# Complementarity Demystified and Holography as a Model of Observation


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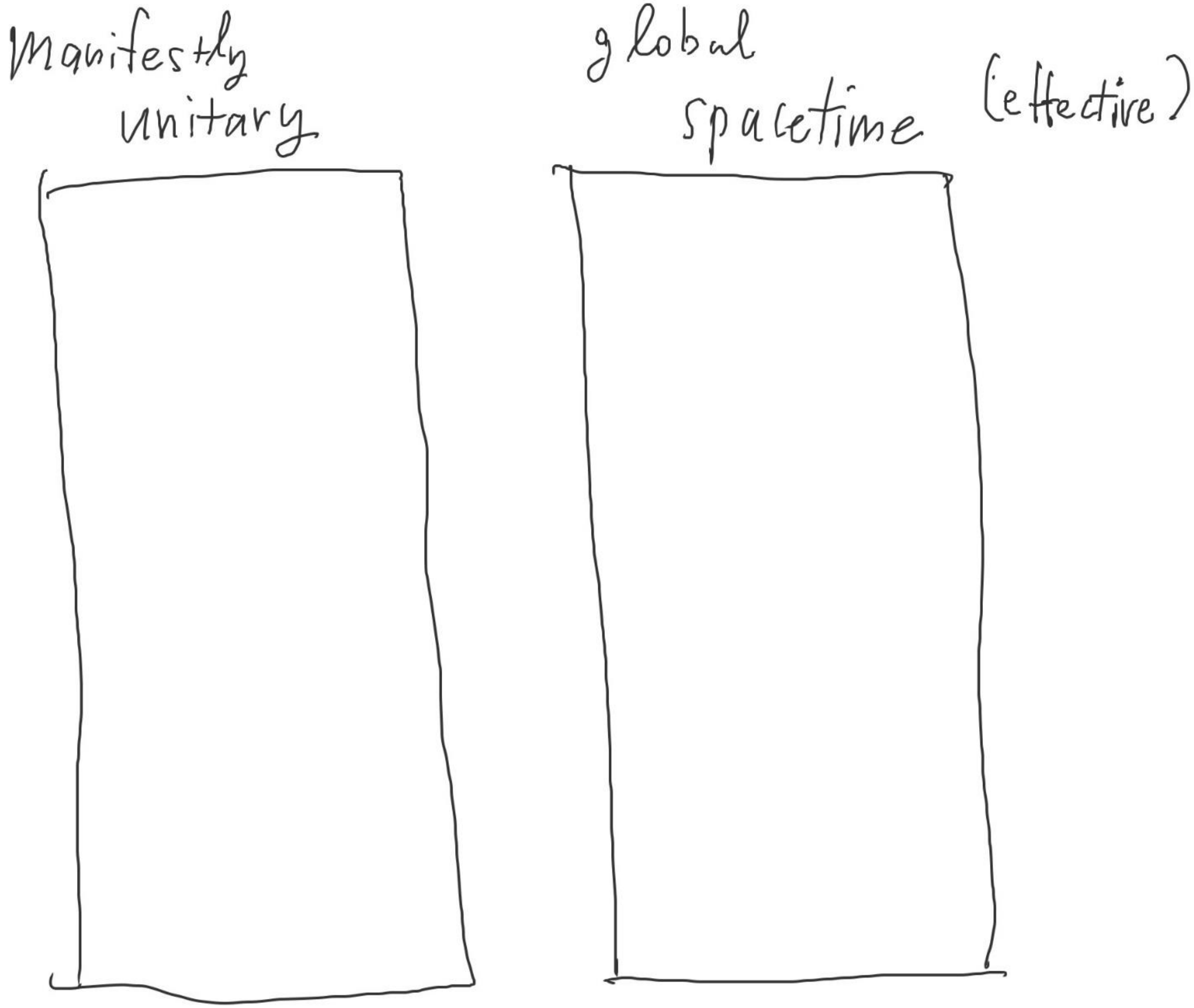
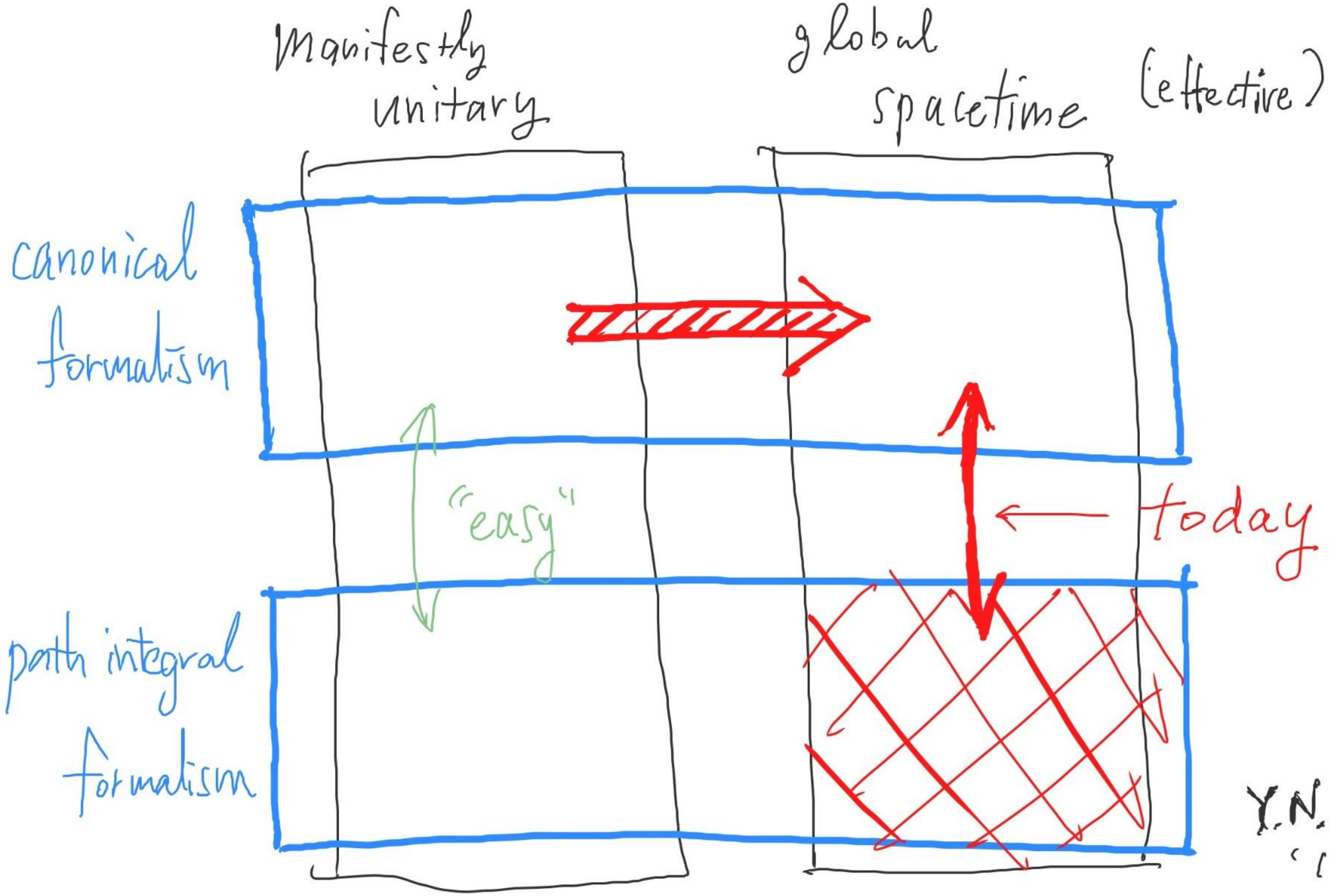
# Complementarity Demystified


Manifestly unitary
global spacetime
(effective)

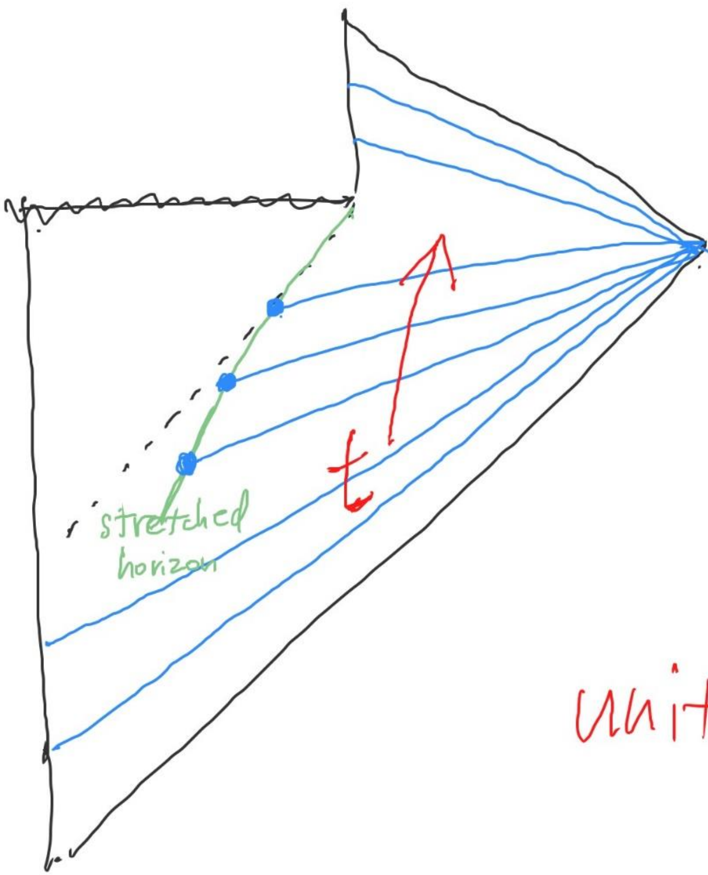



Y.N.
'18 – 23

— Murdia, Y.N., Ritchie, 2207.01625

— with Concepcion, Ritchie, Weiss ('23)

# Manifestly Unitary Description in the Canonical Formalism



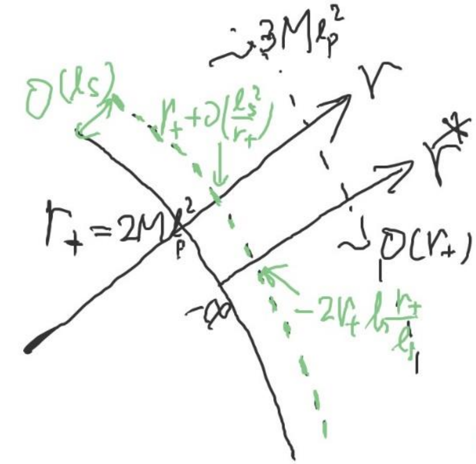

... as viewed from the exterior

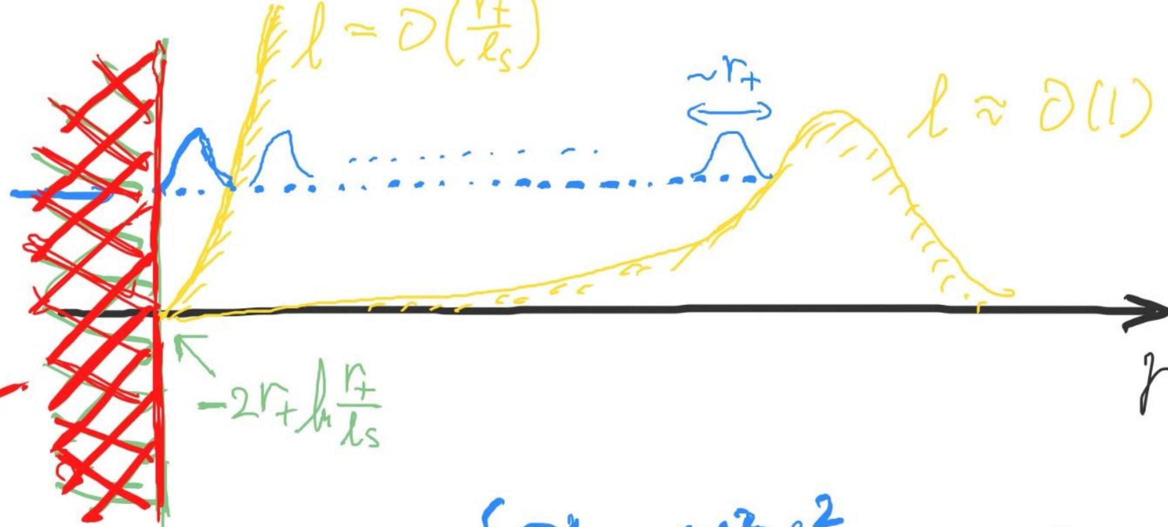
unitary with all d.o.f.s

# Quantum Gravity Bottom-up

"string" scale

- UV cutoff for spacetime $\sim \mathcal{O}(l_s)$
- universal chaos




$$\Delta\omega^{(\ell)} \sim \frac{1}{r_+ \ln\frac{r_+}{\ell\, l_s}}$$

$$E \sim M \Rightarrow \begin{cases} S_{BH} \sim M^2 l_p^2 \\ T_H \sim \frac{1}{M l_p^2} \end{cases}$$

using $l_s^2 \sim N l_p^2$

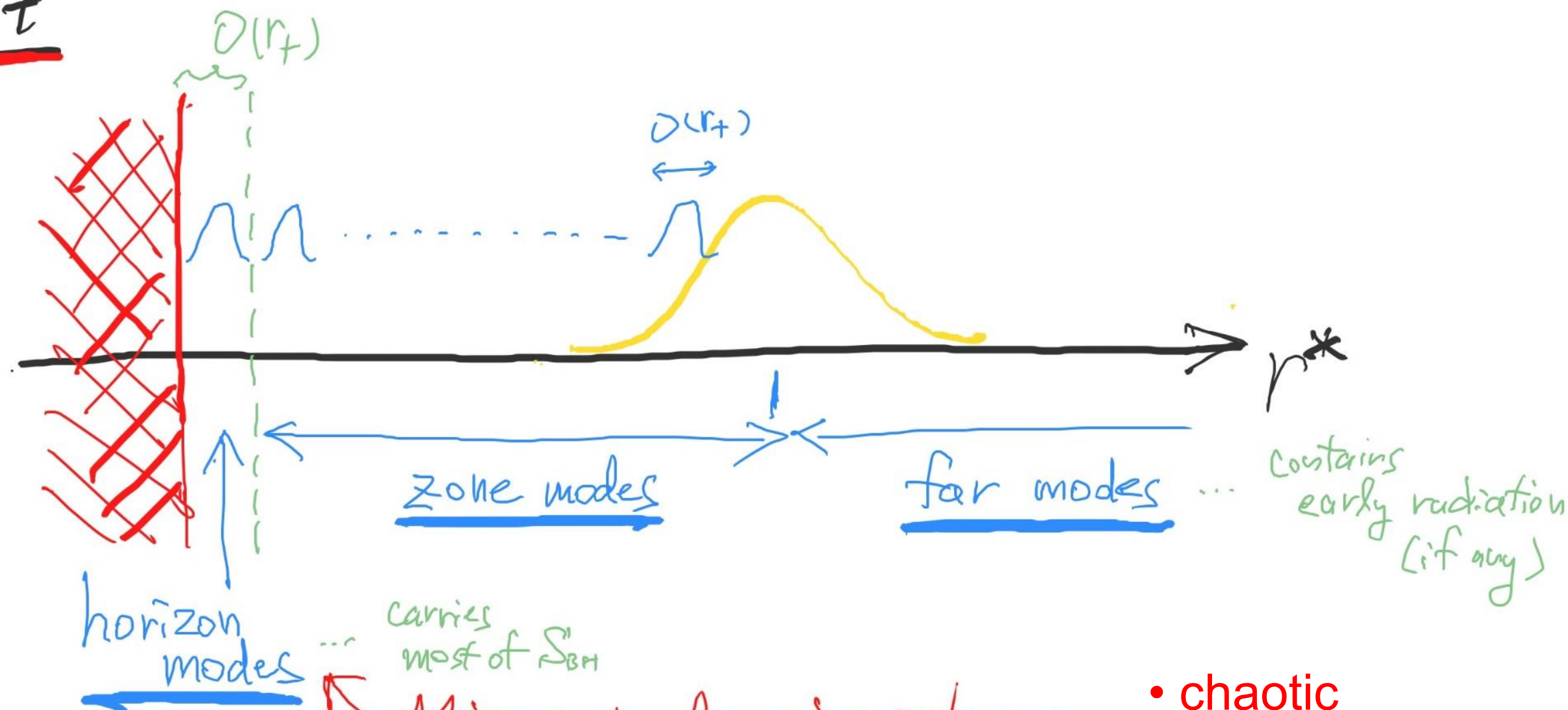
$N$: # of low energy species

# Quantum Gravity Bottom-up

$$S = \cancel{\frac{A}{4 l_p^2}}^{\,?} + S_{bulk}$$

Distribution is scheme dependent

## "Cost"

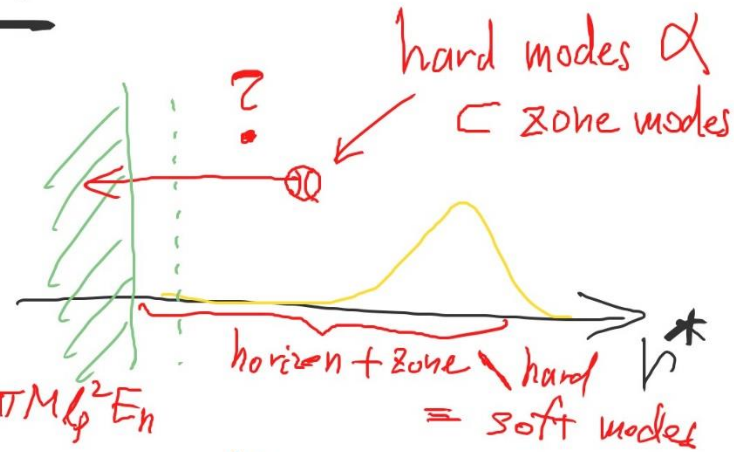



Microscopic dynamics unknown

- chaotic
- complex (all-to-all)

# Universal Chaos Is "Enough"

At a time $t$,
the vacuum state of BH of mass $M$
(within $\Delta M \sim T_H$)




$$4\pi(M-E_n)^2 l_p^2 = 4\pi M^2 l_p^2 - 8\pi M l_p^2 E_n$$

$$|\Psi(M)\rangle = \sum_{\{n_\alpha\}} \sum_{i_n=1}^{e^{S_{BH}(M-E_n)}} \sum_a \underbrace{c_{n i_n a}}_{\text{random}} \underbrace{\overbrace{|\{n_\alpha\}\rangle}^{E_n} \overbrace{|\psi^{(n)}_{i_n}\rangle}^{M-E_n}}_{M} |\phi_a\rangle$$

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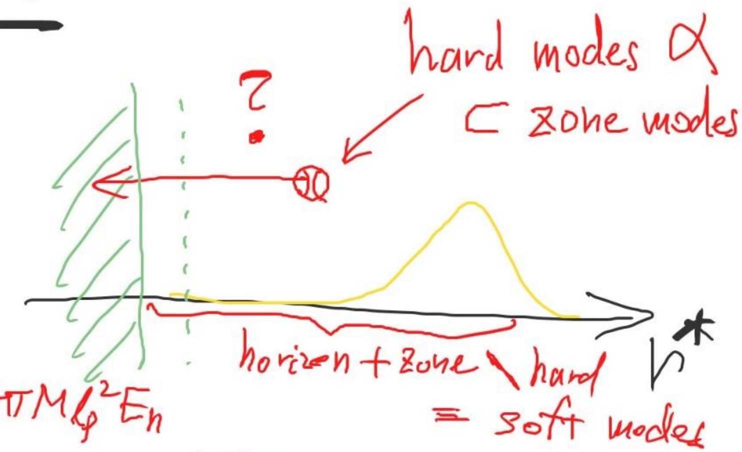


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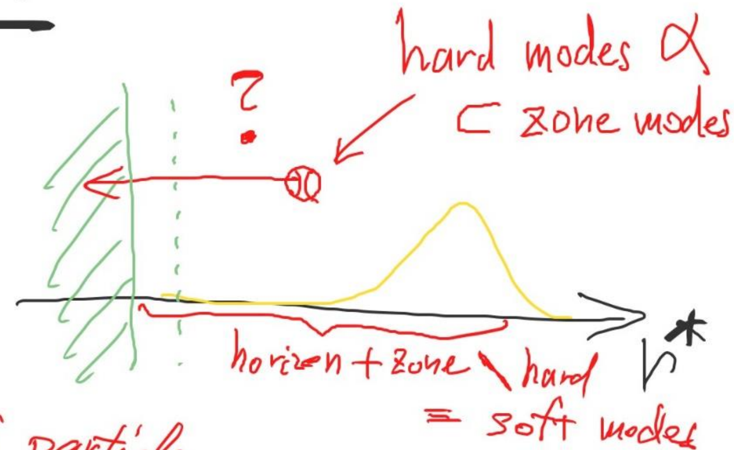
$$\|\{n_\alpha\}\rangle\rangle = \sqrt{Z}\, e^{4\pi M l_p^2 E_n} \sum_{i_n=1}^{e^{S_{BH}(M-E_n)}} \sum_a c_{n i_n a} |\psi^{(n)}_{i_n}\rangle |\phi_a\rangle$$

$$\Rightarrow |\Psi(M)\rangle = \frac{1}{\sqrt{Z}} \sum_{\{n_\alpha\}} e^{-\frac{E_n}{2T_H}} |\{n_\alpha\}\rangle \|\{n_\alpha\}\rangle\rangle \quad \text{.. TFD}$$

# Universal Chaos Is "Enough"




$$b,\ b^\dagger,\ \tilde{b},\ \tilde{b}^\dagger$$

$$\left(\begin{aligned}
b_\gamma &= \sum_n \sqrt{n_\gamma}\, |\{n_\alpha - \delta_{\alpha\gamma}\}\rangle\langle\{n_\alpha\}| \\
b_\gamma^\dagger &= \sum_n \sqrt{n_\gamma + 1}\, |\{n_\alpha + \delta_{\alpha\gamma}\}\rangle\langle\{n_\alpha\}| \\
\tilde{b}_\gamma &= \sum_n \sqrt{n_\gamma}\, \|\{n_\alpha - \delta_{\alpha\gamma}\}\rangle\rangle\langle\langle\{n_\alpha\}\| \\
\tilde{b}_\gamma^\dagger &= \sum_n \sqrt{n_\gamma + 1}\, \|\{n_\alpha + \delta_{\alpha\gamma}\}\rangle\rangle\langle\langle\{n_\alpha\}\|
\end{aligned}\right)$$

quasi particle of soft & far (early radiation)

$\xrightarrow{\text{Bogoliubov transf.}}$ $a,\ a^\dagger$ ... infalling modes

(*) Papadodimos, Raju; N. Varela; Verlinde-Verlinde; Maldacena, Susskind, ...

$$\|\{n_\alpha\}\rangle\rangle = \sqrt{Z}\, e^{4\pi M l_P^2 E_n} \sum_{i_n=1}^{e^{S_{BH}(M-E_n)}} \sum_a c_{n i_n a} |\psi_{i_n}^{(n)}\rangle |\phi_a\rangle$$

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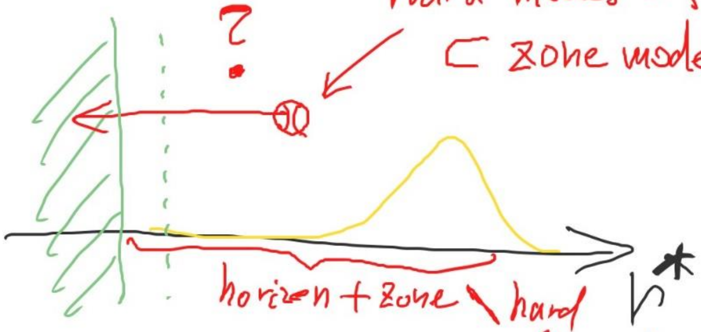


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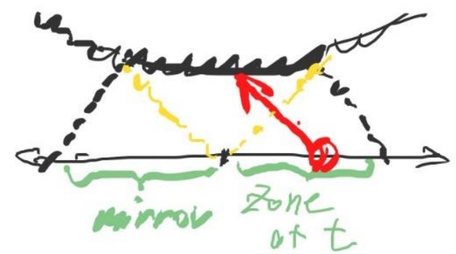



## Fate of a falling object

$(\prod b^\dagger)|\Psi(M)\rangle \longrightarrow$ semiclassical correlation func. with in-in formalism

$e^{-iH_{inf}\tau}$ ; $H_{inf} = \sum_\Omega \Omega\, a_\Omega^\dagger a_\Omega + \cdots$

Different time evolution

$(H = \sum_\omega \omega\, b_\omega^\dagger b_\omega + \cdots)$

# Universal Chaos Is "Enough"

— "Universal" chaos
across all low energy species

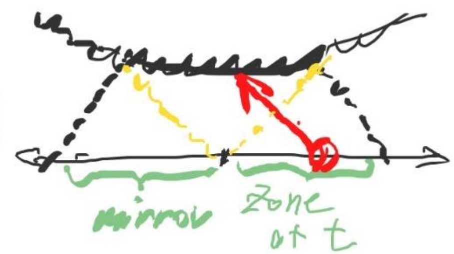
⋯> single out BH horizon


— For a young BH,

$\tilde{b}, \tilde{b}^\dagger$ can be constructed only from soft modes

$$|\Psi(M)\rangle \approx \sum_{\{n_\alpha\}} \underbrace{\sum_{i_n=1}^{e^{S(M-E_n)}} c_{n i_n} |\{n_\alpha\}\rangle |\psi^n_{i_n}\rangle}_{\| \{n_\alpha\} \rangle\rangle}$$

(Related with "full" $b, b^\dagger$ by the Petz map)



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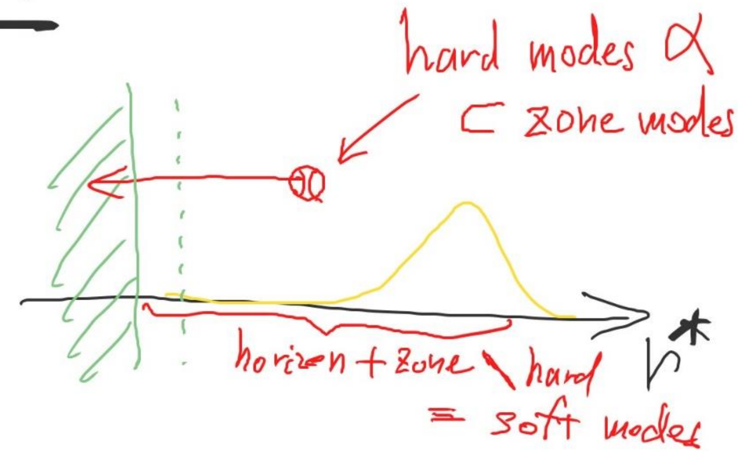


— "Universal" chaos across all low energy species

⇝ single out BH horizon

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$\to \|\{n_\alpha\}\rangle\rangle$

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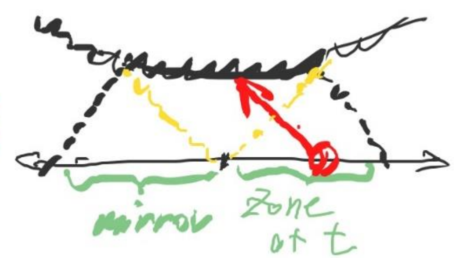



⇓

— Intrinsically semiclassical

… ambiguity of $O(e^{-S/2})$

— (mildly) state dependent

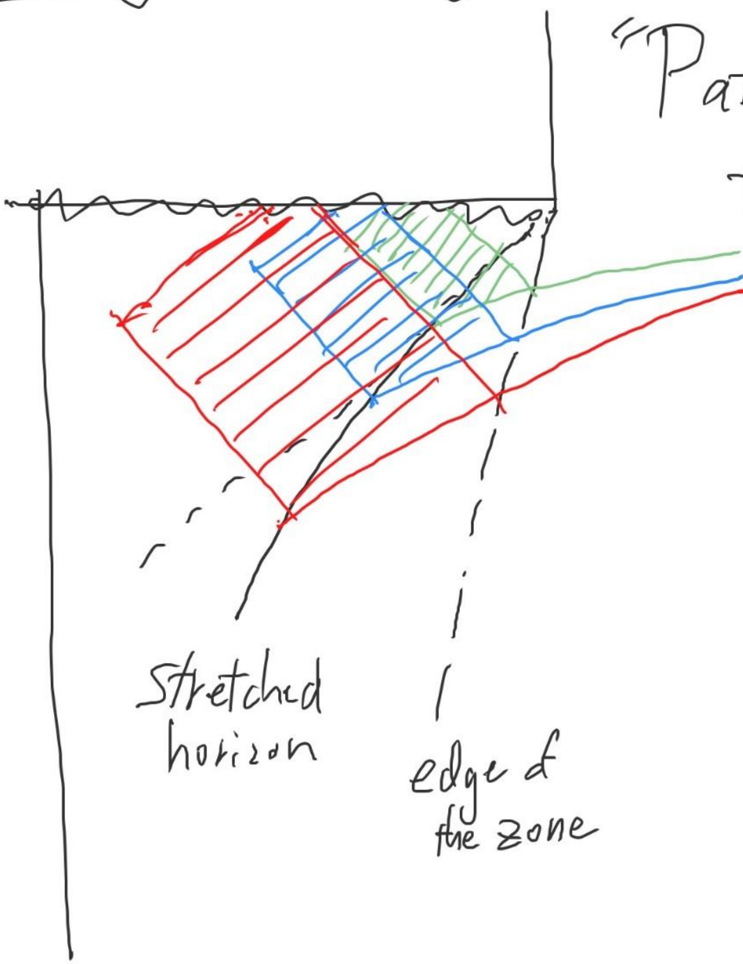
… $\tilde{b}, \tilde{b}^\dagger$ applicable for $e^{cS}$ $(c<1)$

cf.) α bits

Hayden-Penington

# Emergence of global spacetime

"Patching" effective theories of the interior erected at different $t$'s.



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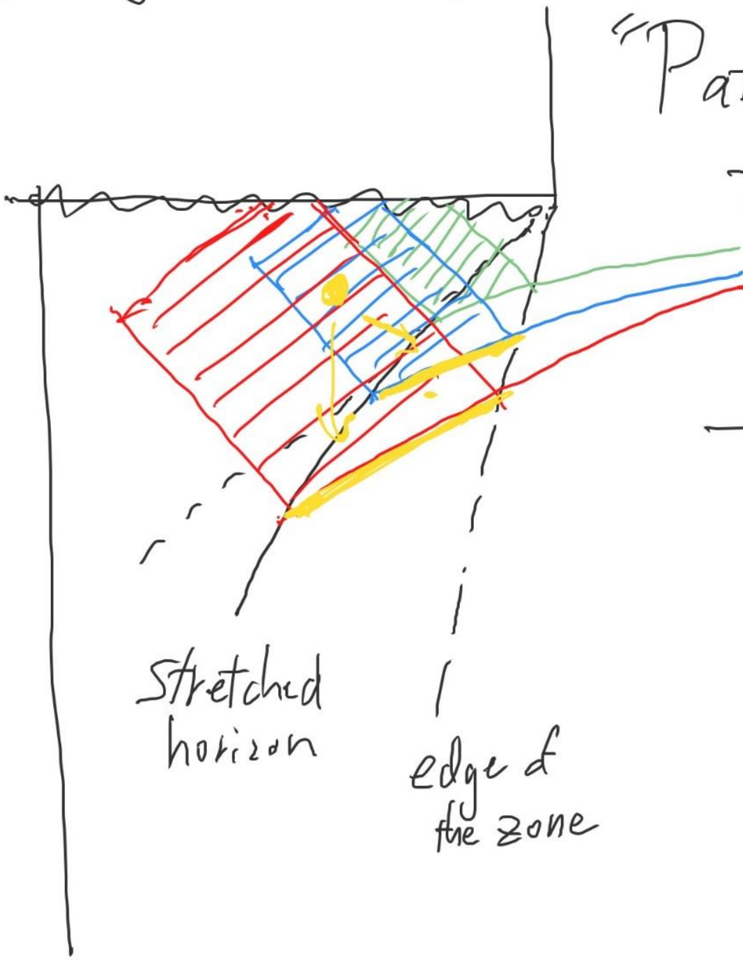


"Patching" effective theories of the interior erected at different t's.

— QEC nature of int. op's in terms of fund. d.o.f.

(horizon, zone, far)

Almheiri - Dong - Harlow

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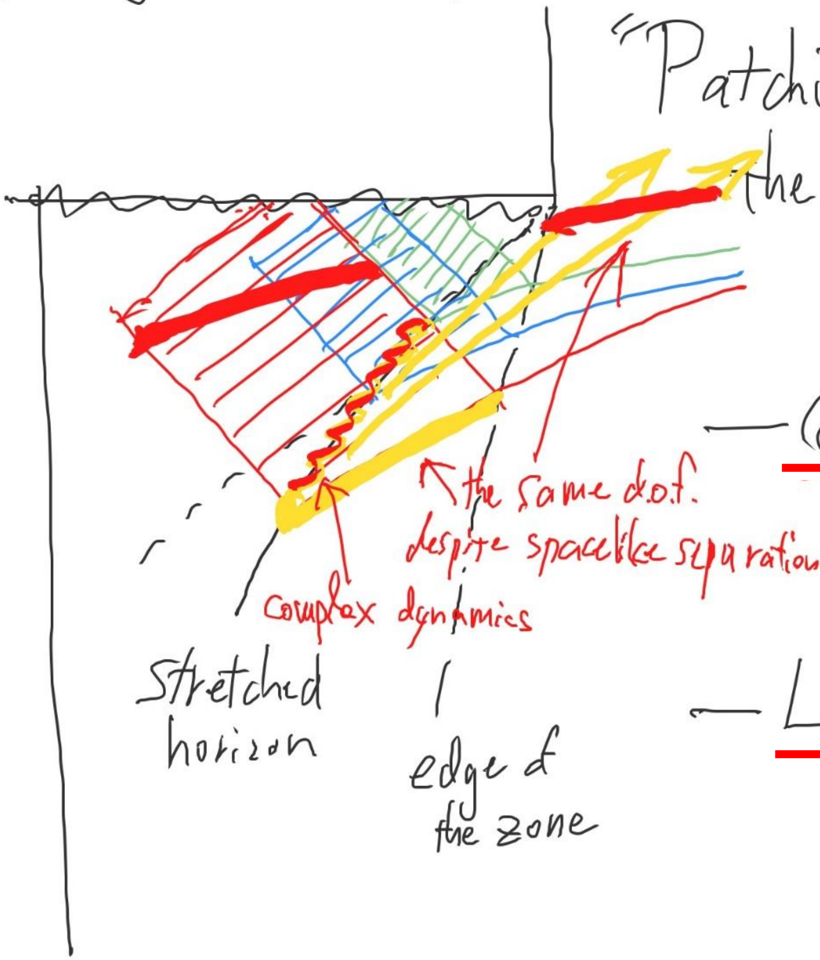


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— Locality from complexity

Kim - Tang - Preskill

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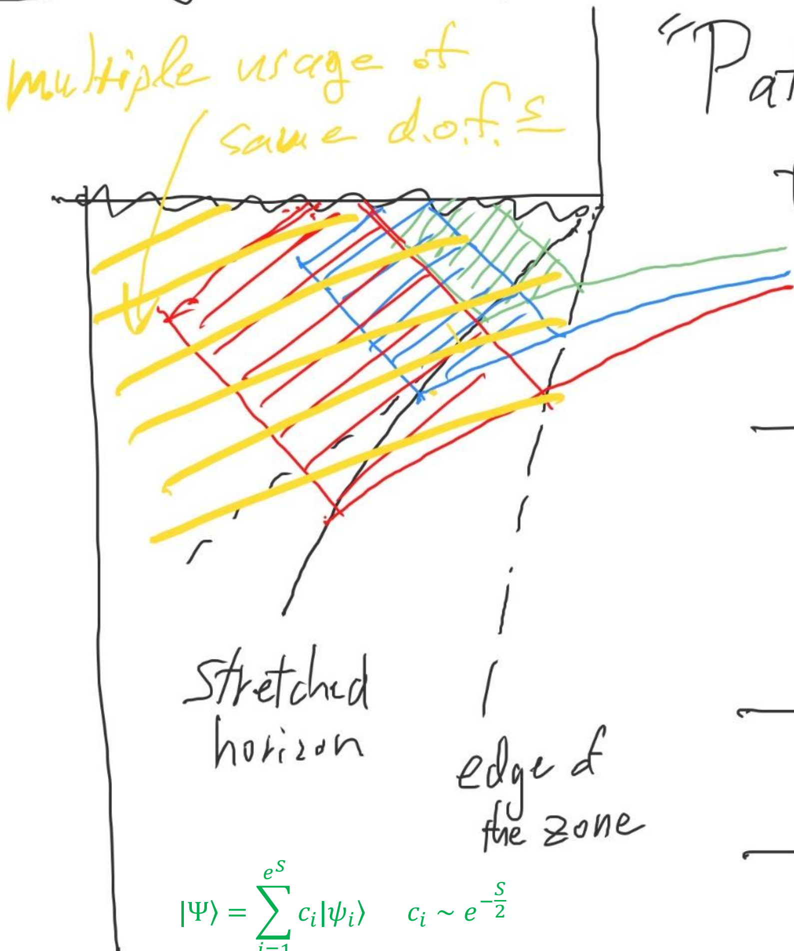


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— QEC nature of int. ops in terms of fund. d.o.f. (horizon, zone, far)

Almheiri-Dong-Harlow

— Locality from complexity

Kim-Tang-Preskill

— Non-isometric nature of interior encoding

Langhoff-Y.N.

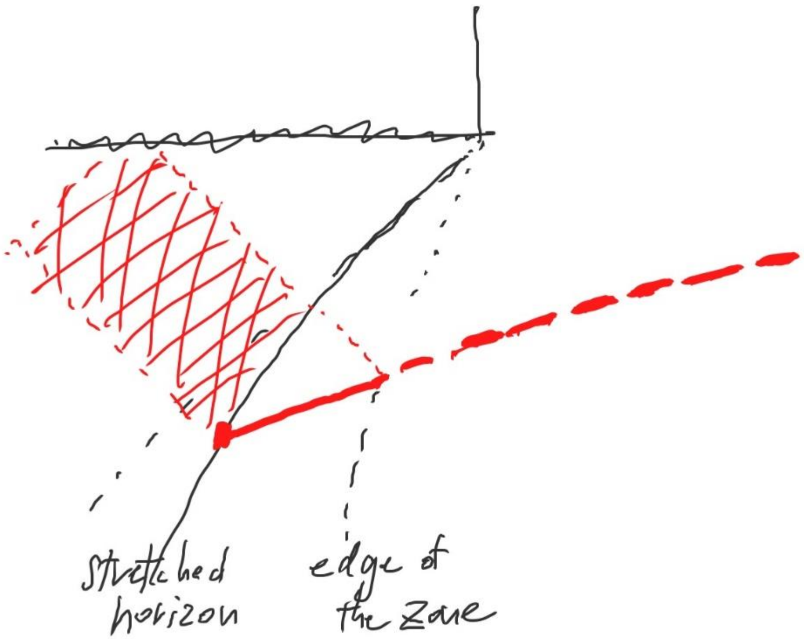
Akers-Engelhardt-Harlow-Penington-Vardhan

$$|\Psi\rangle = \sum_{i=1}^{e^S} c_i |\psi_i\rangle \qquad c_i \sim e^{-\frac{S}{2}}$$

$$\langle\Psi_1|\Psi_2\rangle = \sum_{i=1}^{e^S} c_{1,i}^* c_{2,i} \sim e^{\frac{S}{2}} e^{-S} \sim e^{-\frac{S}{2}}$$

$\rightarrow e^{e^S}$ approximately orthogonal states

# Entanglement Wedge Reconstruction in the Canonical Formalism



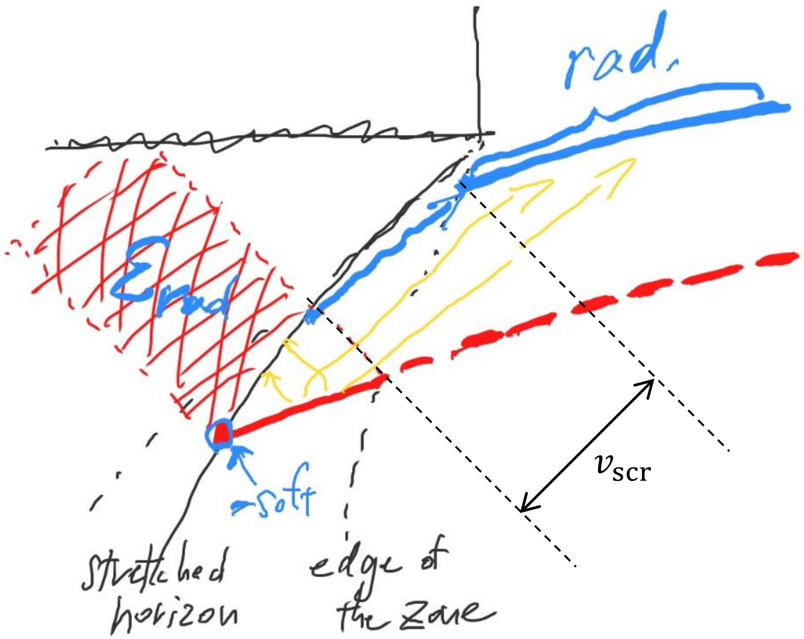

## Effective theory of the interior

use hard & soft + rad.

(For young BH hard & soft)

... always involves soft (BH) d.o.f.

# Entanglement Wedge Reconstruction in the Canonical Formalism

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(nothing other than Hayden-Preskill)

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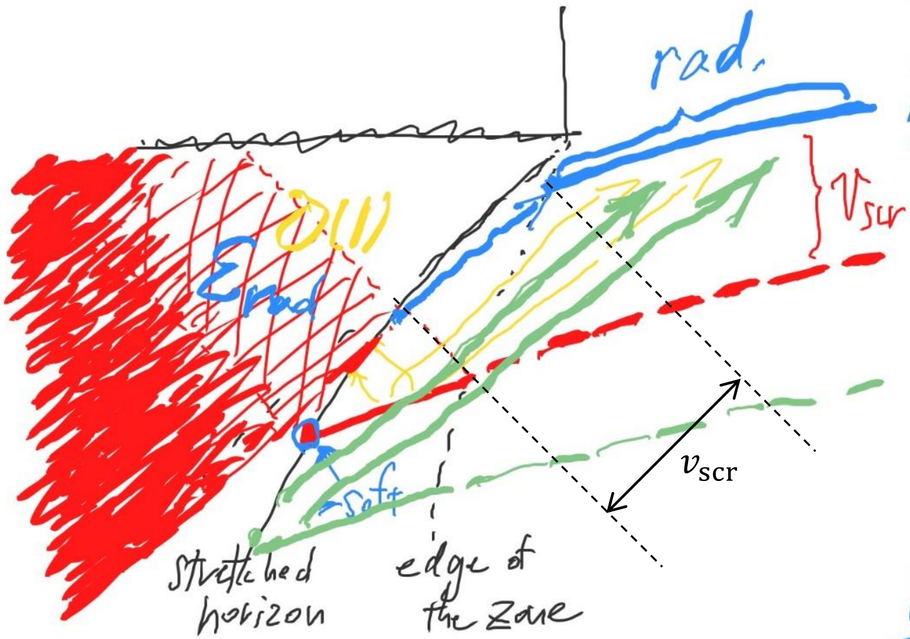


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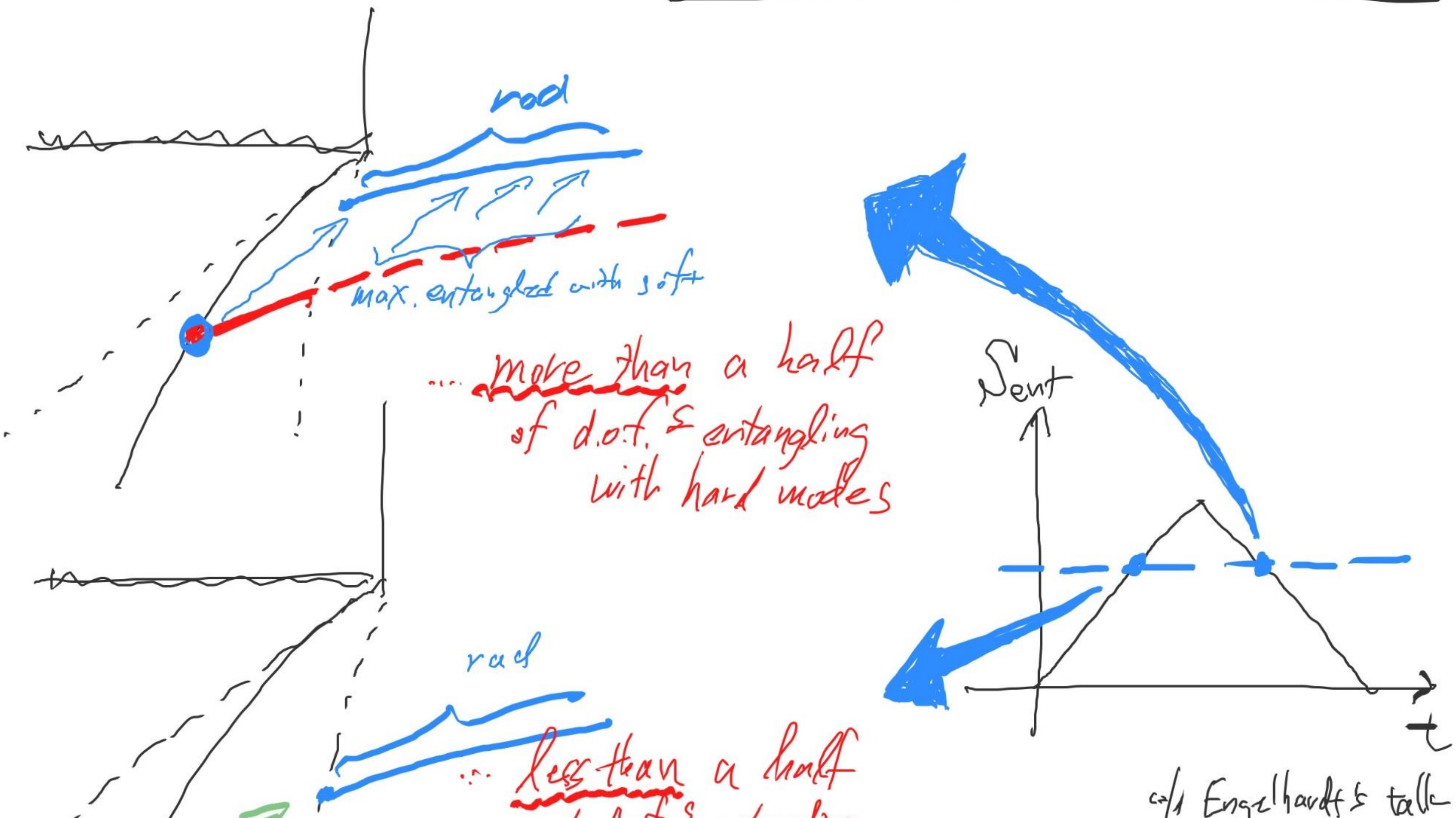
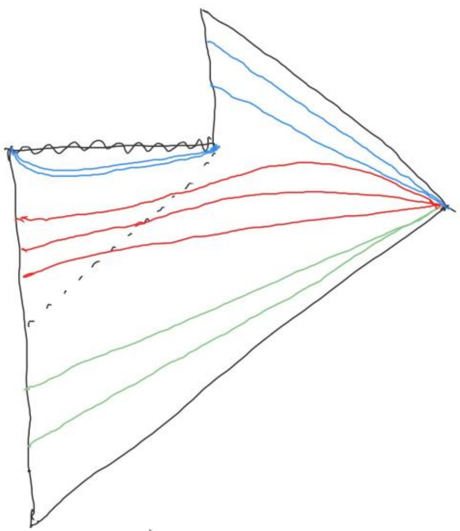



c/f Engelhardt's talk

Simple criterion? — see later

# Global Spacetime as a Coarse-Grained Description

- Semiclassical evolution involving Cauchy surfaces



— ensemble average over microstates

- A generic state has smooth horizon

  cf.) Marolf-Polchinski

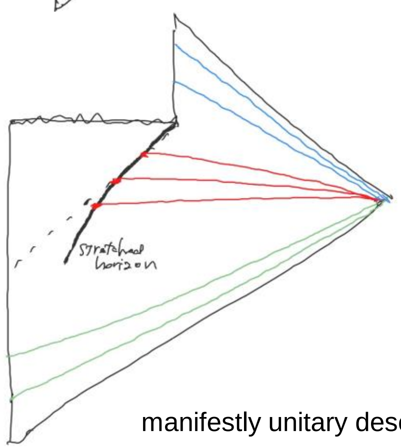
  $$|\Psi(M)\rangle = \sum_n \sum_{i_n} \sum_a C_{n i_n a} \, |\{n_i\}\rangle |\psi^{(n)}_{i_n}\rangle |\phi_a\rangle$$

  Soft excitations near the horizon
  = BH microstates
  ⋯ play the role of the second exterior

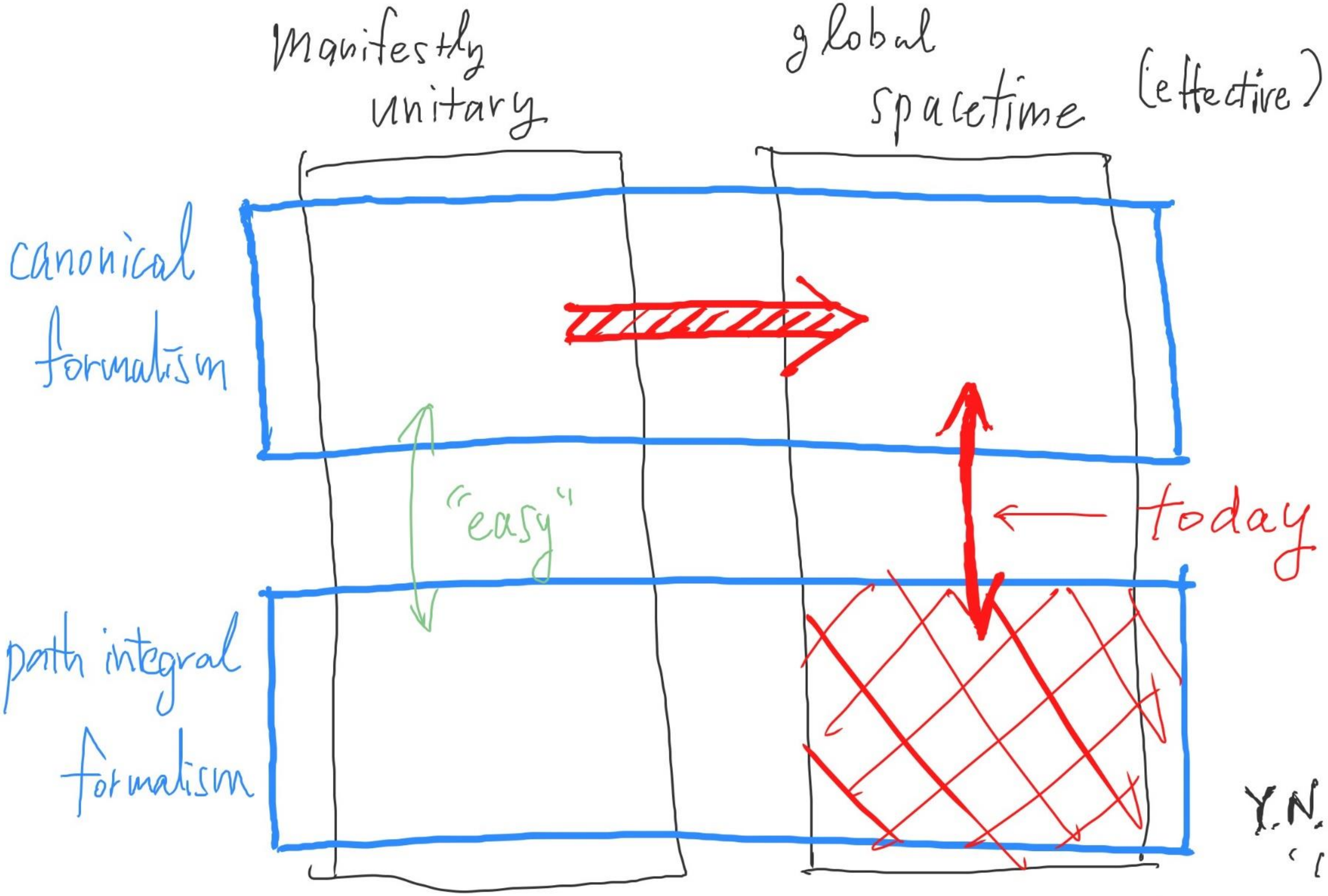
- Spacetime below $\sim O(l_s)$ near the horizon does not exist.

  cf.) Quantum singularity (Bousso-Shahbazi-Moghaddam)

cf.)




manifestly unitary description

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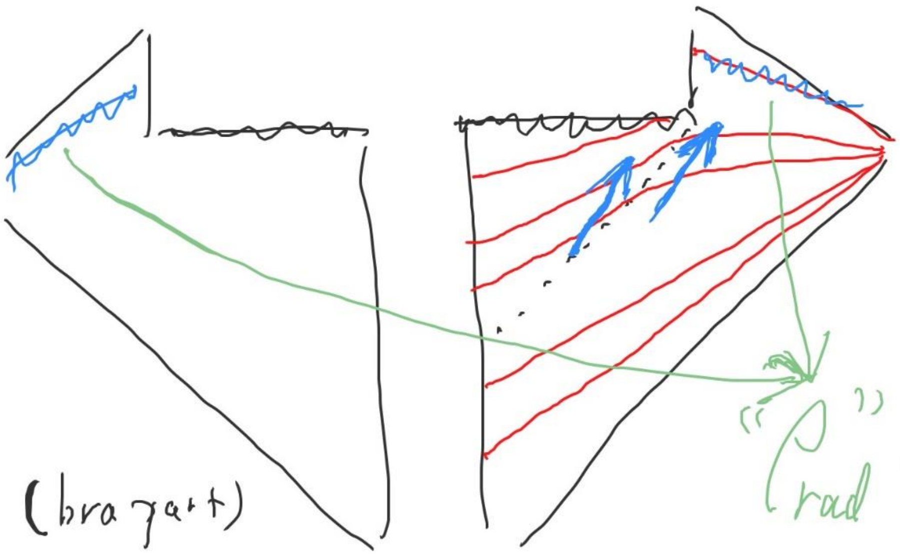
# Gravitational Path Integral as a Coarse-Grained Description




- **ensemble averaged**
  (represents generic properties)
- **complexity cutoff imposed**
  (intrinsically semiclassical even outside BH)

$$\text{“}\rho_{rad}\text{”} = \bar{\rho}_{rad}$$

$$S(\text{“}\rho_{rad}\text{”}) = S(\bar{\rho}_{rad}) = -\mathrm{Tr}\left[\bar{\rho}_{rad} \ln \bar{\rho}_{rad}\right]$$

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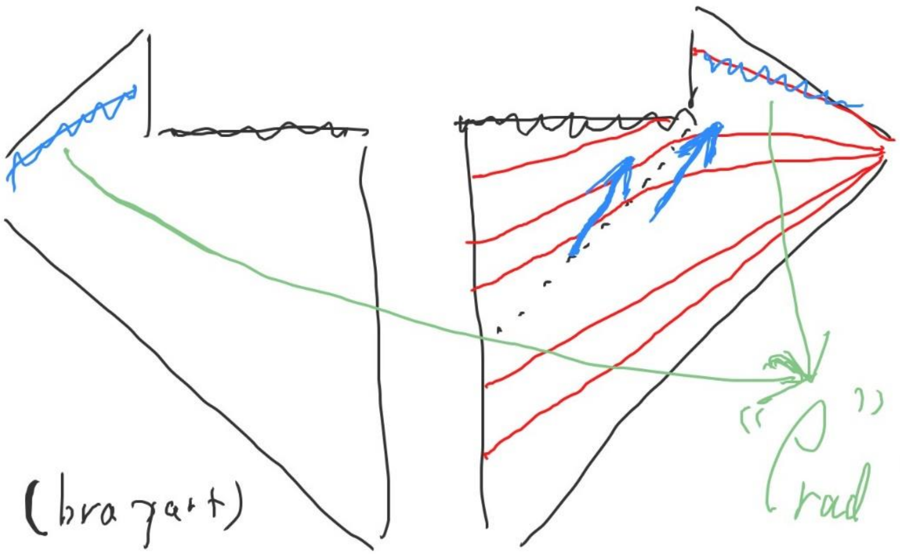

(bra part)

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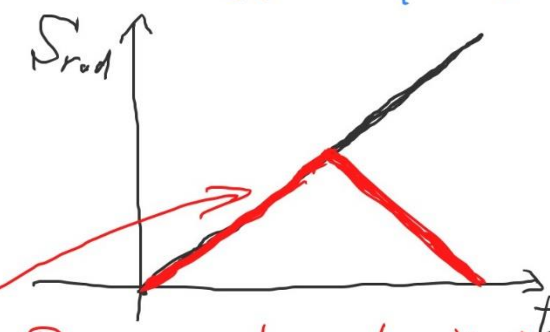
"$\rho_{rad}$" $= \overline{\rho}_{rad}$

$S(``\rho_{rad}") = S(\overline{\rho}_{rad}) = -\mathrm{Tr}\left[\overline{\rho}_{rad} \ln \overline{\rho}_{rad}\right]$

Though always averaged,
we can compute many different quantities

Replica wormhole

$$\to \overline{\mathrm{Tr}\,\rho_{rad}^{n}} \to -\lim_{n\to 1}\frac{\partial}{\partial n}\overline{\mathrm{Tr}\,\rho_{rad}^{n}} = \overline{S(\rho_{rad})}$$

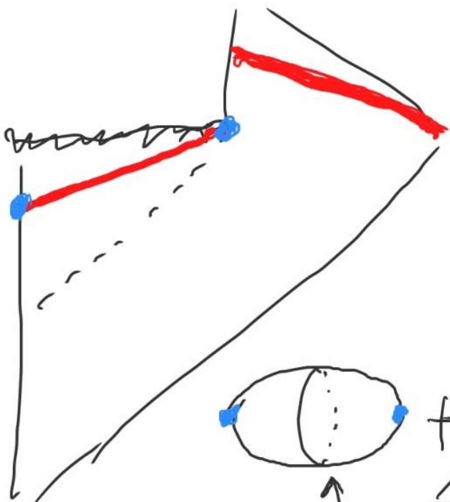



Page curve but not unitarity

(when dimensionally reduced, the average of theories)

# Gravitational Path Integral as a Coarse-Grained Description

After the full evaporation?



cf.) Marolf-Maxfield
(but with different interpretation)

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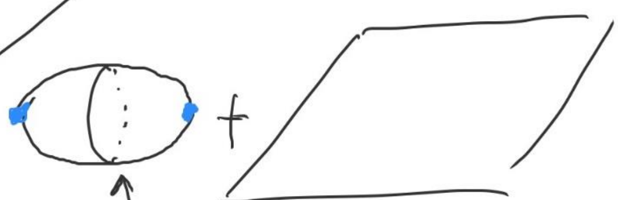
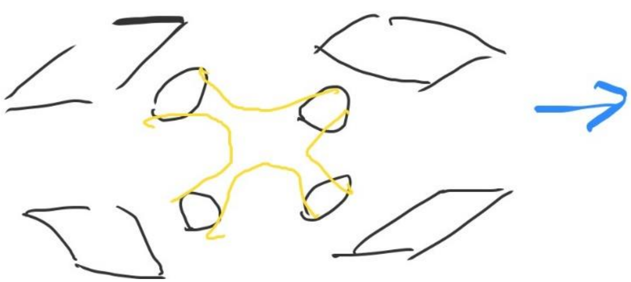


baby universe

“n-foldize”



$\alpha$ states
... superselection sectors

Physical (only) with $n$ BH's

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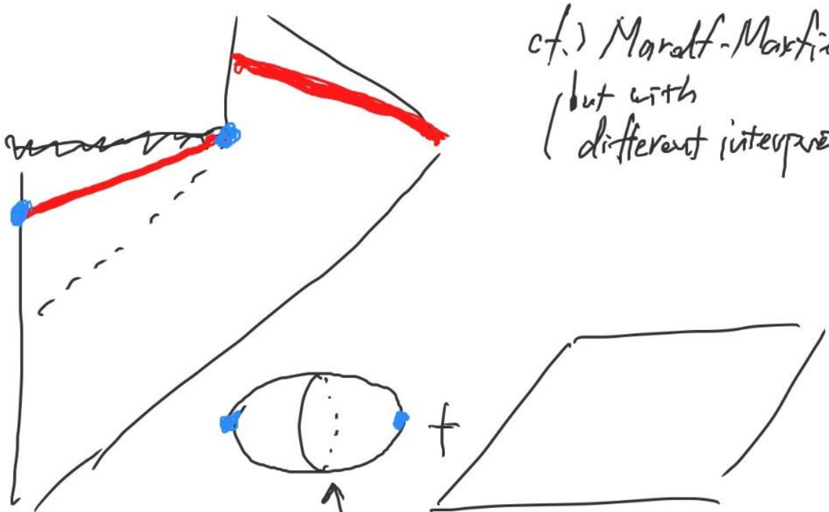


– ensemble averaged (represents generic properties)

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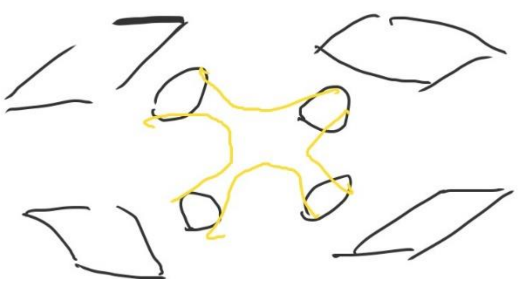
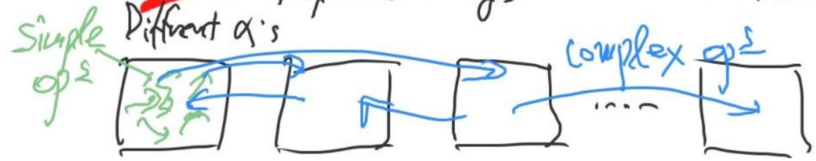
$\alpha$ states = microstates

~~… superselection sectors~~

~~Physical (only) with $n$ BH's~~

“$n$-foldize”





# Gravitational Path Integral as a Coarse-Grained Description

$\alpha$ states in (global) grav. path integral — **ensemble averaged**

(represents generic properties)

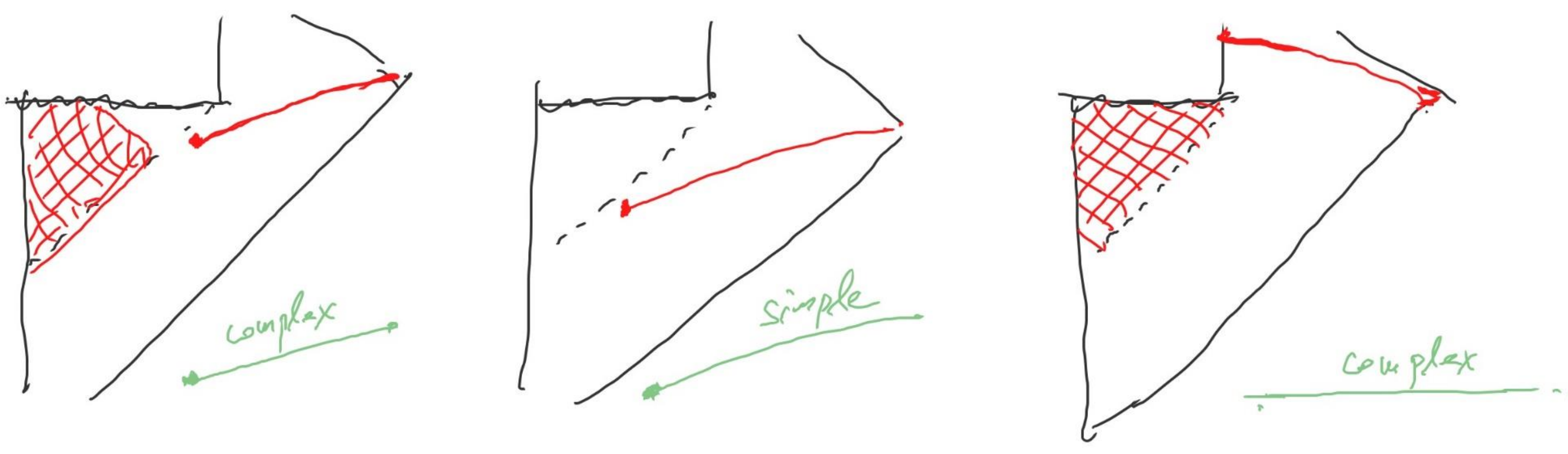
→ complexity separated sectors in the manifestly unitary description — **complexity cutoff imposed**

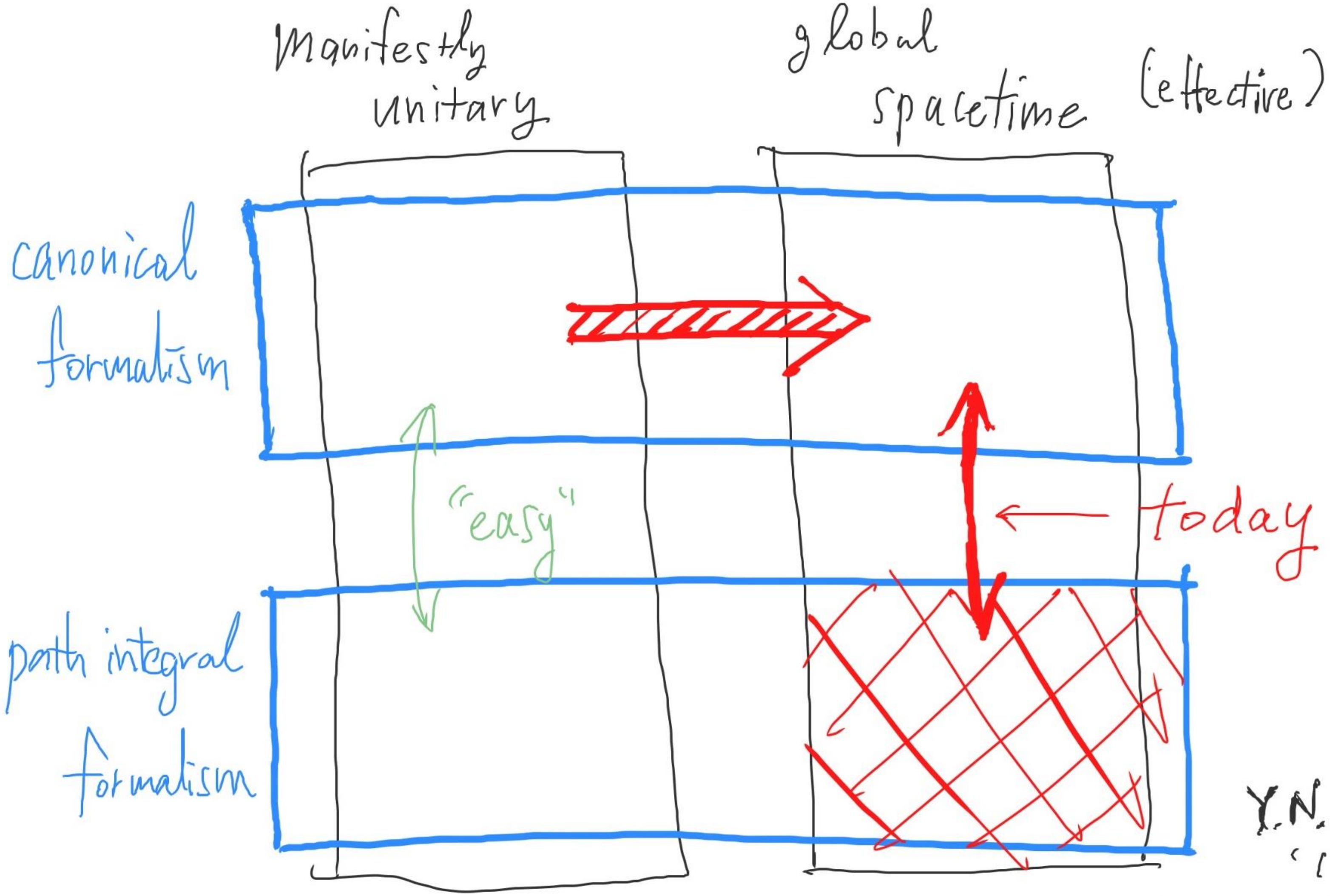
(intrinsically semiclassical even outside BH)

**Proposal**

Complexity of the radiation in the unitary desc.

→ volume/action of the island/baby universe



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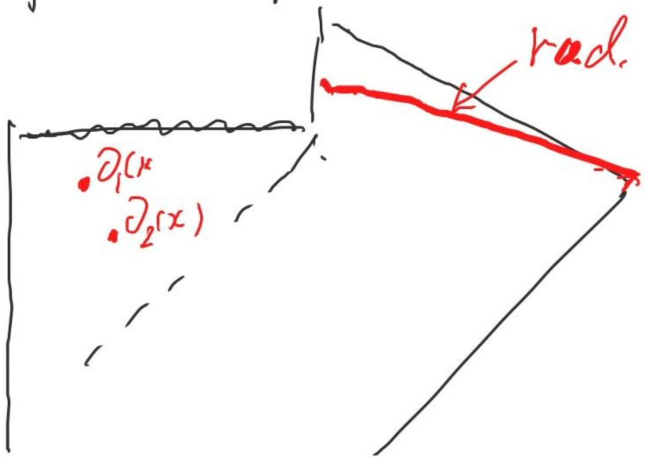
# Example (apparent puzzle)




represents $O_1(x)$ & $O_2(x)$ in Rad.

(entanglement wedge reconstruction)

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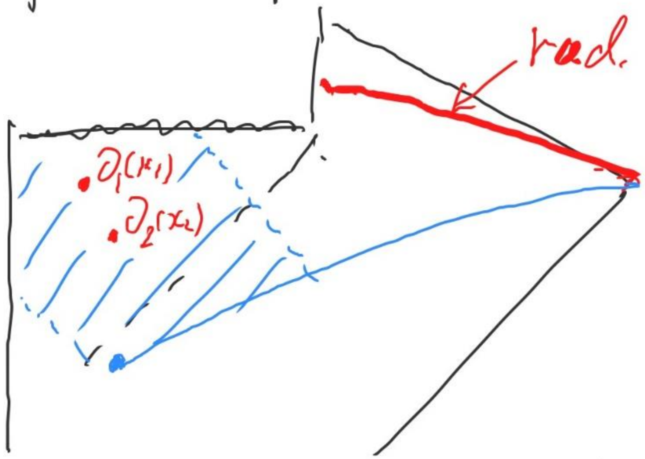



represents $O_1(x_1)$ & $O_2(x_2)$ in Rad.
(entanglement wedge reconstruction)

Suppose we want to compute $\underline{\langle\psi|O_1(x_1)O_2(x_2)|\psi\rangle}$ or $\underline{\langle\psi|O_2(x_2)O_1(x_1)|\psi\rangle}$.

represent observations by an infalling observer

match

$|\psi\rangle_{semiclassical} \rightarrow |\psi\rangle_{full} \rightarrow |\phi\rangle_{rad,full} = e^{-iHt}|\psi\rangle_{full}$

$O_1(x)O_2(x)|\psi\rangle_{semiclassical} \rightarrow O|\psi\rangle_{full} \rightarrow \boxed{R}|\phi\rangle_{rad,full}$ $\qquad R = e^{-iHt}Oe^{iHt}$

$O_2(x)O_1(x)|\psi\rangle_{semiclassical} \rightarrow O'|\psi\rangle_{full} \rightarrow \boxed{R'}|\phi\rangle_{rad,full}$ $\qquad R' = e^{-iHt}O'e^{iHt}$

($O \approx O'$ if $x_1$ & $x_2$ are spacelike separated)

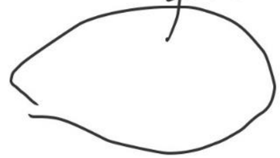
Determined s.t. when ${}_{rad,full}\langle\phi|$ reproduces $\langle\psi|O_{1,2}(x)O_{2,1}(x)|\psi\rangle_{semiclassical}$

# Holography as a Model of Observation

# Quantum Mechanics

"observer"
classical
(infinitely powerful)

quantum
system



closed universe

dim = 1

cf.) Jacobson


$S_{ent} \approx$ the dim. of the observer

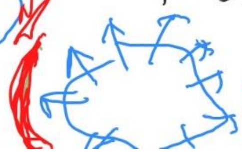
The dimension of the system

$\approx$ Maximal value of $S_{ent}$

$\equiv S_{max}$

For $S_{ent} < S_{max}$,
simple operators in 
cannot descriminate various systems
beyond $e^{S_{ent}}$ (Python's lunch)

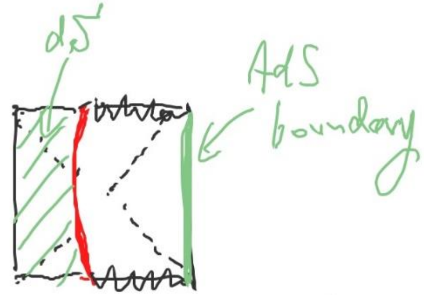
If $S_{ent} > S_{max}$, the system can be
specified uniquely.



infinitely
large $\rightarrow$ faithful holography
(AdS, flat space)

## Example/Test

Probing dS grav. with AdS BH grav.




dS/AdS wormhole

$$S_{ent} = \begin{cases} S_{BH} & (S_{BH} < S_{dS}) \\ 0 & (S_{BH} > S_{dS}) \end{cases}$$

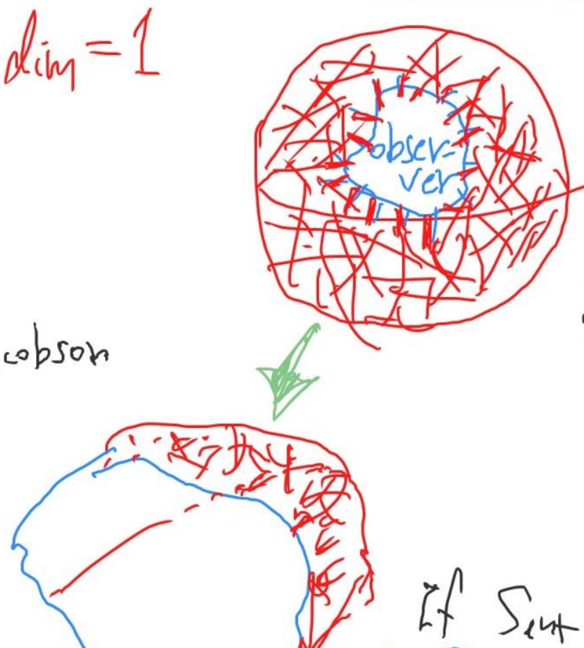
Balasubramanian - Y.N. - Ugajin

Vijay's talk

## closed universe

dim = 1

cf.) Jacobson

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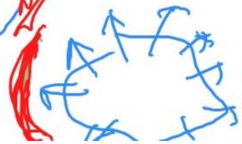

infinitely large → faithful holography (AdS, flat space)