Entanglement Partner and Monogamy in de Sitter Universe

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Based on the paper: Y.Nambu and K.Yamaguchi PRD108, 045002 (2023)

Workshop on Quantum Information, Quantum Matter, and Quantum Gravity@YITP

Sep. 4, 2023

In this talk:

I discuss on entanglement of quantum field in cosmological situation:

primordial quantum fluctuations generated by cosmic inflation

- Introduction
- Entanglement partner in de Sitter universe
- Separability and monogamy in de Sitter universe
- Summary

Introduction

Classical property of primordial fluctuations

$$\hat{\mathcal{R}} = -\frac{H}{\dot{\phi}}\delta\hat{\phi}$$

curvature

- Cosmic inflation provides initial seed fluctuations for structure formation as quantum fluctuations of inflaton fields.
- Usually, quantum expectation value of the fluctuations is adopted as initial value for classical evolution

Assumption of classicality

$\langle\hat{\mathcal{R}}\hat{\mathcal{R}}\rangle$ ➡ $\langle\cdots\rangle$ as statistical average

$\hat{\mathcal{R}}$ ➡ c-number random Gaussian variable

Quantum mechanical variable

- Condition of classicality

Existence of local probability distribution reproducing quantum correlations

Correlations can be represented as local hidden variable (LHV) theory

➡ Bell's inequality is satisfied

For a bipartite system, the condition is equivalent to the state is separable (no bipartite entanglement)

Although we want to prepare classical initial condition for structure formation, seed fluctuation is quantum! We must confirm validity of assumption of classicality.

Entanglement of Local Modes in de Sitter universe

quantum field

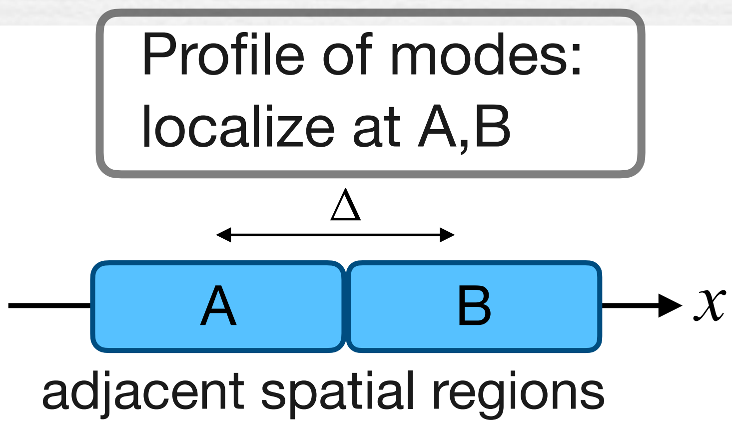
$$\hat{\varphi}(x) = \int_{-\infty}^{\infty} \frac{dk}{\sqrt{2\pi}} \hat{\varphi}_k e^{ikx}, \quad \hat{\varphi}_k = f_k(\eta)\hat{a}_k + f_k^*(\eta)\hat{a}_{-k}^\dagger,$$

$$\hat{\pi}(x) = \int_{-\infty}^{\infty} \frac{dk}{\sqrt{2\pi}} \hat{\pi}_k e^{ikx}, \quad \hat{\pi}_k = (-i)(g_k(\eta)\hat{a}_k - g_k(\eta)^*\hat{a}_{-k}^\dagger),$$

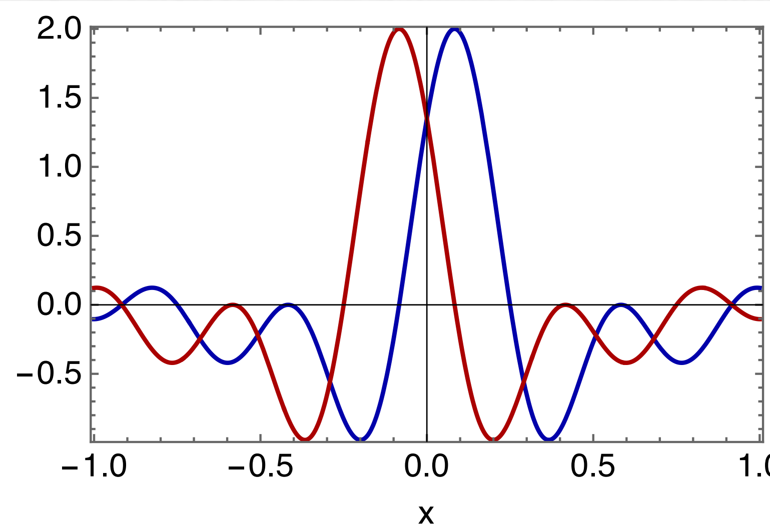
We introduce local modes using a window function

- **Local oscillator modes** $j = \mathrm{A,B}$

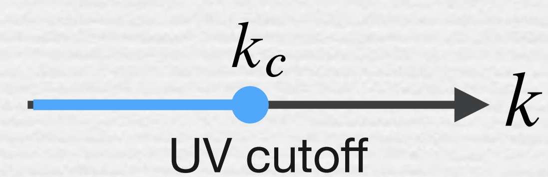
$$\hat{q}_j = \int_{-\infty}^{\infty} dx w_j(x)\hat{\varphi}(x) = \int_{-\infty}^{\infty} dk\, \hat{\varphi}_k\, e^{ikx_j} w_k, \quad \hat{p}_j := \int_{-\infty}^{\infty} dx w_j(x)\hat{\pi}(x) = \int_{-\infty}^{\infty} dk\, \hat{\pi}_k\, e^{ikx_j} w_k,$$



$$\Delta \sim k_c^{-1}$$



$$w_k = w_0\theta(k_c - |k|)$$



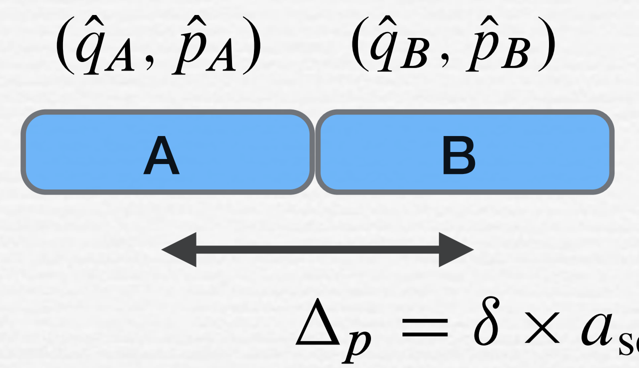
- Bipartite Gaussian state associated with region AB

Covariance matrix

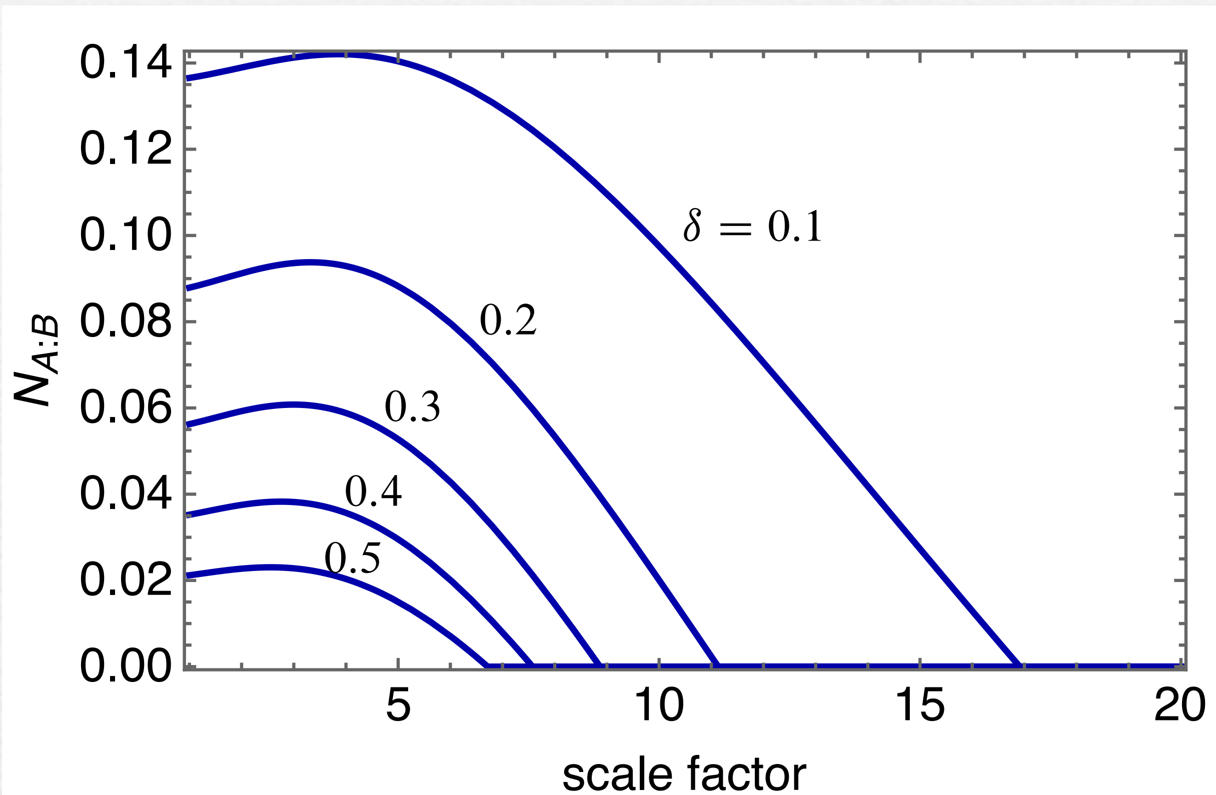
$$\boldsymbol{m}_{\mathrm{AB}} = \begin{bmatrix} a_1 & a_3 & c_1 & c_3 \\ a_3 & a_2 & c_3 & c_2 \\ c_1 & c_3 & a_1 & a_3 \\ c_3 & c_2 & a_3 & a_2 \end{bmatrix}$$

$$c_1(i,j) := \langle\{\hat{q}_i, \hat{q}_j\}\rangle$$
$$c_2(i,j) := \langle\{\hat{p}_i, \hat{p}_j\}\rangle$$
$$c_3(i,j) := \langle\{\hat{q}_i, \hat{p}_j\}\rangle$$

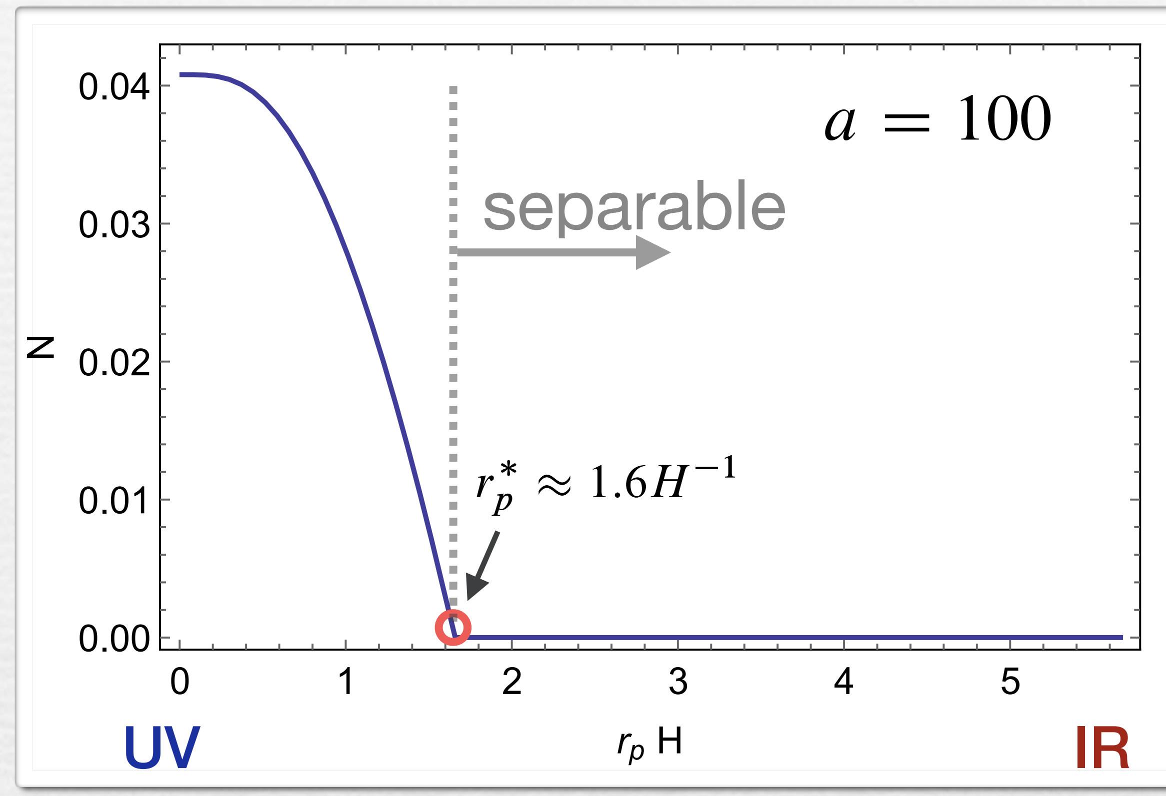
Entanglement negativity



de Sitter background $ds^2 = -dt^2 + e^{2Ht}dx^2$



$\delta =$ size of local regions/size of total universe



coarse graining scale

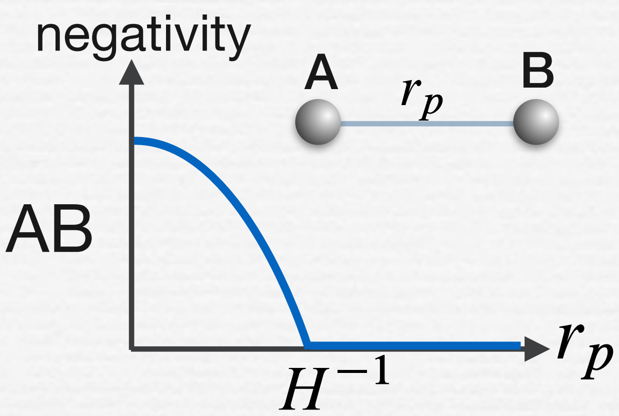
The local modes A and B become separable for distance larger than the Hubble horizon

$$\Delta_p = a_{\rm sc}\delta \sim H^{-1}$$

Y.Nambu 2008,2013

Y.Nambu and Y.Ohsumi 2009,2011

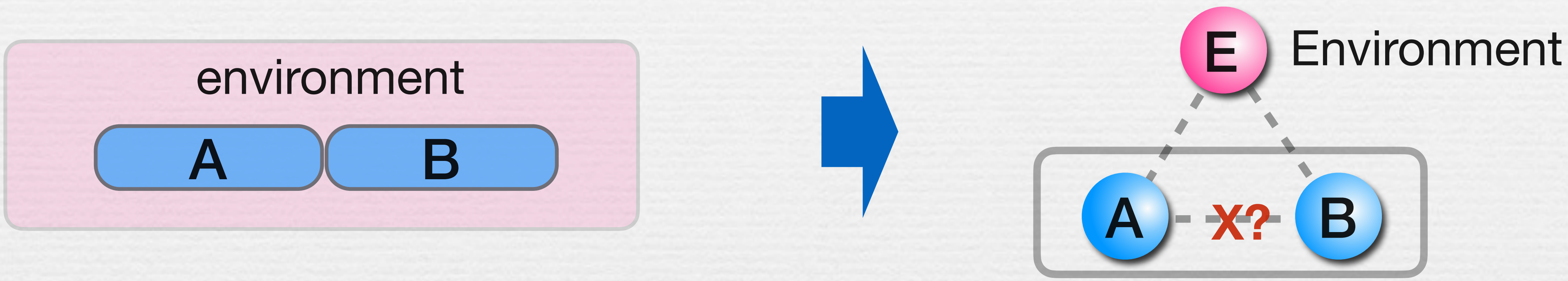
Why the state becomes separable on large scale?



From these observation, we want to understand why the local modes AB become separable for super-horizon scales.

- "Thermal noise" with Gibbons-Hawking temperature $T_{\rm H}=H/2\pi$ destroys quantum correlations

We examine this behavior based on multipartite entanglement



- This separability can be understood based on entanglement monogamy

(sharing and tradeoff relation of entanglement)

We expect that as correlation between AB and E (partner) increases, correlation between A and B decreases

- What is the role of "partner" mode E that purifies AB.

To answer these questions,
we obtain explicit form of the partner of the bipartite system AB.

Partner modes and entanglement

First, we shortly review concept of purification and entanglement partner
(basic concept in QM)

- **Purification**

Let us consider a mode A with mixed state:

$$\hat{\rho}_A = \sum_{n_A} p_{n_A} |n_A\rangle \langle n_A|$$

It is always possible to embed $\hat{\rho}_A$ in a pure state by adding an ancillary mode $\bar{A}$:

$$|A\bar{A}\rangle = \sum_{n} \sqrt{p_n}\, |n_A\rangle\, |n_{\bar{A}}\rangle \qquad \text{pure state}$$

$$\begin{aligned} \mathrm{tr}_{\bar{A}}\left(|A\bar{A}\rangle\langle A\bar{A}|\right) &= \sum_{m,n} \sqrt{p_m p_n}\, |m_A\rangle \langle n_A|\, \mathrm{tr}\left(|m_{\bar{A}}\rangle\langle n_{\bar{A}}|\right) \\ &= \sum_{n} p_n |n_A\rangle \langle n_A| \\ &= \hat{\rho}_A \end{aligned}$$

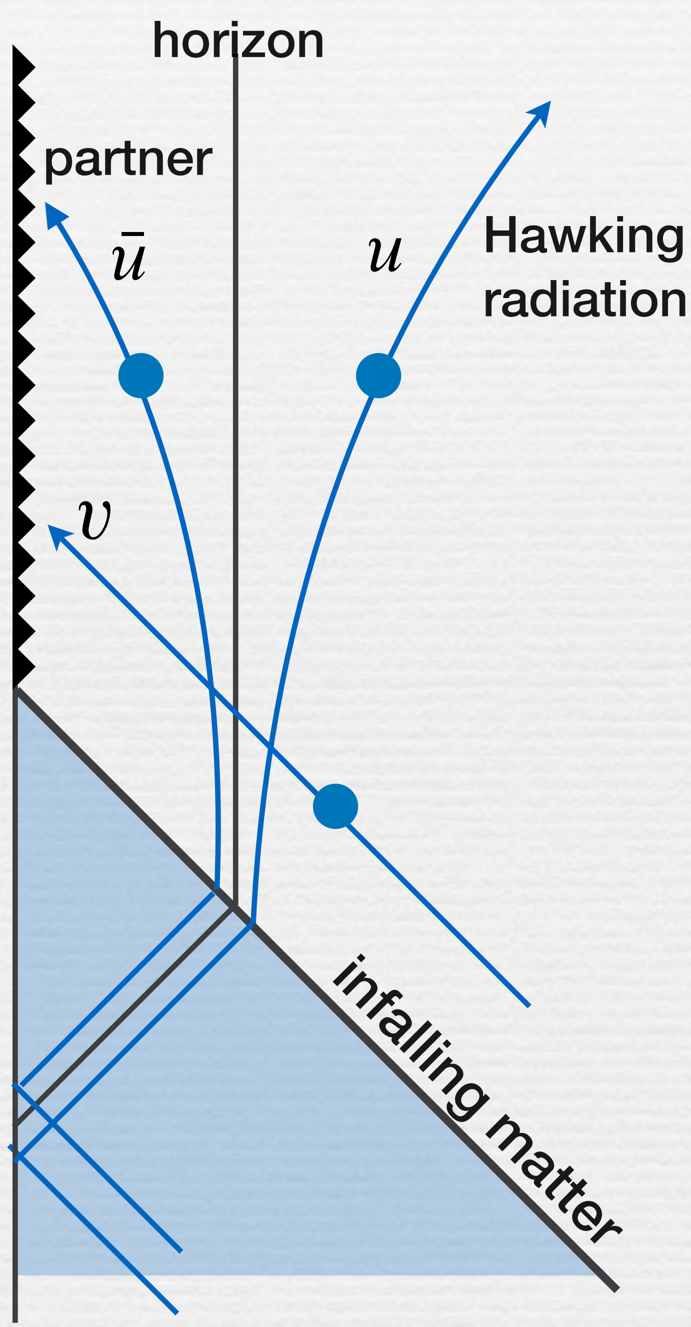
- $\bar{A}$ is purification (entanglement) partner of A
- If A is pure, $A\bar{A}$ is a product state (no correlation between A and $\bar{A}$)

Entanglement of Hawking radiation

It is well known that a partner mode appears in Hawking radiation.

- **Gravitational collapse**

S.W.Hawking 1975



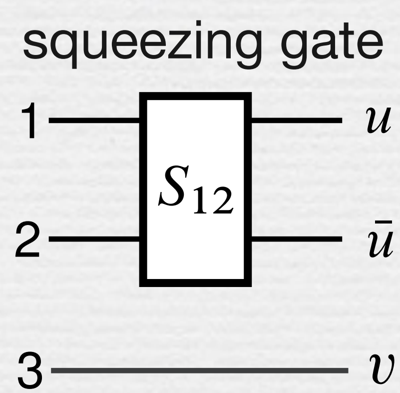
In-vacuum state two mode squeezed sate (entangled state)

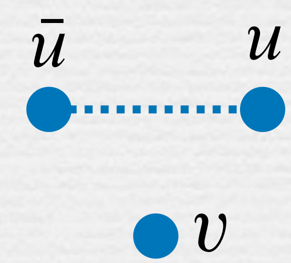
$$|\psi_0\rangle \propto \sum_n e^{-n\omega/(2T_H)} |n\rangle_u |n\rangle_{\bar{u}}$$

Reduced state

$$\hat{\rho}_0 \propto \sum_n e^{-n\omega/T_H} |n_u\rangle \langle n_u|$$

Thermal state with Hawking temperature $T_H = \dfrac{\kappa}{2\pi}$





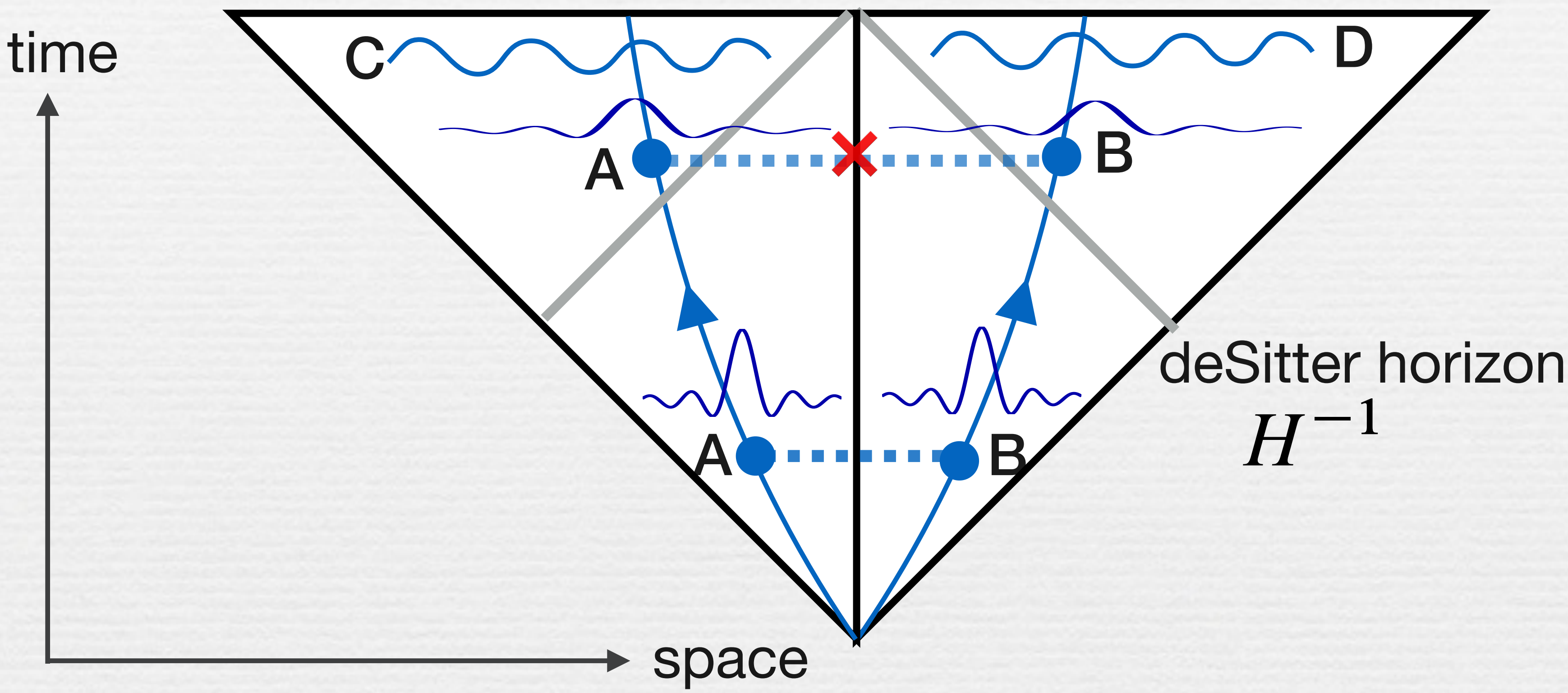
Infalling $\bar{u}$ is a purification partner of u

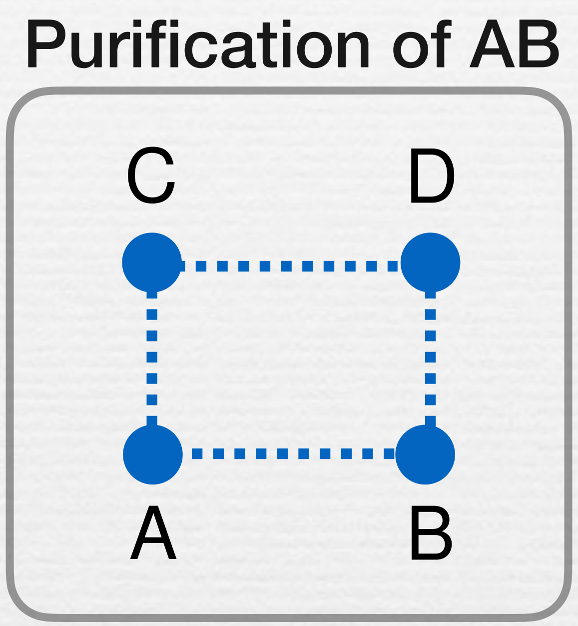
(Hawking radiation)

Origin of $u\,\bar{u}$ is vacuum fluctuation before horizon formation

What is partner mode in de Sitter universe ?

For de Sitter case, we want to identify partner modes for the bipartite system AB, which is defined as two spatially local modes at points A and B.





4-mode pure Gaussian state

- Fact: entanglement between mode A and mode B is lost on super horizon scale
- By embedding the original modes AB in a 4-mode pure Gaussian state, it is possible to understand structure of entanglement sharing of these 4 modes (looking for partner modes of AB)

Purification of Gaussian State

Construction of partner mode

We adopt the method formulated in these papers (partner formula)

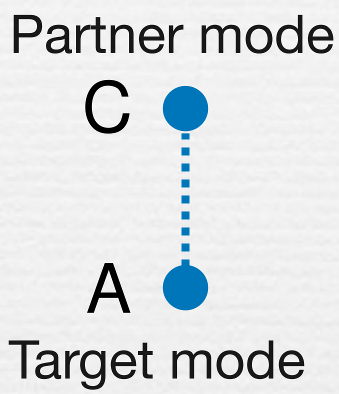
K.Yamaguchi et al. Phys.Lett.A 383 (2019)1255

K.Yamaguchi et al. PRD 101(2020)105009

Y.Nambu and K.Yamaguchi PRD108 (2023) 045002

- **Single mode case: purification of a mixed state mode A**

Covariance matrix of mode A in the standard form (mixed)



$$M_A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}, \quad a \geq 1$$

symplectic eigenvalue

$$\hat{\xi}_A = (\hat{q}_A, \hat{p}_A)$$

$$M_A = \left\langle \{\hat{\xi}_i, \hat{\xi}_j\} \right\rangle$$

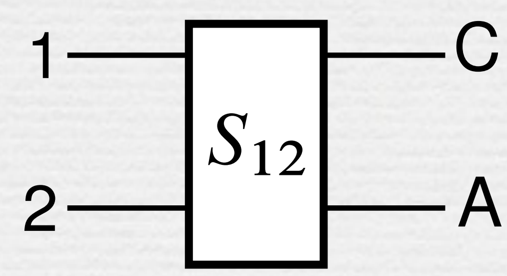
Covariance matrix of total system (pure)

$$M_{AC} = \begin{bmatrix} aI & \sqrt{a^2-1}Z \\ \sqrt{a^2-1}Z & aI \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

This is the standard form of **two mode squeezed state** (pure state)

Partner of A is represented as $\hat{\xi}_C = (\hat{q}_C, \hat{p}_C)$

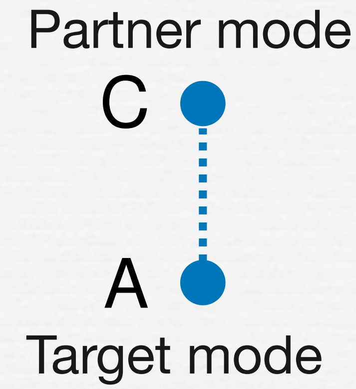
$a = \cosh r$



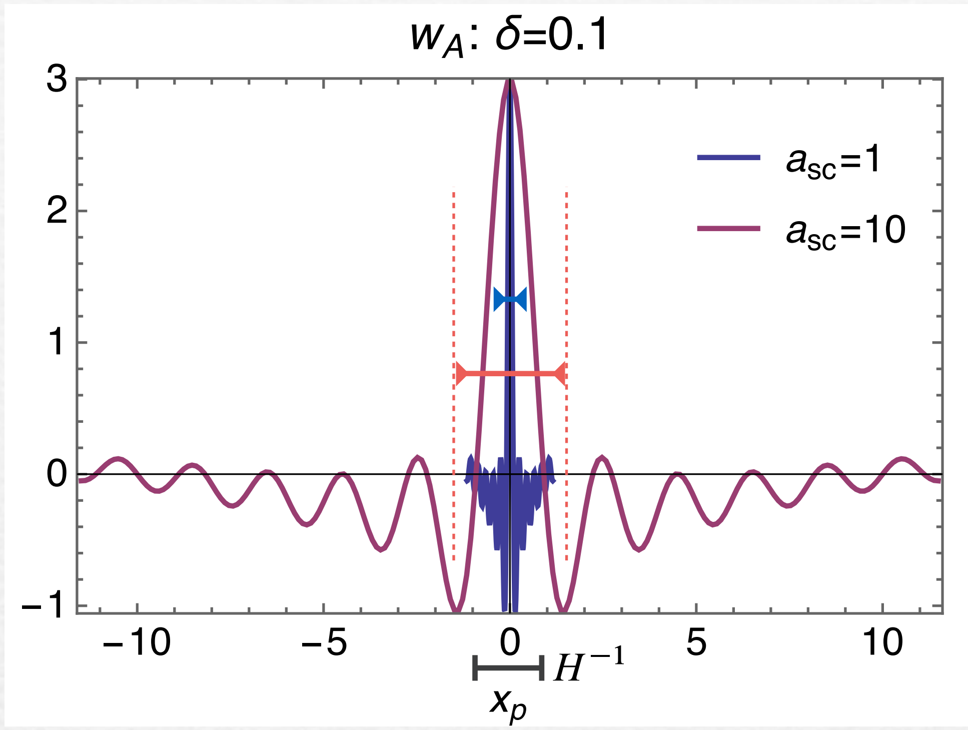
We can obtain spatial profile of the partner mode C :

$$\hat{q}_c = \int dx (w_q(x)\hat{\phi}(x) + w_p(x)\hat{\pi}(x))$$

Spatial profile of mode A and its partner C

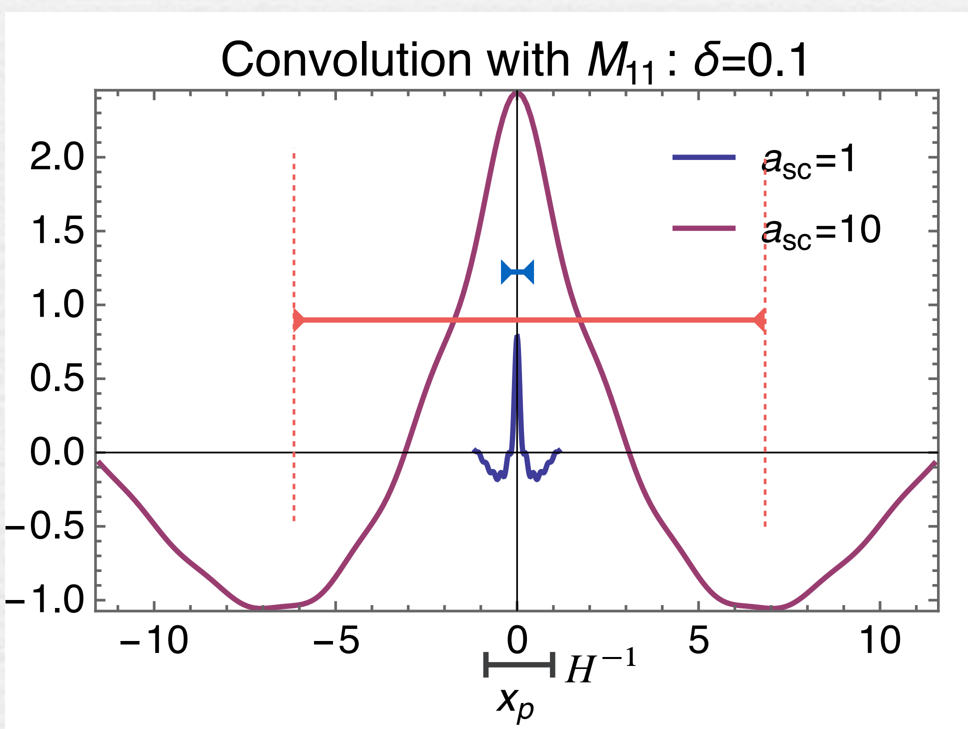


Mode A



Width of mode A increases
as $\delta \times a_{\rm sc}$ (proportional to scale factor)

Mode C



Width of the partner mode C increases
more rapidly compared to the target mode A,
its profile spreads over super-horizon scale

For large size of region A $\delta_p > H^{-1}$, its partner mode spreads
over super horizon scale (de-localize)

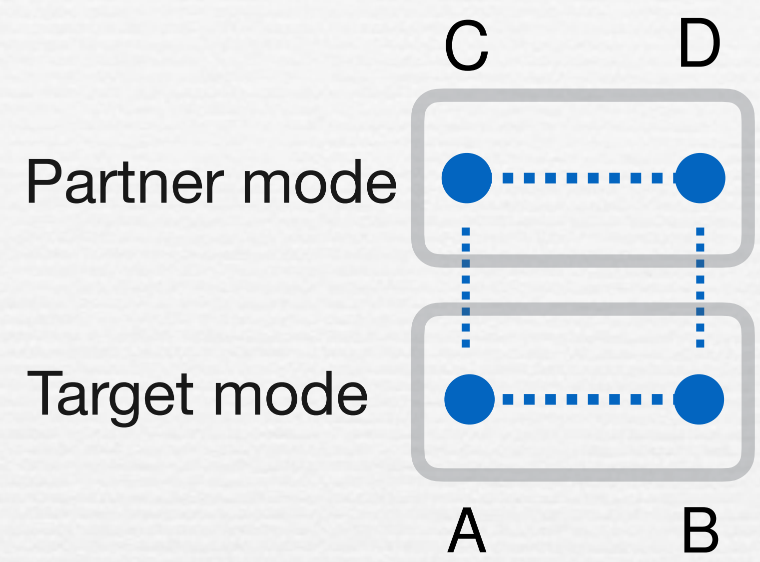
increase of S_A

• Two mode case: purification of symmetric bipartite state AB

Covariance matrix of target mode AB
(we assume standard form of symmetric bipartite state)

$$M_{AB} = \begin{array}{c} \begin{array}{cc} \quad A \quad & \quad B \quad \end{array} \\ \begin{bmatrix} a & 0 & d_1 & 0 \\ 0 & a & 0 & d_2 \\ d_1 & 0 & a & 0 \\ 0 & d_2 & 0 & a \end{bmatrix} \end{array} \begin{array}{c} A \\ \\ B \end{array}$$

3 parameters



Covariance matrix of total 4 mode system (pure state)

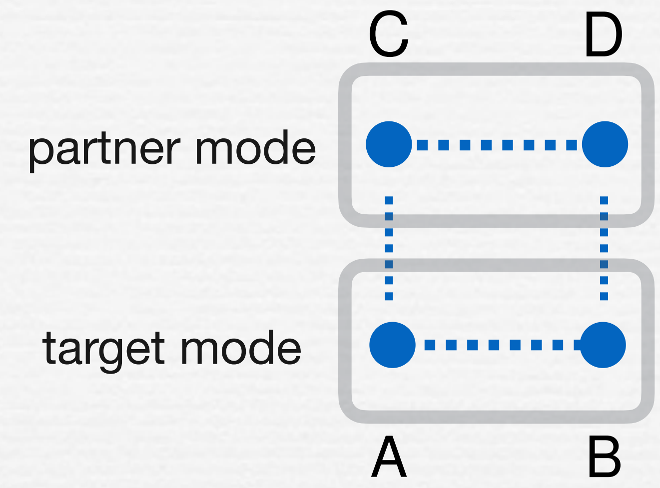
$$M_{ABCD} = \begin{array}{c} \begin{array}{cc} \qquad AB \qquad & \qquad CD \qquad \end{array} \\ \begin{bmatrix} a & 0 & d_1 & 0 & g & 0 & h & 0 \\ 0 & a & 0 & d_2 & 0 & -g & 0 & -h \\ & & a & 0 & h & 0 & g & 0 \\ & & 0 & a & 0 & -h & 0 & -g \\ & & & & a & 0 & d_2 & 0 \\ & & & & 0 & a & 0 & d_1 \\ & & & & & & a & 0 \\ & & & & & & 0 & a \end{bmatrix} \end{array} \begin{array}{c} AB \\ \\ CD \end{array}$$

$$g = \frac{1}{2}(\sqrt{x+y} + \sqrt{x-y}),$$

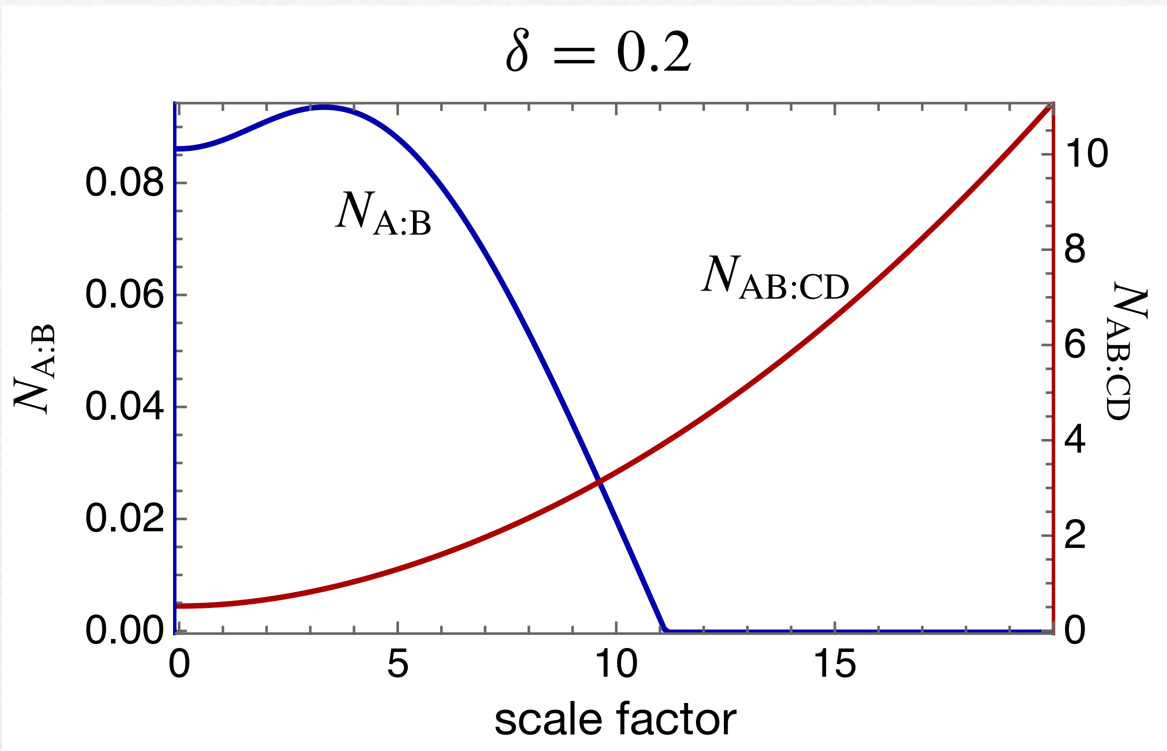
$$h = \frac{1}{2}(\sqrt{x+y} - \sqrt{x-y}),$$

$$x = a^2 + d_1 d_2 - 1, \quad y = a(d_1 + d_2).$$

Behavior of entanglement



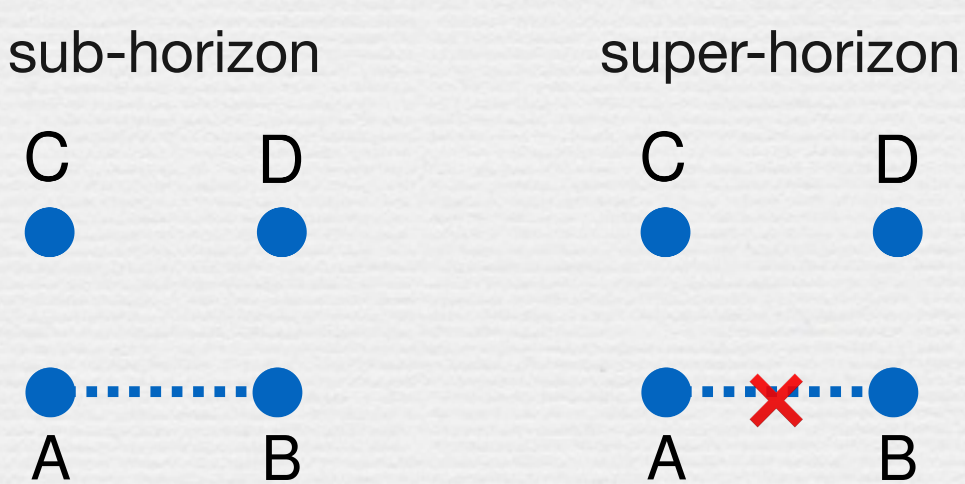
As we obtained the explicit form of 4-mode state ABCD, we can calculate behavior of negativity associated with this state:



$N_{A:B}$

$N_{AB:CD}$

- Target modes AB become separable for super horizon scale
- Entanglement between AB and CD increases monotonically as universe expands
- Trade-off relation between N(A:B) and N(AB:CD) (monogamy property of entanglement)

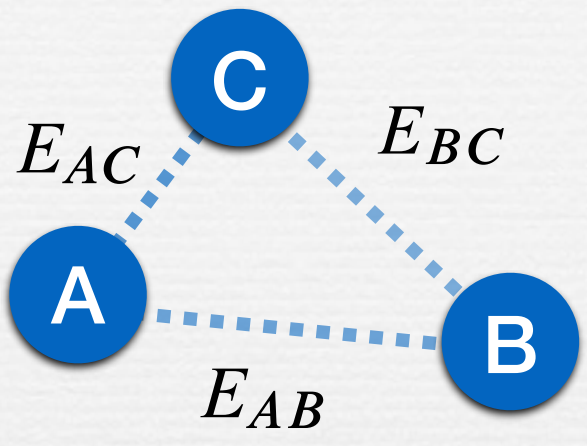


We investigate structure of monogamy for this system in more detail

Monogamy and Separability

Entanglement Monogamy

Basic properties of entanglement shared by multiple modes



E : entanglement measure

qubit system:

square of concurrence, negativity

Gaussian system:

square of negativity

Monogamy relation of entanglement

$$E_{\mathrm{C:A}} + E_{\mathrm{C:B}} \leq E_{\mathrm{C:AB}}$$

V.Coffman, J.Kundu, W.K.Wootters 1998

T.J.Osbone, F.Verstraete 2006

G.Adesso, F.Illuminati 2006

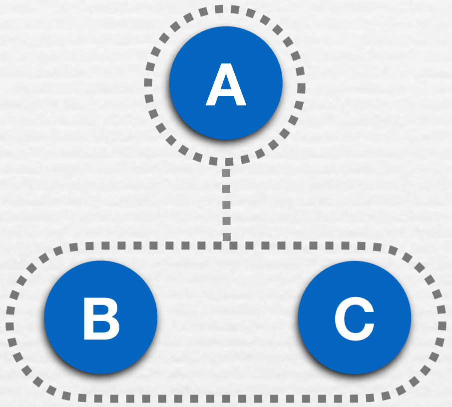
Trade-off relation between $E_{\mathrm{C:A}}$ and $E_{\mathrm{C:B}}$

- Universal relation characterizing multi-partite entanglement
- This relation may provide upper bound of $E_{\mathrm{C:A}}$ and $E_{\mathrm{C:B}}$
- Sharing of quantum information, fidelity of quantum teleportation, etc

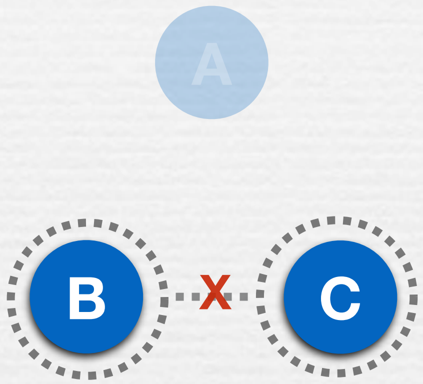
Monogamy and Separability

Is it possible to say something about emergence of separable state just applying this monogamy inequality?

$$|\text{GHZ}\rangle = \frac{1}{\sqrt{2}}(|111\rangle + |000\rangle)$$



$\mathcal{N}(\text{A} : \text{BC}) = 1/2$



$\mathcal{N}(\text{B} : \text{C}) = 0$

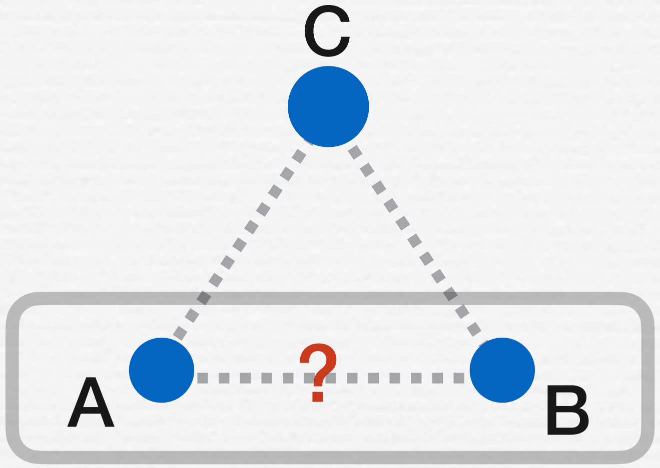
Monogamy inequalities

$$\mathcal{N}^2_{BC} + \mathcal{N}^2_{AB} \leq \mathcal{N}^2_{B|CA}$$

$$\mathcal{N}^2_{AB} + \mathcal{N}^2_{AC} \leq \mathcal{N}^2_{A|BC}$$

These inequalities are trivially satisfied for GHZ state and do not provide any useful information on entanglement sharing and relation between separability and strength of entanglement.

Standard monogamy relation:



target bipartite mode

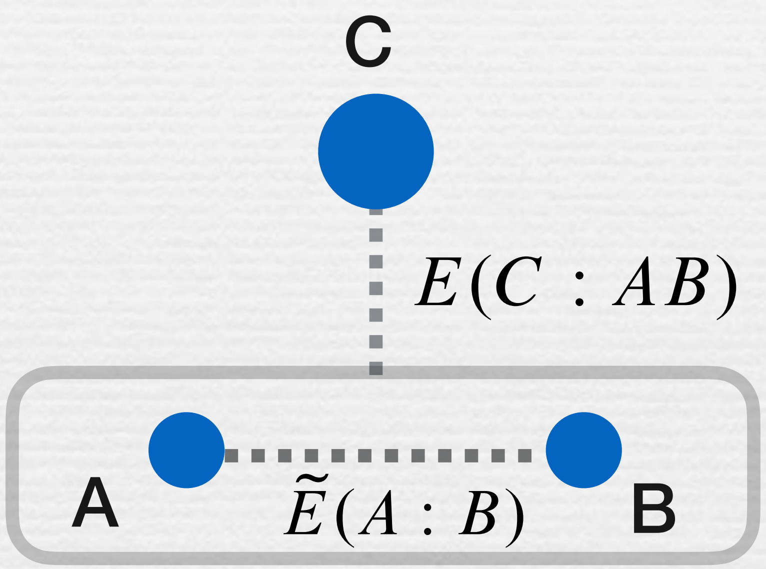
Conventional monogamy inequality:

$$\mathcal{N}^2_{C|A} + \mathcal{N}^2_{C|B} \leq \mathcal{N}^2_{C|AB}$$

This inequality cannot bound strength of correlation between A and B as a function of correlation between C and AB

A new type of monogamy inequality:

S. Camalet 2017



target bipartite mode

$$\tilde{E}(A:B) + E(C:AB) \leq \tilde{E}_{\max}$$

internal ent. external ent.

$\tilde{E}, E$: correlation measures equipped with appropriate conditions

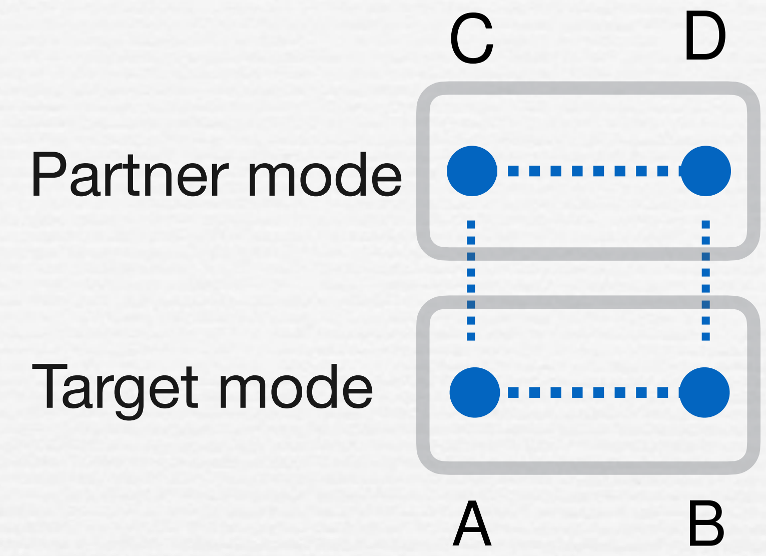
Finite dimensional Hilbert space

- For maximally entangled system AB, $\tilde{E}(A:B) = \tilde{E}_{\max}$ and this inequality says $E(C:AB) = 0$
- This inequality shows trade-off relation between "internal" and "external" entanglement

We have shown this monogamy inequality indeed holds for Gaussian systems

 Y.Nambu and K.Yamaguchi PRD108, 045002 (2023)

Bipartite Gaussian state AB



Covariance matrix of symmetric bipartite state AB:

3 parameters

$$\boldsymbol{m}'_{\mathrm{AB}} = \begin{bmatrix} a & 0 & d_1 & 0 \\ 0 & a & 0 & d_2 \\ d_1 & 0 & a & 0 \\ 0 & d_2 & 0 & a \end{bmatrix} = \boldsymbol{S}\,\boldsymbol{m}_{\mathrm{AB}}\boldsymbol{S}^T$$

$$x = a^2 + d_1 d_2 - 1, \quad y = a(d_1 + d_2)$$

For convenience, we adopt parametrization with a, x, y

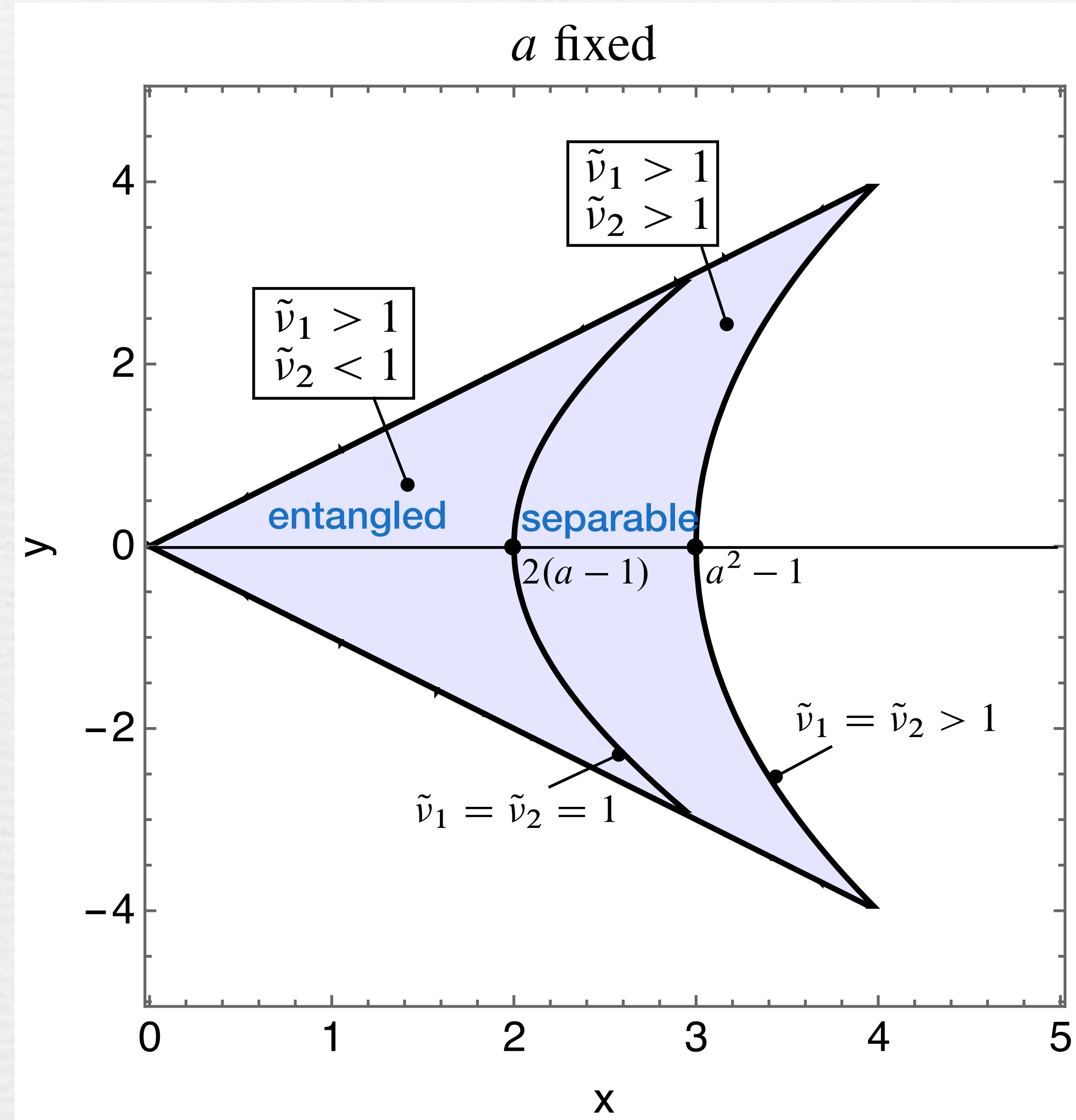
For fixed value of the parameter a, physical states are located in a bounded region in (x,y)-plane

$$\nu_1^2 = (a + d_1)(a + d_2) = x + y + 1 \geq 1$$

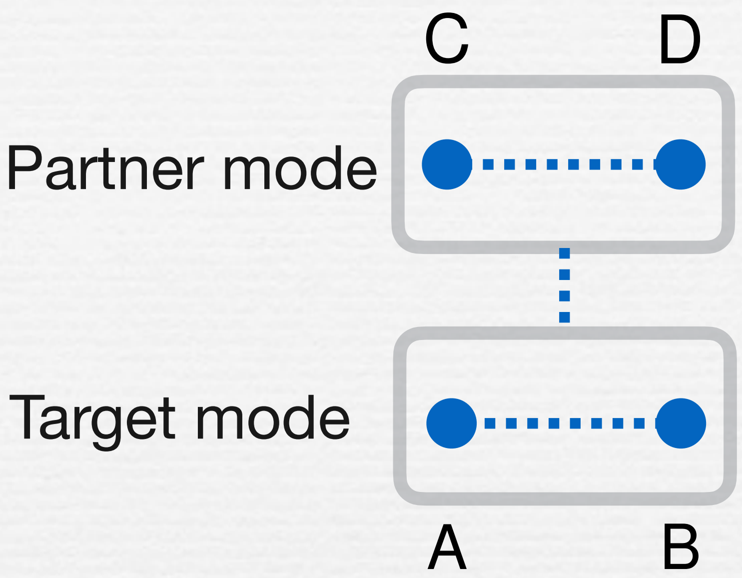
$$\nu_2^2 = (a - d_1)(a - d_2) = x - y + 1 \geq 1,$$

$$\tilde{\nu}_1^2 = (a + d_1)(a - d_2) = 2a^2 - x - 1 + \sqrt{y^2 - 4a^2(x - a^2 + 1)},$$

$$\tilde{\nu}_2^2 = (a - d_1)(a + d_2) = 2a^2 - x - 1 - \sqrt{y^2 - 4a^2(x - a^2 + 1)}.$$



Monogamy for Bipartite Gaussian state



We have embedded a symmetric bipartite state AB in a pure 4 mode Gaussian state ABCD

Internal entanglement: $N_{\mathrm{A:B}}$

External entanglement: $N_{\mathrm{AB:CD}}$

Covariance matrix

$$
\begin{array}{cc}
\begin{array}{cc} \mathrm{AB} & \mathrm{CD} \end{array} & \\
M_{ABCD} = \begin{bmatrix}
a & 0 & d_1 & 0 & g & 0 & h & 0 \\
0 & a & 0 & d_2 & 0 & -g & 0 & -h \\
 & & a & 0 & h & 0 & g & 0 \\
 & & 0 & a & 0 & -h & 0 & -g \\
 & & & & a & 0 & d_2 & 0 \\
 & & & & 0 & a & 0 & d_1 \\
 & & & & & & a & 0 \\
 & & & & & & 0 & a
\end{bmatrix} & \begin{array}{c} \mathrm{AB} \\ \\ \mathrm{CD} \end{array}
\end{array}
$$

$$g = \frac{1}{2}(\sqrt{x+y} + \sqrt{x-y}),$$

$$h = \frac{1}{2}(\sqrt{x+y} - \sqrt{x-y}),$$

$$x = a^2 + d_1 d_2 - 1, \quad y = a(d_1 + d_2).$$

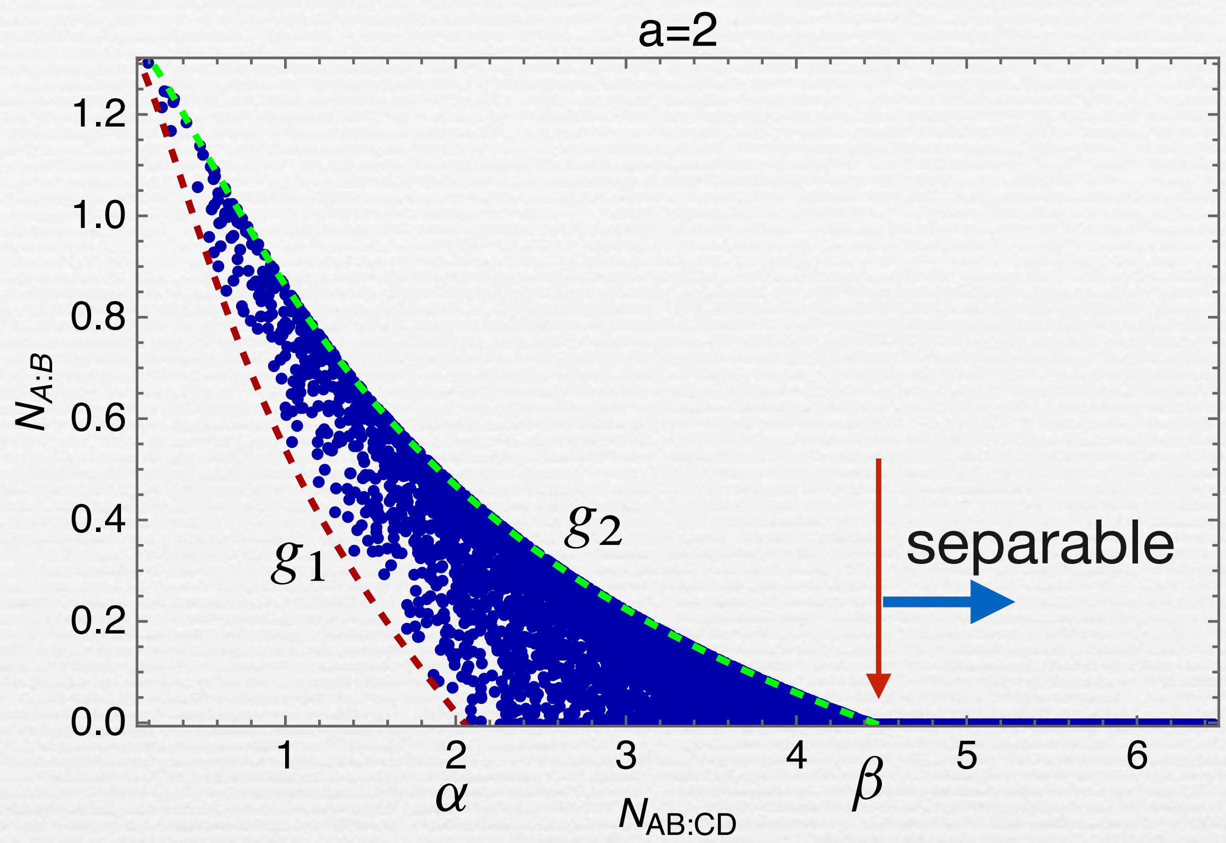
Negativity

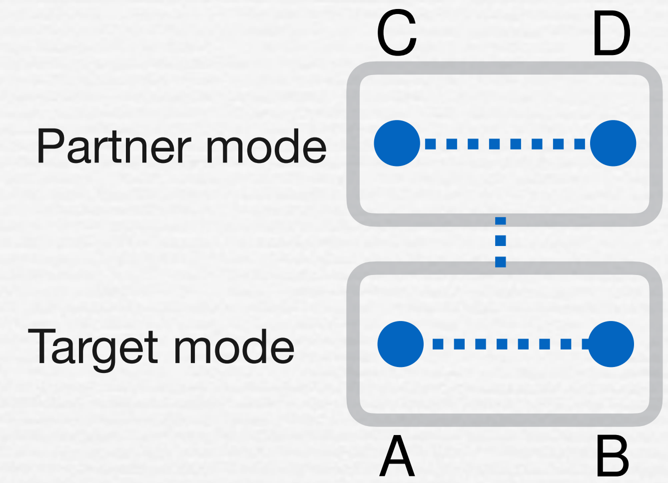
$$N_{\mathrm{A:B}}(x, y, a) = \frac{1}{2}\left(\frac{1}{\tilde{\nu}_2} - 1\right),$$

$$\tilde{\nu}_2^2 = 2a^2 - x - 1 - \sqrt{y^2 - 4a^2(x - a^2 + 1)}.$$

$$N_{\mathrm{AB:CD}}(x, y) = \frac{1}{2}\left(\sqrt{x+y+1} + \sqrt{x+y}\right)\left(\sqrt{x-y+1} + \sqrt{x-y}\right) - \frac{1}{2}.$$

Randomly generated symmetric bipartite Gaussian States





$N_{A:B}$ is bounded by $g_2(N_{AB:CD})$

$$g_2 = \frac{1}{2}\left(-1+\left(-1+\left(a-\sqrt{a^2-\frac{(N_{\mathrm{AB:CD}}+1)^2}{2N_{\mathrm{AB:CD}}+1}}\right)^2\right)^{-1/2}\right),$$

$$\beta = -\frac{1}{2}+\left(\sqrt{a-1/2}+\sqrt{a-1}\right)^2.$$

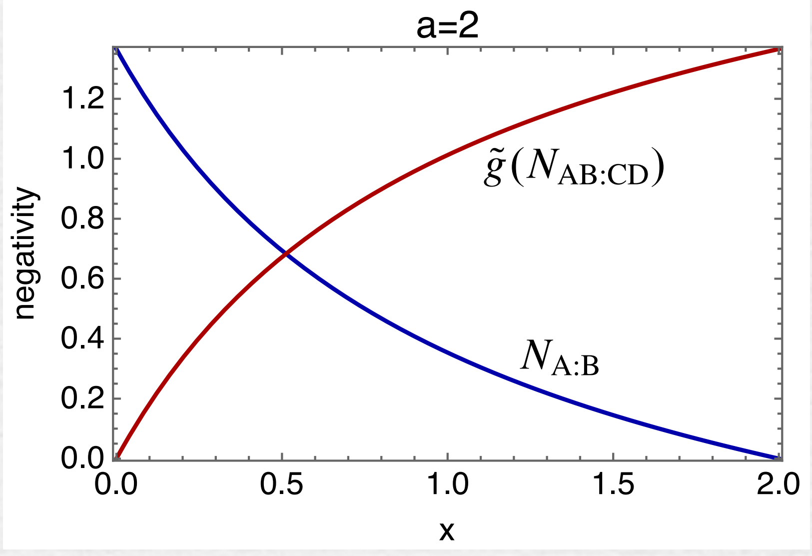
Monogamy relation between "internal" and "external" quantum correlations for bipartite Gaussian states

$$\boxed{N_{\mathrm{A:B}} \leq g_2(N_{\mathrm{AB:CD}}, a)}$$

or

$$\boxed{\begin{aligned} &N_{\mathrm{A:B}} + \tilde{g}(N_{\mathrm{AB:CD}}, a) \leq N_{\mathrm{A:B}}|_{\max} \\ &\tilde{g}(N_{\mathrm{AB:CD}}, a) := N_{\mathrm{A:B}}|_{\max} - g_2(N_{\mathrm{AB:CD}}, a) \end{aligned}}$$

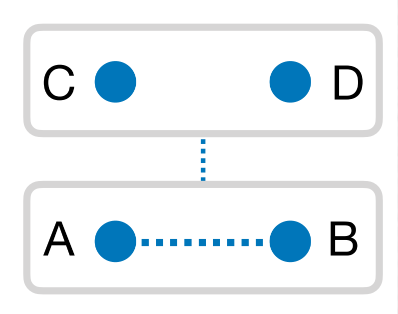
Trade-off relation between $N_{A:B}$ and $N_{AB:CD}$



Local modes in de Sitter universe

We can confirm that the trade-off relation between $N_{A:B}$ and $N_{AB:CD}$ holds:

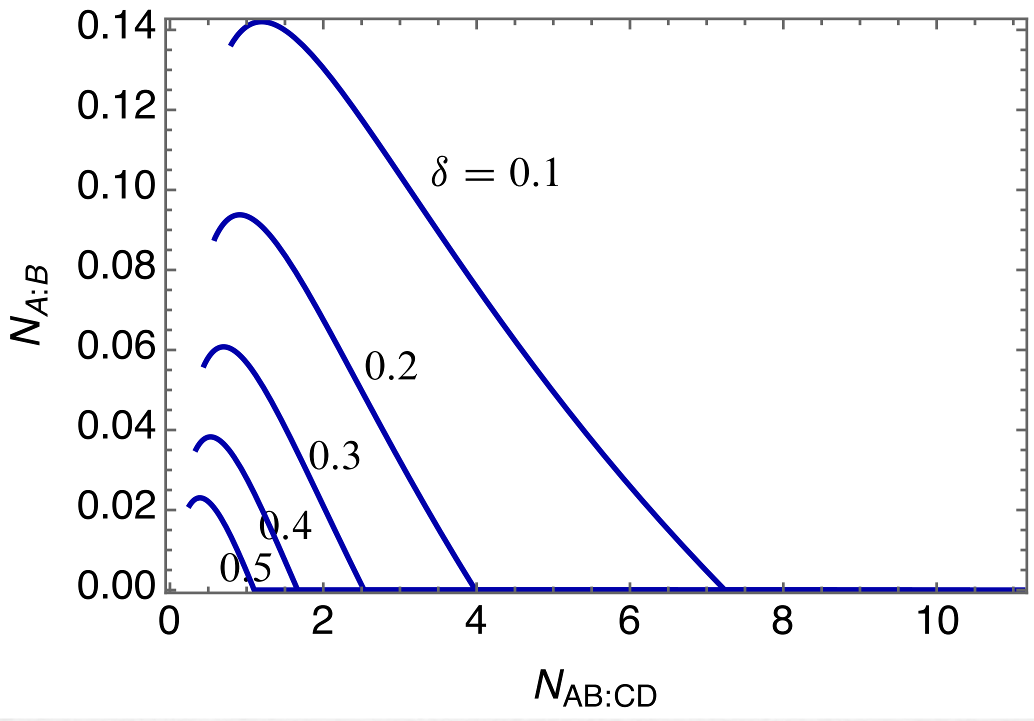
$$N_{\mathrm{A:B}}(a_{\mathrm{sc}},\delta) \le g_2(N_{\mathrm{AB:CD}}(a_{\mathrm{sc}},\delta), a_{\mathrm{sc}},\delta)$$

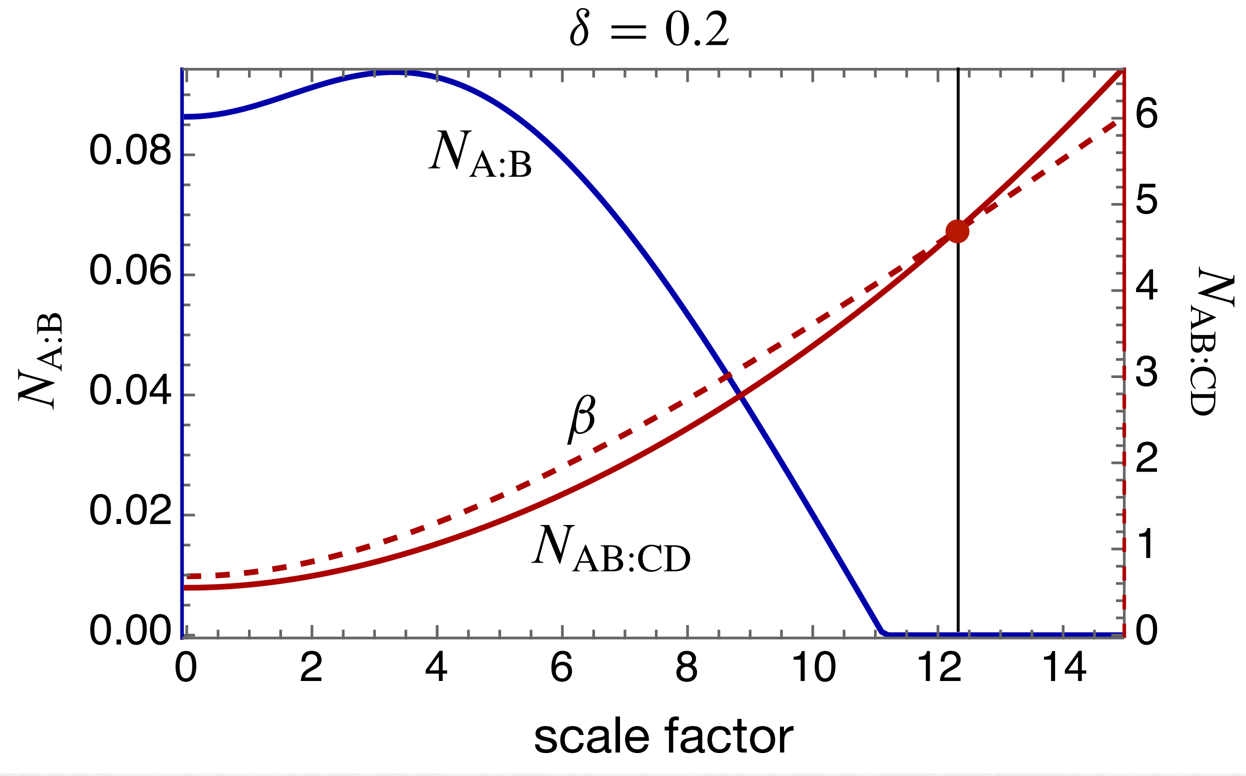


If $N_{AB:CD} > \beta$ ➡ $N_{A:B}=0$ (separable)

When $N_{AB:CD}$ reaches a "maximal" value which satisfies $g_2(N_{AB:CD})=0$, $N_{A:B}=0$ and AB becomes separable

Monogamy relation provides a sufficient condition of separability





Summary

Structure of entanglement in de Sitter universe

- Entanglement of bipartite system associated with spatial regions
  - Two modes AB is becomes separable for super horizon scale
  - This behavior can be understood from monogamous property by introducing partner modes
  - We have proved monogamy relation for 4-mode Gaussian states, and explained emergence of separability of bipartite system AB from the perspective of trade-off between “internal” and “external” entanglement

- Aspect of information
  - Spatial profile of partner modes de-localize over super horizon scale
  - Information scrambling through partner modes
  - Entropy bound in deSitter universe

Back Up

Construction of partner mode and QIC

- As already introduced, we define local mode of the quantum field using window function, which determines spatial profile of the modes.
- "Information" of the local mode at x_j can be measured by Unruh-DeWitt type detectors:

$$\hat{O}_j = \int d^3x \left(\underbrace{w_1(x, x_j)}\hat{\varphi}(x) + \underbrace{w_2(x, x_j)}\hat{\pi}(x)\right) \qquad H_{\text{int}} = \lambda \hat{\mu}(x) \otimes \sum_j \hat{O}_j$$

window functions

- For given set of operators $\{\hat{O}_j\}$ $(j = 1, \cdots, 2k)$, it is possible to identify 2k independent modes

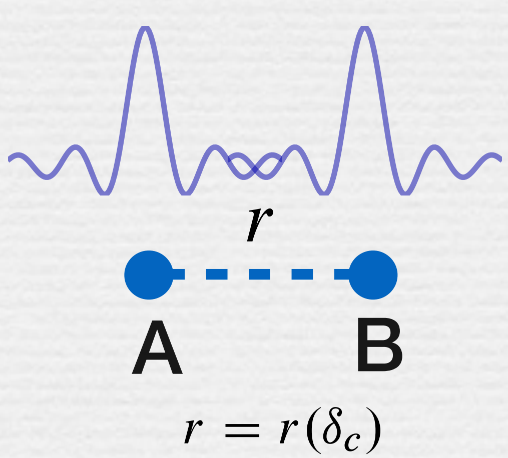
(1) $\hat{\xi}_j = (\hat{q}_j, \hat{p}_j),\ j = 1, \cdots k \qquad [\hat{q}_j, \hat{p}_j] = i\delta_{kj}$

(2) Modes $\{\hat{\xi}_j\}$ constitute a pure state (provides a method of purification)

QIC : quantum information capsule (by M.Hotta and K.Yamaguchi)

Information associated with original mode $\{\hat{O}_j\}$ is completely contained in QIC (purification of target modes)

target modes: localize at A,B



We adopt the method formulated in these papers

J.Trevision et al. PTEP 2018, 103A03
K.Yamaguchi et al. Phys.Lett.A 383 (2019)1255
K.Yamaguchi et al. PRD 101(2020)105009

Construction of partner mode

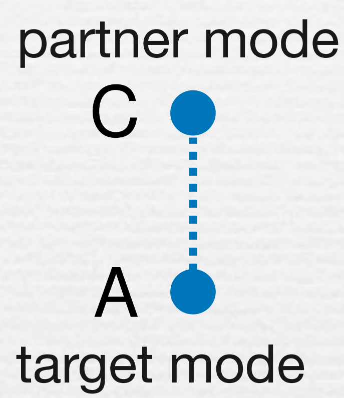
As a demonstration, we first consider purification of local mode A in de Sitter space

- Single mode case

Using coarse-grained field, we assign canonical operator at a spatial point x_1

$$\hat{\xi}_A = (\hat{q}_1, \hat{p}_1)^T \qquad \hat{q}_1 = \int_{-\infty}^{\infty} dk W_k (f_k \hat{a}_k + f_k^* \hat{a}_{-k}{}^{\dagger}) e^{ikx_1},$$

$$[\hat{q}_1, \hat{p}_1] = i \qquad \hat{p}_1 = \int_{-\infty}^{\infty} dk W_k (-i)(g_k \hat{a}_k - g_k^* \hat{a}_{-k}{}^{\dagger}) e^{ikx_1},$$



We define the following linear map for operators:

$$\boxed{f_\psi(\hat{a}_k) = -i\hat{a}_k, \quad f_\psi(\hat{a}_k^{\dagger}) = i\hat{a}_k^{\dagger}.}$$

$$f_\psi(\hat{q}_1) := \int dk W_k (-i)(f_k \hat{a}_k - f_k^* \hat{a}_{-k}{}^{\dagger}) e^{ikx_1},$$

$$f_\psi(\hat{p}_1) := -\int dk W_k (g_k \hat{a}_k + g_k^* \hat{a}_{-k}{}^{\dagger}) e^{ikx_1}.$$

Thus we introduced 2-mode system characterized by the following 4 operators:

$$\hat{q}_1, \hat{p}_1, f_\psi(\hat{q}_1), f_\psi(\hat{p}_1).$$

The map f preserves commutation relation and covariance:

target mode: $\hat{\xi}_i = (\hat{q}_1, \hat{p}_1)^T \qquad \hat{\xi}_i,\ f_\psi(\hat{\xi}_i)$

commutators $[\hat{\xi}_i, \hat{\xi}_j] = [f_\psi(\hat{\xi}_i), f_\psi(\hat{\xi}_j)] = i(J)_{ij}, \quad [\hat{\xi}_i, f_\psi(\hat{\xi}_j)] = i(m)_{ij} = ia\delta_{ij}.$

covariance $\left\langle \{\hat{\xi}_i, \hat{\xi}_j\} \right\rangle = \left\langle \{f_\psi(\hat{\xi}_i), f_\psi(\hat{\xi}_j)\} \right\rangle = (m)_{ij}, \quad \left\langle \{\hat{\xi}_i, f_\psi(\hat{\xi}_j)\} \right\rangle = -(J)_{ij}$

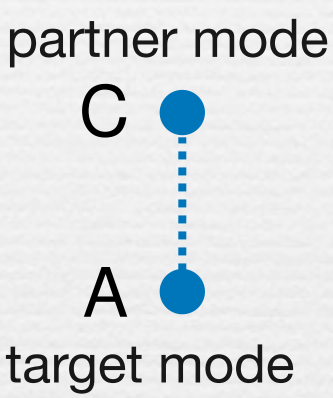
covariance matrix of target mode

$$\langle \hat{q}_1^2 \rangle = \langle \hat{p}_1^2 \rangle = a/2 \qquad m = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} = aI, \quad J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

By re-defining combination of 4 ops., it is possible to identify independent two mode

$$\boxed{\begin{gathered} \underbrace{\hat{\xi}_A}_{\text{target mode}} = \hat{\xi}, \quad \underbrace{\hat{\xi}_C}_{\text{partner mode}} = \frac{1}{\sqrt{a^2-1}} X(f_\psi(\hat{\xi}) - mJ\hat{\xi}) \\ [\hat{\xi}_A, \hat{\xi}_A] = iJ, \quad [\hat{\xi}_C, \hat{\xi}_C] = iJ, \quad [\hat{\xi}_A, \hat{\xi}_C] = 0 \end{gathered}}$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



Covariance of total 2 mode system (pure)

$$M_{AC} = \begin{bmatrix} aI & \sqrt{a^2-1}Z \\ \sqrt{a^2-1}Z & aI \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

This is the standard form of two mode squeezed state (pure)

Partner of A is represented as $\hat{\xi}_C = (\hat{q}_C, \hat{p}_C)$

As we will see, this method provides spatial profile of partner mode