Krylov complexity in the IP matrix model

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[arXiv:2306.04805]

[arXiv:2308.07567]

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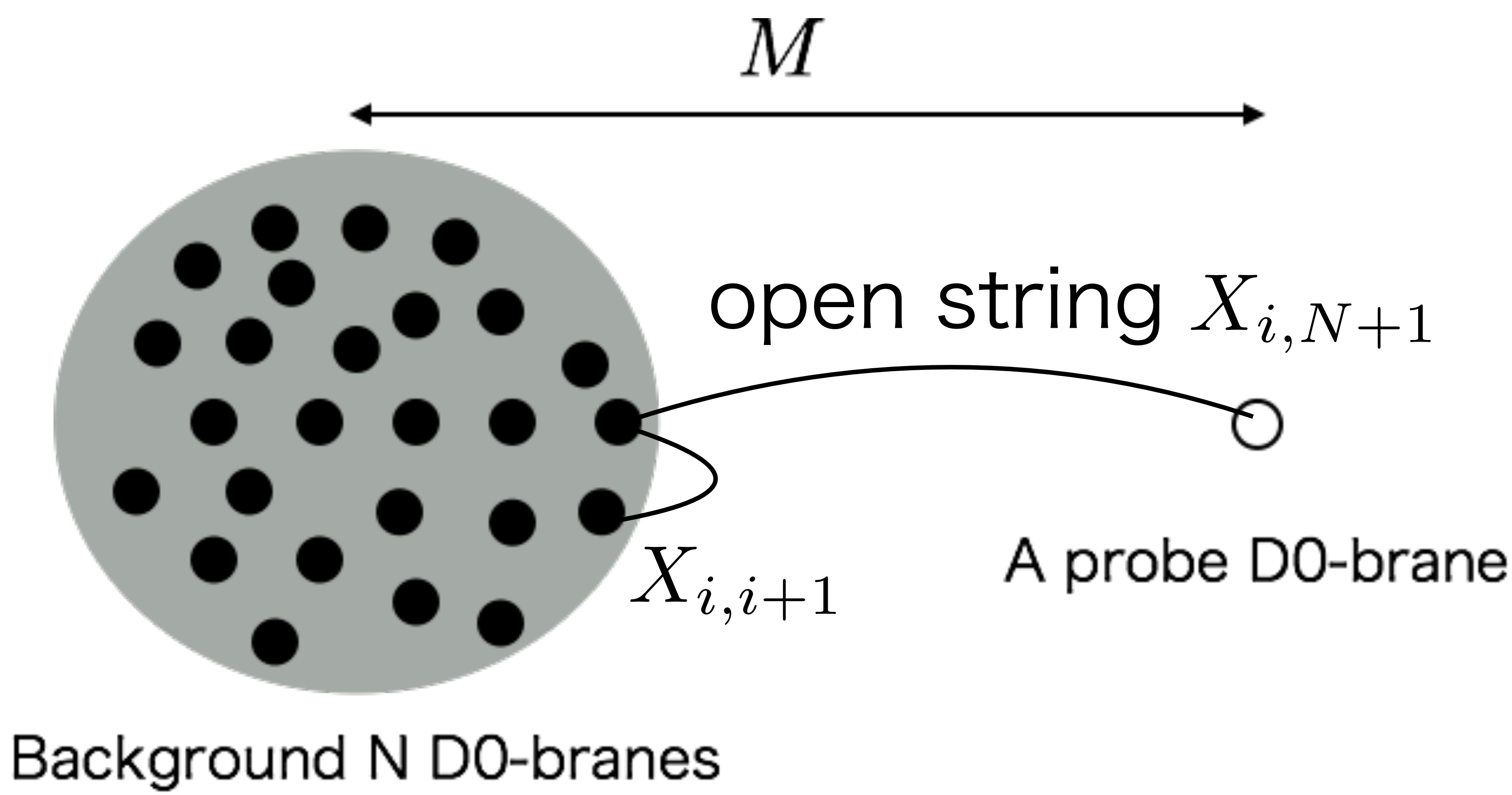
Matrix model for black hole

[T. Banks, W. Fischler, S.H. Shenker, L. Susskind, 1996],
[N. Iizuka, D. Kabat, G. Lifschytz, D. A. Lowe, 2001], ⋯



N D0-brane's positions are X_{ii} .

A probe D0-brane's position is $X_{N+1,N+1} = M$.

Matrix models are toy models
for the gauge theory dual of black hole.

Iizuka-Polchinski (IP) matrix model is a simple large N matrix model

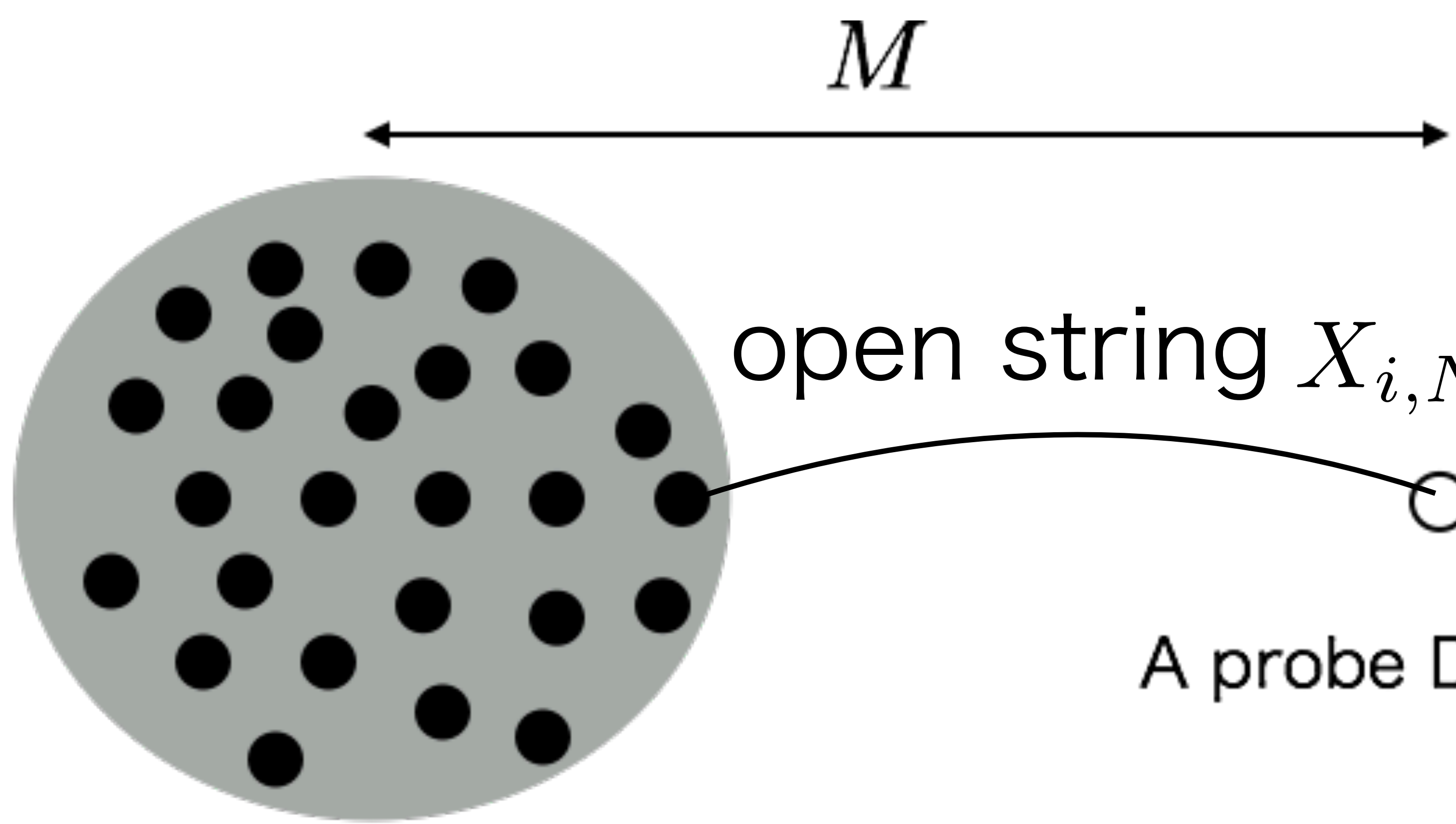
[N. Iizuka, J. Polchinski, 2008]

Interaction

$$H = \frac{1}{2}\mathrm{Tr}(\Pi^2) + \frac{m^2}{2}\mathrm{Tr}(X^2) + M(a^\dagger a + \bar{a}^\dagger \bar{a}) + \underline{g(a^\dagger X a + \bar{a}^\dagger X^T \bar{a})}$$

$a_i^\dagger, \bar{a}_i^\dagger$: creation operators for complex vector ϕ_i

$$\begin{array}{c|cc} & N & 1 \\ \hline N & X_{ij} & \phi_i \\ 1 & \phi_j^\dagger & M \end{array}$$



Background N D0-branes X_{ij}

Effective mass m

Our interest

Quantum chaos and Krylov complexity in the IP model

- Krylov complexity has been proposed as a new measure for quantum chaos.
  [D. E. Parker, X. Cao, A. Avdoshkin, T. Scaffidi, E. Altman, 2018]

- It has been conjectured that Krylov complexity grows exponentially in non-integrable systems with large DoF.

- Krylov complexity in large N SYK grows exponentially.
  What about the IP model?

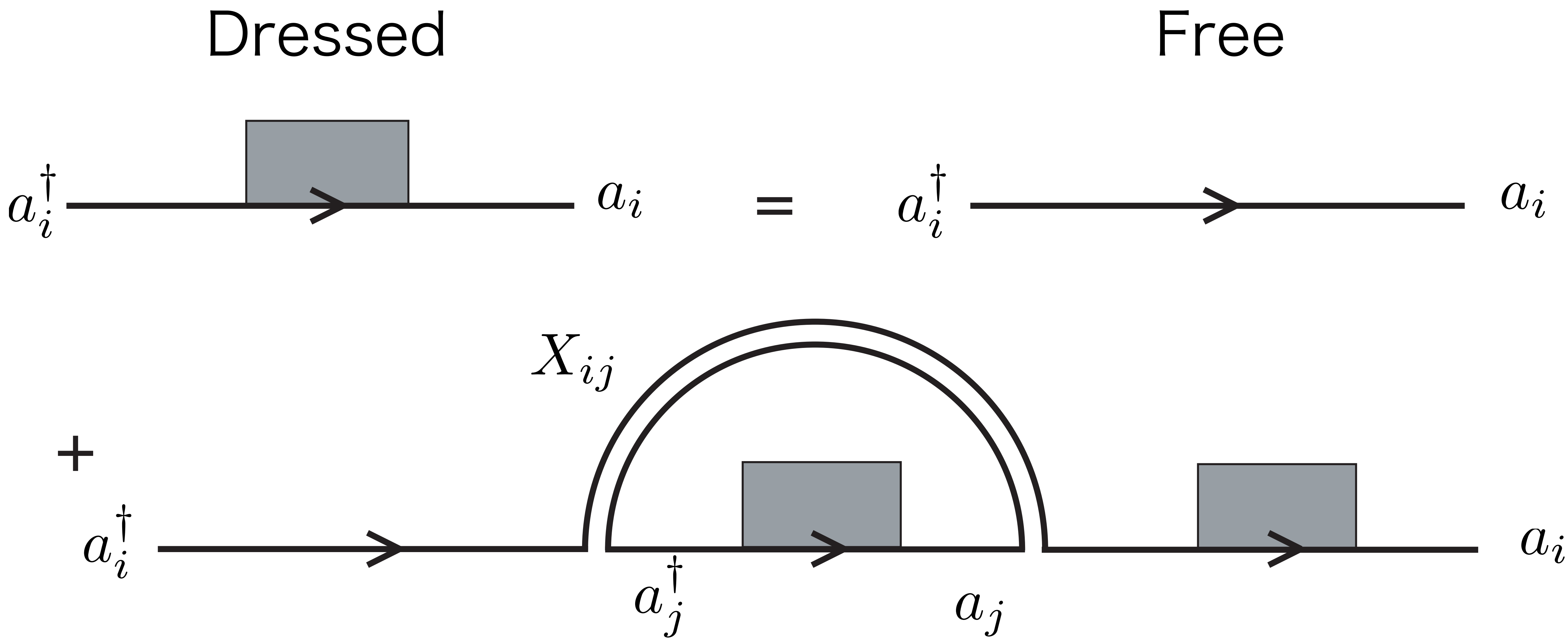
Our result

- In the IP model with nonzero mass m at nonzero T, Krylov complexity grows exponentially $K(t) \sim e^{\mathcal{O}(\sqrt{t})}$.

- This exponential growth implies that the IP model is chaotic.

- As far as we know, this is the first example of exponential growth found in large N matrix models.

Spectrum of the IP model can be solved by Schwinger-Dyson equation



2-pt function $\delta_{ij} G(T, t-t') := e^{iM(t-t')} \left\langle \mathrm{T}\, a_i(t)\, a_j^\dagger(t') \right\rangle_T$

$H_{int} = g a^\dagger X a$ fixed t' Hooft coupling $\lambda = g^2 N$

Large N, Large M

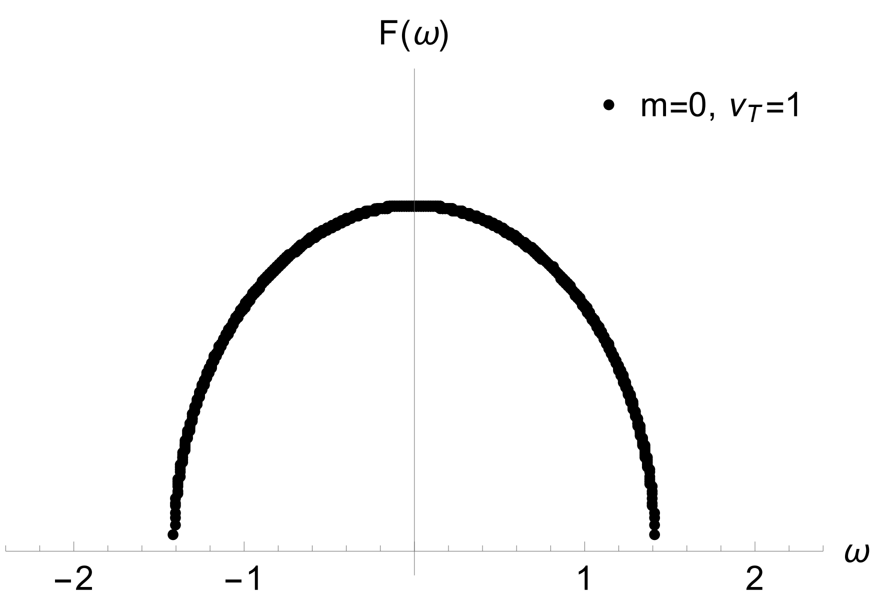
Three types of power spectrum

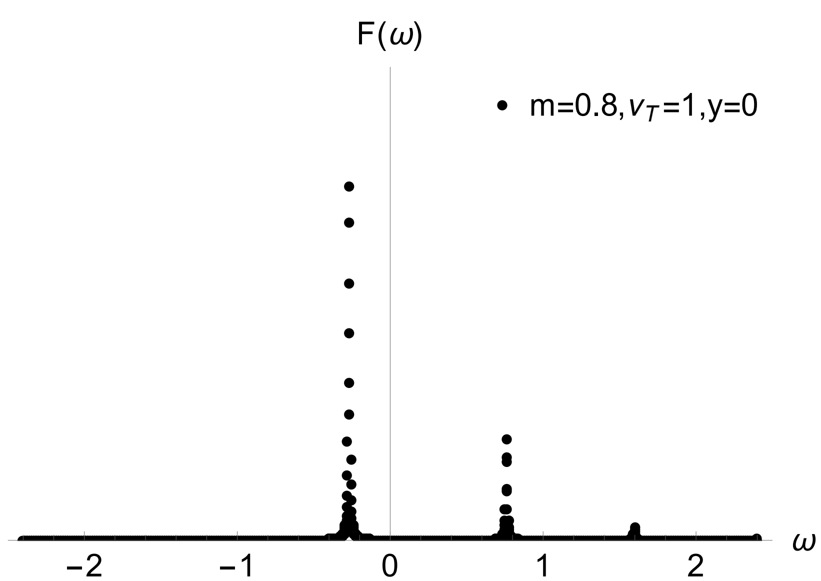
[N. Iizuka, J. Polchinski, 2008]

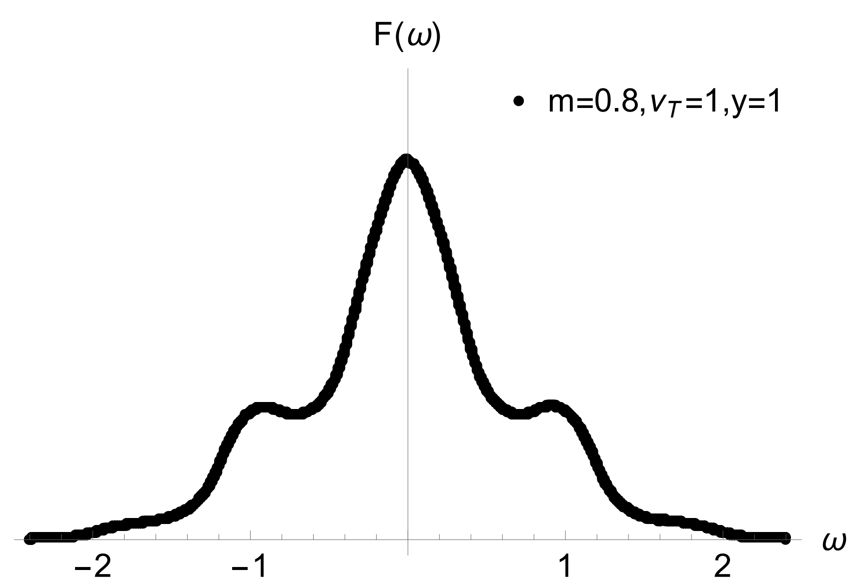
SD eq
$$\tilde{G}(T,\omega-m) - \frac{4}{\nu_T^2}\frac{1}{\tilde{G}(T,\omega)} + e^{-m/T}\tilde{G}(T,\omega+m) = \frac{4i\omega}{\nu_T^2}$$

m : mass T : temperature $\nu_T^2 := \dfrac{2\lambda}{m(1-e^{-m/T})}$

$m=0$	$m\neq 0, T=0$	$m \neq 0, T=\infty$
Continuous bounded	Discrete	Continuous unbounded







Lanczos coefficients a_n, b_n and Krylov complexity $K(t)$

$$\mathcal{O}(t) = e^{iHt}\mathcal{O}e^{-iHt} = \sum_{n=0} \frac{(it)^n}{n!}\mathcal{L}^n\mathcal{O} \qquad \mathcal{L}\mathcal{O} := [H, \mathcal{O}]$$

$$|\mathcal{O}_{-1}) := 0,\ |\mathcal{O}_0) := |\mathcal{O}),\ \mathcal{L}|\mathcal{O}_n) = a_n|\mathcal{O}_n) + b_n|\mathcal{O}_{n-1}) + b_{n+1}|\mathcal{O}_{n+1})$$

$$(\mathcal{O}_m|\mathcal{O}_n) = \delta_{mn} \quad \mathcal{O}(t) = \sum_{n=0} i^n\varphi_n(t)\mathcal{O}_n$$

Krylov complexity $K(t) := \sum_n n|\varphi_n(t)|^2$

[D. E. Parker, X. Cao, A. Avdoshkin, T. Scaffidi, E. Altman, 2018]

Krylov complexity is a measure for operator growth of the initial operator $\mathcal{O}(0)$.

Lanczos coefficients a_n, b_n and Krylov complexity $K(t)$

- They depend on the initial operator and inner product and can be computed from the power spectrum.

- Lanczos coefficient b_n of non-integrable systems in the thermodynamic limit would grow as fast as possible.

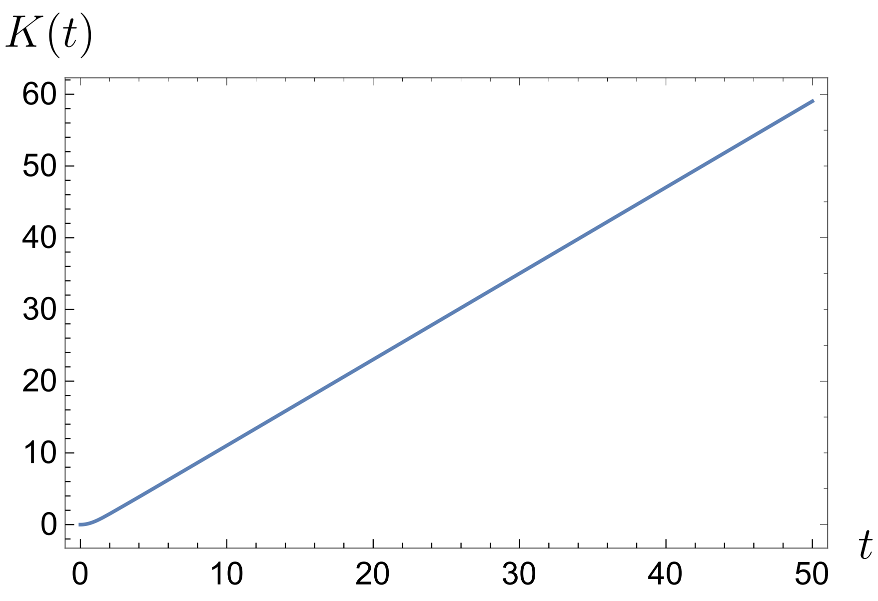
- Krylov complexity $K(t)$ would grow exponentially.

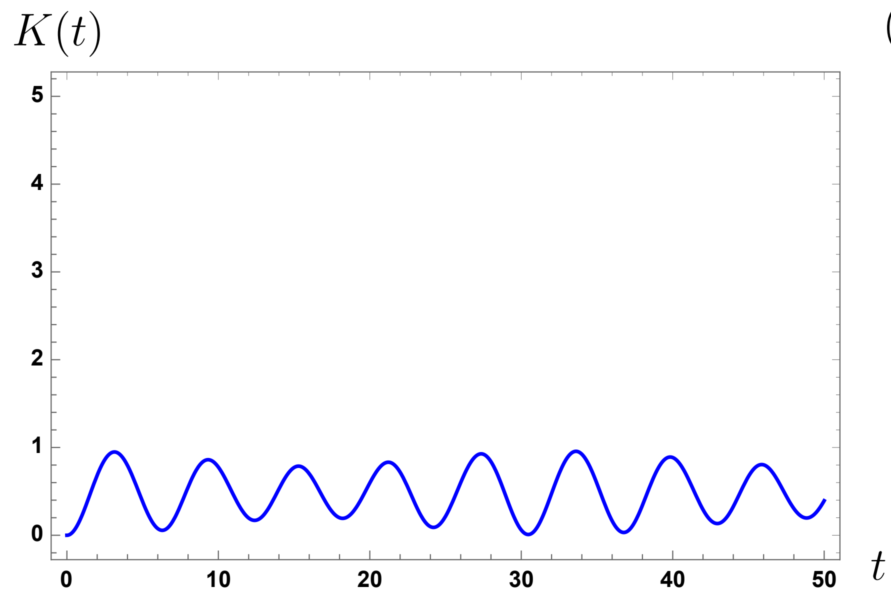
- We analyze $a_n, b_n, K(t)$ associated to 2-pt function $C(T,t) := e^{iM(t-t')} \Big\langle a_i(t)\, a_j^\dagger(0) \Big\rangle_T$.

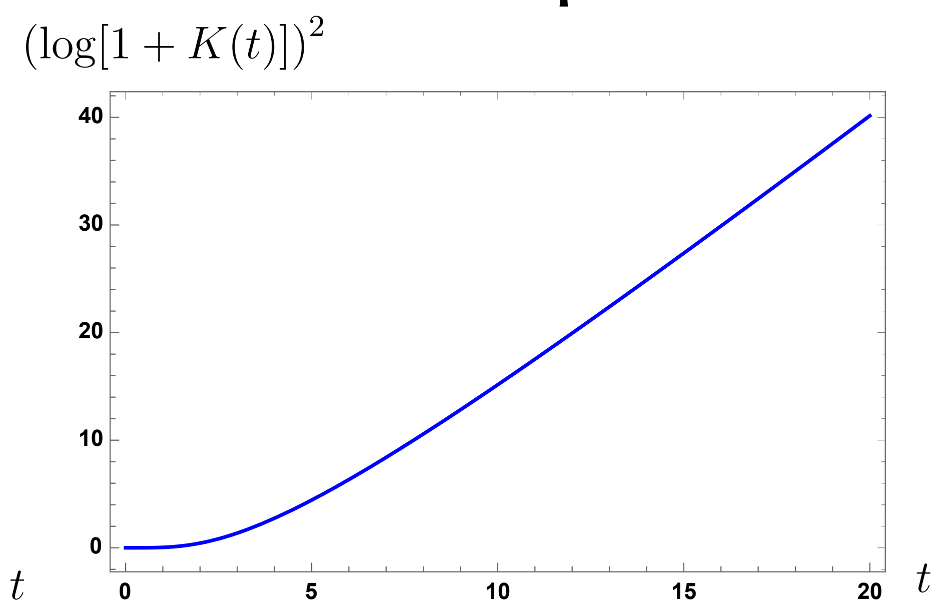
Krylov complexity in the IP model

[N. Iizuka, M.N., 2023]

$m = 0$	$m \neq 0, T = 0$	$m \neq 0, T = \infty$
Continuous bounded spectrum	Discrete spectrum	Continuous unbounded spectrum







Linear growth	Oscillation	Exponential growth
$K(t) \sim \nu_T t$		$K(t) \sim e^{\sqrt{m\pi t}}$

With nonzero mass m at nonzero T ,
Krylov complexity grows exponentially.

a_n, b_n with $m \neq 0, T = \infty$

Asymptotic behavior of the spectrum

$$F(\omega) \sim \exp\left[-\frac{2|\omega|}{m}\log\left(\frac{2|\omega|}{\nu_T}\right)\right] \quad (|\omega| \to \infty)$$

exponential decay with log correction

$$b_n \sim \frac{m\pi n}{4W(2m\pi n/\nu_T)} \sim \frac{m\pi n}{4\log n} \quad (n \to \infty)$$

linear growth with log correction

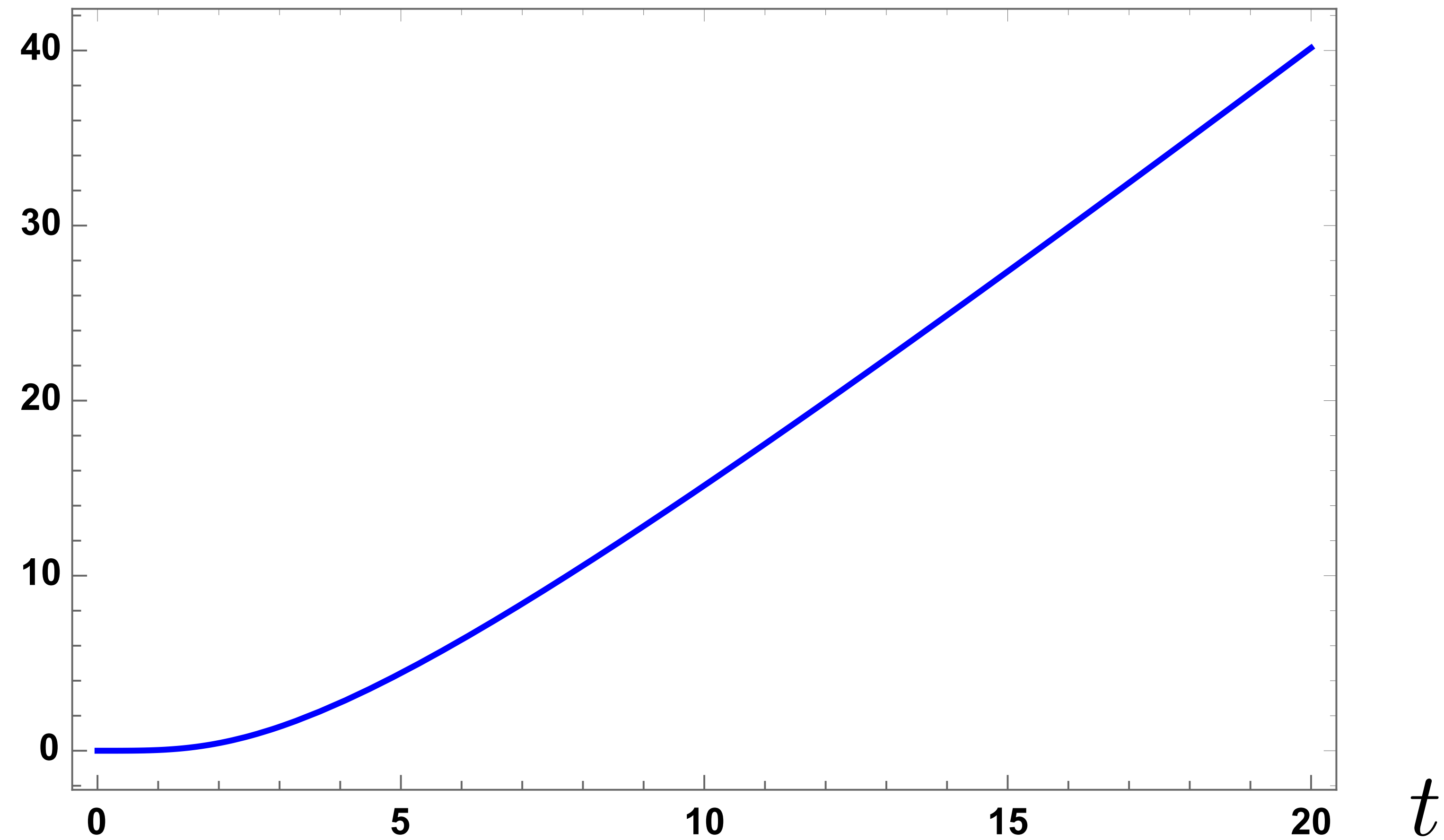
$a_n = 0$ because the spectrum is symmetric.

$K(t)$ **with** $m \neq 0, T = \infty$

$$K(t) \sim e^{\sqrt{m\pi t}}$$ exponential growth

$$a_n = 0,$$
$$b_n = \frac{m\pi n}{4W(2m\pi n/\nu_T)}$$

$(\log[1+K(t)])^2 \ (m = 0.8, \nu_T = 1)$



Summary

- The Iizuka-Polchinski (IP) model is a simple large N QM matrix model for black hole.

- Krylov complexity has been proposed as a new measure for quantum chaos.

- In the IP model with nonzero mass m at nonzero T, Krylov complexity grows exponentially $K(t) \sim e^{\mathcal{O}(\sqrt{t})}$.

- As far as we know, this is the first example of exponential growth found in large N matrix models.