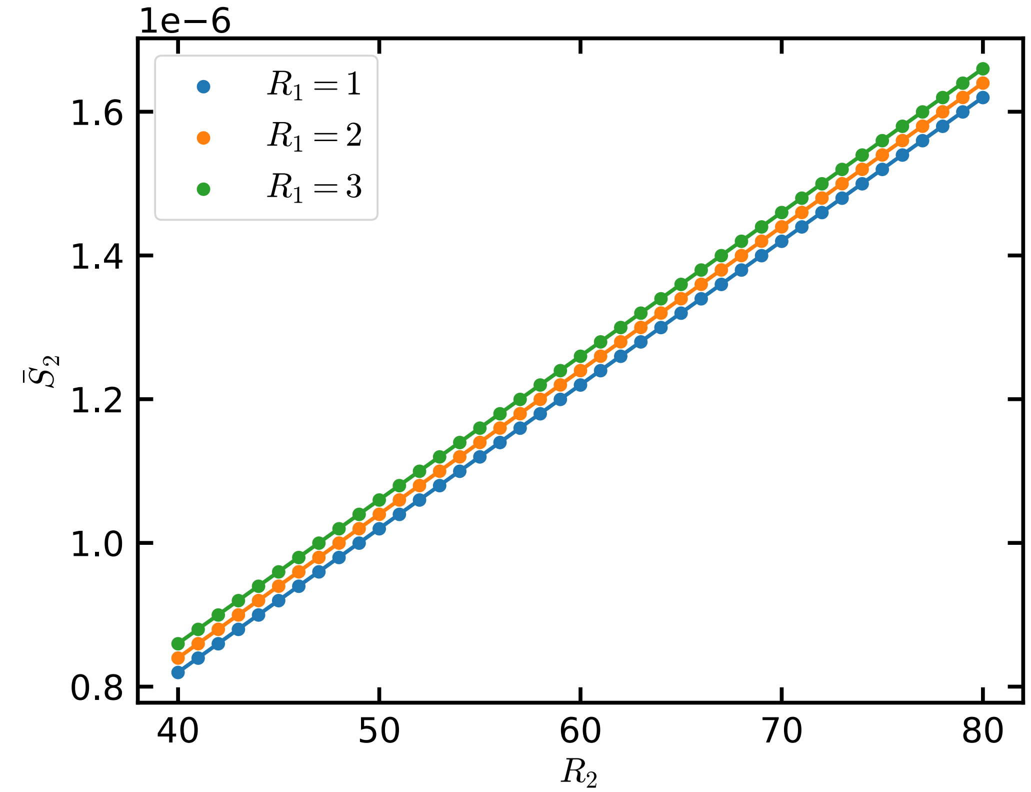
$$\bar{S}_2 = 2\ln K - (R_1 + R_2)\ln\left(\frac{\rho_1'^{(2)}}{\rho_1^{(2)}}\right)$$

- valid in the continuum
- representation of **entanglement Area law**

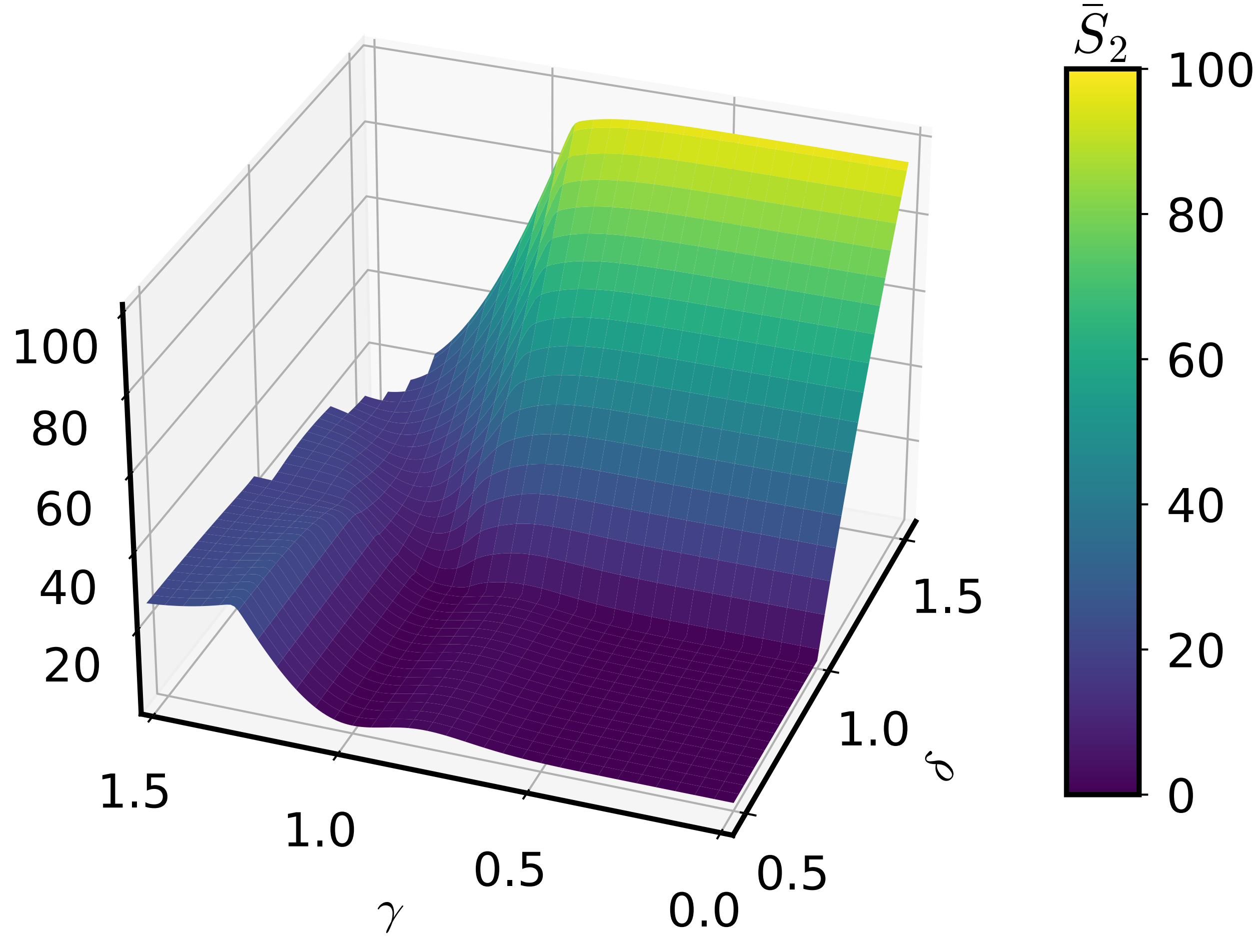
Numerical illustration: $\mathbb{Z}_2$ LGT

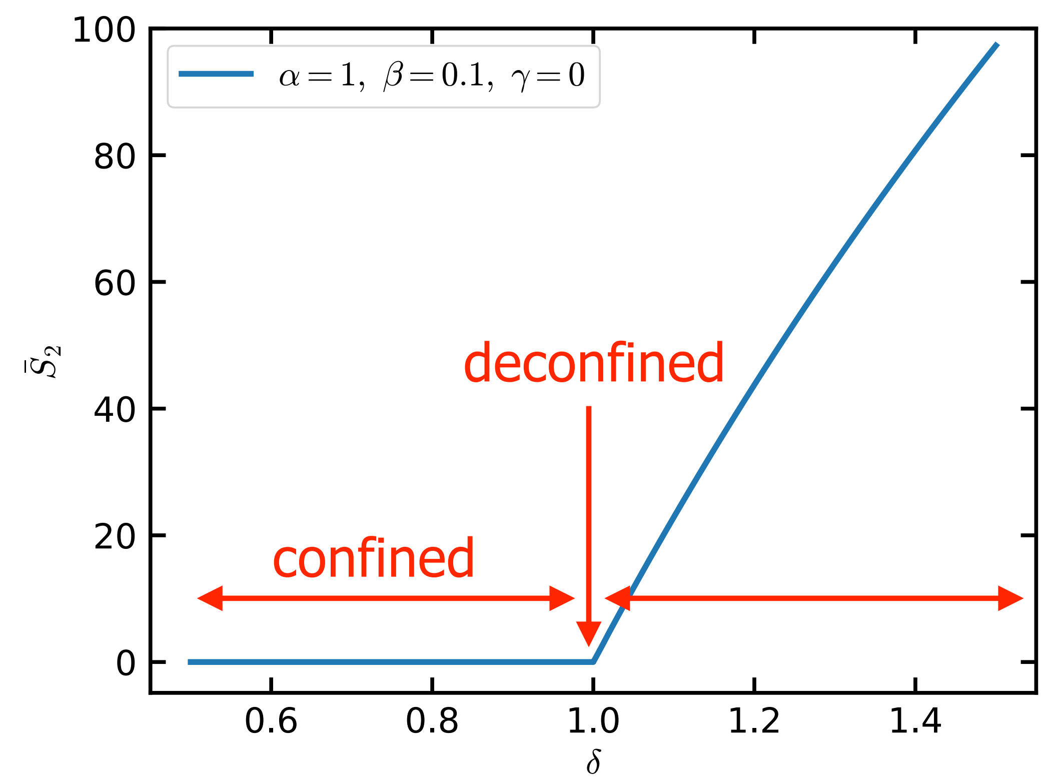






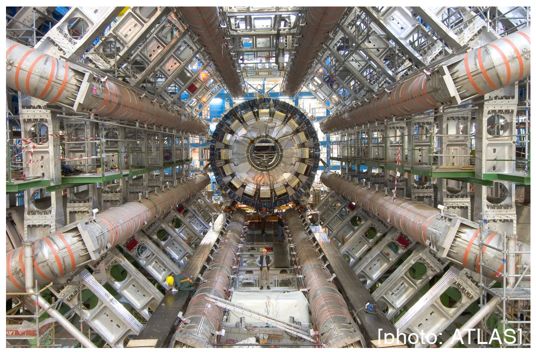
$\alpha = 1, \beta = 0.1$

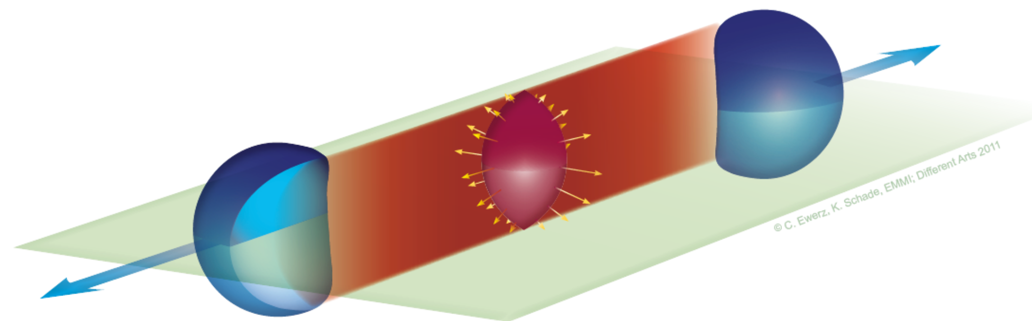




Summary and Conclusions

- Tensor network simulations provide access to entanglement properties as a key intrinsic feature of emergent phenomena such as confinement





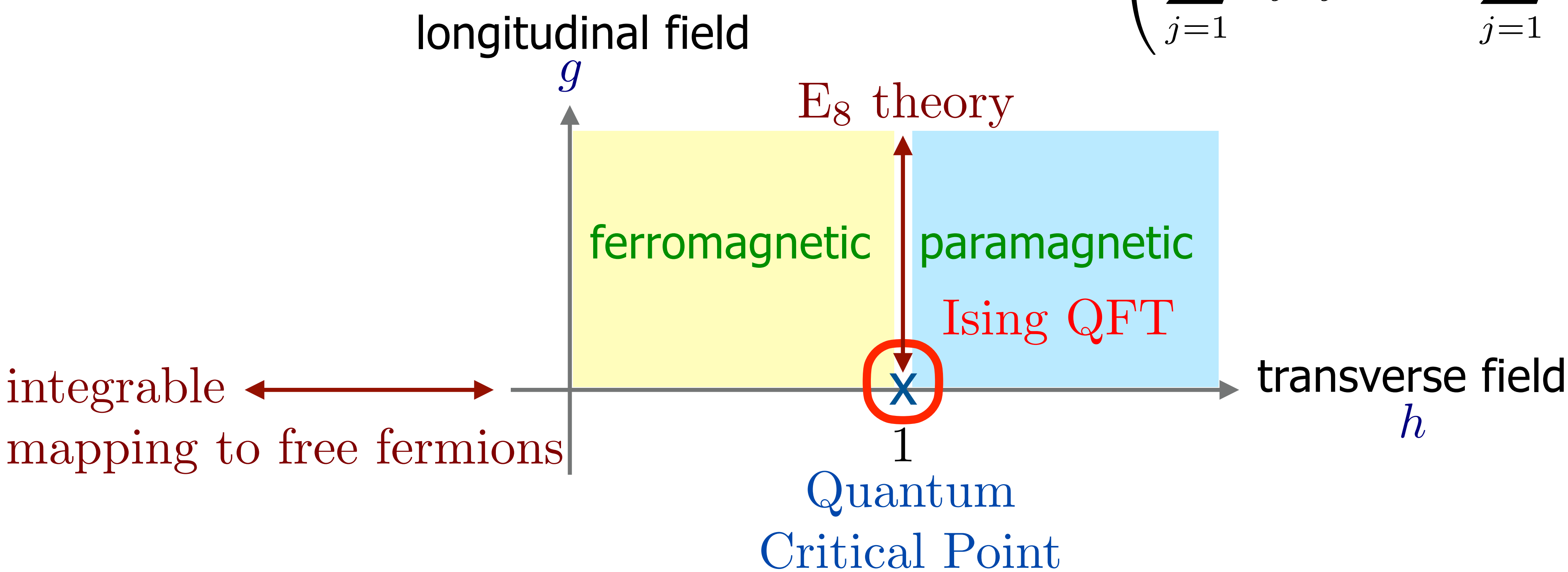
- Entropic entanglement measures witness the melting of mesons and characterize quantum quenches in the (1+1)D Ising QFT
- PEPS can be used to study entanglement and confinement properties of arbitrary 2D LGTs
- Outlook: a symmetry-resolved approach can give further insights to entanglement

  [Feldman, JK, Zohar, Goldstein]

Thank you to the organizers!

Testbed: The quantum Ising model and Ising QFT

$$H = -J\left(\sum_{j=1}^{N-1}\sigma_j^z\sigma_{j+1}^z + h\sum_{j=1}^{N}\sigma_j^x + g\sum_{j=1}^{N}\sigma_j^z\right)$$



- emergence of Ising QFT in continuum limit:

$$H = \int_{-\infty}^{\infty} dx \left\{ \frac{i}{4\pi}\left[\left(\psi\partial_x\psi - \bar{\psi}\partial_x\bar{\psi}\right) - \frac{M_h}{2\pi} i\bar{\psi}\psi\right] + \mathcal{C} M_g^{15/8}\,\sigma(x)\right\}$$

- $M_h = 0, M_g \neq 0$: interacting integrable E_8 QFT: 8 massive stable fermionic bound states [Zamolodchikov (1989)]
- $M_h \neq 0, M_g \neq 0$: interacting non-integrable QFT: stable and unstable bound states

gauge-invariant PEPS: