"Extreme Magnet, scale without conformal in dipolar fixed point"

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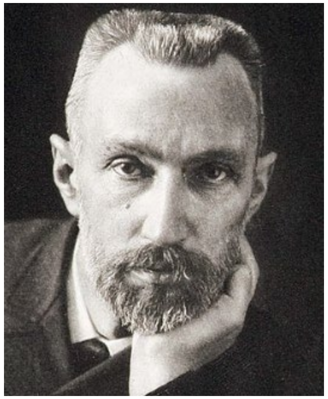
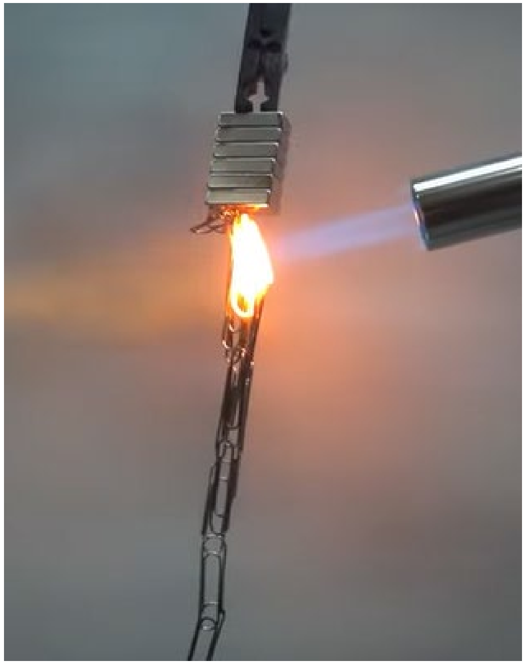
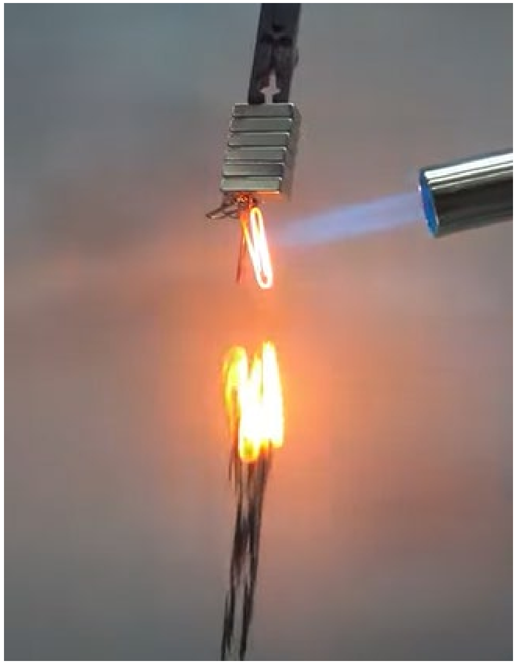
“Extreme magnet”

At Curie temperature, ferromagnet loses magnetism







(photo taken from youtube video: https://www.youtube.com/watch?v=sGPuDtzz_s8)

We know Curie Phase transition

We believe we know the physics of the Curie transition of an isotropic magnet (e.g. Fe, Ni)

- It is second order phase transition
- It is effectively described by the Heisenberg model or O(3) invariant Landau-Ginzburg-Wilson model in the continuum limit
- We can study critical exponents in epsilon expansions or 1/N expansions with RG
- It is conformal invariant
- Conformal bootstrap gives

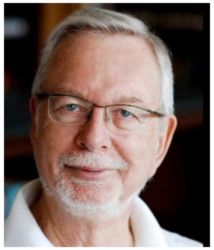
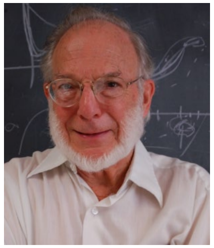
$$\eta = 2\Delta_\phi - 1 = 0.03788(15) \qquad \nu = \frac{1}{3 - \Delta_{\phi^2}} = 0.7117(5)$$

These are extremely wrong

- It is second order phase transition
- It is NOT described by the Heisenberg model or O(3) invariant Landau-Ginzburg-Wilson model in the continuum limit
- It is NOT conformal invariant
- Conformal bootstrap does NOT work
- At this point, you may regret you come to

  a crackpot talk;)

Then what is really happening?

- It is second order phase transition
- It is described by the dipolar fixed point

(first pointed out by Aharony and Fisher back in 1973)

- It is scale invariant but not conformal invariant (new!)

- Guru of conformal bootstrap was convinced so I hope you will be, too

Plan of my talk

- Dipolar fixed point of isotropic magnet
  – Theory and Experiment

- Scale invariance w/o conformal invariance
  – Importance of hidden shift symmetry

Dipolar fixed point instead of Heisenberg

Landau-Ginzburg theory

- Order parameter is magnetization vector ϕ_i
- Effective Hamiltonian

$$H = \int d^3x \left(\partial_\mu \phi_i \partial_\mu \phi_i + (T - T_c)\phi_i^2 + \lambda(\phi_i^2)^2 + B_i \phi_i\right)$$

- (Without external source B) Fixed point is (believed to be) a CFT with global O(3) symmetry

- Index i is treated as internal O(3), not as a spatial vector O(3) → O(3) x O(3) enhanced
- Spin-orbit decoupling: At the O(3) symmetric conformal fixed point, all the local deformation that breaks O(3) x O(3) → O(3) are irrelevant

Aharony-Fisher theory

- Stable under local deformation, but in reality

$$H = \int d^3x \left(\partial_\mu \phi_i \partial_\mu \phi_i + (T - T_c)\phi_i^2 + \lambda(\phi_i^2)^2 + B_i \phi_i\right)$$

- Magnetic field is dynamical in nature → dipolar interaction

$$\int d^3q \phi_i(-q) \frac{q_i q_j}{q^2} \phi_j(q) = \int d^3x d^3y \phi(x) \frac{\delta_{ij} - 3\hat{n}_i \hat{n}_j}{(x-y)^3} \phi(y)$$

- In the IR limit, the net effect is to make the magnetization vector transverse (AF theory 1973)

$$\partial_i \phi_i(x) = q_i \phi_i(q) = 0$$

- If you prefer "microscopic theory" take any of your favorite model of magnet e.g. Heisenberg model and just add B_i^2

Aharony-Fisher theory

- Net result of the dipolar interaction (in IR limit) is to make the magnetization vector transverse

$$\partial_i \phi_i(x) = q_i \phi_i(q) = 0$$

- (Eventually, this transverse condition is what breaks conformal symmetry down to scale symmetry only)
- Perturbative critical exponents are different from Heisenberg fixed point due to the change of the propagator

$$\langle \phi_i(q)\phi_j(-q)\rangle_0 = \frac{1}{q^2}\left(\delta_{ij} - \frac{q_i q_j}{q^2}\right)$$

Aharony-Fisher

$$\Delta_t = 2 - \frac{8}{17}\epsilon$$

$$\Delta_\phi = \frac{2-\epsilon}{2} + \frac{10}{867}\epsilon^2 \ .$$

O(N)

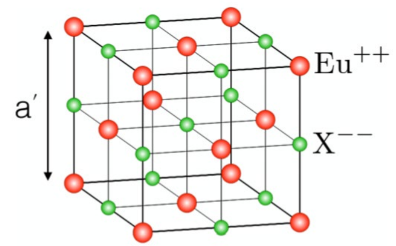
$$\Delta_t = 2 - \frac{6}{N+8}\epsilon$$

$$\Delta_\phi = \frac{2-\epsilon}{2} + \frac{(N+2)}{4(N+8)^2}\epsilon^2 \ .$$

- Only limited theoretical attempt to compute critical exponents (3-loop Kudlis-Pikelner, functional RG Nakayama etc)

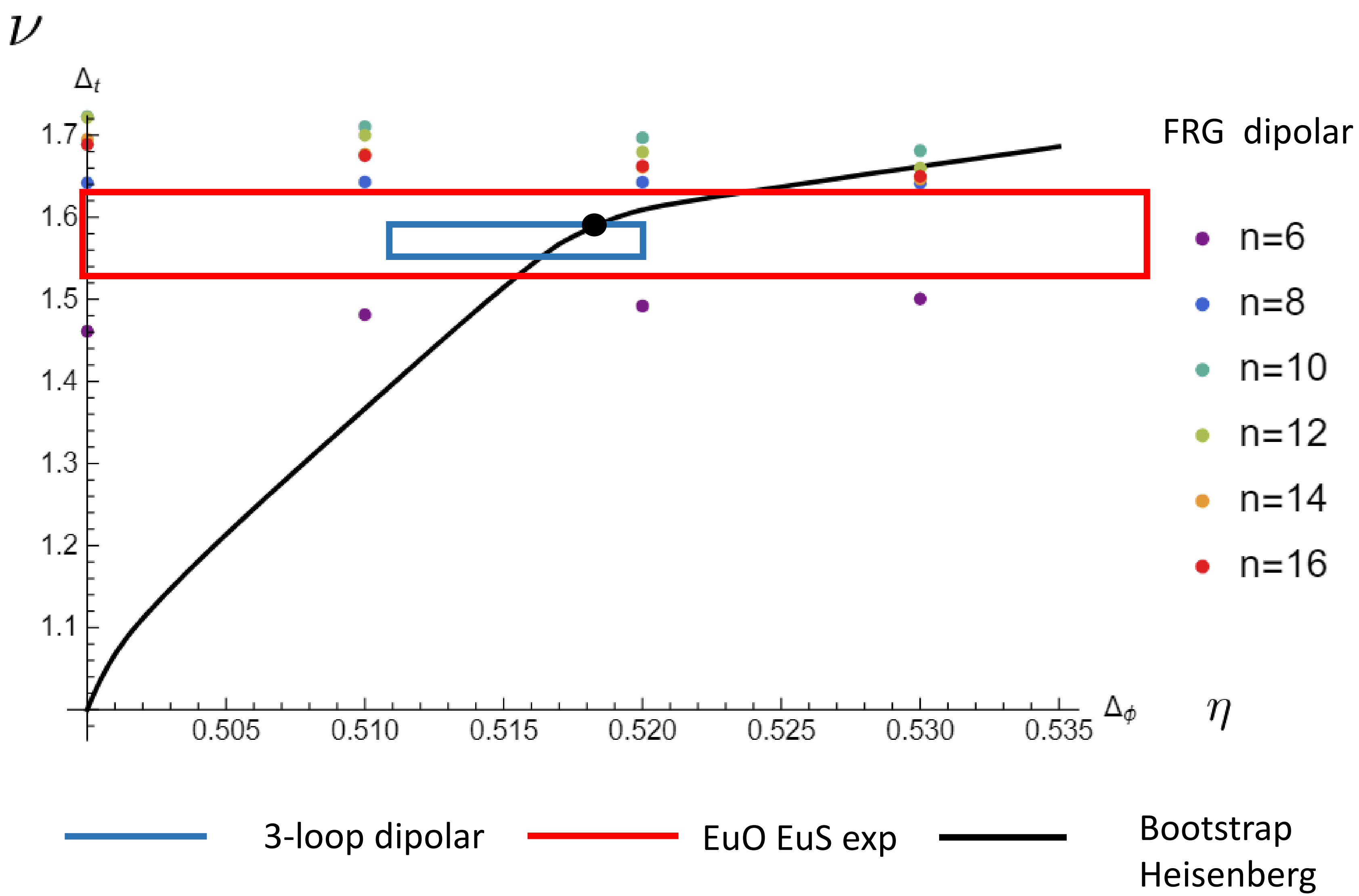
Experimental evidence?

- In EuO or EuS, neutron scattering experiments (Kotzler-Mezei et al) suggest a suppression of the longitudinal correlation (in two-point functions)



- Eu compounds are semi-conductor and Ferromagnet
- Critical exponents in EuO or EuS are measured but not clear if they are away from the Heisenberg value (after all, we do not have a precise theoretical value for the dipolar fixed point as well)
- Not yet(?) observed in Fe or Ni

Critical exponents in d=3



Why in Eu compound but not in Fe (yet)?

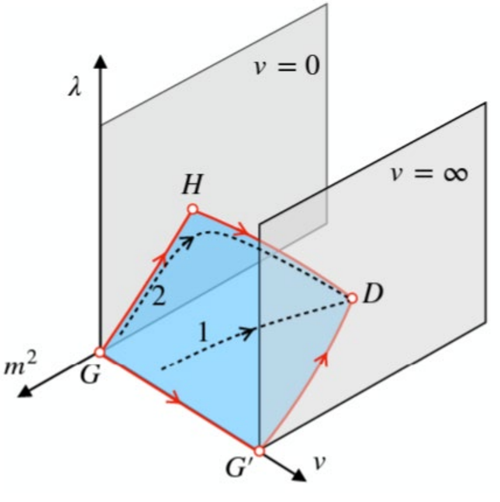
$$\mathcal{H}[\phi] = \int d^3x \left(\frac{1}{2}a(\partial_i\phi_j)^2 + \frac{1}{2}b\phi_i^2\right) + \frac{1}{2}\int d^3x \int d^3y\, U_{ij}(x-y)\phi_i(x)\phi_j(y)\,.$$

- From Experiment, we can determine EFT parameters (a,b) awav from the fixed point

$$G_{ij}^{-1}(q) \propto (q^2+\xi^{-2})\delta_{ij} + q_d^2\,\frac{q_iq_j}{q^2}\,, \qquad \phi_i = \frac{B_i^{(0)}}{b+4\pi D_i}\,, \qquad \xi = (a/b)^{1/2}\,, \qquad q_d = (4\pi/a)^{1/2}\,.$$

- Then we can predict "critical region" t $t = \dfrac{T-T_c}{T_C}$

	EuS	EuO	Fe	Ni
C, 10^{-3}	16	5.2	0.13	0.040
t_d	0.31	0.14	0.010	0.0044
f^+, Å	1.8	1.6	0.91	1.27
q_d, Å^{-1}	0.24	0.16	0.045	0.018



Summary of first part

- The critical property of isotropic magnet in nature should be different from Heisenberg fixed point

- Higher order perturbative computation of critical exponents should be done
- Any non-perturbative method? FRG, Monte Carlo, (non-conformal) bootstrap?
- Experiments should be done (dipolar behavior in Fe, Ni? More precise critical exponents?)

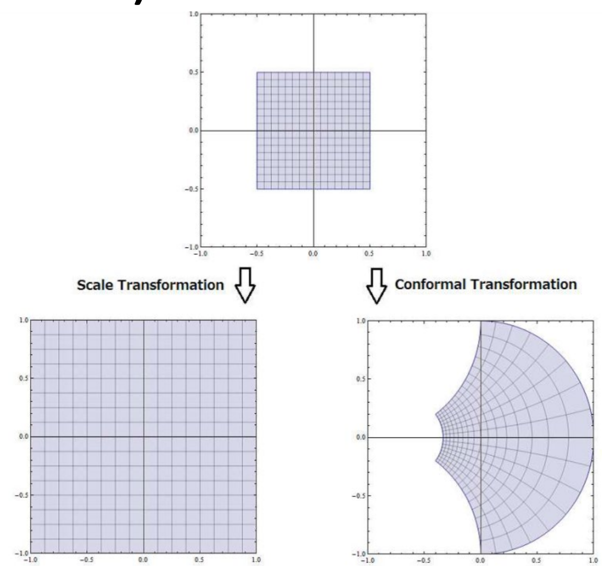
- But the extreme property of dipolar fixed point is scale symmetry without conformal

Scale without Conformal – Is it possible? Yes, if shift-symmetric

Scale or Conformal, that's the question

- It is not a fancy name for scale symmetry!
- It has extra generators called special conformal transformation
- Scale does not automatically mean conformal, but many critical systems are conformal (e.g. Ising, superfluid, algebraic spin liquid etc)

$$\mathcal{P}_\mu = -\partial_\mu$$
$$\mathcal{J}_{\mu\nu} = x_\mu \partial_\nu - x_\nu \partial_\mu$$
$$\mathcal{D} = x^\mu \partial_\mu$$
$$\mathcal{K}_\mu = (2x_\mu x^\rho \partial_\rho - x^2 \partial_\mu)$$



Scale or Conformal, that's the question

- In field theory, there is a good diagnosis
- Consider stress tensor $T_{ij} = T_{ji}\ , \partial^i T_{ij} = 0$
- Scale invariance implies

$$\int d^d x \sigma T^i{}_i = 0 \qquad T^i{}_i = \partial^i V_i$$

- If the trace of the stress tensor is somehow zero, it is conformal

$$T^i{}_i = 0 \qquad k^i = -T^i_j (2 v_k x^k x^j - v^j x^2)$$

$$\partial^i k_i = 0$$

- Scale invariance is weaker (more generic???)

Scale or Conformal, that’s the question

- Scale without conformal needs V in

$$T^i_{\ i} = \partial^i V_i$$

- Scaling dimension of stress tensor is exactly d, so scaling dimension of V must be exactly d-1

- In interacting (non-Gaussian) theories, most operators get anomalous dimensions (unless conserved)
- We do not know any mechanism to protect non-conserved vector operators like V
- Scale invariance without conformal invariance is very unlikely (see e.g. Rychkov’s lecture note on CFT)

First, I show it is not conformal

- A dipolar fixed point cannot be conformal because the magnetization vector is transverse

$$\partial_i \phi_i(x) = q_i \phi_i(q) = 0$$

- Take two-point function

$$\langle \phi_i(x) \phi_j(0) \rangle = \frac{A}{|x|^{2\Delta_\phi}} \left(\delta_{ij} - \alpha \frac{x_i x_j}{x^2} \right) \;, \qquad \alpha = \frac{2\Delta_\phi}{2\Delta_\phi - (d-1)} \,,$$

- Conformal invariance fixes tensor structure from conformal symmetry: it must be $\frac{A}{x^{2\Delta_\phi}} \left(\delta_{ij} - 2 \frac{x_i x_j}{x^2} \right)$
- (equivalently in CFT conserved current must be dimension d-1)
- There is no chance that the dipolar fixed point is conformal

Indeed we have V

- To compute stress tensor, we need unconstrained fields, so we introduce Lagrange multiplier U ("magnetic potential")

$$H = \int d^3x \left(\partial_j \phi_i \partial_j \phi_i + \lambda(\phi_i^2)^2 + U\partial_i \phi_i\right)$$

- Trace of the (renormalized) stress tensor

$$T_i^i = \partial^i V_i + \beta_\lambda (\phi_i^2)^2$$
$$V_i = \Delta_U U \phi_i$$

- Explicit computation shows dimensions of V is protected (after understanding careful renormalization, operator mixing etc) although U and ϕ_i are renormalized
- Does this violate the genericity argument?

Non-renormalization from shift symmetry

- Key observation is Hamiltonian is invariant under the shift of U (constant shift of magnetic potential)

$$H = \int d^3x \left(\partial_j \phi_i \partial_j \phi_i + \lambda(\phi_i^2)^2 + U\partial_i\phi_i\right)$$

$$U \to U + \mathrm{const}$$

- There exists a conserved shift current, which is nothing but magnetization vector

$$\partial^i \phi_i = 0 \qquad Q_U = \int d^{d-1} \Sigma_i \phi_i$$

- Shift charge has the commutation relation

$$i[Q_U, U] = 1 \qquad i[Q_U, U\phi_i] = \phi_i$$

- Comparing scaling dimensions, we have non-renormalization of $V_i = \Delta_U U \phi_i$

$$\Delta_\phi - (d-1) + \Delta_V = \Delta_\phi$$

Shift-symmetry, reflection positivity, and Nambu-Goldstone theorem

- Current for the shift-symmetry has anomalous dimensions, but it protects renormalization of other composite operators
- Shift-symmetry is spontaneously broken

$$\langle [Q_U, U] \rangle = 1$$

- In unitary QFT (reflection positive statistical models) Nambu-Goldstone theorem applies and we have massless (IR free) particles, but not here
- Reflection positivity is violated. Indeed, any interacting shift-symmetric scale invariant model (w/o conformal) must violate reflection positivity

Shift symmetry and scale but non-conformal critical points

- Is this particular to the dipolar fixed point?
- Surprisingly, all the known (but very rare) interacting field theories with scale invariance w/o conformal invariance have (hidden) shift symmetry
  - Fluctuating Membrane theory
  - Higher derivative (interacting) scalar theory
- The shift symmetry is the only known mechanism to protect non-conserved vector operator from acquiring anomalous dimensions

- We may conjecture all the (interacting) scale invariant but non-conformal field theories have a (hidden) shift symmetry

General lesson of the second part

- Generically at interacting (non-Gaussian) RG fixed point, scale invariance implies conformal invariance

- The only (known) exception is if the theory is shift-symmetric

- Generically shift-symmetric interacting RG fixed point is scale invariant but not-conformal invariant

- (I said generic but I do not know exceptions)

Summary

- 2^{nd} order phase transition of isotropic magnet must be reconsidered
- It is not Heisenberg but dipolar fixed point
- It is scale invariant but not conformal
- Holographic realization (c.f. large N O(N) = higher spin gravity)?
- Any application of interacting shift-symmetric field theory in cosmology or asymptotic safety (issue of unitarity)? Derivative constraint?
- Quantum information? Measurement phase transition?