Islands and quantum focusing conjecture

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Quantum focusing conjecture is not violated even after Page time

Does quantum focusing conjecture (QFC) hold after Page time?

QFC ⇨ S_{gen} is non-decreasing $\quad S_{\mathrm{gen}} = \dfrac{\mathrm{Area}}{4G_N} + S_{\mathrm{rad}}$

After Page time $\begin{cases} \mathrm{Area}\text{: decreasing} \\ S_{\mathrm{rad}}\text{: decreasing} \end{cases}$ ⇨ Violation of QFC?

Results from the island rule (formula)

Page time is an approximately null surface

Entanglement entropy is increasing after Page time

Quantum focusing conjecture is not violated

Plan of Talk

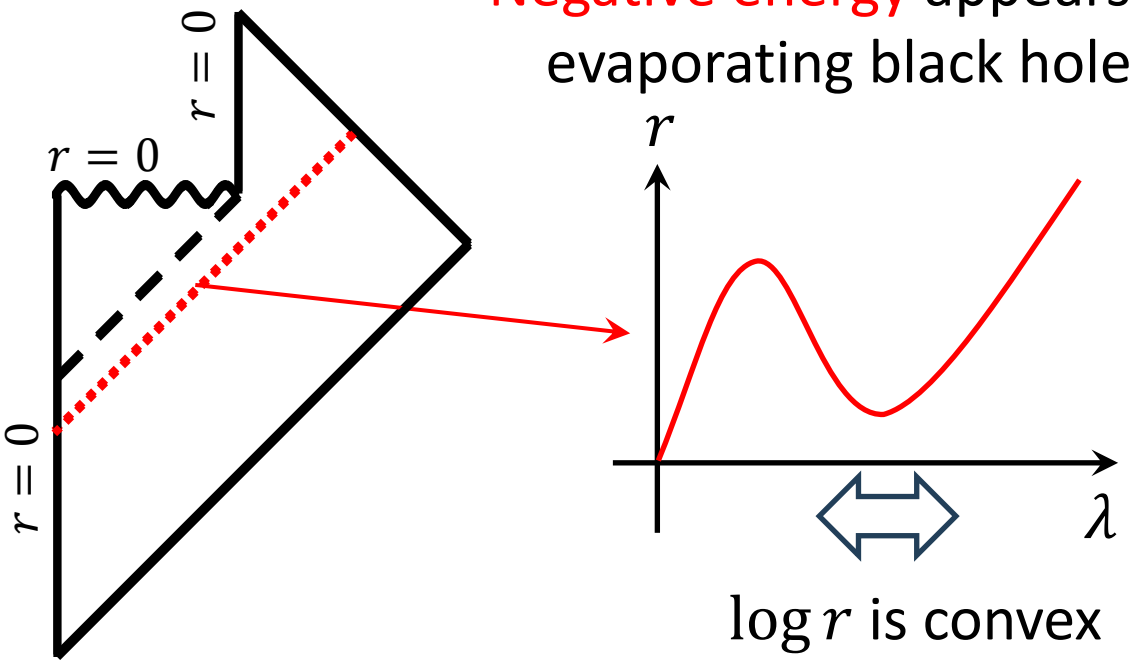
1. Quantum focusing conjecture (QFC)
   - Classical focusing is violated by quantum effects
   - QFC is inspired by generalized second law
   - Question: does QFC hold even after Page time?
2. Results from the island rule

Classical focusing is violated by quantum effects

Classical focusing theorem holds if energy is positive

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma^2 - R_{\mu\nu}k^\mu k^\nu \leq 0 \qquad \theta = \frac{1}{r^2}\frac{dr^2}{d\lambda}$$

Negative energy appears in evaporating black hole geometry



$\log r$ is convex ⇨ Violation of classical focusing

Quantum focusing conjecture is proposed inspired by generalized second law

[Bousso-Fisher-Leuchenauer-Wall,'15]

Classical focusing ⇨ θ is positive or ~~caustics occur~~

Event horizon has no caustics

Area of event horizon is non-decreasing

$$S_{\mathrm{BH}} = \frac{\mathrm{Area}}{4G_N}$$

BH entropy satisfies second law for classical black hole

Black hole emits Hawking radiation

Total entropy satisfies second law $S_{\mathrm{gen}} = \frac{\mathrm{Area}}{4G_N} + S_{\mathrm{rad}}$

Classical focusing $\frac{d\theta}{\mathrm{d}\lambda} \leq 0$ ⇨ Quantum focusing $\frac{d\Theta}{\mathrm{d}\lambda} \leq 0$

$$\theta = \frac{1}{r^2}\frac{dr^2}{d\lambda} \qquad \Theta = \frac{1}{r^2}\frac{dS_{\mathrm{gen}}}{d\lambda}$$

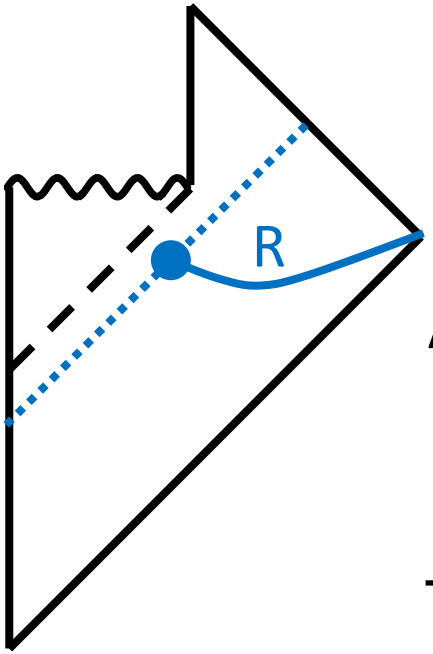
Question: does quantum focusing conjecture hold even after Page time?

Quantum focusing conjecture: $\frac{d\Theta}{d\lambda} \leq 0$ (along null surface)

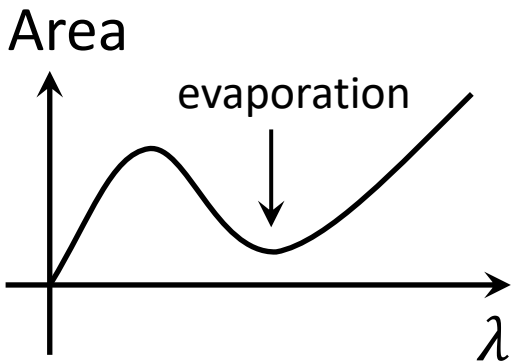
$$\Theta = \frac{1}{r^2}\frac{dS_{\text{gen}}}{d\lambda} \qquad S_{\text{gen}} = \frac{\text{Area}}{4G_N} + S_{\text{rad}}$$

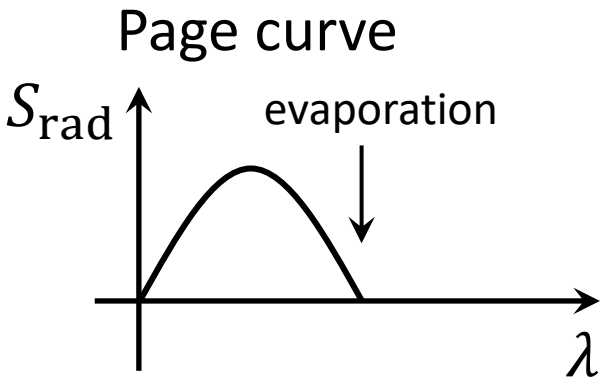
Entanglement entropy of matters on R

Gravity part of entanglement entropy on R



S_{gen} does not satisfy generalized second law if S_{gen} is defined by von Neumann entropy



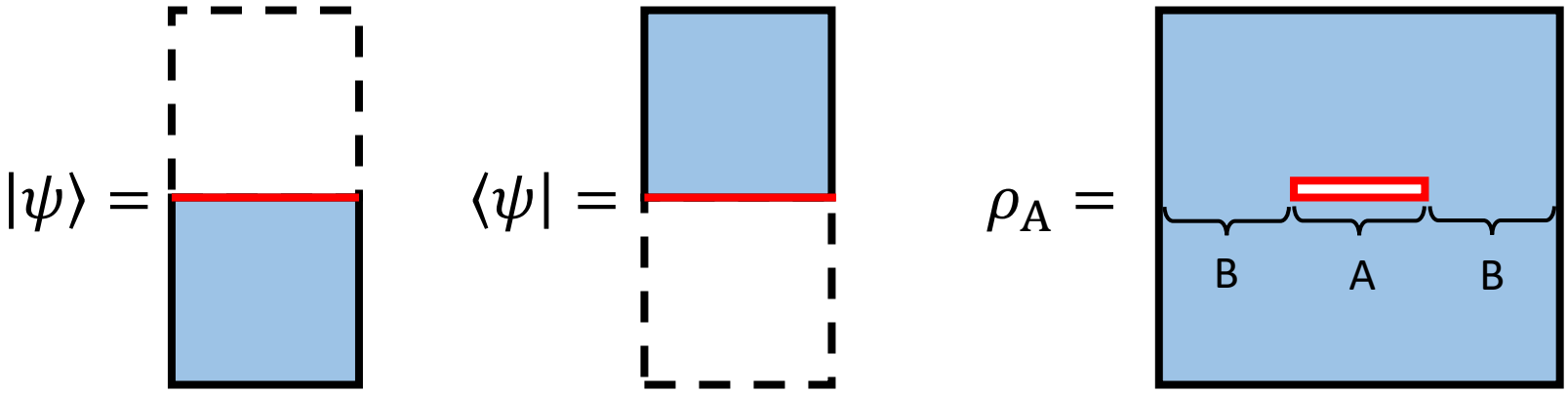


Plan of Talk

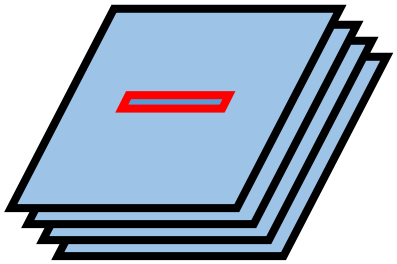
1. Quantum focusing conjecture
2. Results from the island rule
   - Island appears after Page time
   - Page time is an approximately null surface
   - Entanglement entropy decreases for timelike surface
   - Entanglement entropy increases for null surface

Entanglement entropy is calculated by replica trick

How to calculate $\rho_{\mathrm{A}} = \mathrm{tr}_{\mathrm{B}} |\psi\rangle\langle\psi|$



Entanglement entropy $S = -\lim_{n\to 1} \frac{\partial}{\partial n} \mathrm{tr}\, \rho_{\mathrm{A}}^n$

$\mathrm{tr}\, \rho_{\mathrm{A}}^n =$  $= \frac{Z_n}{Z_1^n}$

Z_n: Partition function of n-sheeted spacetime

Path integral for replicas ⇨ Configuration with minimum S

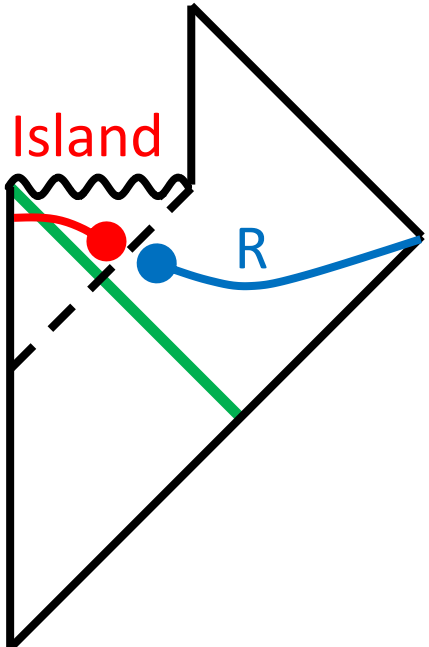
Island appears after Page time

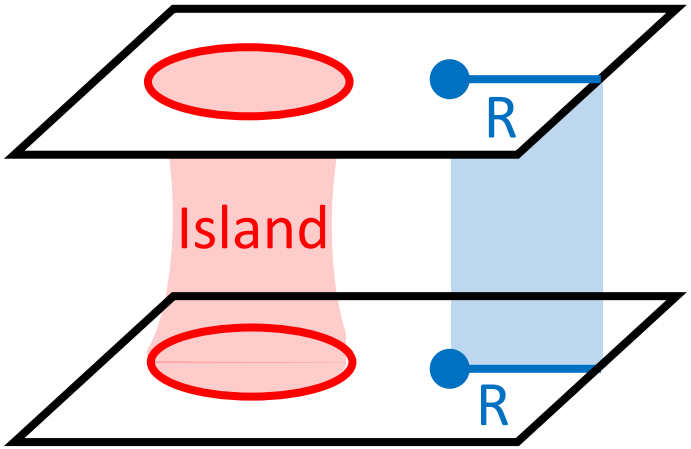
Replica trick with gravity ⇩

Wormhole geometry between replicas is allowed = Island

Configuration with smaller S is dominant ⇨ { Early time: no island

late time: island appears

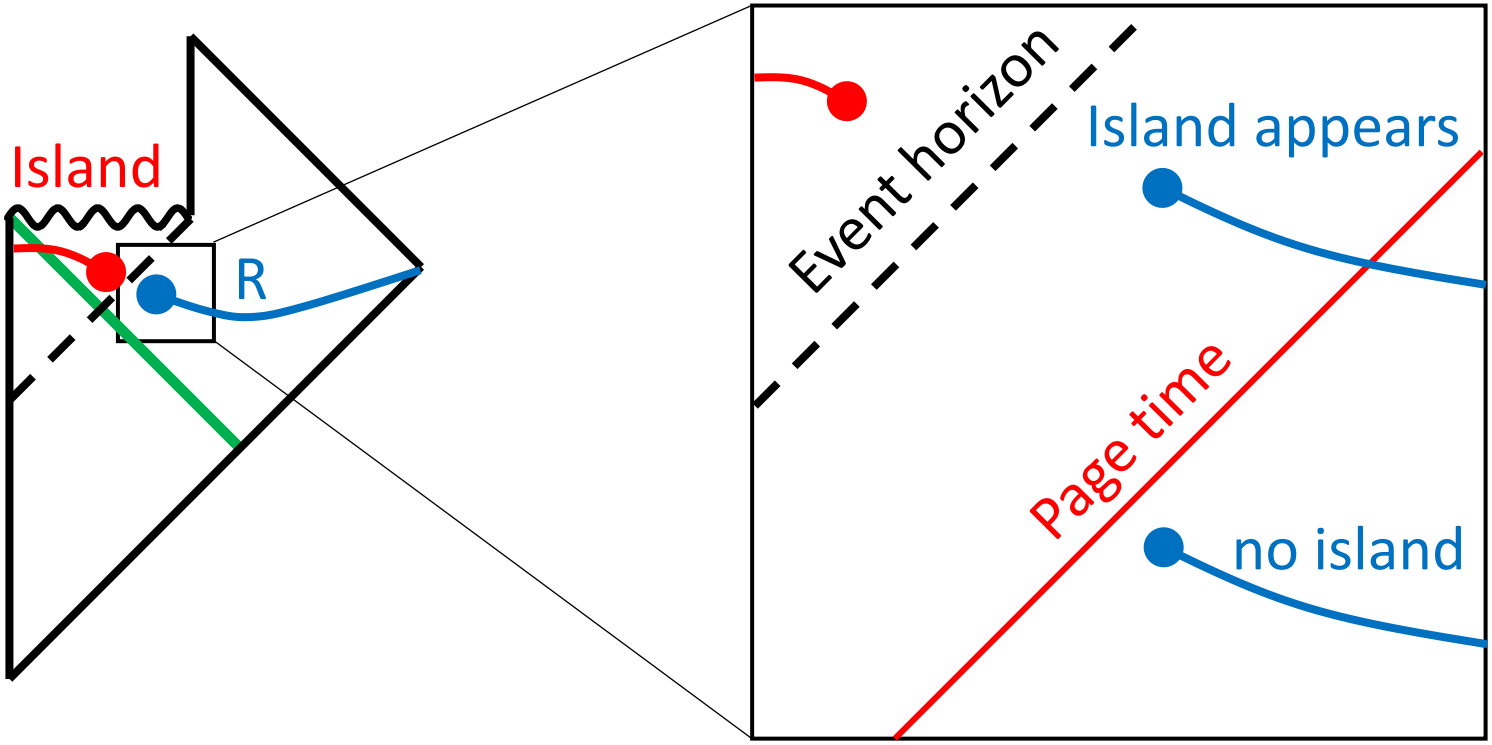




Page time is approximately null surface

Endpoint of R is before Page time ⇨ No island appears

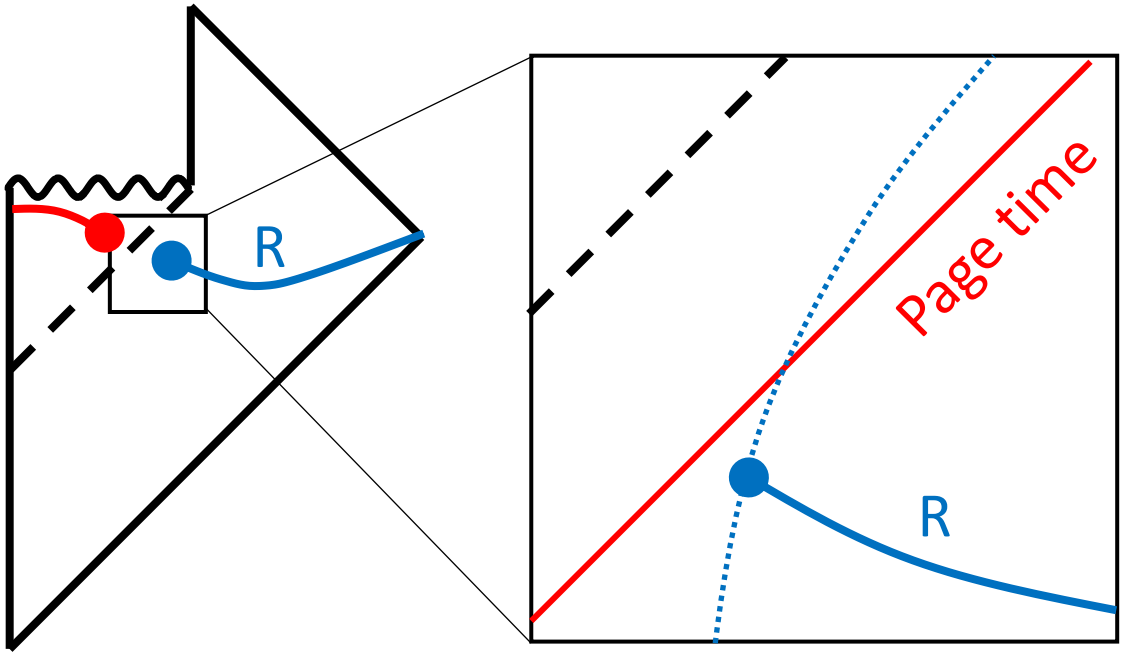
Endpoint of R is after Page time ⇨ Island appears



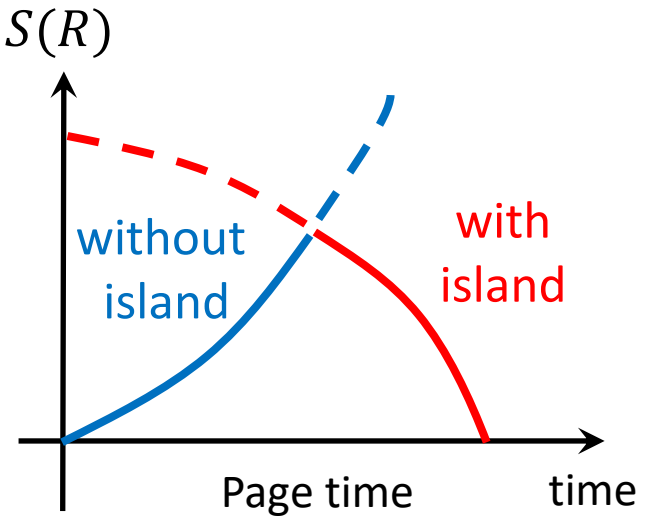
Entanglement entropy decreases along timelike surface after Page time

If endpoint of R moves along timelike surface:

- Island appears after endpoint of R get inside Page time
- Entanglement entropy decreases inside(after) Page time



Page curve reproduced

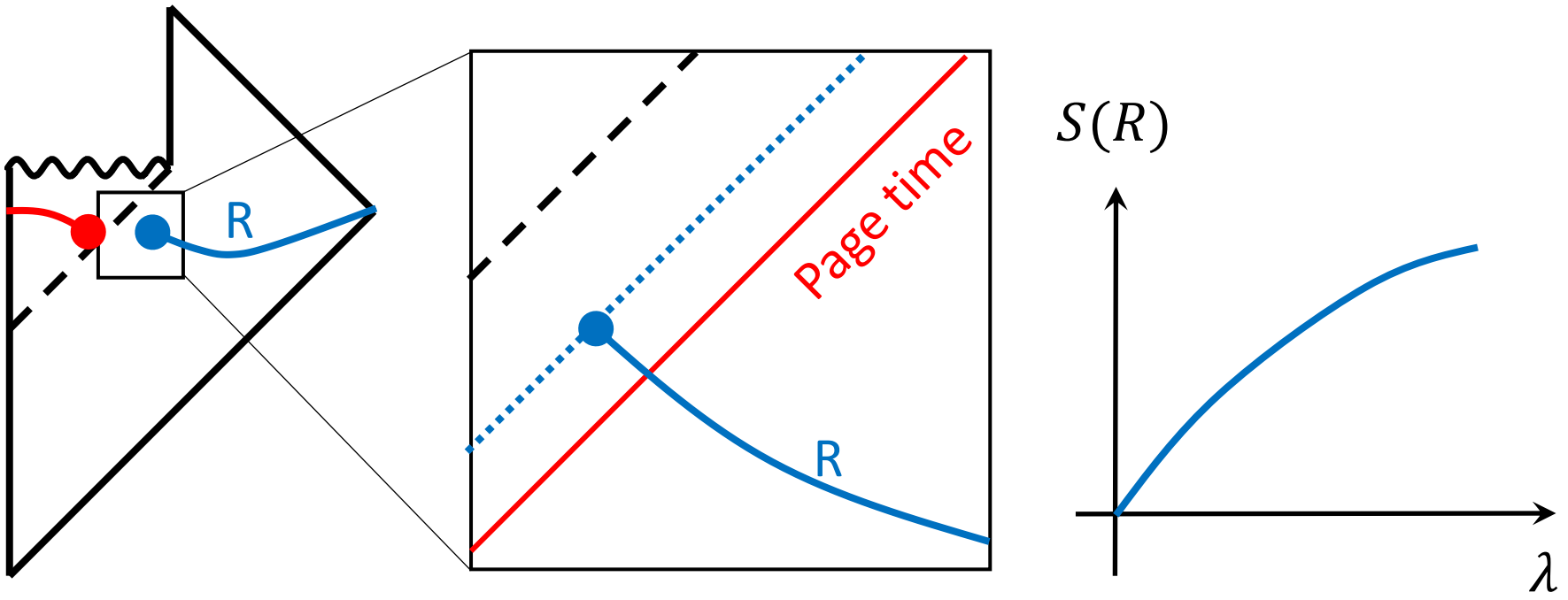


Entanglement entropy increases along outgoing null surface

If endpoint of R moves along null surface,

entanglement entropy increases with time even inside Page time

⇨ Quantum focusing conjecture is not violated



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Thank you