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Complexity and Topology in Quantum Matter

Symmetry-resolved entanglement in AdS and BCFT

Priv. Doz. Dr. René Meyer
Lehrstuhl für Theoretische Physik III
Institut für Theoretische Physik & Astrophysik
Julius Maximilians Universität Würzburg

People behind the research

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Symmetry-resolved entanglement in AdS_3/CFT_2 coupled to $U(1)$ Chern-Simons theory

Suting Zhao, Christian
Institut für Theoretische
Würzburg-Dresden Cluste
Am Hubland, 97074 Wür

Charged Moments in W_3 Higher Spin Holography

Suting Zhao,[a,1,] Christian Northe,[a,2,] Konstantin Weisenberger,[a,3] René Meyer, [a,4]
[a] Institut für Theoretische Physik und Astrophysik

On the boundary conformal field theory approach to symmetry-resolved entanglement

Giuseppe Di Giulio, René Meyer, Christian Northe, Henri Scheppach, Suting Zhao
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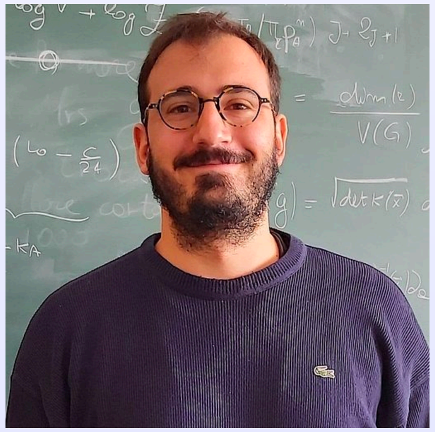
Symmetry-resolved entanglement for excited states and two entangling intervals in AdS_3/CFT_2

Konstantin Weisenberger,[1] Suting Zhao,[1] Christian Northe and René Meyer[2]
Institut für Theoretische Physik und Astrophysik
and Würzburg-Dresden Cluster of Excellence ct.qmat,
Julius-Maximilians-Universität Würzburg,
Am Hubland, 97074 Würzburg, Germany



Dr. G. Di Giulio

S. Zhao

Dr. C. Northe

K. Weisenberger

H. Scheppach

Symmetry Resolved Entanglement

Entanglement entropy in each charge sector, e.g. U(1)

charge operator Q $\quad Q = Q_{\mathcal{A}} \oplus Q_{\mathcal{B}}.$

eigenstate of Q, $\quad [\rho, Q] = 0.$ → $[\rho_{\mathcal{A}}, Q_{\mathcal{A}}] = 0.$

Block decomposition: $\quad \rho_{\mathcal{A}} = \oplus_q \rho_{\mathcal{A}}(q)$

Symmetry Resolved Renyi and Entanglement Entropy:

$$S_n(q) = \frac{1}{1-n} \log \mathrm{Tr} \left(\frac{\rho_{\mathcal{A}}(q)}{P_{\mathcal{A}}(q)} \right)^n$$

$$P_{\mathcal{A}}(q) = \frac{\mathrm{Tr}\rho_{\mathcal{A}}(q)}{\mathrm{Tr}\rho_{\mathcal{A}}} = \mathrm{Tr}\rho_{\mathcal{A}}(q)$$

$$S_1(q) = \lim_{n \to 1} S_n(q) = -\mathrm{Tr} \left(\frac{\rho_{\mathcal{A}}(q)}{P_{\mathcal{A}}(q)} \log \frac{\rho_{\mathcal{A}}(q)}{P_{\mathcal{A}}(q)} \right)$$

Sela, Goldstein PRL 2018

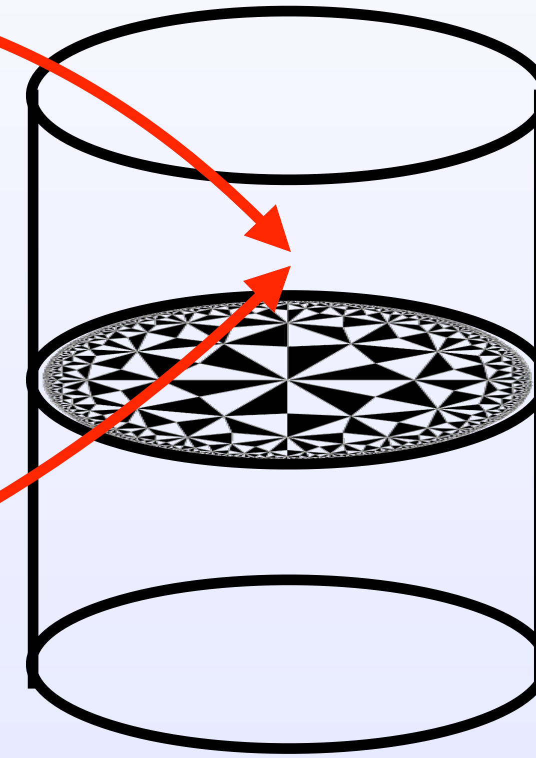
AdS_3 dual to $U(1)_k$ Kac-Moody CFT

3D Einstein-Hilbert gravity

$$S_g = \frac{1}{16\pi G_3} \int d^3x \sqrt{g} \left(R + \frac{2}{L^2}\right)$$

L… Curvature Radius of AdS3 space-time

To suppress quantum gravity effects: $c = \frac{3L}{2G_3} \gg 1$



$U(1)_k$ Chern-Simons theory

$$S_{CS} = \frac{ik}{8\pi} \int A \wedge dA$$

k… Chern-Simons level

Asymptotic symmetry analysis:

Boundary theory has $U(1)_k$ Kac-Moody symmetry

Hilbert space factorizes into gravitational and U(1) part

P. Kraus arXiv:hep-th/0609074

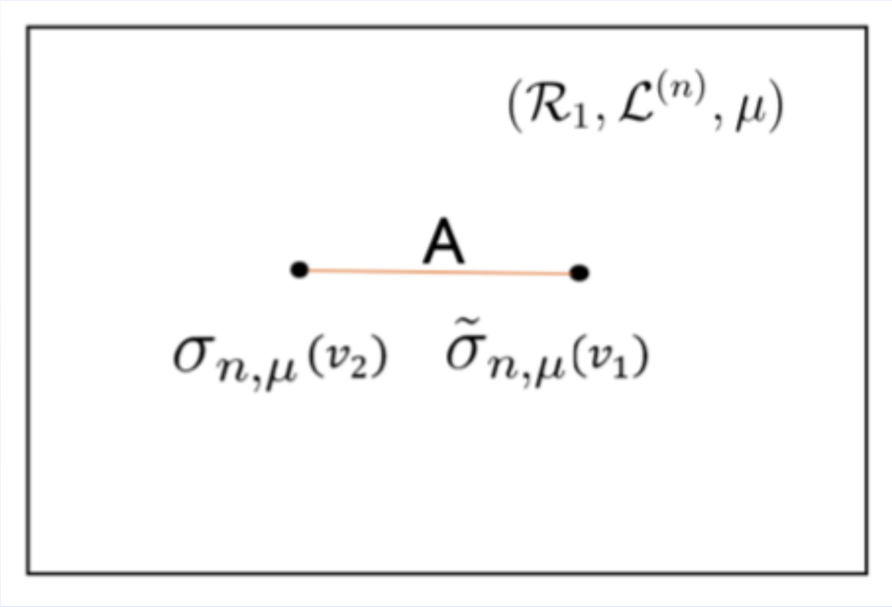
S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2012.11274)

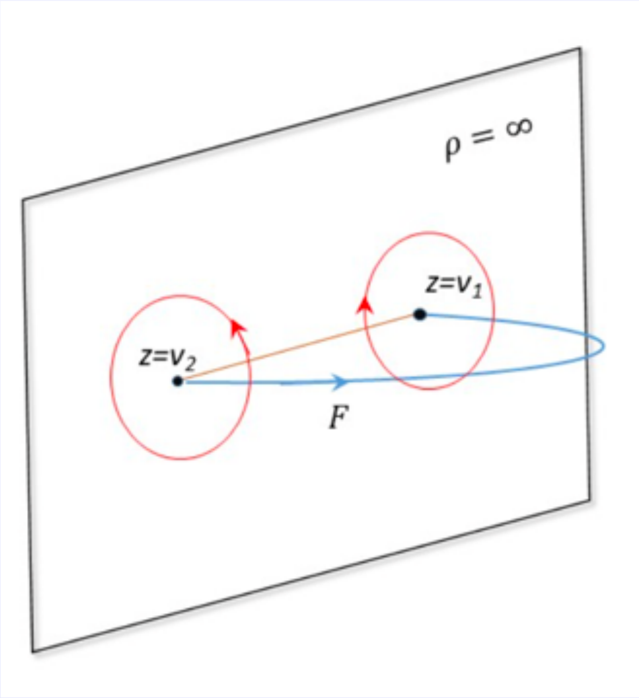
K. Weisenberger, S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2108.09210)

Single Interval SREE in AdS_3/CFT_2

Charged twist operator induces flux:
Sela, Goldstein PRL 2018

Wilson line following the Ryu-Takayanagi geodesic





CFT_2 result matches AdS_3 result for c>>1

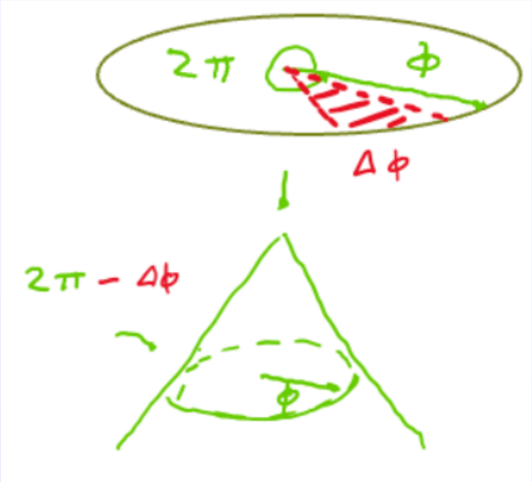
$$S_1(q) = \frac{c}{6}\ell - \frac{1}{2}\log\left(\frac{k\ell}{2\pi}\right) \quad \text{with} \quad \ell = 2\log\frac{|v_1 - v_2|}{\epsilon}$$

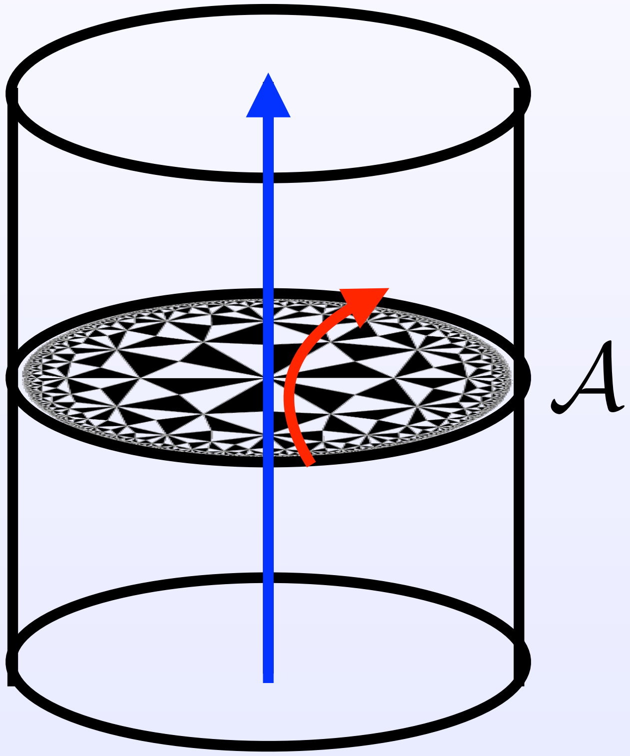
Equipartiton of entanglement!

Suting Zhao, Christian Northe, RM, JHEP 2021 (arXiv: 2012.11274)

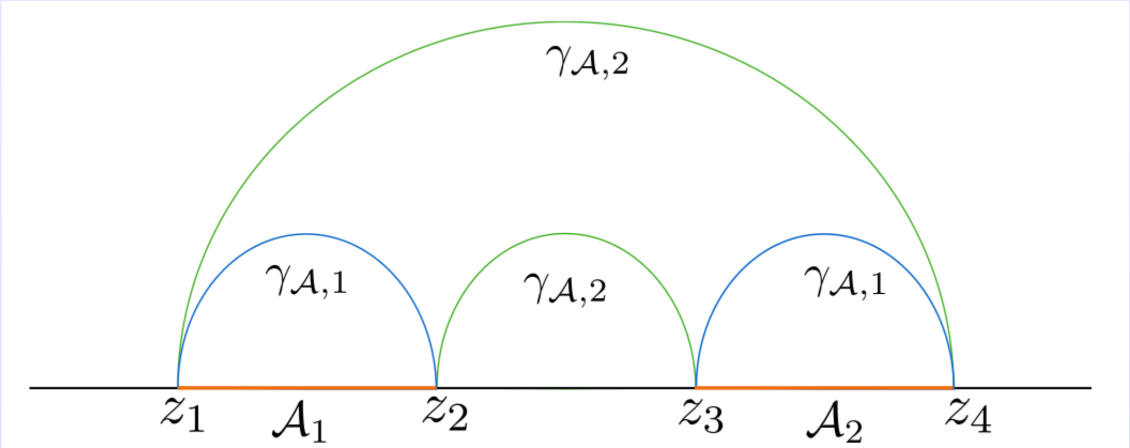
Further Checks

- Single interval with uncharged/charged heavy primary insertions





- Two intervals in the ground state



S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2012.11274)
K. Weisenberger, S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2108.09210)

Breakdown of Equipartition

SL(3,R) Higher Spin Gravity in 3D: W_3 symmetric CFT

Energy-momentum tensor plus Spin 3 current

$$T(z)W(w) = \frac{3W(w)}{(z-w)^2} + \frac{\partial W(w)}{z-w} + \cdots$$

Charged moments for a single interval:

Topological black hole grand canonical partition function

Perturbative result in μ

Kraus+Perlmutter (2011)
Gaberdiel+Hartman+Jin (2012)

$$\log \operatorname{Tr}\left(e^{-2\pi n\mathcal{H}+2\pi i\mu Q_{\mathcal{A}}}\right) = \frac{c\ell}{6n}\left(-\frac{1}{3}\frac{\mu^2}{n^4} + \frac{10}{27}\frac{\mu^4}{n^8} + \cdots\right)$$

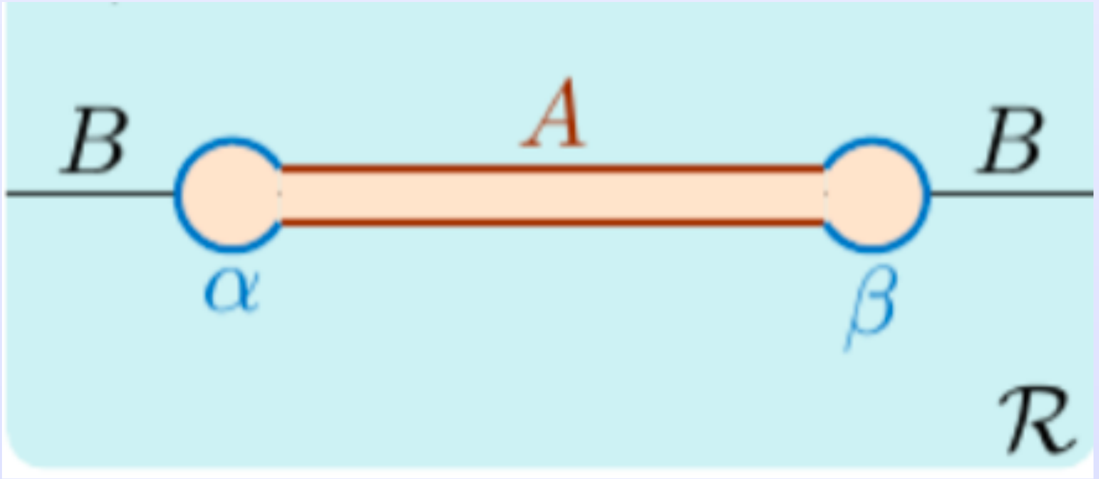
Fourier transformation and taking the replica limit yields breakdown of equipartition in SREE at large c.

Suting Zhao, Christian Northe, Konstantin Weisenberger, RM, 2202.11111

Why BCFT?

BCFT corrections to symmetry resolved entanglement

Boundary conditions are important

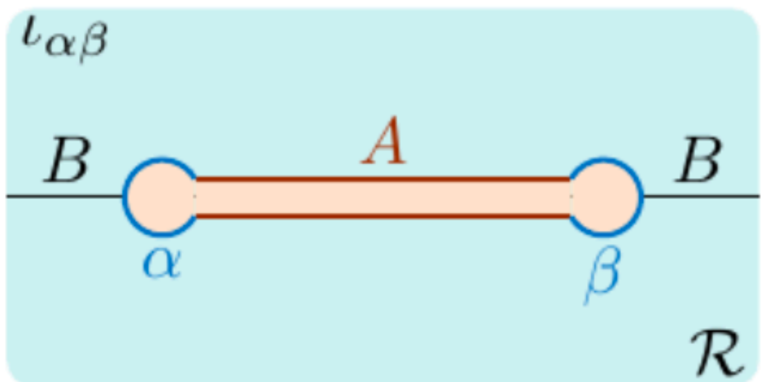


Exact equipartition of entanglement for free boson
to all orders in the UV cutoff expansion

Di Giulio, RM, Northe, Scheppach, Zhao, SciPost Physics Core, arXiv: 2212.09767

Boundary Conformal Field Theory

Boundary Conditions: Define Hilbert space factorization



Choose boundary conditions which preserve conformal symmetry and $U(1)$

$$T = \bar{T}|_{bdy} \qquad J = \pm \bar{J}|_{bdy}$$

Dirichlet

Neumann

Example: Compact Free Boson

$$S = \frac{g}{2}\int_{\Sigma} d^2x \, \partial_\mu \varphi \partial^\mu \varphi, \qquad \varphi = \varphi + 2\pi R$$

$$J^\mu = \partial^\mu \varphi \qquad \tilde{J}^\mu = \epsilon^{\mu\nu}\partial_\nu \varphi$$

NN & DD conditions preserve a diagonal Vir x U(1)

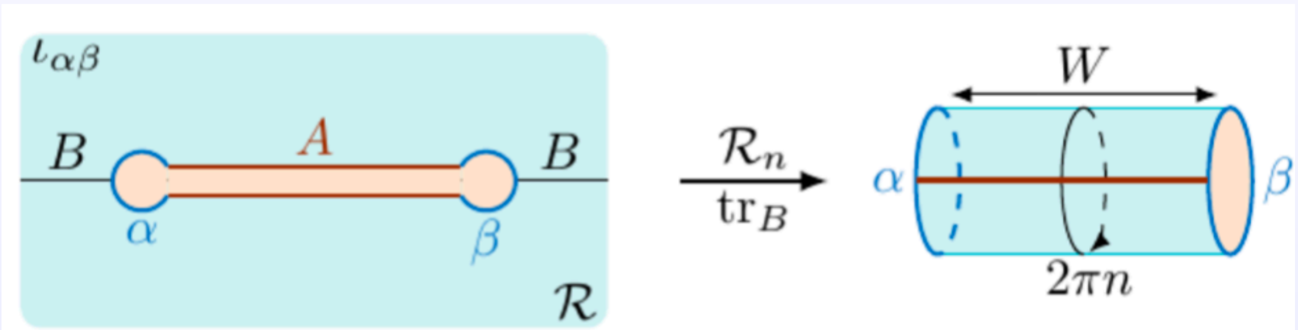
ND & DN conditions break both U(1), preserve Z_2 & twisted U(1)

Ohmori & Tachikawa (2015), Cardy & Tonni (2016)

Di Giulio, RM, Northe, Scheppach, Zhao, SciPost Physics Core, arXiv: 2212.09767

SRE in U(1)$_k$ BCFT

Exact equipartition to all orders in UV cutoff



$$\rho = |0\rangle\langle 0|$$

$$\rho_A = \frac{q^{L_0 - c/24}}{Z_{\alpha\beta}(q)}$$

$$q = e^{-2\pi^2/W}$$

Partition function splits into representations $\mathcal{H}_i$ of chiral algebra with characters $\chi_i(q) = \mathrm{tr}_{\mathcal{H}_i} q^{L_0 - c/24}$

$$Z_{\alpha\beta}(q) = \sum_{i\in\sigma} \mathrm{n}^i_{\alpha\beta}\, \chi_i(q) \approx \chi_\Omega(\tilde{q}) \underbrace{\times \mathrm{g}_\alpha \mathrm{g}_\beta}_{S^{\alpha\beta} = \log[\mathrm{g}_\alpha \mathrm{g}_\beta]} ,$$

$$\chi_Q(q) = \frac{q^{Q^2}}{\eta(q)}$$

Charge-projected partition functions select one representation

$$\mathcal{Z}_n(Q) = \mathrm{tr}_A[\Pi_Q \rho_A^n] = \frac{\chi_Q(q^n)}{(Z_\pm(q))^n} \qquad \mathrm{p}_Q = \frac{\chi_Q(q)}{Z_\pm(q)}$$

$$S_n(Q) = \frac{1}{1-n} \log\left[\frac{q^{nQ^2}}{\left(q^{Q^2}\right)^n} \frac{\eta^n(q)}{\eta(q^n)} \right]$$

$$S_n(Q) = \underbrace{\frac{W}{12}\frac{n+1}{n} - \frac{1}{2}\log\left[\frac{W}{\pi}\right] + \frac{1}{2}\frac{\log n}{1-n}}_{\text{Known terms}} + \frac{1}{1-n}\sum_{k=1}^{\infty} \log\left[\frac{\left(1-e^{-2Wk}\right)^n}{1-e^{-2Wk/n}}\right]$$

Di Giulio, RM, Northe, Scheppach, Zhao, SciPost Physics Core, arXiv: 2212.09767

Conclusions & Outlook

- New insight into AdS/CFT from quantum information, e.g. Entanglement entropy, Symmetry resolved entanglement

- New tests of AdS_3/CFT_2 plus Chern-Simons
- First example of breakdown of equipartition at large c
- BCFT approach yields exact results due to symmetries, U(1) equipartition to all orders for free bosons

- SREE Analysis of further instances of AdS/CFT
- Symmetry resolving other quantum information measures
- Higher dimensions? **P. Bueno, P.A. Cano, A. Murcia, A.R. Sanchez, PRL 2022**
- Other symmetry groups? **Y. Kusuki et. al., arXiv:2309.03287**
- Explore implications on bulk entanglement, in particular AdS/CFT duals of higher order corrections?
- Field theory and Condensed Matter Applications of SRE? **C. Northe, arXiv:2303.07724**