QIMG2023 at YITP @ Oct. 6th.

Applications of inhomogeneous deformations in 2d CFTs: from the revival of non-local correlation to curved spacetime

Masahiro Nozaki (KITS, UCAS & iTHEMS, RIKEN)

This talk is based on collaboration with

Kanato Goto, Weibo Mao, Akihiro Miyata, Shinsei Ryu, Mao Tian Tan, Kotaro Tamaoka, and Masataka Watanabe

Based on

arXiv:2112.14388, arXiv:2302.08009,

arXiv:23XX.XXXXX and arXiv:23XX.XXXX

Short summary of my talk

What we want to study:

1. The non-equilibrium process opposite to the **quantum thermalization** and **scrambling**.

Short summary of my talk

What we want to study:

1. The non-equilibrium process opposite to the **quantum thermalization** and **scrambling**.

In other words, the process can endow the state with **quantum properties such as non-local correlation**.

Short summary of my talk

What we want to study:

1. The non-equilibrium process opposite to the **quantum thermalization** and **scrambling**.

In other words, the process can endow the state with **quantum properties such as non-local correlation**.

For example, **the preparation for the vacuum state**.

Short summary of my talk

What we want to study:

1. The non-equilibrium process opposite to the **quantum thermalization** and **scrambling**.

In other words, the process can endow the state with **quantum properties such as non-local correlation**.

This is **the main topic** in this talk.

Short summary of my talk

What we want to study (if I have time):

2. Thermodynamics of QFT on the curved background.

Short summary of my talk

What we want to study (if I have time):

2. Thermodynamics of QFT on the curved background.

Two-dimensional conformal field theories (2d CFTs)

Short summary of my talk

What we want to study:
2. Thermodynamics of QFT on the curved background.

Phase transition induced by curvature of spacetime?

How about entanglement?

Short summary of my talk

1.We explored the dynamical property of the system evolved with inhomogeneous Hamiltonians:

$$H_{\text{Inho}} = \int_0^L dx f(x)h(x)$$

SSD Hamiltonian: $f_{\text{SSD}}(x) = 2\sin^2\left(\frac{\pi x}{L}\right)$

Mobius Hamiltonian: $f_{\text{Möbius}}(x) = 1 - \tanh 2\theta \cos\left(\frac{2\pi x}{L}\right)$

1. Start from the boundary state (unentangled state),

   and the thermal state (mixed state).

Short summary of my talk

Circumstance of the system considered

1.We explored the dynamical property of the system evolved with inhomogeneous Hamiltonians:

$$H_{\text{Inho}} = \int_0^{L} dx\, f(x)h(x)$$

SSD Hamiltonian: $f_{\text{SSD}}(x) = 2\sin^2\left(\frac{\pi x}{L}\right)$

Mobius Hamiltonian: $f_{\text{Möbius}}(x) = 1 - \tanh 2\theta \cos\left(\frac{2\pi x}{L}\right)$

1. Start from the boundary state (unentangled state),
   and the thermal state (mixed state).

Short summary of my talk

1.We explored the dynamical property of the system evolved with inhomogeneous Hamiltonians:

$$H_{\mathrm{Inho}} = \underline{\int_0^L dx\, f(x) h(x)}$$

SSD Hamiltonian: $f_{\mathrm{SSD}}(x) = 2\sin^2\left(\frac{\pi x}{L}\right)$

Mobius Hamiltonian: $f_{\text{Möbius}}(x) = 1 - \tanh 2\theta \cos\left(\frac{2\pi x}{L}\right)$

1. Start from the boundary state (unentangled state),

and the thermal state (mixed state).

For $f(x) = 1$,

this is undeformed Hamiltonian.

Short summary of my talk

1.We explored the dynamical property of the system evolved with inhomogeneous Hamiltonians:

$$H_{\mathrm{Inho}} = \int_0^L dx\, f(x) h(x)$$

SSD Hamiltonian: $f_{\mathrm{SSD}}(x) = 2\sin^2\left(\frac{\pi x}{L}\right)$

Mobius Hamiltonian: $f_{\mathrm{Möbius}}(x) = 1 - \tanh 2\theta \cos\left(\frac{2\pi x}{L}\right)$

1. Start from the boundary state (unentangled state),
   and the thermal state (mixed state).

➡

Vacuum entanglement or correlation approximately emerge or are recovered.

(also quantum revival.)

Short summary of my talk

1.We explored the dynamical property of the system evolved with inhomogeneous Hamiltonians:

$$H_{\mathrm{Inho}} = \int_0^L dx\, f(x) h(x)$$

SSD Hamiltonian: $f_{\mathrm{SSD}}(x) = 2\sin^2\left(\frac{\pi x}{L}\right)$

Mobius Hamiltonian: $f_{\text{Möbius}}(x) = 1 - \tanh 2\theta \cos\left(\frac{2\pi x}{L}\right)$

1. Start from the boundary state (unentangled state),
   and the thermal state (mixed state).

➡

Vacuum entanglement or correlation approximately emerge or are recovered.

(also quantum revival.)

2. Start from the thermofield double state or page state.

Short summary of my talk

1.We explored the dynamical property of the system evolved with inhomogeneous Hamiltonians:

$$H_{\mathrm{Inho}} = \int_0^L dx\, f(x) h(x)$$

SSD Hamiltonian: $f_{\mathrm{SSD}}(x) = 2\sin^2\left(\frac{\pi x}{L}\right)$

Mobius Hamiltonian: $f_{\text{Möbius}}(x) = 1 - \tanh 2\theta \cos\left(\frac{2\pi x}{L}\right)$

1. Start from the boundary state (unentangled state), and the thermal state (mixed state).

➡ **Vacuum entanglement or correlation approximately emerge or are recovered.**

(also quantum revival.)

2. Start from the thermofield double state or page state.

➡ **Non-local correlation emerges. State posses interesting properties.**

Short summary of my talk

2.We explored the dynamical property of the thermal state whose distribution is determined by the Mobius Hamiltonian:

$$H_{\text{Inho}} = \int_0^L dx f(x) h(x)$$

Mobius Hamiltonian: $f_{\text{Möbius}}(x) = 1 - \tanh 2\theta \cos\left(\frac{2\pi x}{L}\right)$

⇕ **Equivalent**

CFT Hamiltonian on the curved background.

$$\boxed{\begin{aligned} ds^2 &= -f^2_{\text{Möbius}}(x,\theta)dt^2 + dx^2 \\ R(x,\theta) &= \frac{8\pi^2 \tanh 2\theta \cos\left(\frac{2\pi x}{L}\right)}{L^2\left(\tanh 2\theta \cos\left(\frac{2\pi x}{L}\right) - 1\right)} \end{aligned}}$$

Short summary of my talk

2.We explored the dynamical property of the thermal state whose distribution is determined by the Mobius Hamiltonian:

$$H_{\rm Inho} = \int_0^L dx f(x) h(x)$$

Mobius Hamiltonian: $f_{\text{Möbius}}(x) = 1 - \tanh 2\theta \cos\left(\frac{2\pi x}{L}\right)$

Equivalent

CFT Hamiltonian on the curved background.

$$ds^2 = -f^2_{\text{Möbius}}(x,\theta)dt^2 + dx^2$$

$$R(x,\theta) = \frac{8\pi^2 \tanh 2\theta \cos\left(\frac{2\pi x}{L}\right)}{L^2\left(\tanh 2\theta \cos\left(\frac{2\pi x}{L}\right) - 1\right)}$$

By varyingθ ,the system in 2d holographic CFT may exhibit phase transition.

Short summary of my talk

2.We explored the dynamical property of the thermal state whose distribution is determined by the Mobius Hamiltonian:

$$H_{\mathrm{Inho}} = \int_0^L dx f(x) h(x)$$

Mobius Hamiltonian: $f_{\text{Möbius}}(x) = 1 - \tanh 2\theta \cos\left(\frac{2\pi x}{L}\right)$

Equivalent

CFT Hamiltonian on the curved background.

$$ds^2 = -f^2_{\text{Möbius}}(x,\theta) dt^2 + dx^2$$

$$R(x,\theta) = \frac{8\pi^2 \tanh 2\theta \cos\left(\frac{2\pi x}{L}\right)}{L^2\left(\tanh 2\theta \cos\left(\frac{2\pi x}{L}\right) - 1\right)}$$

By varying θ,the system may exhibit phase transition thank to the entanglement phase transition.

Note

All the theories considered in this talk are two-dimensional conformal field theories.

Introduction

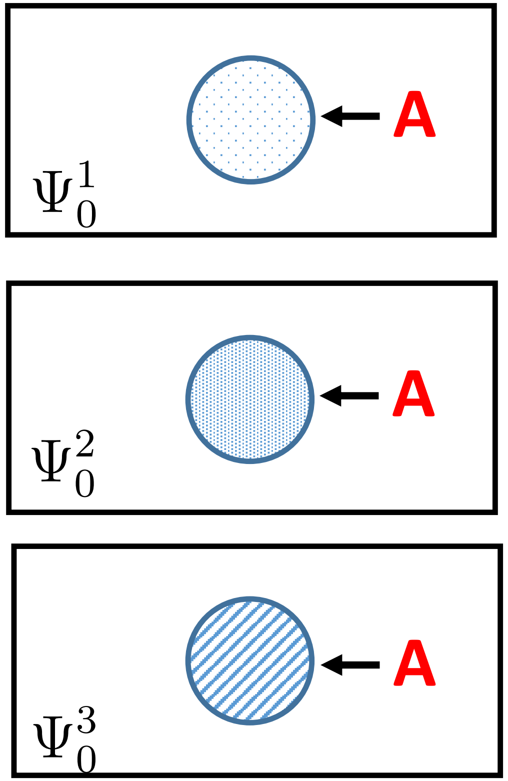
Scrambling is one of the cutting-edge research topics.

This is closely relevant to **quantum thermalization**.

Introduction

Scrambling is one of the cutting-edge research topics.

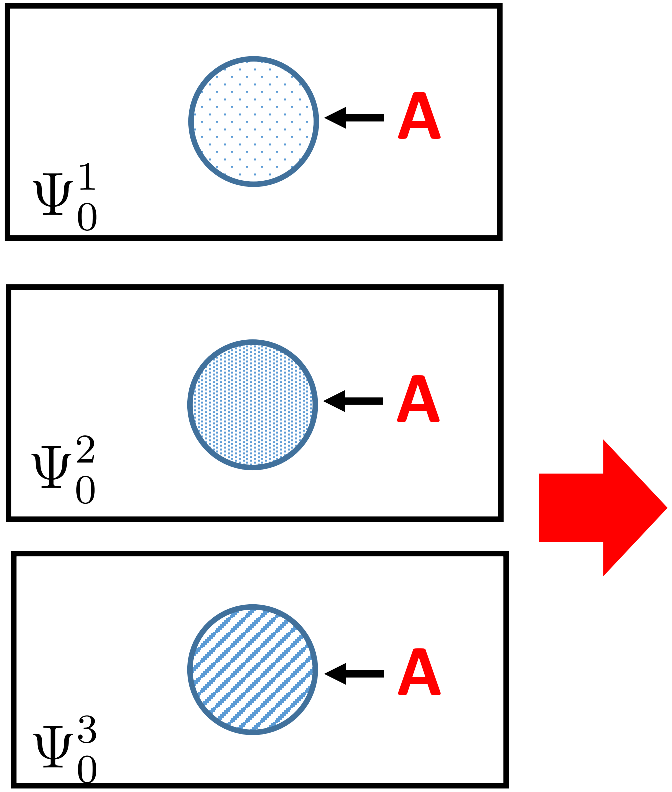
This is closely relevant to **quantum thermalization**.



$\left|\Psi_0^i\right\rangle$ are initial states.

Introduction

Scrambling is one of the cutting-edge research topics.
This is closely relevant to **quantum thermalization**.



$\left|\Psi_0^i\right\rangle$ are initial states.

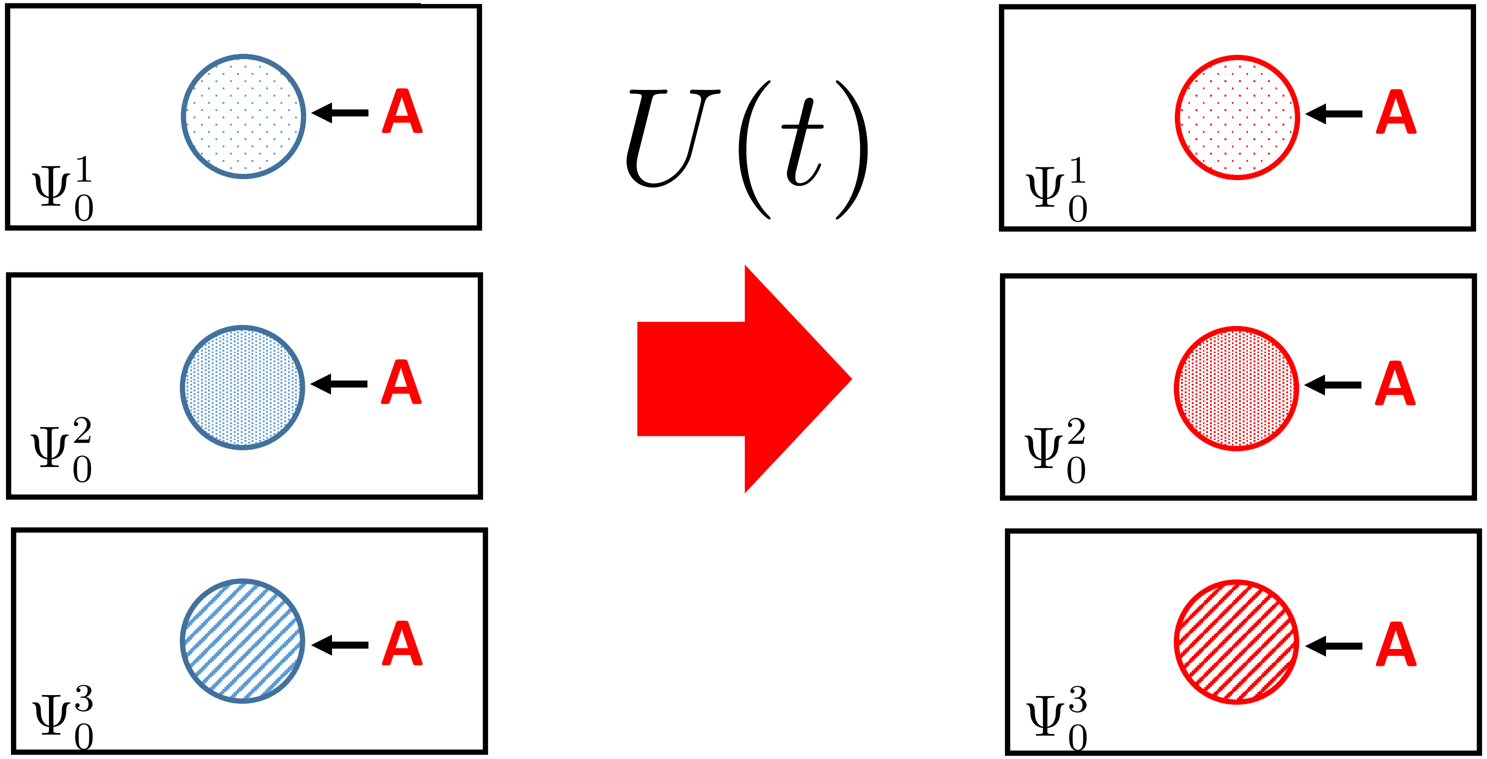
Local observables in A
depend on initial condition.

Introduction

Scrambling is one of the cutting-edge research topics.

This is closely relevant to **quantum thermalization**.

$$\left|\Psi^i(t)\right\rangle = U(t)\left|\Psi^i_0\right\rangle$$

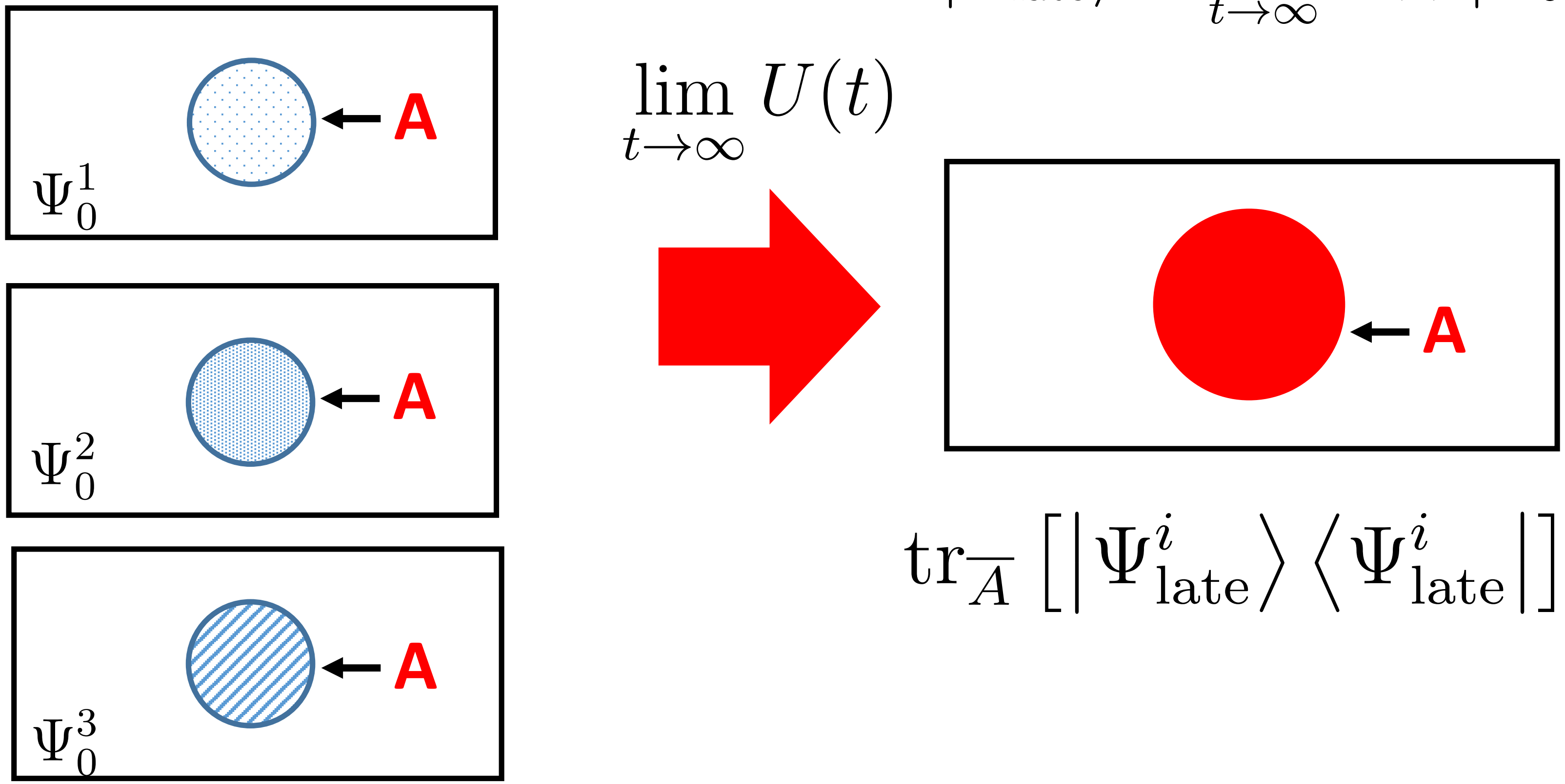


Introduction

Scrambling is one of the cutting-edge research topics.

This is closely relevant to **quantum thermalization**.

$$\left|\Psi^i_{\text{late}}\right\rangle = \lim_{t\to\infty} U(t)\left|\Psi^i_0\right\rangle$$



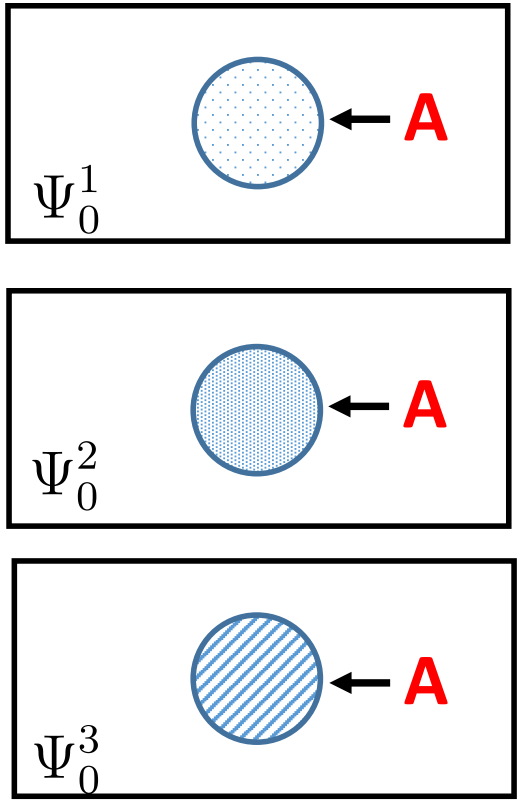
$$\mathrm{tr}_{\overline{A}}\left[\left|\Psi^i_{\text{late}}\right\rangle\left\langle\Psi^i_{\text{late}}\right|\right] \approx \mathrm{tr}_{\overline{A}}e^{-\beta H}$$

Introduction

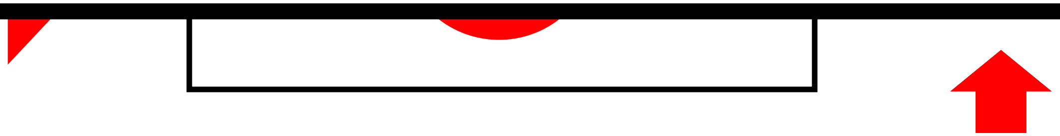
Scrambling is one of the cutting-edge research topics.

This is closely relevant to **quantum thermalization**.

$$\left|\Psi^i_{\rm late}\right\rangle = \lim U(t)\left|\Psi^i_0\right\rangle$$



This late-time reduced density matrix associated to A is **independent of initial state.**



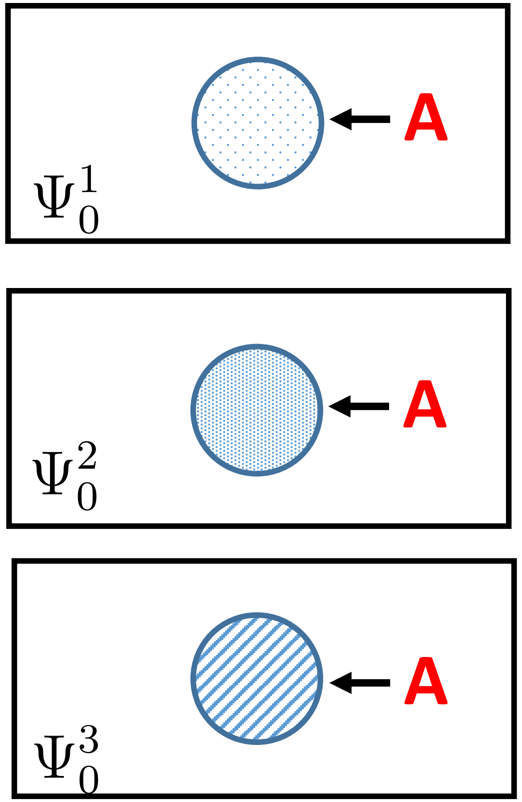
$$\mathrm{tr}_{\overline{A}}\left[\left|\Psi^i_{\rm late}\right\rangle\left\langle\Psi^i_{\rm late}\right|\right] \approx \underline{\mathrm{tr}_{\overline{A}}e^{-\beta H}}$$

Introduction

Scrambling is one of the cutting-edge research topics.

This is closely relevant to **quantum thermalization**.

$$\left|\Psi^i_{\rm late}\right\rangle = \lim U(t)\left|\Psi^i_0\right\rangle$$



This late-time reduced density matrix associated to A is **independent of initial state.**

Information on the initial states is locally hidden (effectively lost).

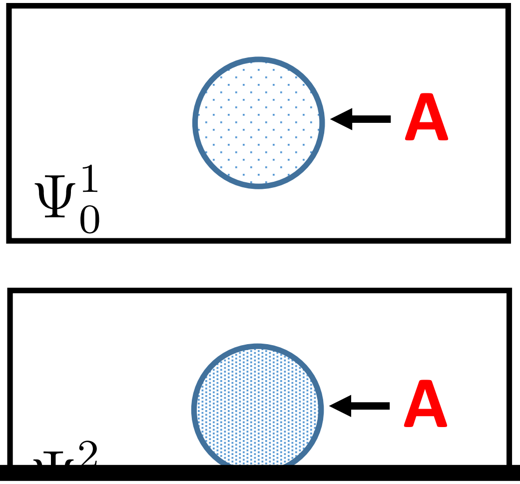
$$\mathrm{tr}_{\overline{A}}\left[\left|\Psi^i_{\rm late}\right\rangle\left\langle\Psi^i_{\rm late}\right|\right] \approx \underline{\mathrm{tr}_{\overline{A}}e^{-\beta H}}$$

Introduction

Scrambling is one of the cutting-edge research topics.

This is closely relevant to **quantum thermalization**.

$$|\Psi^i_{\text{late}}\rangle = \lim U(t)\,|\Psi^i_0\rangle$$



This late-time reduced density matrix associated to A is **independent of initial state.**

Information on the initial states is locally hidden (effectively lost).

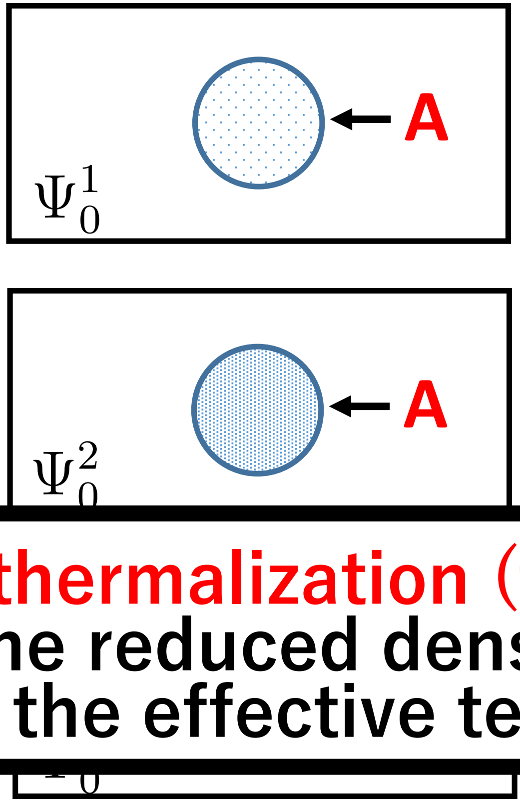
This is the process of (information) scrambling.

Introduction

Scrambling is one of the cutting-edge research topics.

This is closely relevant to **quantum thermalization**.

$$\left|\Psi^{i}_{\mathrm{late}}\right\rangle = \lim U(t)\left|\Psi^{i}_{0}\right\rangle$$



This late-time reduced density matrix associated to A is **independent of initial state.**

Information on the initial states is locally hidden (effectively lost).

Quantum thermalization (the thermalization of the subsystems) occurs because the reduced density matrices are approximated by the thermal state with the effective temperature.

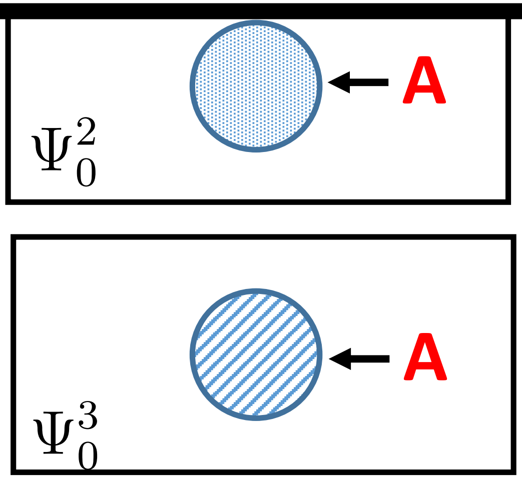
Introduction

Scrambling is one of the cutting-edge research topics.

This is closely relevant to **quantum thermalization**

It is believed that if the Hamiltonian has a strong scrambling ability, information scrambling (quantum thermalization) occurs.

hidden (lost).



$$\mathrm{tr}_{\overline{A}}\left[\left|\Psi^i_{\mathrm{late}}\right\rangle\left\langle\Psi^i_{\mathrm{late}}\right|\right] \approx \mathrm{tr}_{\overline{A}} e^{-\beta H}$$

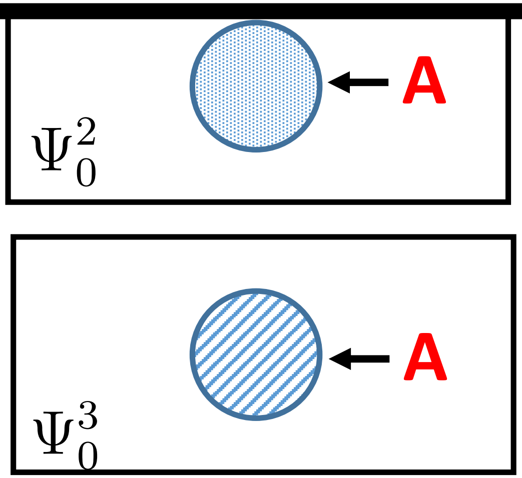
Introduction

Scrambling is one of the cutting-edge research topics.

This is closely relevant to **quantum thermalization**.

It is believed that 2d holographic CFT, CFT having gravity dual has such strong scrambling ability.

hidden (lost).



$$\mathrm{tr}_{\overline{A}}\left[\left|\Psi^i_{\mathrm{late}}\right\rangle\left\langle\Psi^i_{\mathrm{late}}\right|\right] \approx \mathrm{tr}_{\overline{A}}e^{-\beta H}$$

Non-local correlation

Let us consider the behavior of non-local correlation during the quantum thermalization,

The behavior of the reduced density: $\mathrm{tr}_{\overline{A}}\left(e^{-iHt}\left|\Psi\right\rangle\left\langle\Psi\right|e^{iHt}\right)\approx\mathrm{tr}_{\overline{A}}e^{-\epsilon H}$

Non-local correlation

Let us consider the behavior of non-local correlation during the quantum thermalization,

The behavior of the reduced density: $\mathrm{tr}_{\overline{A}}\left(e^{-iHt}\left|\Psi\right\rangle\left\langle\Psi\right|e^{iHt}\right)\approx\mathrm{tr}_{\overline{A}}e^{-\epsilon H}$

Divide the Hilbert space into A, B, and the complement to A and B,

$$\frac{L}{2}>l_A+l_B>0$$

Non-local correlation

Let us consider the behavior of non-local correlation during the quantum thermalization,

The behavior of the reduced density: $\mathrm{tr}_{\overline{A}}\left(e^{-iHt}\left|\Psi\right\rangle\left\langle\Psi\right|e^{iHt}\right)\approx\mathrm{tr}_{\overline{A}}e^{-\epsilon H}$

Divide the Hilbert space into A, B, and the complement to A and B,

$\frac{L}{2}>l_A+l_B>0$

⬇ ***Quantum thermalization occurs.***

$$S_A\approx l_A, S_B\approx l_B, S_{A\cup B}\approx(l_A+l_B)$$

,where $1\gg\epsilon$: an effective inverse temperature

Non-local correlation

Let us consider the behavior of non-local correlation during the quantum thermalization,

The behavior of the reduced density: $\mathrm{tr}_{\overline{A}}\left(e^{-iHt}\left|\Psi\right\rangle\left\langle\Psi\right|e^{iHt}\right) \approx \mathrm{tr}_{\overline{A}}e^{-\epsilon H}$

Divide the Hilbert space into A, B, and the complement to A and B,

$$\frac{L}{2} > l_A + l_B > 0$$

↓ ***Quantum thermalization occurs.***

$$I_{A,B} = S_A + S_B - S_{A\cup B}$$

$$S_A \approx l_A, S_B \approx l_B, S_{A\cup B} \approx (l_A + l_B) \Rightarrow \boxed{I_{A,B} \underset{1\gg\epsilon}{\approx} 0}$$

,where $1 \gg \epsilon$: an effective inverse temperature

Non-local correlation

Let us consider the behavior of non-local correlation during the quantum thermalization,

The behavior of the reduced density: $\mathrm{tr}_{\overline{A}}\left(e^{-iHt}\left|\Psi\right\rangle\left\langle\Psi\right|e^{iHt}\right)\approx\mathrm{tr}_{\overline{A}}e^{-\epsilon H}$

Divide the Hilbert space into A, B, and the complement to A and B,

Quantum thermalization occurs.

$$I_{A,B}=S_A+S_B-S_{A\cup B}$$

$$S_A\approx l_A,S_B\approx l_B,S_{A\cup B}\approx(l_A+l_B)$$

,where $1\gg\epsilon$: an effective inverse temperature

$$\boxed{I_{A,B}\underset{1\gg\epsilon}{\approx}0}$$

$$\boxed{\rho_{A\cup B}\underset{1\gg\epsilon}{\approx}\rho_A\otimes\rho_B}$$

Non-local correlation

Let us consider the behavior of non-local correlation during the quantum thermalization,

The behavi

Non-local correlation is destroyed by the dynamics.

Divide the Hilbert space into A, B, and the complement to A and B

↓ ***Quantum thermalization occurs.***

$I_{A,B} = S_A + S_B - S_{A\cup B}$

$$S_A \approx l_A, S_B \approx l_B, S_{A\cup B} \approx (l_A + l_B) \rightarrow \boxed{I_{A,B} \underset{1 \gg \epsilon}{\approx} 0}$$

$$\rightarrow \boxed{\rho_{A\cup B} \underset{1 \gg \epsilon}{\approx} \rho_A \otimes \rho_B}$$

↑

Relation to our papers

Meaning of $\rho_{A\cup B} \underset{1\gg\epsilon}{\approx} \rho_A \otimes \rho_B$

➡ **No non-local correlation between A and B**

For example, $\langle \mathcal{O}(X_1 \in A)\mathcal{O}(X_2 \in B)\rangle \underset{1\gg\epsilon}{\approx} \langle \mathcal{O}(X_1 \in A)\rangle \times \langle \mathcal{O}(X_2 \in B)\rangle$

Relation to our papers

Meaning of $\rho_{A\cup B} \underset{1 \gg \epsilon}{\approx} \rho_A \bigotimes \rho_B$

➡ **No non-local correlation between A and B**

For example, $\langle \mathcal{O}(X_1 \in A)\mathcal{O}(X_2 \in B)\rangle \underset{1 \gg \epsilon}{\approx} \langle \mathcal{O}(X_1 \in A)\rangle \times \langle \mathcal{O}(X_2 \in B)\rangle$

➡ **Thus, quantum properties (here, non-local correlation) may be destroyed during quantum thermalization**

Relation to our papers

- **Thus, quantum properties (here, non-local correlation) may be completely destroyed during quantum thermalization.**
- **New research topic is to explore the non-equilibrium processes or quantum quench where the state has quantum properties even in the late times.**

for example, Many-body-localization (MBL)

Quantum-many-body-scars

This may lead to the non-equilibrium phenomena beyond statical mechanics.

This may be applicable for the quantum computation.

Why we consider this new topic?

1. These phenomena may be beyond the statical mechanics.

2. These phenomena lead to implementation of quantum computer (quantum computation=non-equilibrium process)

3. **In AdS/CFT, this may lead to new finding about black holes**

Why we consider this new topic?

1. These phenomena may be beyond the statical mechanics.

2. These phenomena lead to implementation of quantum computer (quantum computation=non-equilibrium process)

3. In AdS/CFT, this may lead to new finding about black holes

Thermal state ⟷ **Black hole**

Why we consider this new topic?

1. These phenomena may be beyond the statical mechanics.

2. These phenomena lead to implementation of quantum computer (quantum computation=non-equilibrium process)

3. In AdS/CFT, this may lead to new finding about black hole

Inhomogeneous time evolution of thermal state ↔ ?

Thermal state on the curved background (on-going)

- New thermodynamical properties related to curvature.
- Insight on the quantum matters near the black hole horizon.

Contents

Note

The parameter region considered in this talk is

$$L \gg l_{\mathcal{V}}, t \gg \epsilon \gg 1,$$

where these parameters are dimensionless and their unit is the lattice spacing.

Note

The parameter region considered in this talk is

$$\underbrace{L}_{\text{System size}} \gg \underbrace{l_{\mathcal{V}}}_{\text{Subsystem size}}, \underbrace{t}_{\text{Time}} \gg \underbrace{\epsilon}_{\text{Inverse temperature}} \gg 1.$$

Note

All the theories considered in this talk are two-dimensional conformal field theories.

Motivation on these papers

Setup for the time dependent case:

Theories considered are 2d CFTs on spatial circle.

(Circumstance $= L$)

Start from

$$\rho = \frac{e^{-2\epsilon H}}{\mathrm{tr} e^{-2\epsilon H}}, \; |\Psi\rangle = \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H}}} \sum_a e^{-\epsilon H} |a\rangle_1 \otimes |a\rangle_2 \, ,$$

and $|\Psi\rangle = \dfrac{e^{-\epsilon H} |\mathrm{Bdy}\rangle}{\sqrt{\langle \mathrm{Bdy}| e^{-2\epsilon H} |\mathrm{Bdy}\rangle}}.$

Motivation on these papers

Thermal state

Setup for the time dependent case:

Theories considered are 2d CFTs on spatial circle.

(Circumstance $= L$)

Start from

$$\rho = \frac{e^{-2\epsilon H}}{\mathrm{tr} e^{-2\epsilon H}}, \left|\Psi\right\rangle = \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H}}} \sum_a e^{-\epsilon H} \left|a\right\rangle_1 \otimes \left|a\right\rangle_2 ,$$

and $$\left|\Psi\right\rangle = \frac{e^{-\epsilon H}\left|\mathrm{Bdy}\right\rangle}{\sqrt{\left\langle \mathrm{Bdy}\right| e^{-2\epsilon H}\left|\mathrm{Bdy}\right\rangle}}.$$

Motivation on these papers

Thermofield double state

Setup for the time dependent case:

Theories considered are 2d CFTs on spatial circle.

(Circumstance $= L$)

Start from

$$\rho = \frac{e^{-2\epsilon H}}{\mathrm{tr} e^{-2\epsilon H}}, \boxed{|\Psi\rangle = \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H}}} \sum_a e^{-\epsilon H} |a\rangle_1 \otimes |a\rangle_2},$$

and $|\Psi\rangle = \dfrac{e^{-\epsilon H}|\mathrm{Bdy}\rangle}{\sqrt{\langle \mathrm{Bdy}| e^{-2\epsilon H} |\mathrm{Bdy}\rangle}}.$

Motivation on these papers

Setup for the time dependent case:

Theories considered are 2d CFTs on spatial circle.

(Circumstance $= L$)

Start from

$$\rho = \frac{e^{-2\epsilon H}}{\mathrm{tr} e^{-2\epsilon H}}, \ |\Psi\rangle = \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H}}} \sum_a e^{-\epsilon H} |a\rangle_1 \otimes |a\rangle_2 ,$$

and $|\Psi\rangle = \frac{e^{-\epsilon H} |\mathrm{Bdy}\rangle}{\sqrt{\langle \mathrm{Bdy}| e^{-2\epsilon H} |\mathrm{Bdy}\rangle}}.$

Boundary state with regularization

Motivation on these papers

Setup for the time dependent case:

Theories considered are 2d CFTs on spatial circle.

(Circumstance = L)

Start from

$$\rho = \frac{e^{-2\epsilon H}}{\mathrm{tr} e^{-2\epsilon H}}, \quad |\Psi\rangle = \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H}}} \sum_a e^{-\epsilon H} |a\rangle_1 \otimes |a\rangle_2 ,$$

and $$|\Psi\rangle = \frac{e^{-\epsilon H} |\mathrm{Bdy}\rangle}{\sqrt{\langle \mathrm{Bdy}| e^{-2\epsilon H} |\mathrm{Bdy}\rangle}}.$$

They don't have non-local correlation.
Therefore, we can check whether or not Inhomogeneous evolution endow these states with non-local correlations.

Motivation on these papers

Setup for the time dependent case:

Theories considered are 2d CFTs on spatial circle.

Start from (Circumstance $= L$)

$$\rho = \frac{e^{-2\epsilon H}}{\mathrm{tr} e^{-2\epsilon H}}, \quad |\Psi\rangle = \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H}}} \sum_a e^{-\epsilon H} |a\rangle_1 \otimes |a\rangle_2 ,$$

and $|\Psi\rangle = \dfrac{e^{-\epsilon H} |\mathrm{Bdy}\rangle}{\sqrt{\langle \mathrm{Bdy}| e^{-2\epsilon H} |\mathrm{Bdy}\rangle}}$.

This has strong non-local correlation between Hilbert space one and two. We may be able to explore other property of this dynamics.

Motivation on these papers

Setup for the time dependent case:

Theories considered are 2d CFTs on spatial circle.

Start from

(Circumstance $=L$)

Undeformed(Uniform)

$$\rho = \frac{e^{-2\epsilon H}}{\mathrm{tr}e^{-2\epsilon H}}, \qquad |\Psi\rangle = \frac{1}{\sqrt{\mathrm{tr}e^{-2\epsilon H}}}\sum_a e^{-\epsilon H} \underline{|a\rangle}_1 \otimes \underline{|a\rangle}_2,$$

Eigenstates of H

$$\text{and } |\Psi\rangle = \frac{e^{-\epsilon H}|\mathrm{Bdy}\rangle}{\sqrt{\langle \mathrm{Bdy}| e^{-2\epsilon H}|\mathrm{Bdy}\rangle}}.$$

Motivation on these papers

Setup for the time dependent case:

Theories considered are 2d CFTs on spatial circle.

(Circumstance $= L$)

Start from

$$\rho = \frac{e^{-2\epsilon H}}{\mathrm{tr} e^{-2\epsilon H}}, \; |\Psi\rangle = \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H}}} \sum_a e^{-\epsilon H} |a\rangle_1 \otimes |a\rangle_2 ,$$

and $$|\Psi\rangle = \frac{\boxed{e^{-\epsilon H} |\mathrm{Bdy}\rangle}}{\sqrt{\langle \mathrm{Bdy}| e^{-2\epsilon H} |\mathrm{Bdy}\rangle}} .$$

Short entangled state where entanglement entropy does not depend on the subsystem size.

Motivation on these papers

Setup for the time dependent case:

Theories considered are 2d CFTs on spatial circle.

Start from (Circumstance $= L$)

$$\rho = \frac{e^{-2\epsilon H}}{\mathrm{tr} e^{-2\epsilon H}}, \; |\Psi\rangle = \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H}}} \sum_a e^{-\epsilon H} |a\rangle_1 \otimes |a\rangle_2 \,,$$

and $|\Psi\rangle = \dfrac{\boxed{e^{-\epsilon H} |\mathrm{Bdy}\rangle}}{\sqrt{\langle \mathrm{Bdy}| e^{-2\epsilon H} |\mathrm{Bdy}\rangle}}.$

This is easily preparable in the lab..

Motivation on these papers

Setup for the time dependent case:

Theories considered are 2d CFTs on spatial circle.

(Circumstance $= L$)

Start from

$$\rho = \frac{e^{-2\epsilon H}}{\mathrm{tr} e^{-2\epsilon H}}, \, |\Psi\rangle = \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H}}} \sum_a e^{-\epsilon H} |a\rangle_1 \otimes |a\rangle_2 \, ,$$

and $|\Psi\rangle = \frac{e^{-\epsilon H} |\mathrm{Bdy}\rangle}{\sqrt{\langle \mathrm{Bdy}| e^{-2\epsilon H} |\mathrm{Bdy}\rangle}}.$

Evolve the system with **the sine-square deformed Hamiltonian and Mobius Hamiltonian.**

Motivation on these papers

Setup for the time dependent case:

Theories considered are 2d CFTs on spatial circle.

(Circumstance $= L$)

Start from

$$\rho = \frac{e^{-2\epsilon H}}{\mathrm{tr} e^{-2\epsilon H}}, \left|\Psi\right\rangle = \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H}}} \sum_a e^{-\epsilon H} \left|a\right\rangle_1 \otimes \left|a\right\rangle_2 ,$$

and $\left|\Psi\right\rangle = \dfrac{e^{-\epsilon H}\left|\mathrm{Bdy}\right\rangle}{\sqrt{\left\langle\mathrm{Bdy}\right| e^{-2\epsilon H}\left|\mathrm{Bdy}\right\rangle}}.$

Explain later

Evolve the system with the sine-square deformed Hamiltonian and Mobius Hamiltonian.

Motivation on these papers

- In this setup, entanglement entropy, two point function and so on **can be analytically computable**.

Motivation on these papers

- In this setup, entanglement entropy, two point function and so on **can be analytically computable**.

⬇

- It is possible to **study non-equilibrium process in the larger system than that numerically-computable**.

Motivation on these papers

- In this setup, entanglement entropy, two point function and so on **can be analytically computable**.

⬇

- It is possible to **study non-equilibrium process in the larger system than that numerically-computable!!**

⬇

- **We can study the dynamical property, independent of the finite-size effect, of the system.**

Motivation on these papers

- In this setup, entanglement entropy, two point function and so on **can be analytically computable**.

This is the reason we consider 2d CFTs.

⬇

- **We can study the dynamical property, independent of the finite-size effect, of the system.**

Why we consider the inhomogeneous time evolution

- The inhomogeneous Hamiltonians considered are Mobius and Sine-square deformed (SSD).

Why we consider the inhomogeneous time evolution

- The inhomogeneous Hamiltonians considered are Mobius and Sine-square deformed (SSD).

↕

- The CFT Hamiltonians on the curved background.

Why we consider the inhomogeneous time evolution

- The inhomogeneous Hamiltonians considered are Mobius and Sine-square deformed (SSD).

↕

- The CFT Hamiltonians on the curved background.

➡ **Quasiparicles (excitations generated during the time evolution) may move with the velocity determined by the geometry, and distributes inhomogeneously.**

Why we consider the inhomogeneous time evolution

- The inhomogeneous Hamiltonians considered are Mobius and Sine-square deformed (SSD).

⇕

- The CFT Hamiltonians on the curved background.

➡ **Quasiparicles (excitations generated during the time evolution) may move with the velocity determined by, and distributes inhomogeneously.**

➡ **Entanglement entropy (EE) and local temperature may depend on the location of the subsystems.**

Why we consider the inhomogeneous time evolution

- The CFT Hamiltonians on the curved background.

➡ **Quasiparicles (excitations generated during the time evolution) may move with the velocity determined by, and distributes inhomogeneously.**

➡ **Entanglement entropy (EE) and local temperature may depend on the location of the subsystems.**

➡ **Mutual information may become non-zero.**

➡ **Non-local correlation may emerge.**

For example

The relation between the thermofield double state and Bell state.

For example

The relation between the thermofield double state and Bell state.

Thermofield double state in high temperature limit is expected to be a product of Bell states.

$$|\mathrm{TFD}\rangle \approx \Pi_{\tilde{x}}\, |\mathrm{Bell};\tilde{x}\rangle_L\, |\mathrm{Bell};\tilde{x}\rangle_R$$

For example

The relation between the thermofield double state and Bell state.

Thermofield double state in high temperature limit is expected to be a product of Bell states.

$$|\mathrm{TFD}\rangle \approx \Pi_{\tilde{x}} \left|\mathrm{Bell};\tilde{x}\right\rangle_L \left|\mathrm{Bell};\tilde{x}\right\rangle_R$$

For example, for the infinite-temperature TFD in the spin system

**In the infinite-temperature limit,
the thermal state is given
by the identity.** ➡

$$e^{0\times H} = \mathbf{1} = \sum_{i_1=\uparrow,\downarrow} \cdots \sum_{i_L=\uparrow,\downarrow} |i_1,\cdots,i_L\rangle \langle i_1,\cdots,i_L|$$

For example

The relation between the thermofield double state and Bell state.

Thermofield double state in high temperature limit is expected to be a product of Bell states.

$$|\mathrm{TFD}\rangle \approx \Pi_{\tilde{x}} \left|\mathrm{Bell};\tilde{x}\right\rangle_L \left|\mathrm{Bell};\tilde{x}\right\rangle_R$$

For example, for the infinite-temperature TFD in the spin system

$$e^{0\times H} = \mathbf{1} = \sum_{i_1=\uparrow,\downarrow}\cdots\sum_{i_L=\uparrow,\downarrow} |i_1,\cdots,i_L\rangle\langle i_1,\cdots,i_L| \xRightarrow[\text{Purification}]{} |\mathrm{TFD}\rangle = \prod_{k=1}^{L}\left(\frac{1}{\sqrt{2}}\sum_{i_k=\uparrow,\downarrow}|i_k\rangle_1|i_k\rangle_2\right)$$

For example

The relation between the thermofield double state and Bell state.

Thermofield double state in high temperature limit is expected to be a product of Bell states.

$$|\mathrm{TFD}\rangle \approx \Pi_{\tilde{x}} \, |\mathrm{Bell};\tilde{x}\rangle_L \, |\mathrm{Bell};\tilde{x}\rangle_R$$

$$\boxed{\langle\cdot| \Rightarrow |\cdot\rangle}$$

For example, for the infinite-temperature TFD in the spin system

$$e^{0\times H} = \mathbf{1} = \sum_{i_1=\uparrow,\downarrow} \cdots \sum_{i_L=\uparrow,\downarrow} |i_1,\cdots,i_L\rangle \underline{\langle i_1,\cdots,i_L|} \xRightarrow[\text{Purification}]{} |\mathrm{TFD}\rangle = \prod_{k=1}^{L} \left(\frac{1}{\sqrt{2}} \sum_{i_k=\uparrow,\downarrow} |i_k\rangle_1 |i_k\rangle_2 \right)$$

For example

The relation between the thermofield double state and Bell state.

Thermofield double state in high temperature limit is expected to be a product of Bell states.

$$|\mathrm{TFD}\rangle \approx \Pi_{\tilde{x}} |\mathrm{Bell};\tilde{x}\rangle_L |\mathrm{Bell};\tilde{x}\rangle_R$$

On k-th site, $|\mathrm{Bell},k\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 |\uparrow\rangle_2 + |\downarrow\rangle_1 |\downarrow\rangle_2\right)$

For example, for the infinite-temperature TFD in the spin system

$$e^{0\times H} = \mathbf{1} = \sum_{i_1=\uparrow,\downarrow}\cdots\sum_{i_L=\uparrow,\downarrow} |i_1,\cdots,i_L\rangle\langle i_1,\cdots,i_L| \xrightarrow[\text{Purification}]{} |\mathrm{TFD}\rangle = \prod_{k=1}^{L}\left(\boxed{\frac{1}{\sqrt{2}}\sum_{i_k=\uparrow,\downarrow} |i_k\rangle_1 |i_k\rangle_2}\right)$$

Motivation on this paper

The relation between the thermofield double state and Bell state.

Thermofield double state in high temperature limit is expected to be a product of Bell states.

$$|\mathrm{TFD}\rangle \approx \Pi_{\tilde{x}} \, |\mathrm{Bell}; \tilde{x}\rangle_L \, |\mathrm{Bell}; \tilde{x}\rangle_R$$

For example, for the infinite-temperature TFD in the spin system

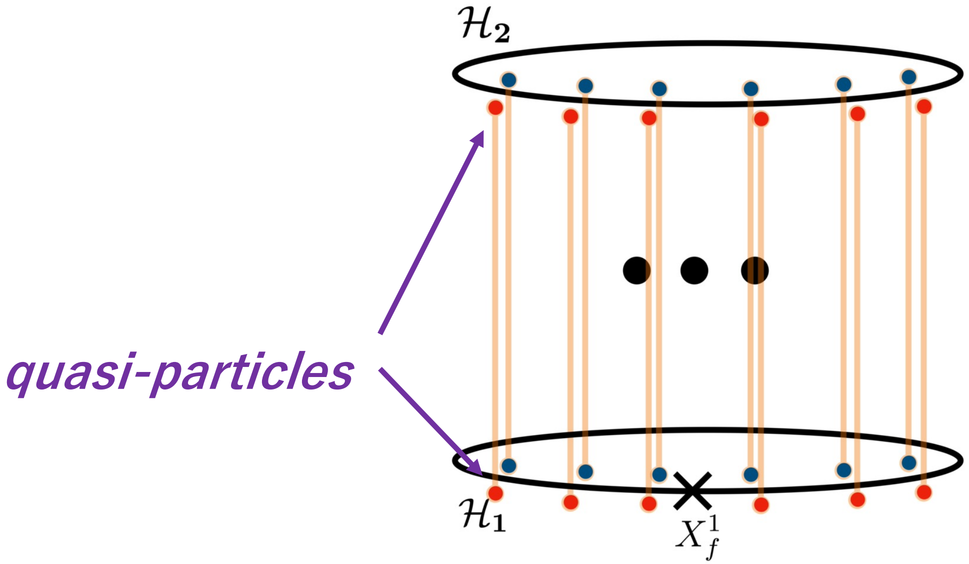
$$e^{0\times H} = \mathbf{1} = \sum_{i_1=\uparrow,\downarrow} \cdots \sum_{i_L=\uparrow,\downarrow} |i_1, \cdots, i_L\rangle \langle i_1, \cdots, i_L| \; \underset{\text{Purification}}{\Longrightarrow} \; |\mathrm{TFD}\rangle = \prod_{k=1}^{L} |\mathrm{Bell}, k\rangle$$

Motivation on this paper

The relation between the thermofield double state and Bell state.

Thermofield double state in high temperature limit is expected to be a product of Bell states.

$$|\mathrm{TFD}\rangle \approx \Pi_{\tilde{x}} \, |\mathrm{Bell}; \tilde{x}\rangle_L \, |\mathrm{Bell}; \tilde{x}\rangle_R$$



For example

For $|\Psi(t)\rangle = e^{-itH^{1}_{\text{inh}}}|\text{TFD}\rangle$

During the inhomogeneous time evolution, the quasiparticles on the first Hilbert space may propagate with the velocity determined by the geometry.

For example

For $|\Psi(t)\rangle = e^{-itH^{1}_{\text{inh}}}|\text{TFD}\rangle$

Acts on the first Hilbert space

During the inhomogeneous time evolution, the quasiparticles on the first Hilbert space may propagate with the velocity determined by the geometry.

For example

For $|\Psi(t)\rangle = e^{-itH^{1}_{\mathrm{inh}}}|\mathrm{TFD}\rangle$

During the inhomogeneous time evolution, the quasiparticles on the first Hilbert space may propagate with the velocity determined by the geometry.

Undeformed

$$ds^2 = -dt^2 + dx^2$$

Velocity: 1

→

Inhomogeneous

$$ds^2 = -f^2(x)dt^2 + dx^2$$

Mobius: $f_{\text{Möbius}}(x) = 1 - \tanh 2\theta \cos\left(\frac{2\pi x}{L}\right)$

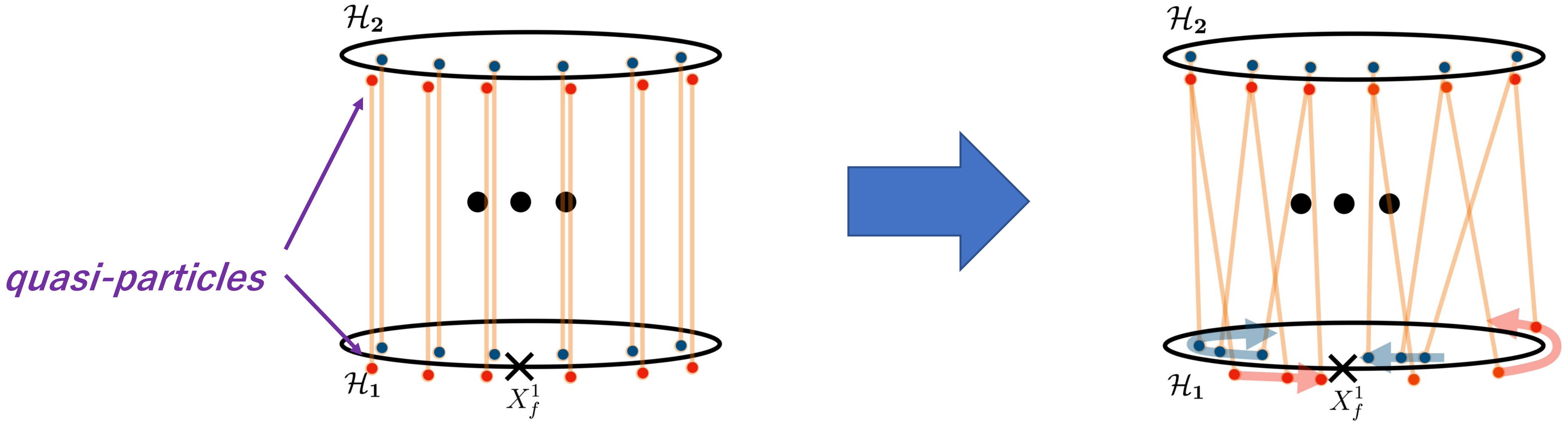
SSD: $f_{\mathrm{SSD}}(x) = 2\sin^2\left(\frac{\pi x}{L}\right)$

Velocity: $\frac{dx}{dt} = f(x)$

For example

For $|\Psi(t)\rangle = e^{-itH^1_{\mathrm{inh}}}|\mathrm{TFD}\rangle$

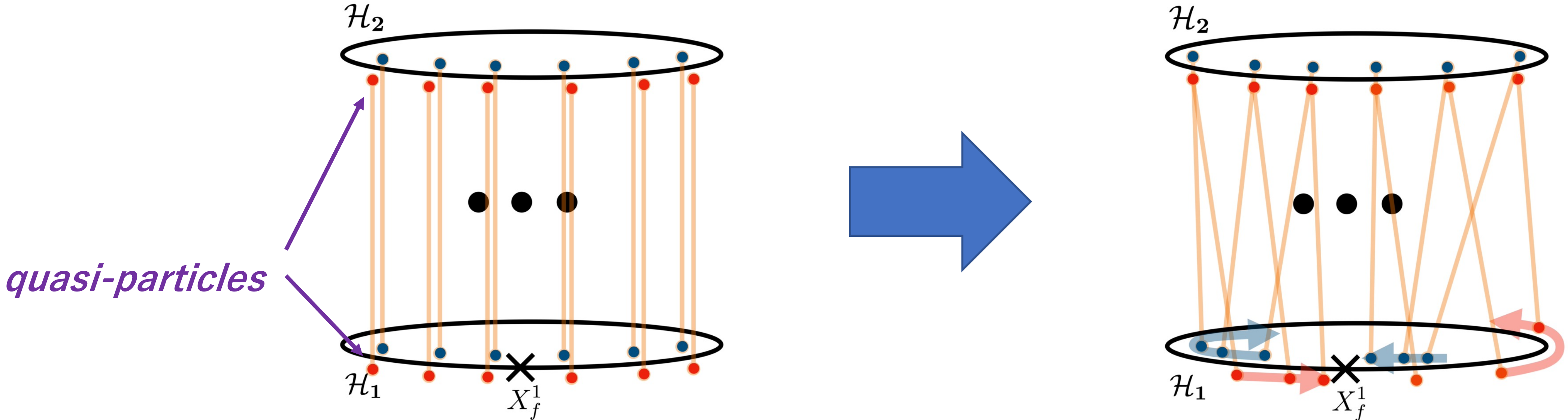
During the inhomogeneous time evolution, the quasiparticles on the first Hilbert space may propagate with the velocity determined by the geometry.



For example

During the inhomogeneous time evolution, the distribution of quasiparticle on the first Hilbert space may inhomogeneously change with time.

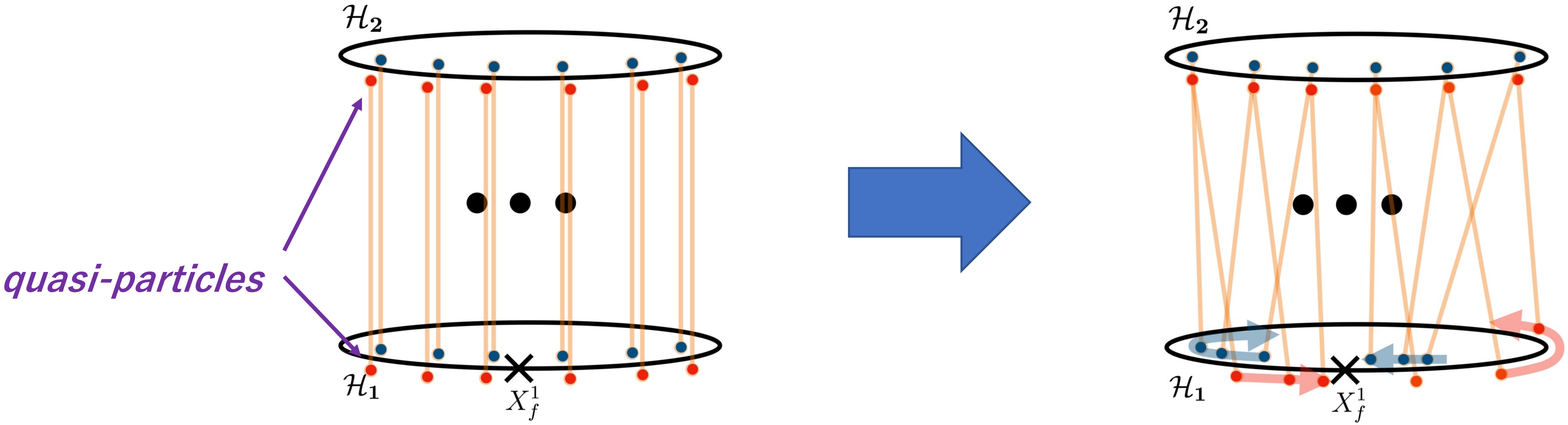
We assume that the number of quasiparticles in the subsystem determines EE. ➡ EE may depend on the position.



For example

During the inhomogeneous time evolution, the distribution of quasi-particle on the first Hilbert space may inhomogeneously change with time.

EE may depend on the position.

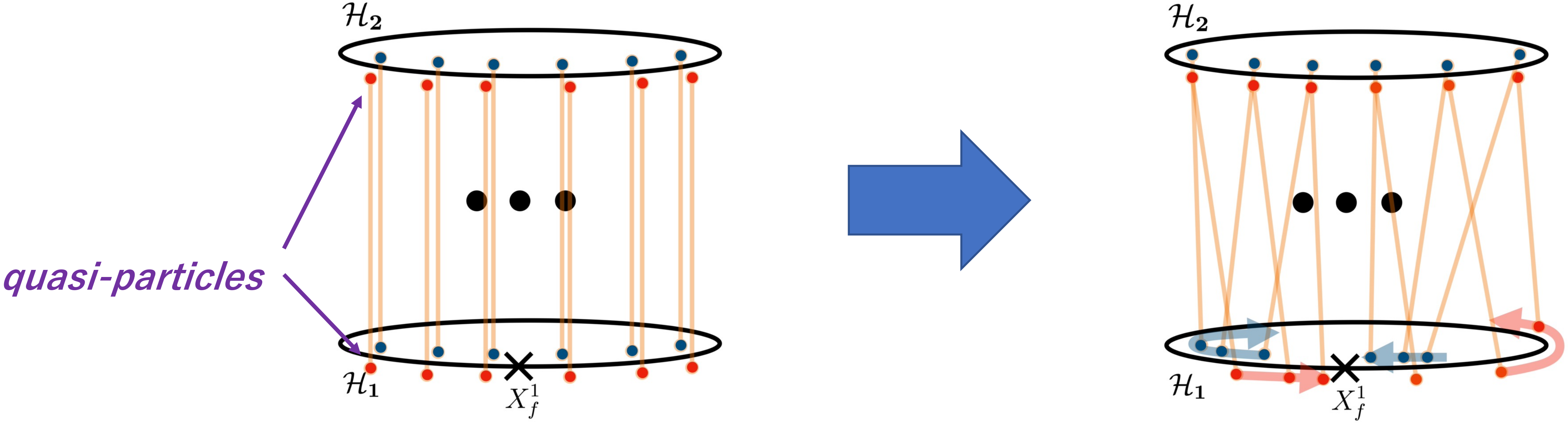


For example

During the inhomogeneous time evolution, the distribution of quasi-particle on the first Hilbert space may inhomogeneously change with time.

EE may depend on the position.

$$\boxed{S_\alpha \neq l_\alpha, \quad I_{A,B} \neq 0}$$

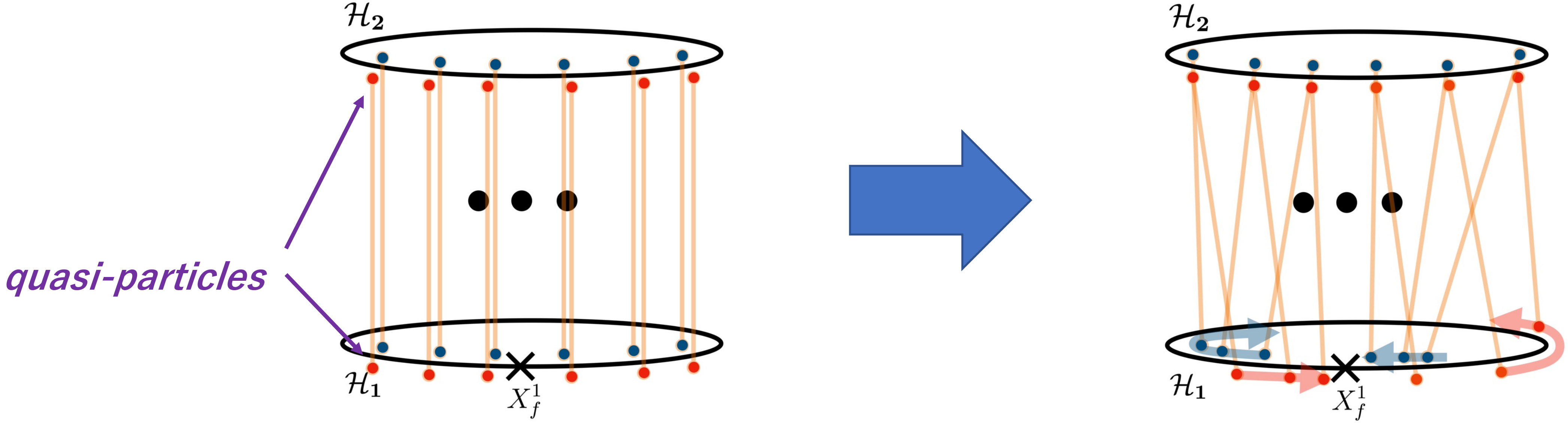


For example

During the inhomogeneous time evolution, the distribution of quasi-particle on the first Hilbert space may inhomogeneously change with time.

EE may depend on the position.

Quantum property (non-local correlation) may locally recover or emerges.



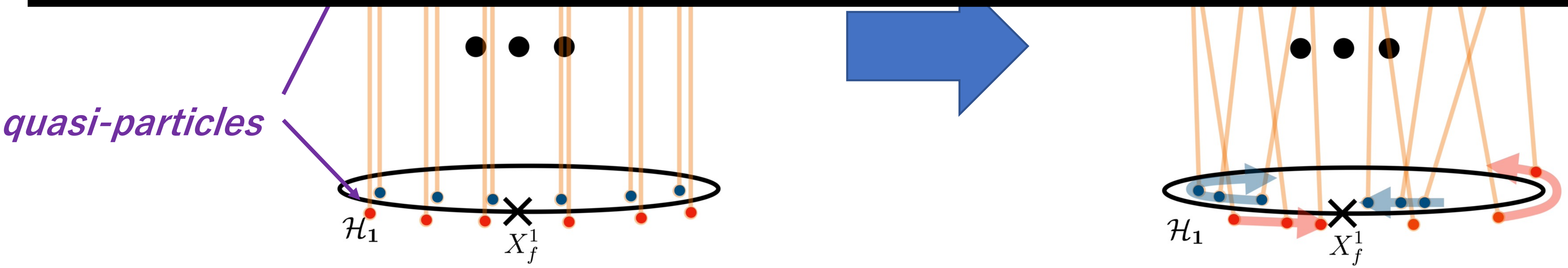
For example

During the inhomogeneous time evolution, the distribution of quasi-particle on the first Hilbert space may inhomogeneously change with time.

Temperature may depend on the

Motivation : SSD/Mobius quenches may make the system have the temperature gradient(inhomogeneity of quasi-particle).
Quantum nature may emerge.

El



Motivation on the thermodynamics

In the time dependent case, **the entanglement entropy depends on the location of the subsystem.**

Motivation on the thermodynamics

In the time dependent case, **the entanglement entropy depends on the location of the subsystem.**

↓

Even for the thermal state, **the entanglement entropy depends on the location of the subsystem.**

Motivation on the thermodynamics

In the time dependent case, **the entanglement entropy depends on the location of the subsystem.**

↓

Even for the thermal state, the entanglement entropy depends on the location of the subsystem.

↓

The local temperature may depend on the location.

Motivation on the thermodynamics

In the time dependent case, **the entanglement entropy depends on the location of the subsystem.**

↓

Even for the thermal state, the entanglement entropy depends on the location of the subsystem.

↓

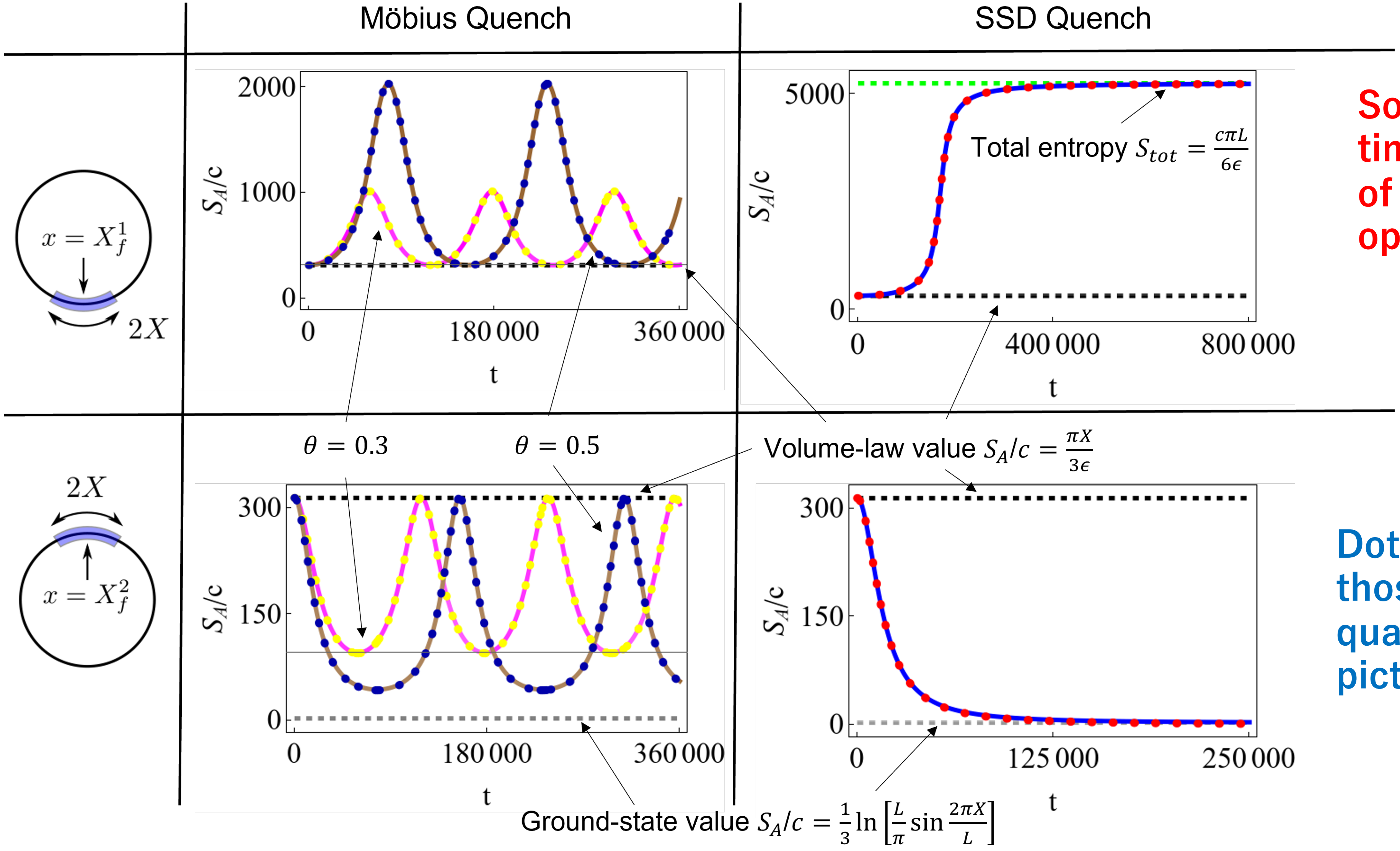
The local temperature may depend on the location.

↓

Some interesting *local phenomena* (for example, entanglement phase transition) occur.

Summary 1: Preparation of nearly vacuum state by checking with EE

Evolution from the thermal state: $\rho = \dfrac{e^{-2\epsilon H}}{\mathrm{tr} e^{-2\epsilon H}}$



Total entropy $S_{tot} = \frac{c\pi L}{6\epsilon}$

$\theta = 0.3$

$\theta = 0.5$

Volume-law value $S_A/c = \frac{\pi X}{3\epsilon}$

Ground-state value $S_A/c = \frac{1}{3}\ln\left[\frac{L}{\pi}\sin\frac{2\pi X}{L}\right]$

Solid lines are the time-dependences of EE in the twist operator formalism.

Dotted lines are those of EE in the quasi-particle picture.

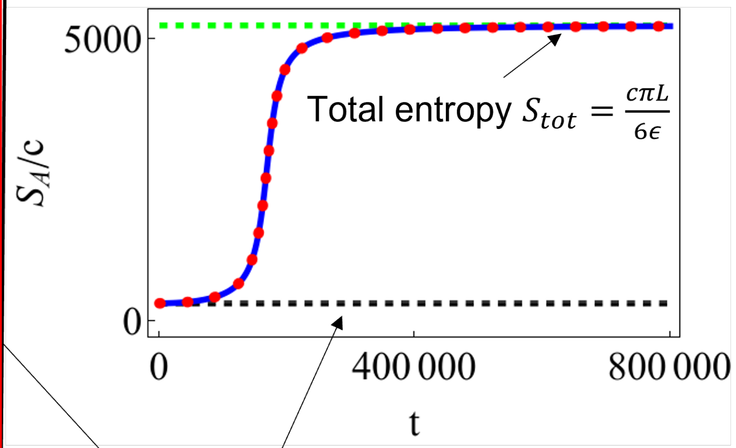
Summary 1: Preparation of nearly vacuum state by checking with EE

Evolution from the thermal state: $\rho = \frac{e^{-2\epsilon H}}{\mathrm{tr} e^{-2\epsilon H}}$

Möbius Quench

SSD Quench

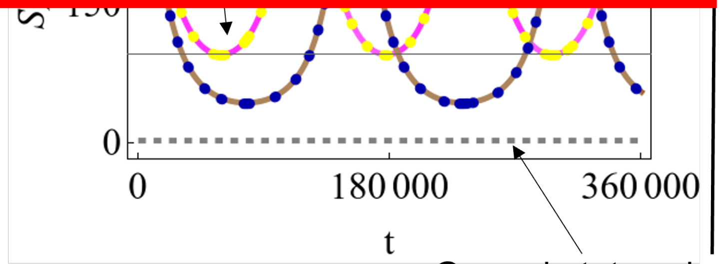
During SSD evolution, if subsystem **doesn’t include the origin（a fixed point）,** the entanglement entropy is approximated by **the vacuum entanglement entropy**.

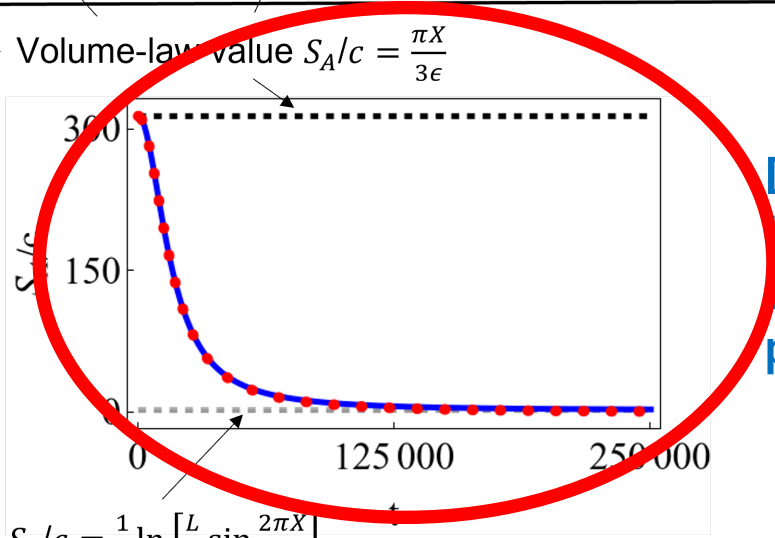


Total entropy $S_{tot} = \frac{c\pi L}{6\epsilon}$

Solid lines are the time-dependences of EE in the twist operator formalism.

Volume-law value $S_A/c = \frac{\pi X}{3\epsilon}$





Dotted lines are those of EE in the quasi-particle picture.

Ground-state value $S_A/c = \frac{1}{3}\ln\left[\frac{L}{\pi}\sin\frac{2\pi X}{L}\right]$

Summary 1: Preparation of nearly vacuum state by checking with EE

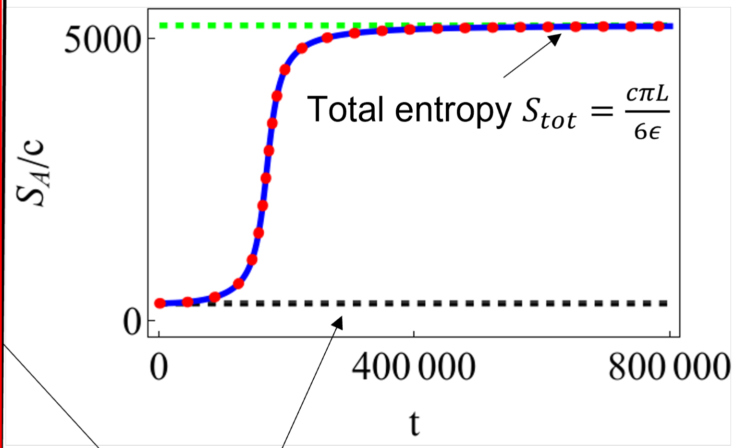
Evolution from the thermal state: $\rho = \dfrac{e^{-2\epsilon H}}{\mathrm{tr} e^{-2\epsilon H}}$

Möbius Quench

SSD Quench

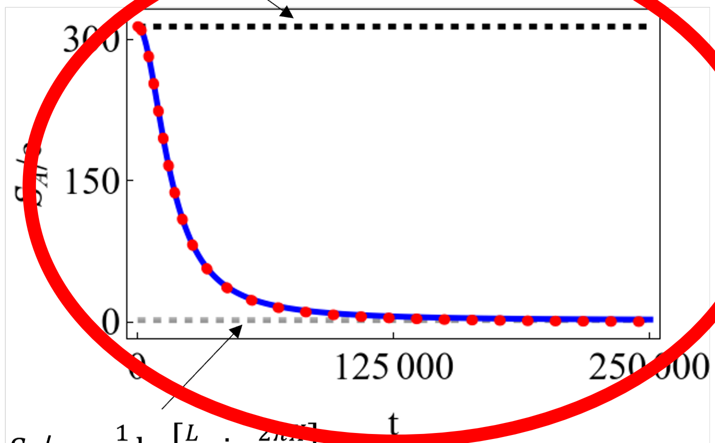
During SSD evolution, if subsystem **doesn’t include the origin（a fixed point）**, the entanglement entropy is approximated by **the vacuum entanglement entropy**.

Reduced density matrix is approximated by vacuum one.



Solid lines are the time-dependences of EE in the twist operator formalism.

Volume-law value $S_A/c = \frac{\pi X}{3\epsilon}$

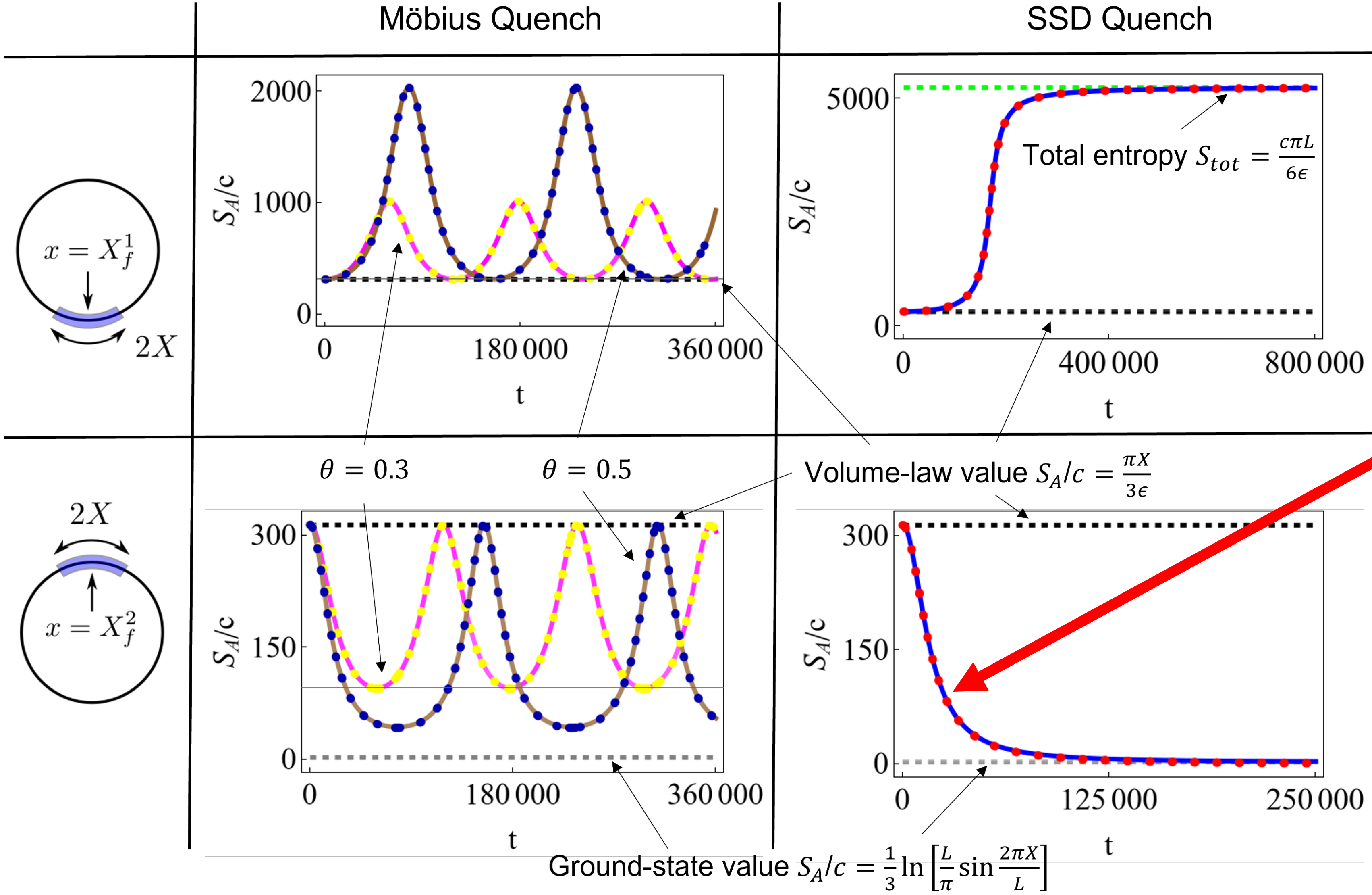


Ground-state value $S_A/c = \frac{1}{3}\ln\left[\frac{L}{\pi}\sin\frac{2\pi X}{L}\right]$

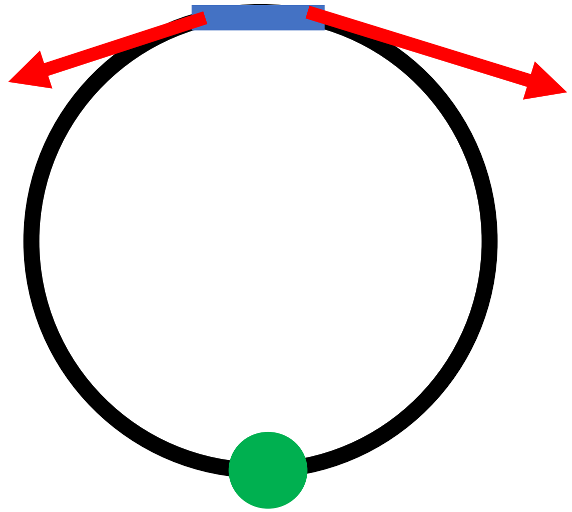
Dotted lines are those of EE in the quasi-particle picture.

Summary 1: Preparation of nearly vacuum state by checking with EE

Evolution from the thermal state: $\rho = \frac{e^{-2\epsilon H}}{\mathrm{tr} e^{-2\epsilon H}}$



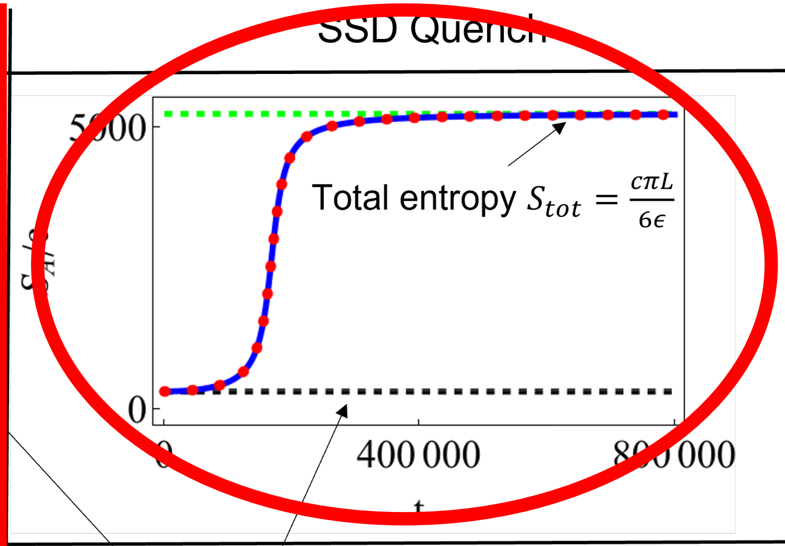
Quasi-particles move out from subsystem. Therefore, EE decreases with time.



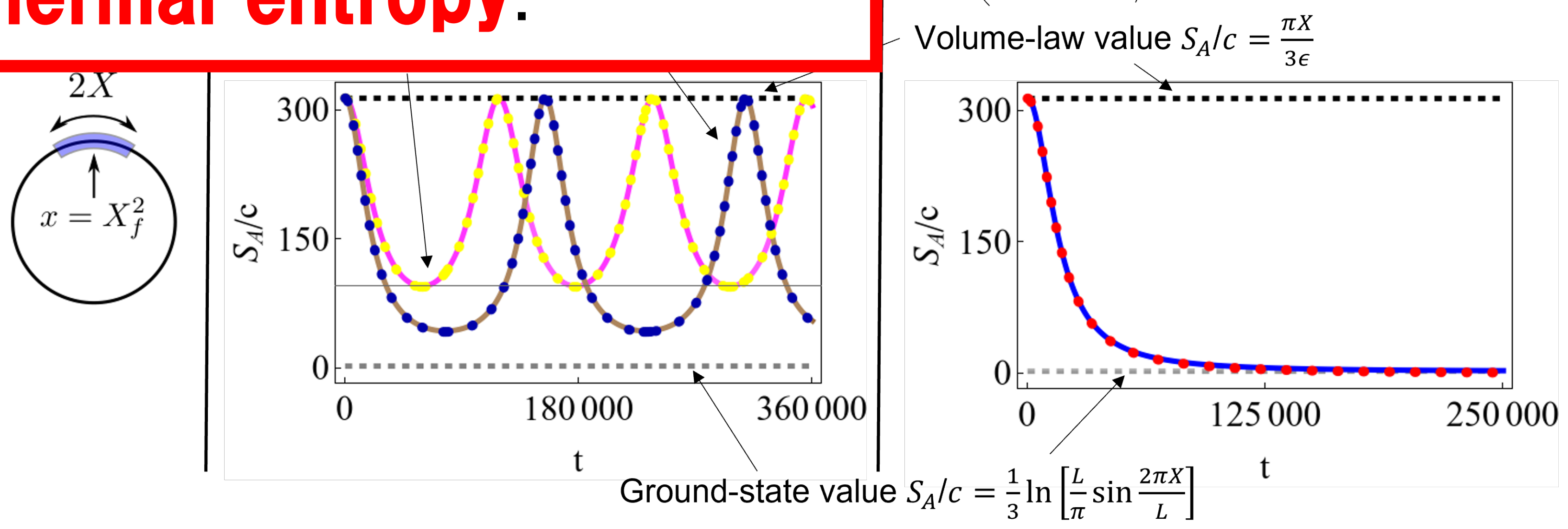
Summary 1: Preparation of nearly vacuum state by checking with EE

Evolution from the thermal state: $\rho = \frac{e^{-2\epsilon H}}{\mathrm{tr} e^{-2\epsilon H}}$

During SSD evolution, if subsystem **does include the origin（a fixed point）,** the entanglement entropy is approximated by **the thermal entropy**.



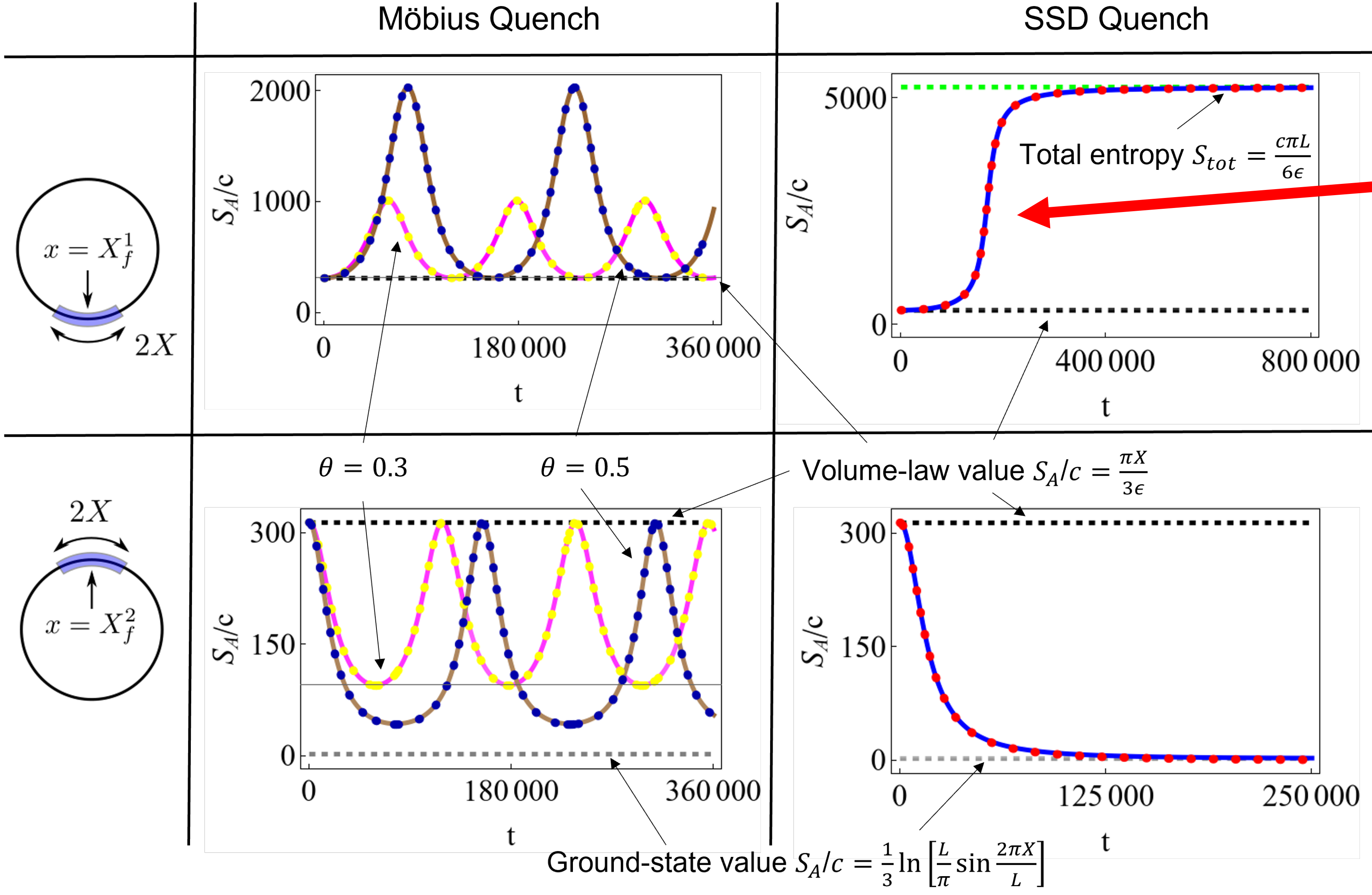
Solid lines are the time-dependences of EE in the twist operator formalism.



Dotted lines are those of EE in the quasi-particle picture.

Summary 1: Preparation of nearly vacuum state by checking with EE

Evolution from the thermal state: $\rho = \frac{e^{-2\epsilon H}}{\mathrm{tr} e^{-2\epsilon H}}$



Möbius Quench

SSD Quench

Total entropy $S_{tot} = \frac{c\pi L}{6\epsilon}$

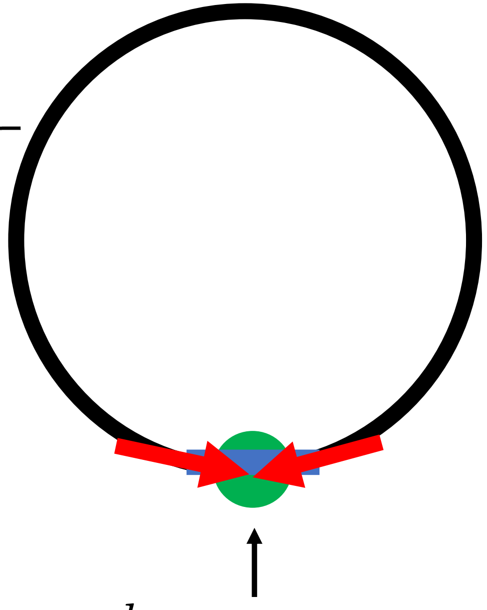
$\theta = 0.3$

$\theta = 0.5$

Volume-law value $S_A/c = \frac{\pi X}{3\epsilon}$

Ground-state value $S_A/c = \frac{1}{3}\ln\left[\frac{L}{\pi}\sin\frac{2\pi X}{L}\right]$

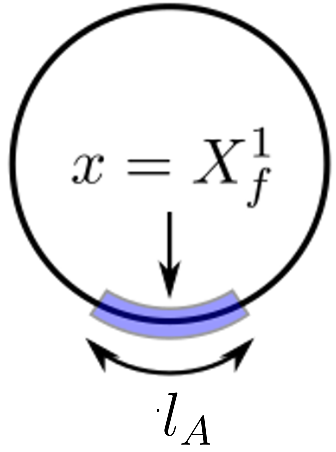
All quasiparticles accumulate around $x = X_f^1$. Therefore, EE approaches to thermal entropy.

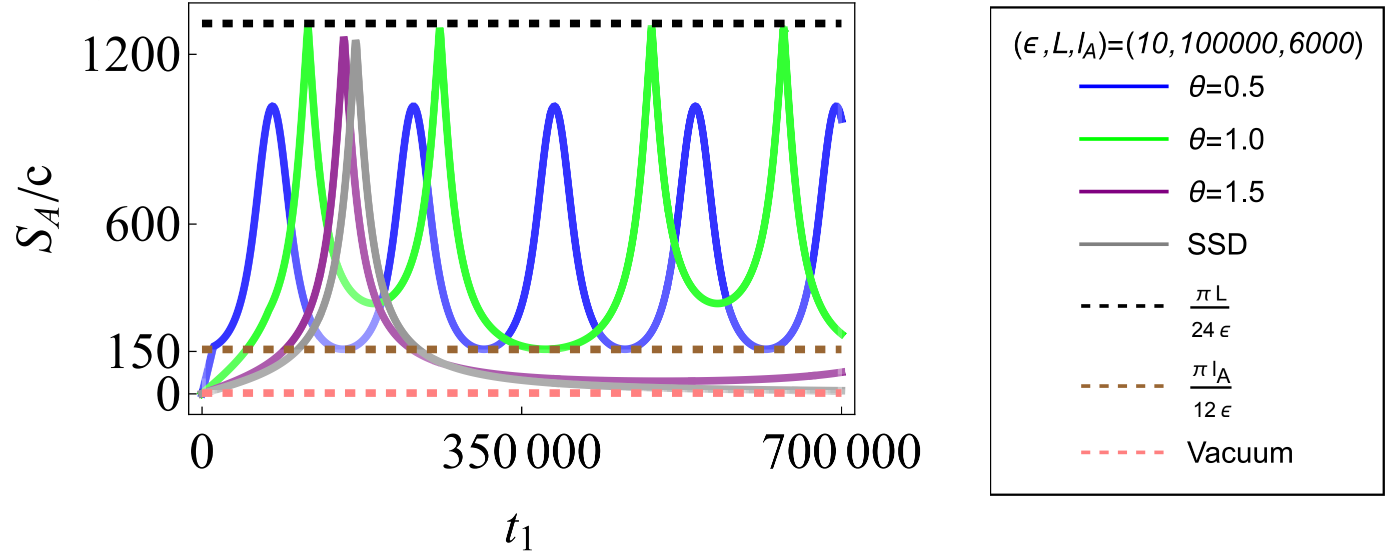


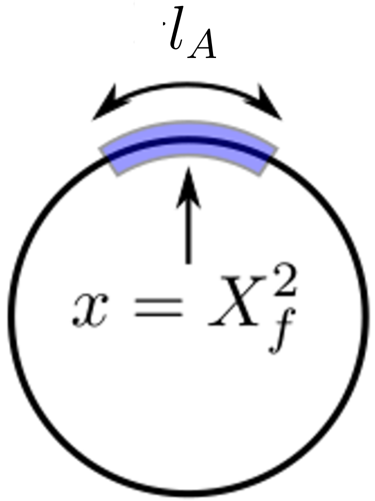
$$\frac{dx}{dt}(x \approx 0) \approx 0$$

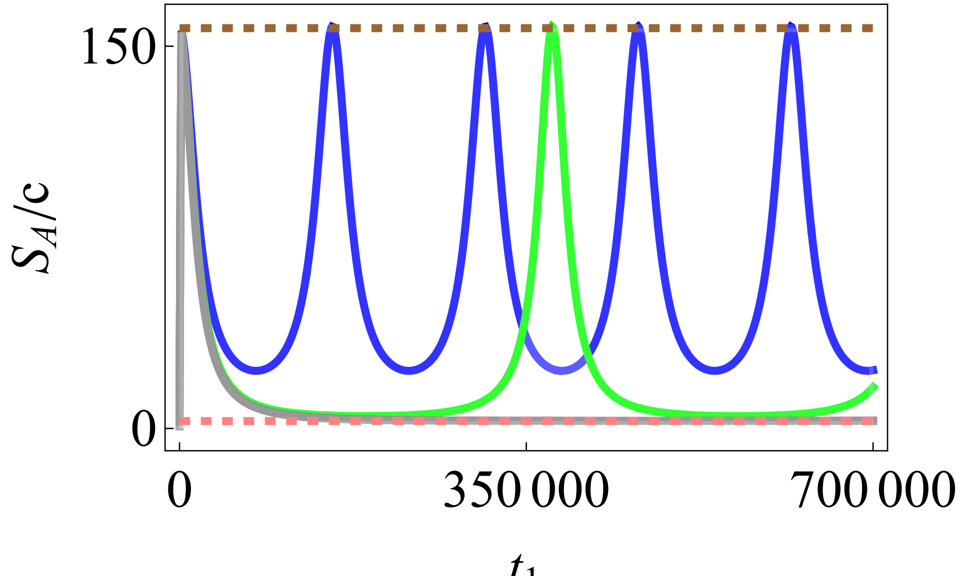
Summary 1: Preparation of nearly vacuum state by checking with EE

Evolution from the boundary state: $|\Psi\rangle = \dfrac{e^{-\epsilon H}\,|\mathrm{Bdy}\rangle}{\sqrt{\langle \mathrm{Bdy}|\, e^{-2\epsilon H}\,|\mathrm{Bdy}\rangle}}$



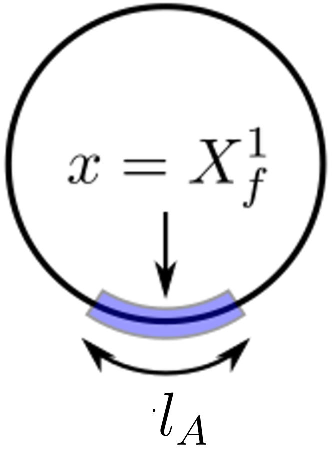


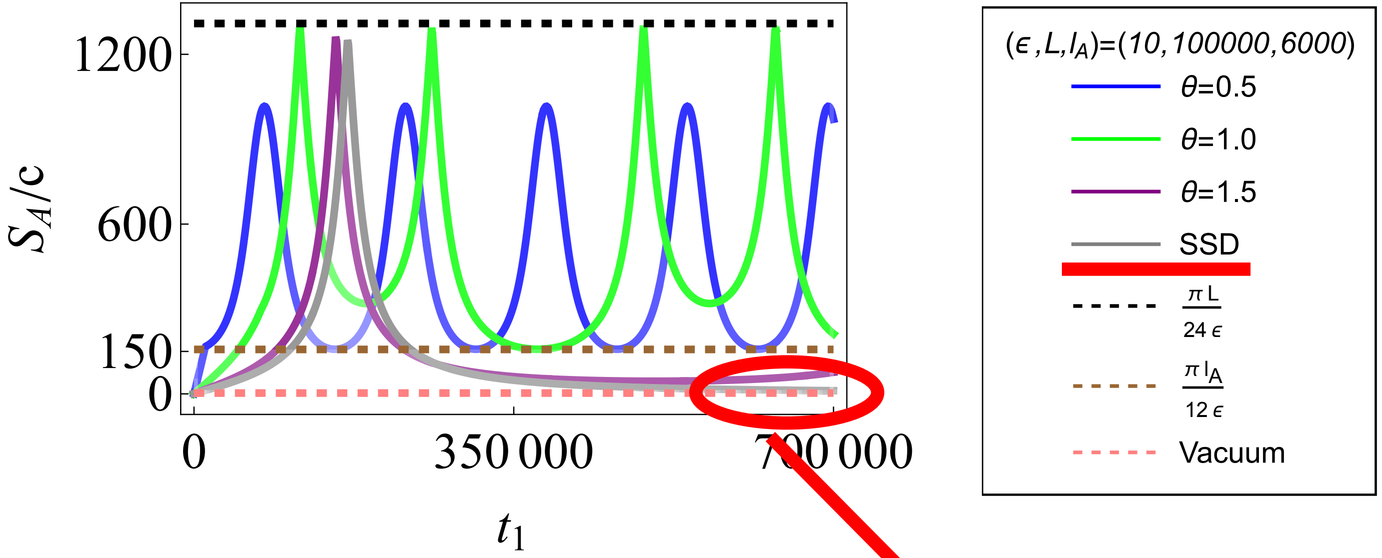


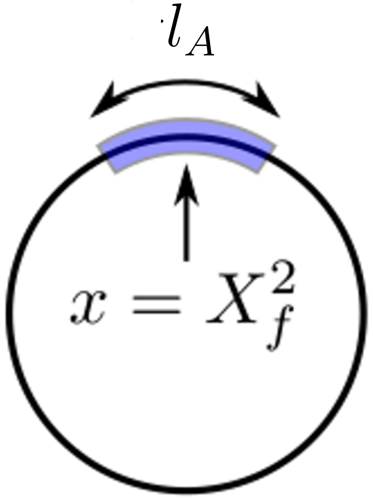


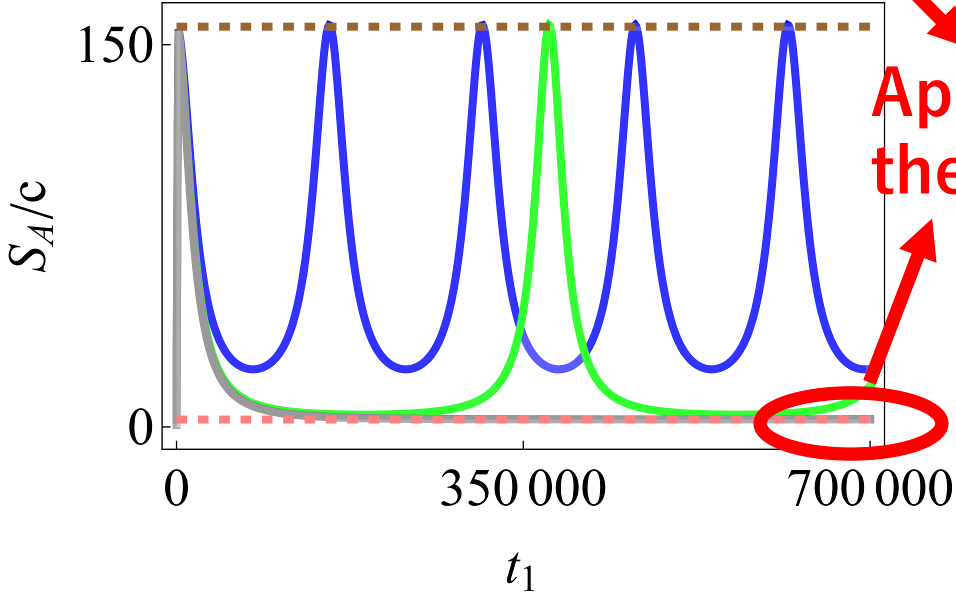
Summary 1: Preparation of nearly vacuum state by checking with EE

Evolution from the boundary state: $|\Psi\rangle = \dfrac{e^{-\epsilon H}\,|\mathrm{Bdy}\rangle}{\sqrt{\langle \mathrm{Bdy}|\, e^{-2\epsilon H}\,|\mathrm{Bdy}\rangle}}$







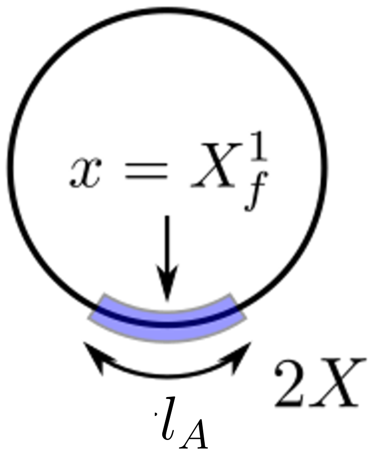


Approaches to the vacuum EE

Summary 1: Preparation of nearly vacuum state by checking with EE

The time dependence of entanglement entropy during SSD time evolution suggests that

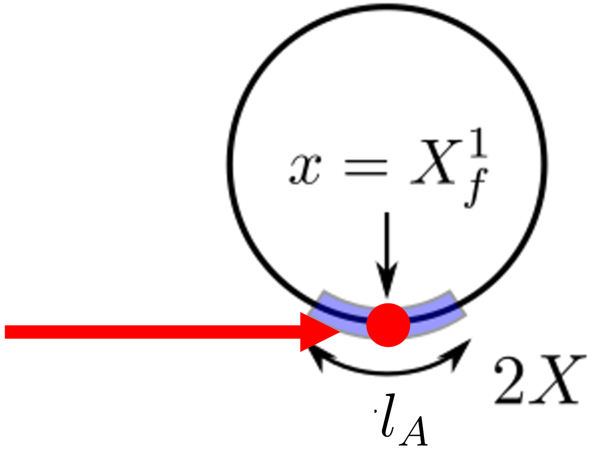
the state approximately approaches to the vacuum state except for $x = X_f^1$.



Summary 1: Preparation of nearly vacuum state by checking with EE

The time dependence of entanglement entropy during SSD time evolution suggests that

the state approximately approaches to the vacuum state except for $x = X_f^1$.



In quasiparticle picture, quasiparticles move to this point.

Summary 1: Preparation of nearly vacuum state by checking with MI

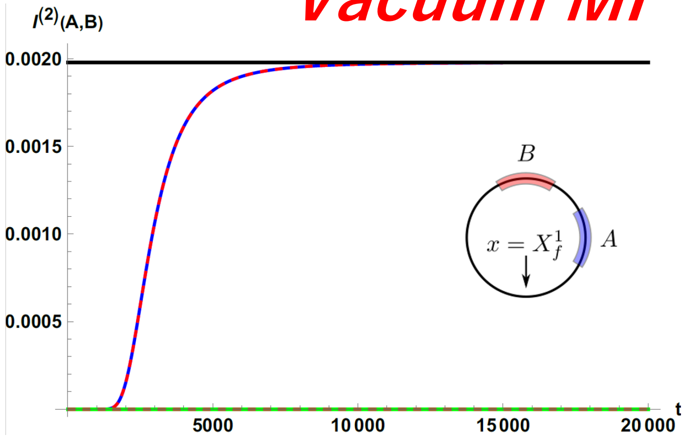
We start from $\rho = \frac{e^{-2\epsilon H}}{\mathrm{tr} e^{-2\epsilon H}}$ and $|\Psi\rangle = \frac{e^{-\epsilon H}|\mathrm{Bdy}\rangle}{\sqrt{\langle \mathrm{Bdy}| e^{-2\epsilon H} |\mathrm{Bdy}\rangle}}$, and then evolve the system with SSD Hamiltonian.

The time dependence of mutual information (MI) show the mutual information approaches to the vacuum one for any subsystems.

$$I_{A,B} = S_A + S_B - S_{A\cup B}$$

For example, free fermion

Vacuum MI



$$\boxed{I_{A,B} \approx 0 \quad \Longrightarrow \quad I_{A,B}^{\mathrm{Vacuum}}}$$

Summary 1: Preparation of nearly vacuum state by checking with MI

We start from $\rho = \frac{e^{-2\epsilon H}}{\mathrm{tr} e^{-2\epsilon H}}$ and. $|\Psi\rangle = \frac{e^{-\epsilon H}|\mathrm{Bdy}\rangle}{\sqrt{\langle \mathrm{Bdy}| e^{-2\epsilon H} |\mathrm{Bdy}\rangle}}$. ,and then evolve the system with SSD Hamiltonian.

The SSD time evolution operator endows the states with the vacuum non-local correlation.

w

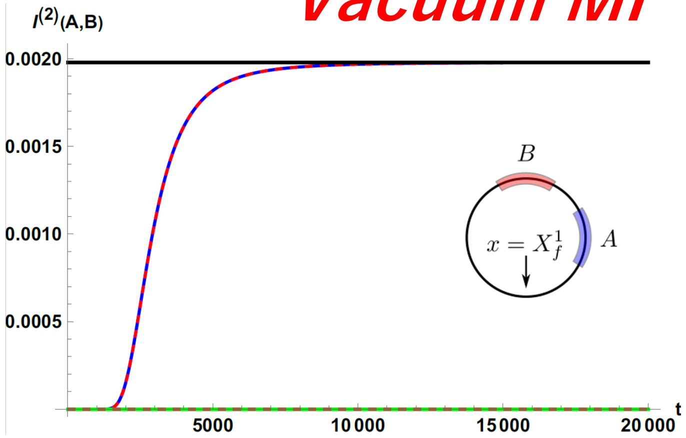
e

ion

$$I_{A,B} = S_A + S_B - S_{A\cup B}$$

$$\boxed{I_{A,B} \approx 0 \quad \Rightarrow \quad I_{A,B}^{\mathrm{Vacuum}}}$$

Vacuum MI



Summary 2: Revival of mutual information from ***the typical state***

The setup considered:

The system in the pure state is **unitarily** evolved to ***the typical state with the strong scrambling Hamiltonian (2d holographic Hamiltonian.).***

The entanglement entropy for this state follows the Page's curve:

$$S_A = -\mathrm{tr}_A \rho_A \log \rho_A \approx \begin{cases} l_A \cdot \log d & \frac{L}{2} > l_A > 0 \\ (L - l_A) \cdot \log d & L > l_A > \frac{L}{2} \end{cases}$$

L: system size

l_A:subsystem size

d:dimension of local Hilbert space

Summary 2: Revival of mutual information from the typical state

The setup considered:

The system in the pure state is **unitarily** evolved to ***the typical state with the strong scrambling non-equilibrium process.***

The entanglement entropy for this state follows the Page's curve:

$$S_A = -\mathrm{tr}_A \rho_A \log \rho_A \approx \begin{cases} l_A \cdot \log d & \frac{L}{2} > l_A > 0 \\ (L - l_A) \cdot \log d & L > l_A > \frac{L}{2} \end{cases}$$

L: system size

l_A:subsystem size

d:dimension of local Hilbert space

$$I_{A,B} \approx 0 \text{ for } \tfrac{L}{2} > l_A, l_B, l_A + l_B > 0$$

Summary 2: Revival of mutual information from the typical state

The setup considered:

The system in the pure state is **unitarily** evolved to ***the typical state with the strong scrambling non-equilibrium process.***

The entanglement entropy for this state follows the Page's curve:

$$S_A = -\mathrm{tr}_A \rho_A \log \rho_A \approx \begin{cases} l_A \cdot \log d & \frac{L}{2} > l_A > 0 \\ (L - l_A) \cdot \log d & L > l_A > \frac{L}{2} \end{cases}$$

L: system size
l_A:subsystem size
d:dimension of local Hilbert space

$$I_{A,B} \approx 0 \text{ for } \tfrac{L}{2} > l_A, l_B, l_A + l_B > 0$$

$$\rho_{A\cup B} \underset{1 \gg \epsilon}{\approx} \rho_A \otimes \rho_B$$

Summary 2: Revival of mutual information from the typical state

There are no non-local correlations of the typical state for the small subsystems $\frac{L}{2} > l_A, l_B, l_A + l_B > 0$ ***.***

Summary 2: Revival of mutual information from the typical state

There are no non-local correlations of the typical state for the small subsystems $\frac{L}{2} > l_A, l_B, l_A + l_B > 0$ **.**

We try to **recover the non-local correlation from the thermofield double state by the SSD time evolution**.

Summary 2: Revival of mutual information from the typical state

Information retrieval by using inhomogeneous quenches

(Non-local correlation)

We evolve the system with the 2d **uniform holographic Hamiltonian,** $e^{-iH_0^1 t_0} \otimes \mathbf{1}_2$.

In the large t_0-regime, the system may be approximated by a typical state.

Summary 2: Revival of mutual information from the typical state

Result 1: Information retrieval by using inhomogeneous quenches

(Non-local correlation)

We evolve the system with the 2d **uniform holographic Hamiltonian,** $e^{-iH_0^1 t_0} \otimes \mathbf{1}_2$.

In the large t_0-regime, the system may be approximated by a typical state.

$$\underline{t_0 \gg \mathcal{O}(L)}$$

EE follows Page's curve.

Summary 2: Revival of mutual information from the typical state

Information retrieval by using inhomogeneous quenches

(Non-local correlation)

We evolve the system with the 2d **uniform holographic Hamiltonian,** $e^{-iH_0^1 t_0} \otimes \mathbf{1}_2$.

In the large t_0-regime, the system may be approximated by a typical state.

As a consequence, $I_{A,B} \approx 0.$

Summary 2: Revival of mutual information from the typical state

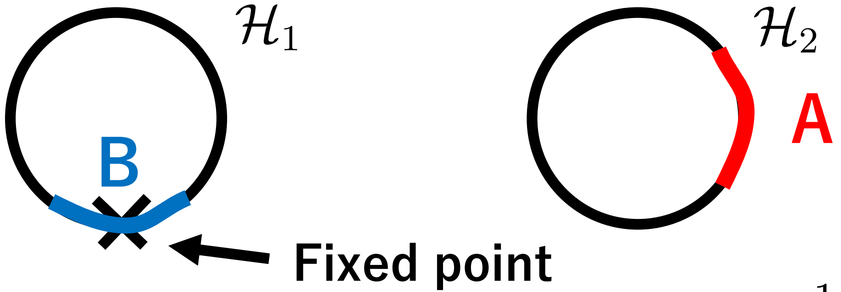
Information retrieval by using inhomogeneous quenches

(Non-local correlation)

We evolve the system with the 2d **uniform holographic Hamiltonian,** $e^{-iH_0^1 t_0} \otimes \mathbf{1}_2$.

In the large t_0-regime, the system may be approximated by a typical state.

As a consequence, $I_{A,B} \approx 0.$



Then, we evolve it with the 2d **SSD holographic Hamiltonian,** $e^{-iH_{\text{SSD}}^1 t_1} \otimes \mathbf{1}_2$.

The non-local correlation is recovered from the typical state: $I_{A,B} \approx \frac{2c\pi l_A}{6\epsilon}$

Summary 2: Revival of mutual information from the typical state

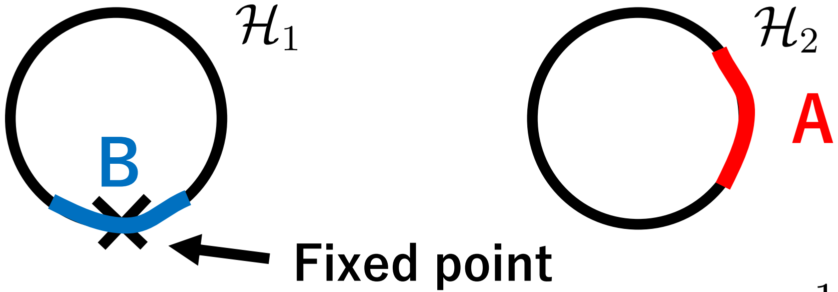
Information retrieval by using inhomogeneous quenches

(Non-local correlation)

We evolve the system with the 2d **uniform holographic Hamiltonian,** $e^{-iH_0^1 t_0} \otimes \mathbf{1}_2$.

In the large t_0-regime, the system may be approximated by a typical state.

As a consequence, $I_{A,B} \approx 0.$



Then, we evolve it with the 2d **SSD holographic Hamiltonian,** $e^{-iH_{\mathrm{SSD}}^1 t_1} \otimes \mathbf{1}_2$.

Size of A

The non-local correlation is recovered from the typical state: $I_{A,B} \approx \frac{2c\tau l_A}{6\epsilon}$

Summary 2: Revival of mutual information from the typical state

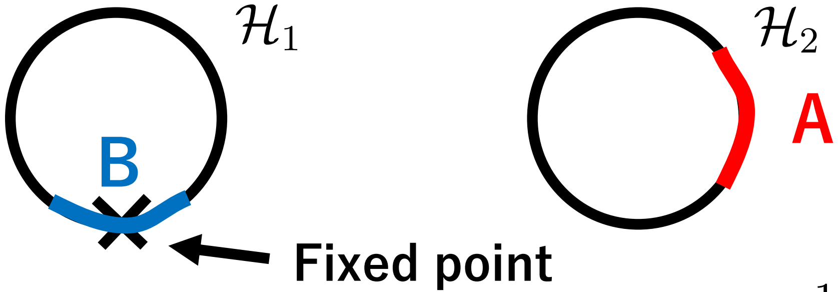
Information retrieval by using inhomogeneous quenches

(Non-local correlation)

We evolve the system with the 2d **uniform holographic Hamiltonian,** $e^{-iH_0^1 t_0} \otimes \mathbf{1}_2$.

In the large t_0-regime, the system may be approximated by a typical state.

As a consequence, $I_{A,B} \approx 0.$



This is proportional to the number of Bell pairs shared between A and B.

Then, we evolve it with the 2d **SSD holographic Hamiltonian,** $e^{-iH_{\mathrm{SSD}}^1 t_1} \otimes \mathbf{1}_2$.

The non-local correlation is recovered from the typical state: $I_{A,B} \approx \frac{2c\tau l_A}{6\epsilon}$

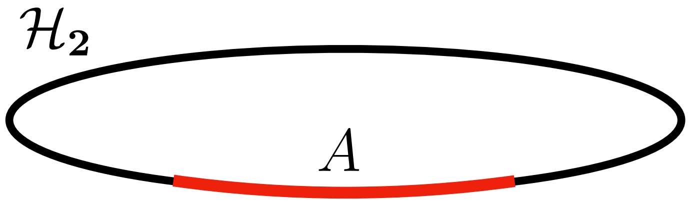
Summary 3: Genuine tripartite entanglement

The system considered is in:

$$|\Psi(t_0,t_1)\rangle = \left(e^{-it_0H_0^1}\otimes \mathbf{1}_{\mathcal{H}_2}\right)\left(e^{-it_1H_{\mathrm{SSD}}^1}\otimes \mathbf{1}_{\mathcal{H}_2}\right)\frac{1}{\sqrt{\mathrm{tr}e^{-2\epsilon H_0}}}\sum_a e^{\frac{-\epsilon}{2}\left(H_0^1+H_0^2\right)}|a\rangle_{\mathcal{H}_1}\otimes|a\rangle_{\mathcal{H}_2}$$

Let us divide $\mathcal{H}_1$ into B_1, B_2, and the complement to them.

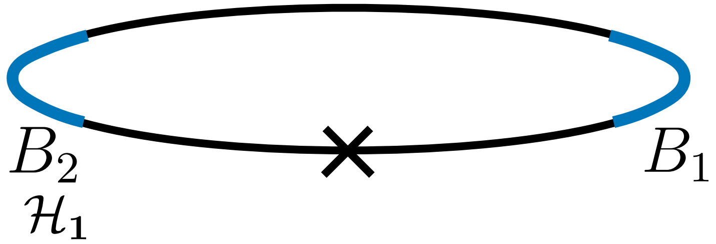
A denotes the subsystem of $\mathcal{H}_2$.



$$B_1 = \left\{x \middle| L > L-Y_1 > x > L-Y_2 > \frac{L}{2}\right\},$$

$$B_2 = \left\{x \middle| \frac{L}{2} > Y_1 > x > Y_2 > 0\right\},$$

where $\frac{L}{2} > Y_1 > Y_2 > 0.$



Summary 3: Genuine tripartite entanglement

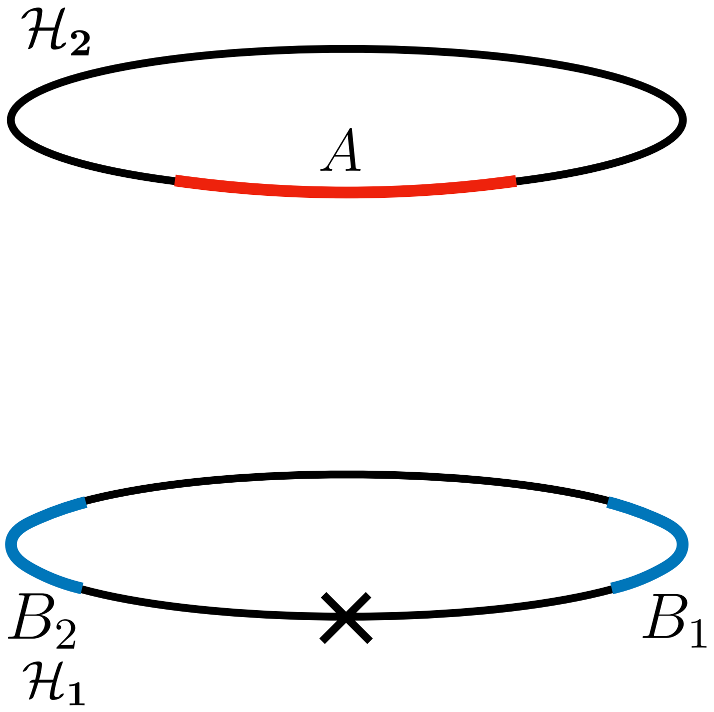
The system considered is in:

$$|\Psi(t_0,t_1)\rangle = \left(e^{-it_0 H_0^1}\otimes \mathbf{1}_{\mathcal{H}_2}\right)\left(e^{-it_1 H_{\mathrm{SSD}}^1}\otimes \mathbf{1}_{\mathcal{H}_2}\right)\frac{1}{\sqrt{\mathrm{tr}e^{-2\epsilon H_0}}}\sum_a e^{\frac{-\epsilon}{2}\left(H_0^1+H_0^2\right)}|a\rangle_{\mathcal{H}_1}\otimes|a\rangle_{\mathcal{H}_2}$$

Inhomogeneous(SSD) **Homogeneous**

Let us divide $\mathcal{H}_1$ into B_1, B_2, and the complement to them.

A denotes the subsystem of $\mathcal{H}_2$.



$$B_1 = \left\{x \middle| L > L - Y_1 > x > L - Y_2 > \frac{L}{2}\right\},$$

$$B_2 = \left\{x \middle| \frac{L}{2} > Y_1 > x > Y_2 > 0\right\},$$

where $\frac{L}{2} > Y_1 > Y_2 > 0.$

Summary 3: Genuine tripartite entanglement

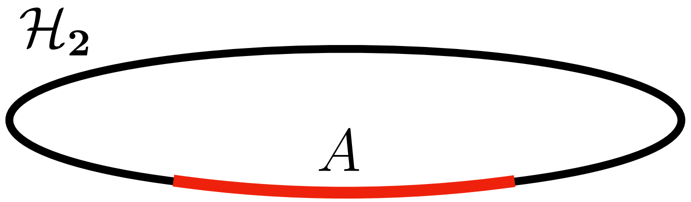
The system considered is in:

Eigenstates of H_0

$$|\Psi(t_0,t_1)\rangle = \left(e^{-it_0 H_0^1} \otimes \mathbf{1}_{\mathcal{H}_2}\right)\left(e^{-it_1 H^1_{\mathrm{SSD}}} \otimes \mathbf{1}_{\mathcal{H}_2}\right) \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H_0}}} \sum_a e^{\frac{-\epsilon}{2}\left(H_0^1+H_0^2\right)} \underline{|a\rangle_{\mathcal{H}_1}} \otimes |a\rangle_{\mathcal{H}_2}$$

Let us divide $\mathcal{H}_1$ into B_1, B_2, and the complement to them.

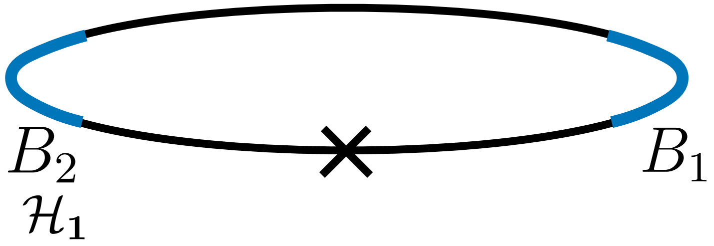
A denotes the subsystem of $\mathcal{H}_2$.



$$B_1 = \left\{ x \middle| L > L - Y_1 > x > L - Y_2 > \frac{L}{2} \right\},$$

$$B_2 = \left\{ x \middle| \frac{L}{2} > Y_1 > x > Y_2 > 0 \right\},$$

where $\frac{L}{2} > Y_1 > Y_2 > 0.$



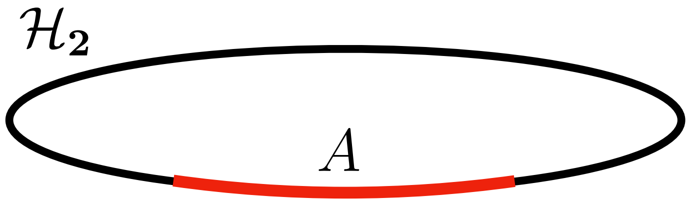
Summary 3: Genuine tripartite entanglement

The system considered is in:

$$|\Psi(t_0,t_1)\rangle = \left(e^{-it_0 H_0^1}\otimes \mathbf{1}_{\mathcal{H}_2}\right)\left(e^{-it_1 H^1_{\mathrm{SSD}}}\otimes \mathbf{1}_{\mathcal{H}_2}\right)\frac{1}{\sqrt{\mathrm{tr}e^{-2\epsilon H_0}}}\sum_a e^{\frac{-\epsilon}{2}\left(H_0^1+H_0^2\right)}|a\rangle_{\mathcal{H}_1}\otimes|a\rangle_{\mathcal{H}_2}$$

Let us divide $\mathcal{H}_1$ into B_1, B_2, and the complement to them.

A denotes the subsystem of $\mathcal{H}_2$.

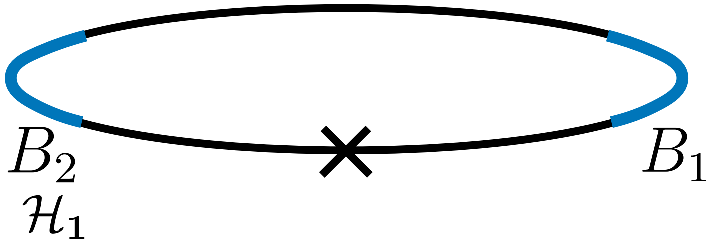


$$B_1 = \left\{ x \middle| L > L - Y_1 > x > L - Y_2 > \frac{L}{2} \right\},$$

$$B_2 = \left\{ x \middle| \frac{L}{2} > Y_1 > x > Y_2 > 0 \right\},$$

B does not include x=0 or x=L/2.

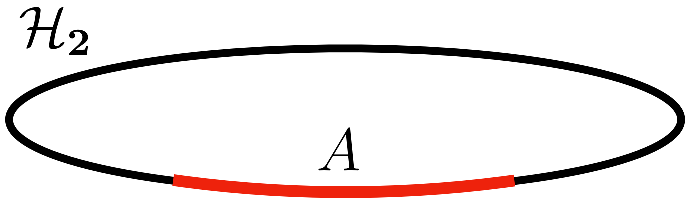
where $\frac{L}{2} > Y_1 > Y_2 > 0.$



Summary 3: Genuine tripartite entanglement

Let us divide $\mathcal{H}_1$ into B_1, B_2, and the complement to them.

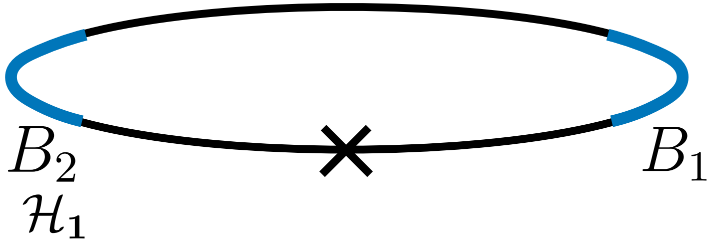
A denotes the subsystem of $\mathcal{H}_2$.



$$B_1 = \left\{ x \middle| L > L - Y_1 > x > L - Y_2 > \frac{L}{2} \right\},$$

$$B_2 = \left\{ x \middle| \frac{L}{2} > Y_1 > x > Y_2 > 0 \right\},$$

where $\frac{L}{2} > Y_1 > Y_2 > 0.$



In 2d Free fermion,

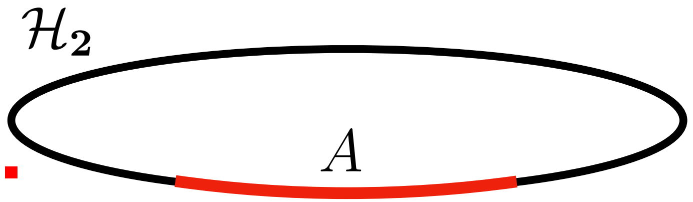
$$\boxed{I_{A,B_{i=1,2}} \geq 0, I_{B_1,B_2} \geq 0}$$

Summary 3: Quasiparticle picture

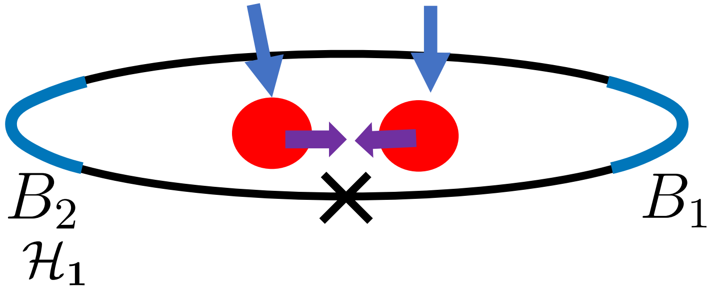
The system considered is in:

$$|\Psi(t_0,t_1)\rangle = \left(e^{-it_0 H_0^1}\otimes \mathbf{1}_{\mathcal{H}_2}\right)\left(\underline{e^{-it_1 H^1_{\text{SSD}}}\otimes \mathbf{1}_{\mathcal{H}_2}}\right)\frac{1}{\sqrt{\text{tr}e^{-2\epsilon H_0}}}\sum_a e^{\frac{-\epsilon}{2}\left(H_0^1+H_0^2\right)}|a\rangle_{\mathcal{H}_1}\otimes|a\rangle_{\mathcal{H}_2}$$

· During the SSD time evolution, **quasiparticles on $\mathcal{H}_1$ move to the fixed point and accumulate there.**



Group of left and right-moving quasi-particles

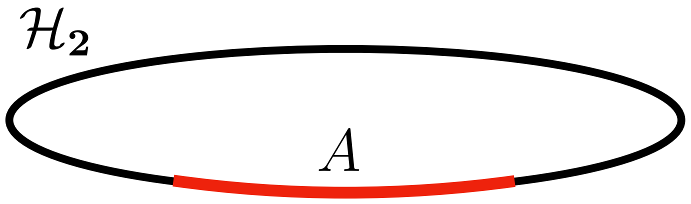


Summary 3: Quasiparticle picture

The system considered is in:

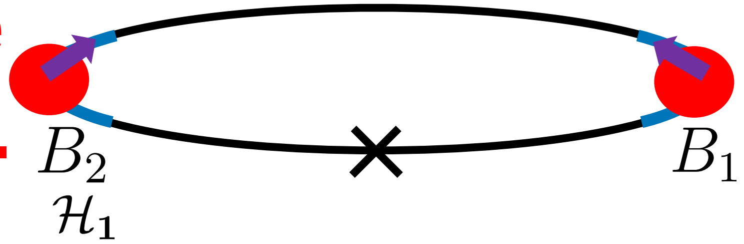
$$|\Psi(t_0,t_1)\rangle = \underline{\left(e^{-it_0 H_0^1}\otimes \mathbf{1}_{\mathcal{H}_2}\right)} \left(e^{-it_1 H^1_{\mathrm{SSD}}}\otimes \mathbf{1}_{\mathcal{H}_2}\right) \frac{1}{\sqrt{\mathrm{tr} e^{-2\epsilon H_0}}} \sum_a e^{\frac{-\epsilon}{2}\left(H_0^1+H_0^2\right)} |a\rangle_{\mathcal{H}_1}\otimes|a\rangle_{\mathcal{H}_2}$$

During the SSD time evolution,
quasiparticles on $\mathcal{H}_1$move to the fixed point and accumulate there.



⬇

During the uniform time evolution,
the groups of quasiparticles move left and right at the speed of light.



Summary 3: Quasiparticle picture

The system considere

$$|\Psi(t_0, t_1)\rangle = \underline{\left(e^{-it_0 H_0^1} \otimes \mathbf{1}_{\mathcal{H}_2}\right)} \left(e^{-it_1 H_{\mathrm{S}}^1}\right.$$

During the SSD time

quasiparticles on $\mathcal{H}_1$

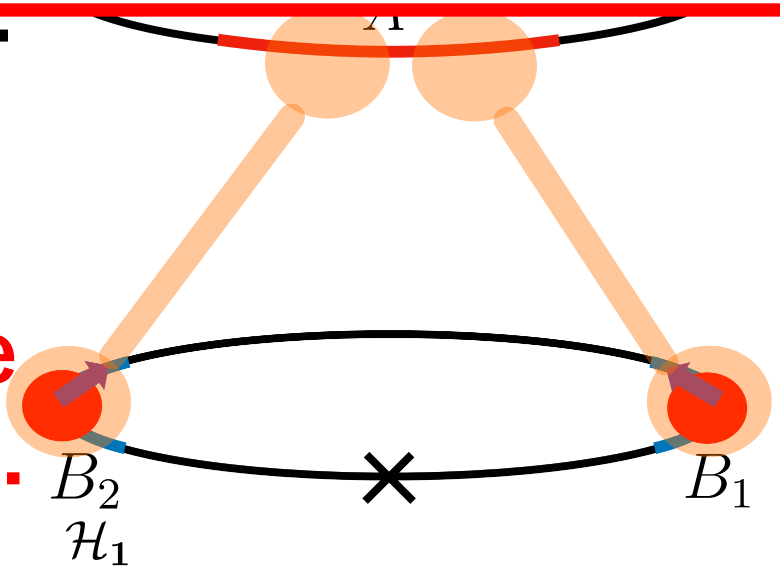
fixed point and accumulate there.

Mutual information is given by the number of Bell pairs shared by two subsystems. In the time interval where the group of quasiparticles are in B_1 or B_2 ,

$$I_{A,B_{i=1,2}} \geq 0,$$

During the uniform time evolution,

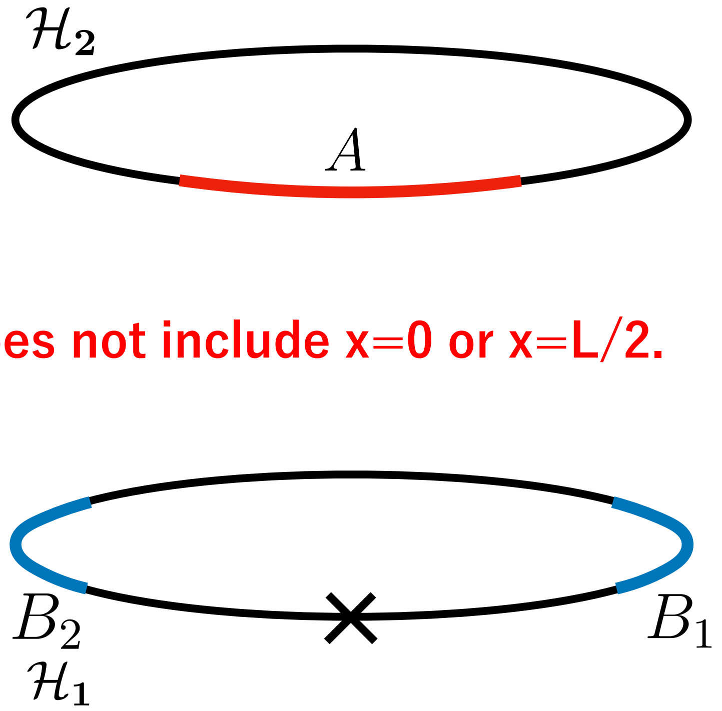
the groups of quasiparticles move left and right at the speed of light.



Summary 3: Genuine tripartite entanglement

Let us divide $\mathcal{H}_1$ into B_1, B_2, and the complement to them.

A denotes the subsystem of $\mathcal{H}_2$.



$$B_1 = \left\{ x \middle| L > L - Y_1 > x > L - Y_2 > \frac{L}{2} \right\},$$

B does not include x=0 or x=L/2.

$$B_2 = \left\{ x \middle| \frac{L}{2} > Y_1 > x > Y_2 > 0 \right\},$$

where $\frac{L}{2} > Y_1 > Y_2 > 0.$

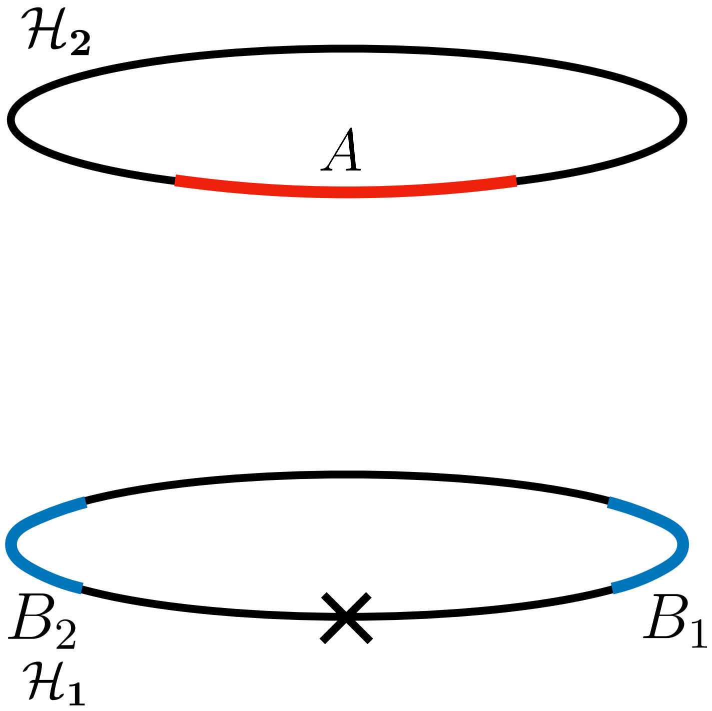
In 2d holographic CFT,

$$\boxed{I_{A,B_{i=1,2}} \underline{\approx 0}, \ I_{B_1,B_2} \underline{\approx 0}}$$

Summary 3: Genuine tripartite entanglement

Let us divide $\mathcal{H}_1$ into B_1, B_2, and the complement to them.

A denotes the subsystem of $\mathcal{H}_2$.



$$B_1 = \left\{ x \middle| L > L - Y_1 > x > L - Y_2 > \frac{L}{2} \right\},$$

$$B_2 = \left\{ x \middle| \frac{L}{2} > Y_1 > x > Y_2 > 0 \right\},$$

where $\frac{L}{2} > Y_1 > Y_2 > 0.$

In 2d holographic CFT,

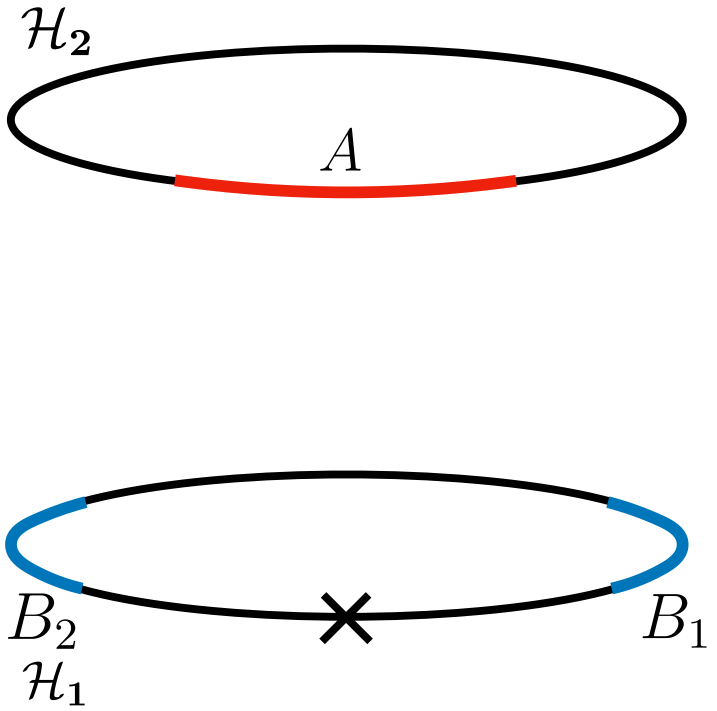
$$\boxed{I_{A,B_{i=1,2}} \underline{\approx 0}, \quad I_{B_1,B_2} \underline{\approx 0}}$$

In 2d holographic CFTs, the strong scrambling effect completely delocalize the quasiparticles in $\mathcal{H}_1$.

Summary 3: Genuine tripartite entanglement

Let us divide $\mathcal{H}_1$ into B_1, B_2, and the complement to them.

A denotes the subsystem of $\mathcal{H}_2$.



$$B_1 = \left\{ x \middle| L > L - Y_1 > x > L - Y_2 > \frac{L}{2} \right\},$$

$$B_2 = \left\{ x \middle| \frac{L}{2} > Y_1 > x > Y_2 > 0 \right\},$$

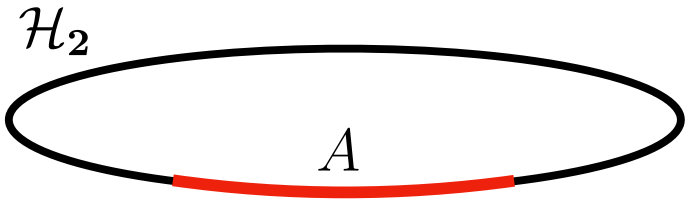
where $\frac{L}{2} > Y_1 > Y_2 > 0.$

How about the mutual information between A and $B_1 \cup B_2$?

Summary 3: Genuine tripartite entanglement

Let us divide $\mathcal{H}_1$ into B_1, B_2, and the complement to them.

A denotes the subsystem of $\mathcal{H}_2$.

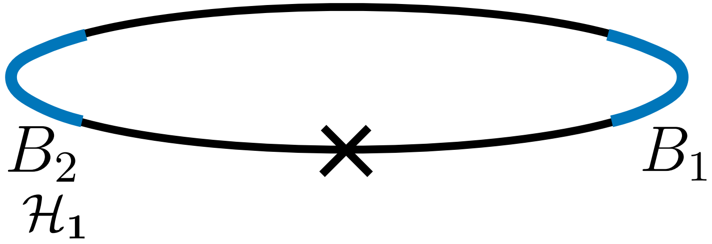


$$B_1 = \left\{ x \middle| L > L - Y_1 > x > L - Y_2 > \frac{L}{2} \right\},$$

$$B_2 = \left\{ x \middle| \frac{L}{2} > Y_1 > x > Y_2 > 0 \right\},$$

B does not include x=0 or x=L/2, then mutual information is approximately zero.

where $\frac{L}{2} > Y_1 > Y_2 > 0.$



There are time-regimes where a *non-local correlation shared by three parties exists*.

$$I_{A,B_1\cup B_2} \approx \begin{cases} 0 & nL + Y_2 > t_0 > nL - Y_2 \\ \frac{c\pi l_A}{3\epsilon} & nL + Y_1 > t_0 > nL + Y_2 \\ 0 & (n+1)L - Y_1 > t_0 > nL + Y_1 \\ \frac{c\pi l_A}{3\epsilon} & (n+1)L - Y_2 > t_0 > (n+1)L - Y_1 \end{cases} .$$

Summary 3: Genuine tripartite entanglement

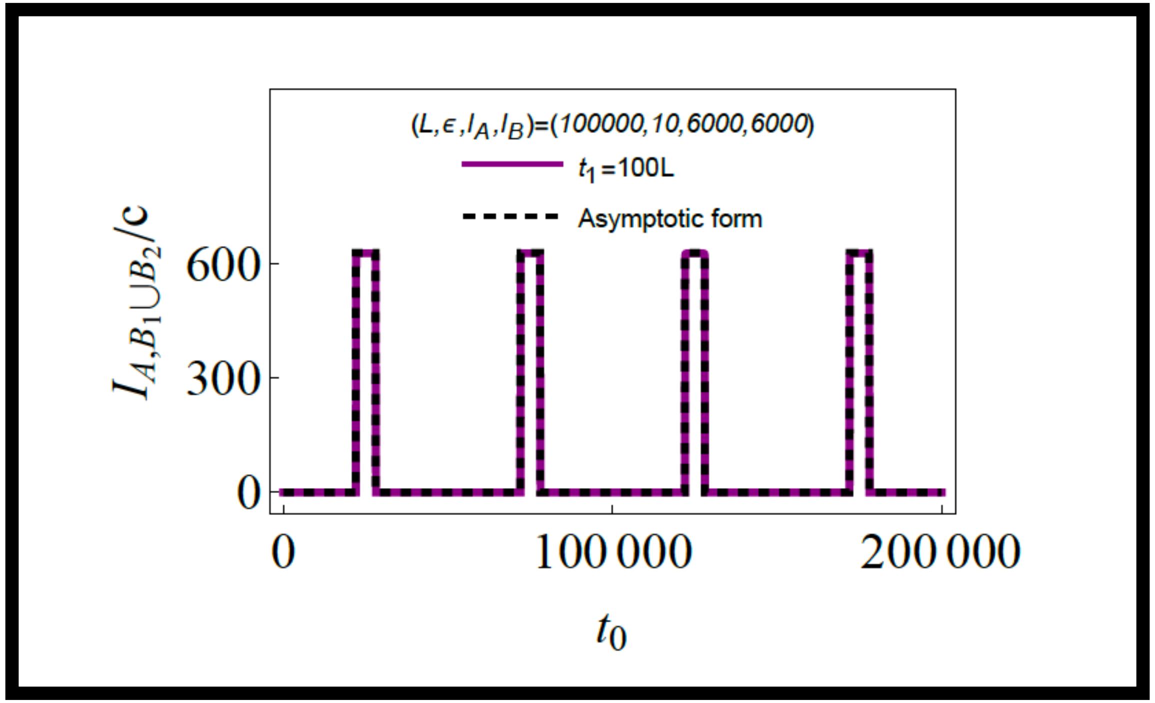
Let us divide $\mathcal{H}_1$ into B_1, B_2, and the complement to them. $P_{C,B_1} = \frac{3L}{4}$, and $P_{C,B_1} = \frac{L}{4}$.

A denotes the subsystem of $\mathcal{H}_2$.

$$B_1 = \left\{ x \middle| L > L - Y_1 > x > L - Y_2 > \frac{L}{2} \right\},$$

$$B_2 = \left\{ x \middle| \frac{L}{2} > Y_1 > x > Y_2 > 0 \right\},$$

where $\frac{L}{2} > Y_1 > Y_2 > 0.$



$\mathcal{H}_1$

There are time-regimes where a *non-local correlation shared by three parties exists*.

$$I_{A,B_1\cup B_2} \approx \begin{cases} 0 & nL + Y_2 > t_0 > nL - Y_2 \\ \frac{c\pi l_A}{3\epsilon} & nL + Y_1 > t_0 > nL + Y_2 \\ 0 & (n+1)L - Y_1 > t_0 > nL + Y_1 \\ \frac{c\pi l_A}{3\epsilon} & (n+1)L - Y_2 > t_0 > (n+1)L - Y_1 \end{cases}.$$

Summary 3: Qu

The system considere

$$|\Psi(t_0,t_1)\rangle = \underline{\left(e^{-it_0 H_0^1} \otimes \mathbf{1}_{\mathcal{H}_2}\right)} \left(e^{-it_1 H_S^1}\right.$$

During the SSD time

quasiparticles on $\mathcal{H}_1$

fixed point and accumulate there.

⬇

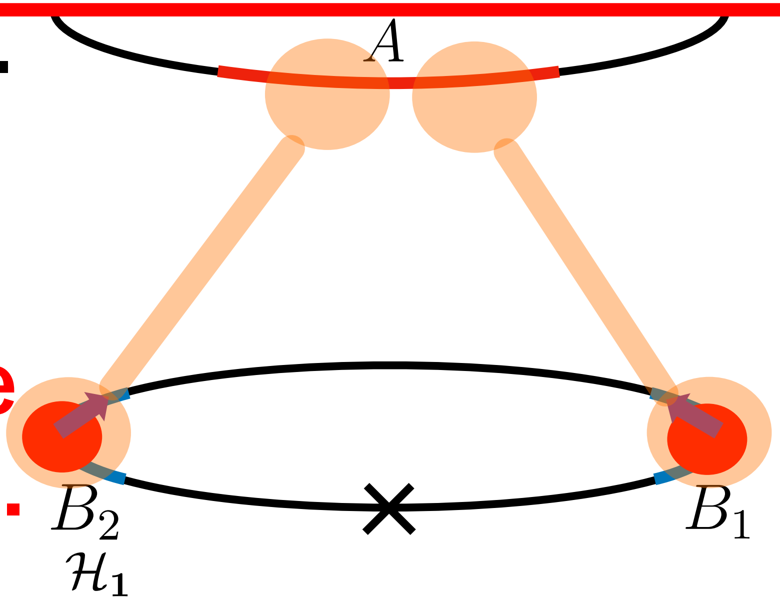
During the uniform time evolution,

the groups of quasiparticles move left and right at the speed of light.

In 2d free fermion and holographic CFT, there are the time intervals where the all quasiparticles on $\mathcal{H}_1$ are in $B_1 \cup B_2$

In these time intervals,

$$I_{A,B_1\cup B_2} \geq 0$$



Summary 3: Genuine tripartite entanglement

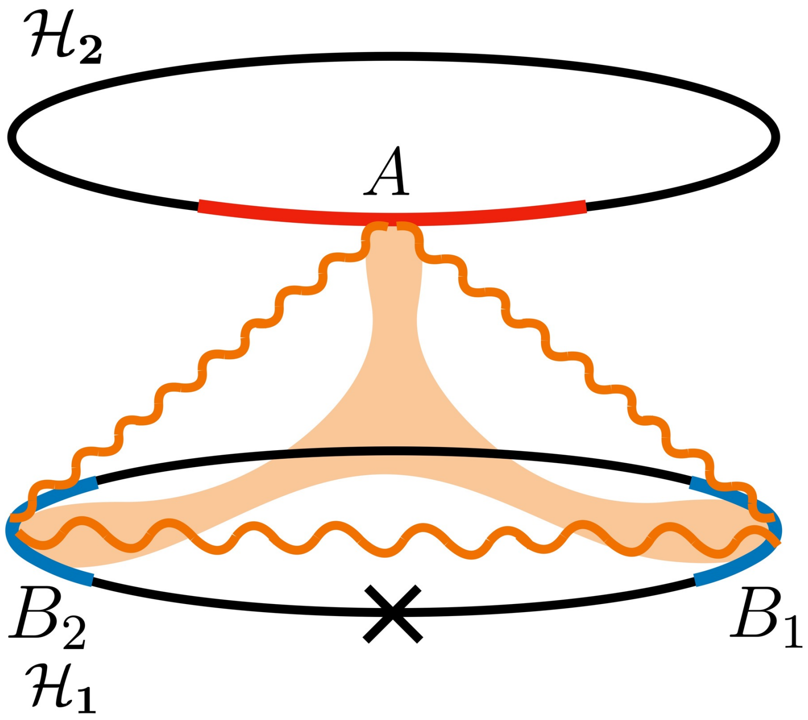
Result 3: Tripartite entanglement

In 2d free fermion (no or weakly scrambling system.)

$$I_{A,B_{i=1,2}} \geq 0, I_{B_1,B_2} \geq 0$$

There are time-regimes where a *non-local correlation shared by three parties exist*.

$$I_{A,B_1\cup B_2} \approx \begin{cases} 0 & nL+Y_2 > t_0 > nL-Y_2 \\ \frac{c\pi l_A}{3\epsilon} & nL+Y_1 > t_0 > nL+Y_2 \\ 0 & (n+1)L-Y_1 > t_0 > nL+Y_1 \\ \frac{c\pi l_A}{3\epsilon} & (n+1)L-Y_2 > t_0 > (n+1)L-Y_1 \end{cases} .$$



Summary 3: Genuine tripartite entanglement

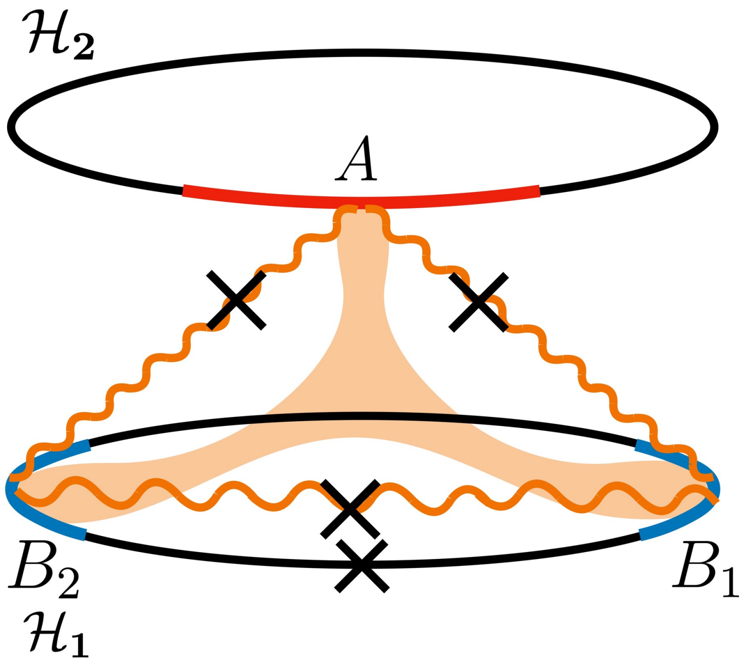
Result 3: Tripartite entanglement

In 2d holographic CFT (strong scrambling system.),

$$I_{A,B_{i=1,2}} \approx 0, \;\; I_{B_1,B_2} \approx 0$$

There are time-regimes where a *non-local correlation shared by three parties exist*.

$$I_{A,B_1\cup B_2} \approx \begin{cases} 0 & nL+Y_2 > t_0 > nL-Y_2 \\ \frac{c\pi l_A}{3\epsilon} & nL+Y_1 > t_0 > nL+Y_2 \\ 0 & (n+1)L-Y_1 > t_0 > nL+Y_1 \\ \frac{c\pi l_A}{3\epsilon} & (n+1)L-Y_2 > t_0 > (n+1)L-Y_1 \end{cases}.$$



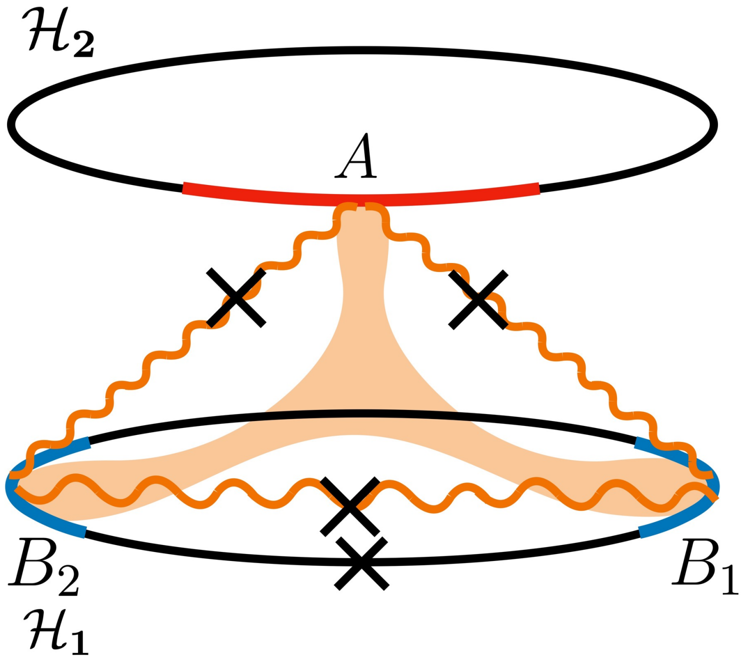
Summary 3: Genuine tripartite entanglement

Result 3: Tripartite entanglement

In 2d holographic CFT（strong scrambling system.)

Key property of this atypical state

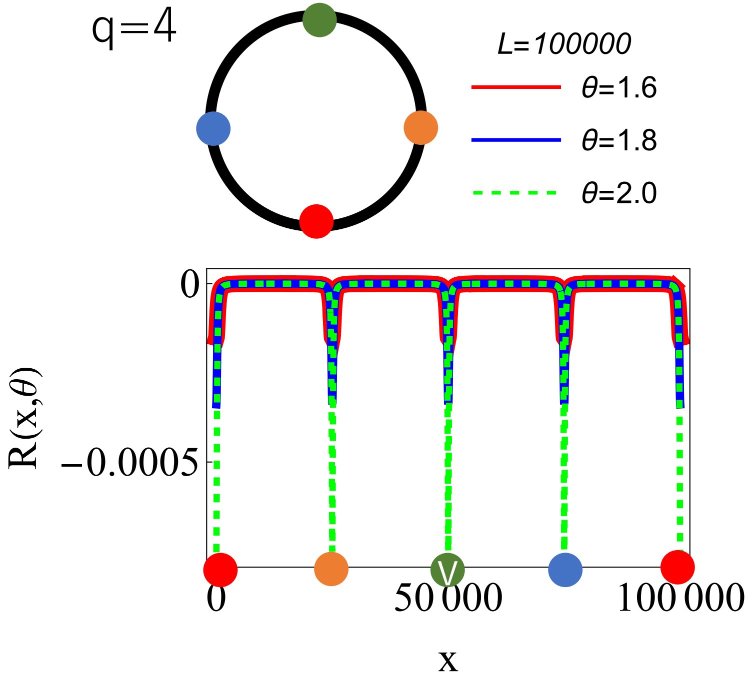
- **There are no correlations shared by the two parties.**
- **There are *a correlation shared by the three parties.***



Summary 4: The thermodynamic on the curved spacetime (on-going)

We consider the **thermodynamic property of the system in 2d holographic CFT on the curved background**.

Our thermal state: $\rho = \frac{e^{-\beta H_{q-\text{Möbius}}}}{\text{tr} e^{-\beta H_{q-\text{Möbius}}}}$



Summary 4: The thermodynamic on the curved spacetime (on-going)

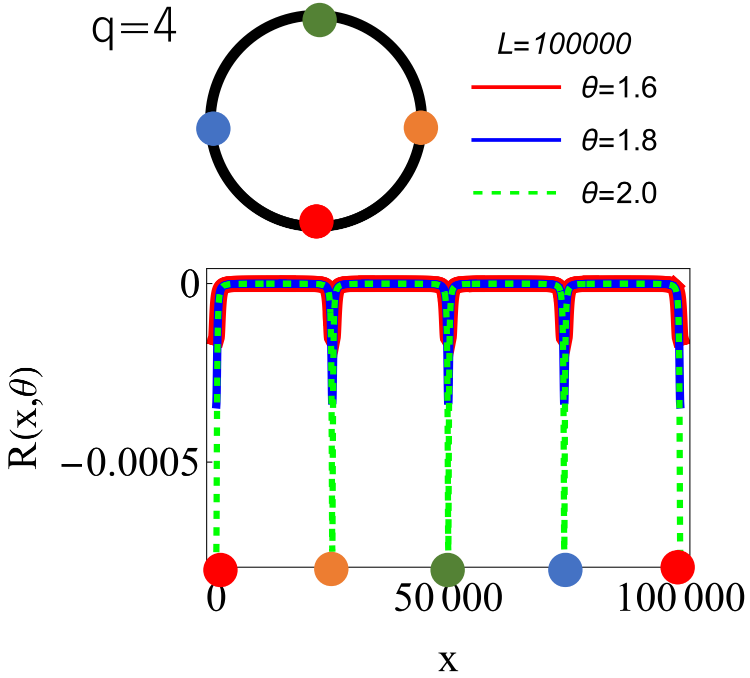
We consider the thermodynamic property of the system in 2d holographic CFT on the curved background.

Our thermal state: $\rho = \frac{e^{-\beta H_{q-\text{Möbius}}}}{\text{tr} e^{-\beta H_{q-\text{Möbius}}}}$

CFT Hamiltonian on the curved spacetime:

$$H_{q-\text{Möbius}} = \int_0^L dx \left[1 - \tanh 2\theta \left(1 - 2\sin^2\left(\frac{q\pi x}{L}\right)\right)\right] (T(x) + \overline{T}(x))$$

$$ds^2 = -f^2(x,\theta)dt^2 + dx^2, \quad f(x,\theta) = 1 - \tanh 2\theta \left(1 - 2\sin^2\left(\frac{q\pi x}{L}\right)\right).$$



Summary 4: The thermodynamic on the curved spacetime (on-going)

We consider the thermodynamic property of the system in 2d

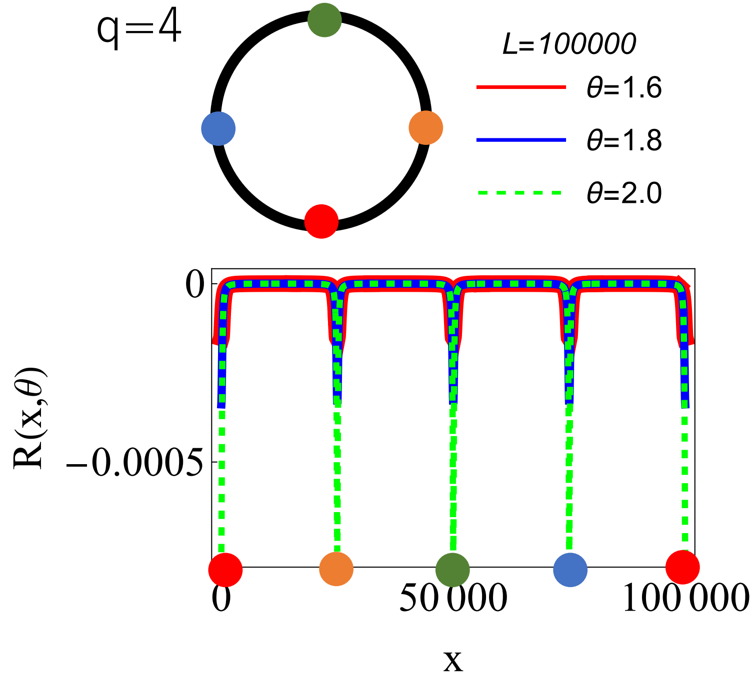
Thermal entropy exhibits **phase transition** with respect to θ .

$\mathrm{tr} e^{-\beta H_{q-\text{Möbius}}}$

CFT Hamiltonian on the curved spacetime:

$$H_{q-\text{Möbius}} = \int_0^L dx \left[1 - \tanh 2\theta \left(1 - 2\sin^2\left(\frac{q\pi x}{L}\right)\right)\right]\left(T(x)+\overline{T}(x)\right)$$

$$ds^2 = -f^2(x,\theta)dt^2 + dx^2, \quad f(x,\theta) = 1 - \tanh 2\theta \left(1 - 2\sin^2\left(\frac{q\pi x}{L}\right)\right).$$



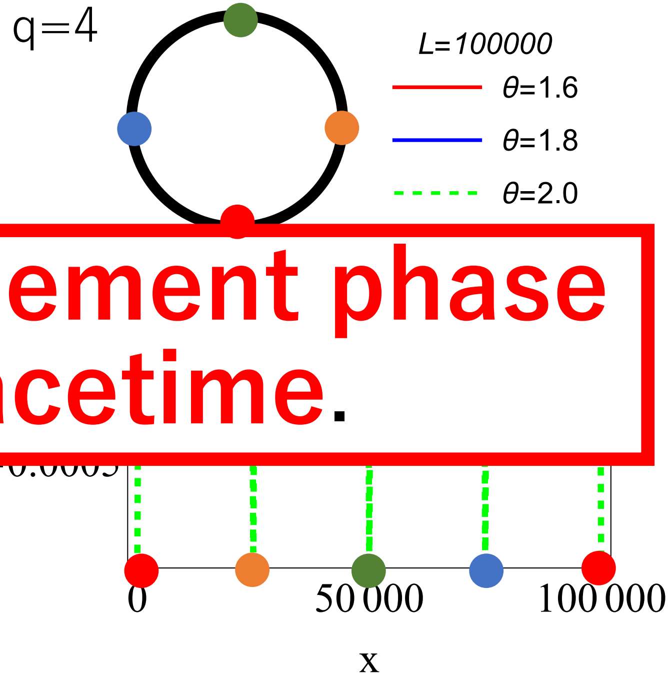
Summary 4: The thermodynamic on the curved spacetime (on-going)

We consider the thermodynamic property of the system in 2d

Thermal entropy exhibits **phase transition** with respect to θ .

CFT Hamiltonian on the curved spacetime:

This may be induced by **the entanglement phase transition (growth) induced by spacetime**.



Details of this study

We will explain the details of

Summary 2

and

Summary 4.

Mobius/SS deformation

The definition of Mobius and sine-square deformed Hamiltonians are

$$H_{\text{Inho}} = \int_0^L dx f(x) h(x) \,,$$

where h(x) is Hamiltonian density of undeformed one: $H = \int_0^L dx h(x)$.

The envelop functions considered are

$$f_{\text{Möbius}}(x) = 1 - \tanh 2\theta \cos\left(\frac{2\pi x}{L}\right), f_{\text{SSD}}(x) = 2\sin^2\left(\frac{\pi x}{L}\right).$$

Mobius/SS deformation

The definition of Mobius and sine-square deformed Hamiltonians are

$$H_{\mathrm{Inho}} = \int_0^L dx f(x) h(x) \quad ,$$

where h(x) is Hamiltonian density of undeformed one: $H = \int_0^L dx h(x)$.

The envelop functions considered are

$$f_{\text{Möbius}}(x) = 1 - \tanh 2\theta \cos\left(\frac{2\pi x}{L}\right), f_{\mathrm{SSD}}(x) = 2\sin^2\left(\frac{\pi x}{L}\right)$$

↓

For $\theta = 0$, $H_{\mathrm{Inho}} = H$. In SSD limit, $\theta \to \infty$, $H_{\mathrm{Inho}} \to H_{\mathrm{SSD}}$.

The evolution of primary operator

The Mobius/SSD Hamiltonians considered are defined on the spatial circle with ***L***, the circumstance.

The evolution of primary operators by these Hamiltonians is given by

$$e^{iH_{\text{Möbius/SSD}}t_1}\sigma_n(w_X,\overline{w}_X)e^{-iH_{\text{Möbius/SSD}}t_1}=\left|\frac{dw_X^{\text{New}}}{dw_x}\right|^{2h_n}\sigma_n\left(w_X^{\text{New}},\overline{w}_X^{\text{New}}\right)$$

where $(w_X,\overline{w}_X)=(iX,-iX)$. h_n is the conformal dimension.

The evolution of primary operator

The Mobius/SSD Hamiltonians considered are defined on the spatial circle with ***L***, the circumstance.

The evolution of primary operators by these Hamiltonians is given by

$$\boxed{e^{iH_{\text{Möbius/SSD}}t_1}\sigma_n(w_X,\overline{w}_X)e^{-iH_{\text{Möbius/SSD}}t_1}=\left|\frac{dw_X^{\text{New}}}{dw_x}\right|^{2h_n}\sigma_n\left(w_X^{\text{New}},\overline{w}_X^{\text{New}}\right)}$$

where $(w_X,\overline{w}_X)=(iX,-iX)$. h_n is the conformal dimension.

This simple transformation makes the computation of EE simpler as explained later.

The evolution of primary operator

The Mobius/SSD Hamiltonians considered are defined on the spatial circle with ***L***, the circumstance.

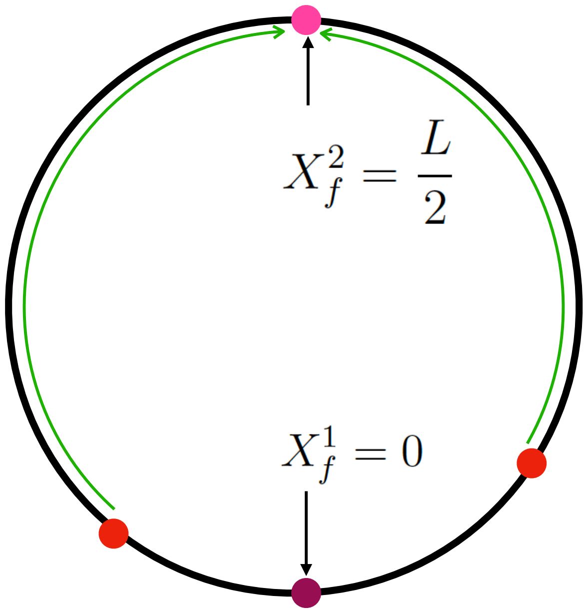
The evolution of primary operators by these Hamiltonians is given by

$$e^{iH_{\text{Möbius/SSD}}t_1}\sigma_n(w_X,\overline{w}_X)e^{-iH_{\text{Möbius/SSD}}t_1}=\left|\frac{dw_X^{\text{New}}}{dw_x}\right|^{2h_n}\sigma_n\left(\underline{w_X^{\text{New}},\overline{w}_X^{\text{New}}}\right)$$

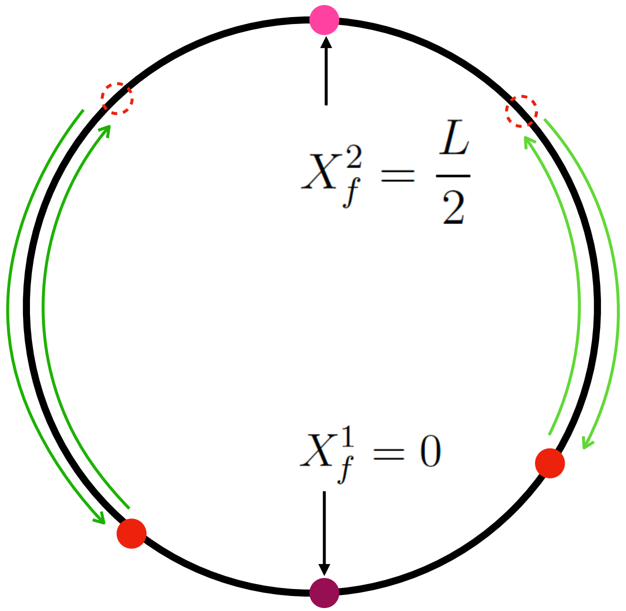
where $(w_X,\overline{w}_X)=(iX,-iX)$. h_n is the conformal dimension.

During the time evolution by the inhomogenous Hamiltonians, the operators move along the spatial circle.

Define the spatial position as $X_X^{\text{New}} = \frac{w_X^{\text{New}} - \overline{w}_X^{\text{New}}}{2i}$.



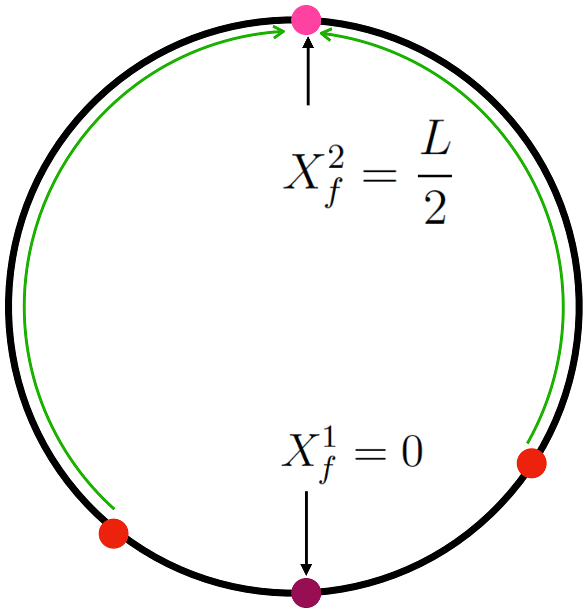
(a) The SSD time evolution



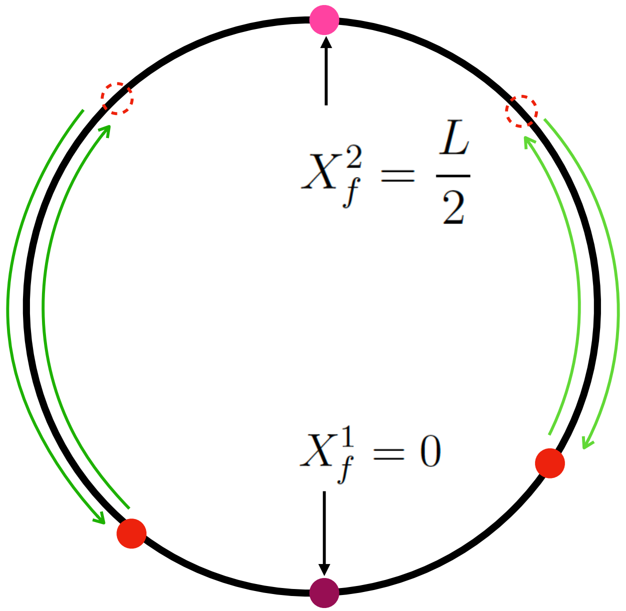
(b) The Möbius time evolution

During the SSD evolution, for $X = X_f^1 = 0,\ X = X_f^2 = \frac{L}{2}$, X_X^{New} doesn't move. We call them **fixed points**.

Define the spatial position as $X_X^{\mathrm{New}} = \dfrac{w_X^{\mathrm{New}} - \overline{w}_X^{\mathrm{New}}}{2i}$.



(a) The SSD time evolution



(b) The Möbius time evolution

During the Mobius evolution, ***the operators periodically move between*** X_f^1 and X_f^2 .

Define the spatial position as $X_X^{\mathrm{New}} = \dfrac{w_X^{\mathrm{New}} - \overline{w}_X^{\mathrm{New}}}{2i}$.

(a) The SSD time evolution

(b) The Möbius time evolution

During the SSD evolution, ***the operators move to*** $X = X_f^2 = \frac{L}{2}$.

Preliminary

- Entanglement entropy (EE)

Definition: $$S_A = \lim_{n\to 1}\frac{1}{1-n}\log \mathrm{tr}_A\rho_A^n = -\mathrm{tr}_A\rho_A\log\rho_A$$

- In the twist operator formalism

$$S_A = \lim_{n\to 1}\frac{1}{1-n}\log \underline{\langle \sigma_n(X)\overline{\sigma}_n(Y)\rangle}$$

By computing two-point function, we can compute (Renyi) entanglement entropy.

How to compute correlator

Suppose that $\langle \sigma_n(X) \overline{\sigma}_n(Y) \rangle$ is given by

$$\langle \sigma_n(X) \overline{\sigma}_n(Y) \rangle = \mathrm{tr}\left[\sigma_n(X) \overline{\sigma}_n(Y) U \rho U^\dagger \right] .$$

How to compute correlator

Suppose that $\langle\sigma_n(X)\overline{\sigma}_n(Y)\rangle$ is given by

$$\langle\sigma_n(X)\overline{\sigma}_n(Y)\rangle = \mathrm{tr}\left[\sigma_n(X)\overline{\sigma}_n(Y)U\rho U^{\dagger}\right].$$

Schrödinger picture: $\langle\sigma_n(X)\overline{\sigma}_n(Y)\rangle = \mathrm{tr}\left[\sigma_n(X)\overline{\sigma}_n(Y)\underline{U\rho U^{\dagger}}\right]$

Depends on the time evolution
(Euclidean geometry is non-trivial)

How to compute correlator

Suppose that $\langle \sigma_n(X)\overline{\sigma}_n(Y)\rangle$ is given by

$$\langle \sigma_n(X)\overline{\sigma}_n(Y)\rangle = \mathrm{tr}\left[\sigma_n(X)\overline{\sigma}_n(Y)U\rho U^\dagger\right].$$

Two pictures: Schrödinger picture:

$$\langle \sigma_n(X)\overline{\sigma}_n(Y)\rangle = \mathrm{tr}\left[\sigma_n(X)\overline{\sigma}_n(Y)U\rho U^\dagger\right]$$

Heisenberg picture: $\langle \sigma_n(X)\overline{\sigma}_n(Y)\rangle = \mathrm{tr}\left[\hat{\sigma}_n(X)\hat{\overline{\sigma}}_n(Y)\underline{\rho}\right]$

Here, $\underline{\hat{\sigma}_n(Y) = U^\dagger\sigma_n(Y)U,\ \hat{\overline{\sigma}}_n(Y) = U^\dagger\overline{\sigma}_n(Y)U}$

Time-independent.
(Euclidean geometry may be simple.)

Depends on the time evolution

How to compute correlator

Suppose that $\langle \sigma_n(X)\overline{\sigma}_n(Y)\rangle$ is given by

$$\langle \sigma_n(X)\overline{\sigma}_n(Y)\rangle = \mathrm{tr}\left[\sigma_n(X)\overline{\sigma}_n(Y)U^\dagger \rho U\right].$$

Two pictures: Schrödinger picture:

$$\langle \sigma_n(X)\overline{\sigma}_n(Y)\rangle = \mathrm{tr}\left[\sigma_n(X)\overline{\sigma}_n(Y)U\rho U^\dagger\right]$$

Heisenberg picture: $\langle \sigma_n(X)\overline{\sigma}_n(Y)\rangle = \mathrm{tr}\left[\hat{\sigma}_n(X)\hat{\overline{\sigma}}_n(Y)\underline{\rho\,}\right]$

Here, $\underline{\hat{\sigma}_n(Y) = U^\dagger \sigma_n(Y) U,\ \hat{\overline{\sigma}}_n(Y) = U^\dagger \hat{\overline{\sigma}}_n(Y) U}$

Time-independent.
(Euclidean geometry may be simple.)

If the operator in Heisenberg picture is simple, then the calculation is simple.

How to compute correlator

Suppose that $\langle \sigma_n(X)\overline{\sigma}_n(Y)\rangle$ is given by

For $U = e^{-iH_{\text{Möbius/SSD}}t_1}$**,**

$$e^{iH_{\text{Möbius/SSD}}t_1}\sigma_n(X)e^{-iH_{\text{Möbius/SSD}}t_1} = \left|\frac{dw_X^{\text{New}}}{dw_x}\right|^{2h_n}\sigma_n\left(w_X^{\text{New}},\overline{w}_X^{\text{New}}\right)$$

where the conformal dimension is $h_n = \frac{c(n^2-1)}{24n}$ **.**

Heisenberg picture: $\langle \sigma_n(X)\overline{\sigma}_n(Y)\rangle = \text{tr}\left[\hat{\sigma}_n(X)\hat{\overline{\sigma}}_n(Y)\underline{\rho\,]}\right.$

Here, $\underline{\hat{\sigma}_n(Y) = U^\dagger\sigma_n(Y)U,\ \hat{\overline{\sigma}}_n(Y) = U^\dagger\hat{\overline{\sigma}}_n(Y)U}$

Time-independent.
(Euclidean geometry may be simple.)

Depends on the time evolution

In AdS/CFT correspondence

In Schrödinger picture, the **dual geometry evolves with time**, while **the locations of operators don’t**.

$$\langle \sigma_n(X)\overline{\sigma}_n(Y)\rangle = \mathrm{tr}\left[\sigma_n(X)\overline{\sigma}_n(Y)U\rho U^{\dagger}\right]$$

In Heisenberg picture, the **dual geometry is static**, while **the locations of operators evolves with time**.

$$\langle \sigma_n(X)\overline{\sigma}_n(Y)\rangle = \mathrm{tr}\left[\hat{\sigma}_n(X)\hat{\overline{\sigma}}_n(Y)\rho\right]$$

In AdS/CFT correspondence

In Schrödinger picture, the **dual geometry evolves with time**, while **the locations of operators don’t**.

$$\langle \sigma_n(X)\overline{\sigma}_n(Y)\rangle = \mathrm{tr}\left[\sigma_n(X)\overline{\sigma}_n(Y)U\rho U^\dagger\right]$$

dual geometry may be complicated

In Heisenberg picture, the **dual geometry is static**, while **the locations of operators evolves with time**.

$$\langle \sigma_n(X)\overline{\sigma}_n(Y)\rangle = \mathrm{tr}\left[\hat{\sigma}_n(X)\hat{\overline{\sigma}}_n(Y)\rho\right]$$

dual geometry may be simple

Results in Summary 2

Entanglement entropy in the twist operator formalism

As a consequence,

$$S_B = -\frac{c}{12} \log \left[\prod_{i=1,2} \left| \frac{dw^{\text{New}}_{Y_i}}{dw_{Y_i}} \right|^2 \right] + \lim_{n \to 1} \frac{1}{1-n} \log \left\langle \sigma_n \left(w^{\text{New}}_{Y_1}, \overline{w}^{\text{New}}_{Y_1} \right) \overline{\sigma}_n \left(w^{\text{New}}_{Y_2}, \overline{w}^{\text{New}}_{Y_2} \right) \right\rangle$$

where B is the subsystem of the $\mathcal{H}_1$ **.**

Entanglement entropy in the twist operator formalism

As a consequence,

$$S_B = -\frac{c}{12}\log\left[\prod_{i=1,2}\left|\frac{dw^{\mathrm{New}}_{\mathrm{Y}_i}}{dw_{\mathrm{Y}_i}}\right|^2\right] + \underline{\lim_{n\to 1}\frac{1}{1-n}\log\left\langle\sigma_n\left(w^{\mathrm{New}}_{\mathrm{Y}_1},\overline{w}^{\mathrm{New}}_{\mathrm{Y}_1}\right)\overline{\sigma}_n\left(w^{\mathrm{New}}_{\mathrm{Y}_2},\overline{w}^{\mathrm{New}}_{\mathrm{Y}_2}\right)\right\rangle}$$

where B is the subsystem of the $\mathcal{H}_1$.

↑

The piece depending on the detail of CFT.

Note

The parameter region considered in this talk is

$$L \gg l_{\mathcal{V}}, t \gg \epsilon \gg 1,$$

where these parameters are dimensionless and their unit is the lattice spacing.

Entanglement entropy in the twist operator formalism

As a consequence,

$$S_B = -\frac{c}{12}\log\left[\prod_{i=1,2}\left|\frac{dw^{\mathrm{New}}_{\mathrm{Y}_i}}{dw_{\mathrm{Y}_i}}\right|^2\right] + \underline{\lim_{n\to 1}\frac{1}{1-n}\log\left\langle\sigma_n\left(w^{\mathrm{New}}_{\mathrm{Y}_1},\overline{w}^{\mathrm{New}}_{\mathrm{Y}_1}\right)\overline{\sigma}_n\left(w^{\mathrm{New}}_{\mathrm{Y}_2},\overline{w}^{\mathrm{New}}_{\mathrm{Y}_2}\right)\right\rangle}$$

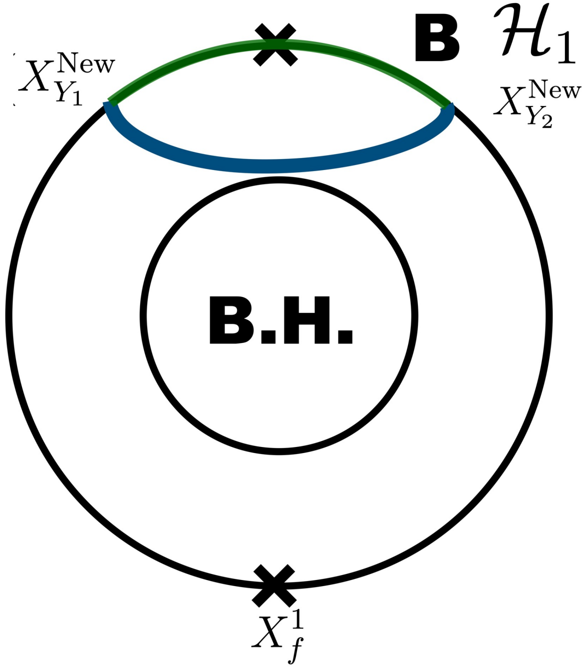
where B is the subsystem of the $\mathcal{H}_1$.

$\downarrow$

Geodesic length associated with B on the BTZ-black-hole geometry.

Vacuum entanglement entropy

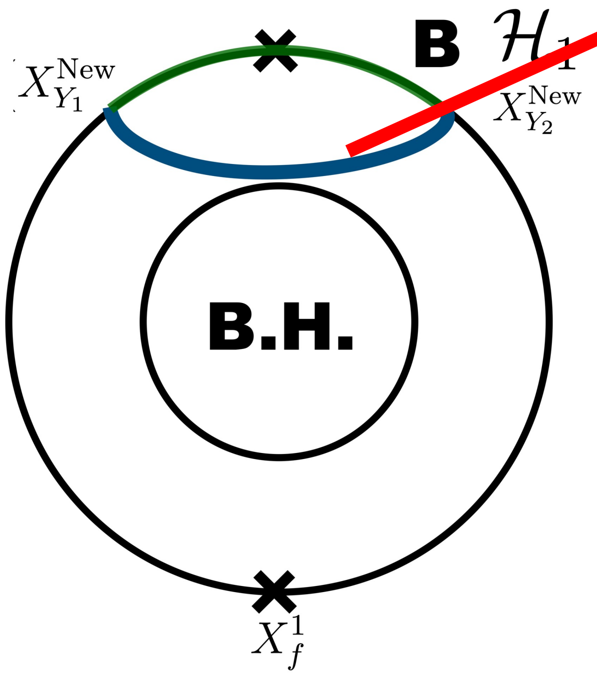
$$S_B = -\frac{c}{12}\log\left[\prod_{i=1,2}\left|\frac{dw^{\mathrm{New}}_{\mathrm{Y}_i}}{dw_{\mathrm{Y}_i}}\right|^2\right] + \lim_{n\to 1}\frac{1}{1-n}\log\left\langle\sigma_n\left(w^{\mathrm{New}}_{\mathrm{Y}_1},\overline{w}^{\mathrm{New}}_{\mathrm{Y}_1}\right)\overline{\sigma}_n\left(w^{\mathrm{New}}_{\mathrm{Y}_2},\overline{w}^{\mathrm{New}}_{\mathrm{Y}_2}\right)\right\rangle_{2\epsilon}$$



Vacuum entanglement entropy

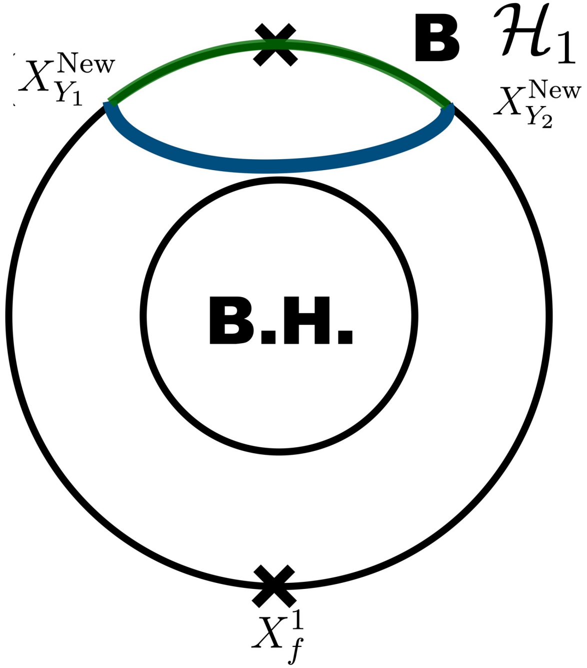
$$S_B = -\frac{c}{12}\log\left[\prod_{i=1,2}\left|\frac{dw_{\mathrm{Y}_i}^{\mathrm{New}}}{dw_{\mathrm{Y}_i}}\right|^2\right] + \underline{\lim_{n\to 1}\frac{1}{1-n}\log\left\langle \sigma_n\left(w_{\mathrm{Y}_1}^{\mathrm{New}}, \overline{w}_{\mathrm{Y}_1}^{\mathrm{New}}\right)\overline{\sigma}_n\left(w_{\mathrm{Y}_2}^{\mathrm{New}}, \overline{w}_{\mathrm{Y}_2}^{\mathrm{New}}\right)\right\rangle_{2\epsilon}}$$

Geodesic length



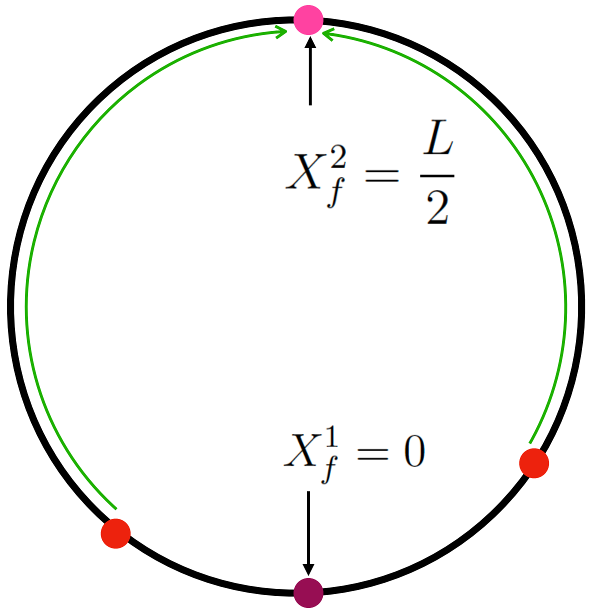
Vacuum entanglement entropy

$$S_B = -\frac{c}{12}\log\left[\prod_{i=1,2}\left|\frac{dw^{\mathrm{New}}_{\mathrm{Y}_i}}{dw_{\mathrm{Y}_i}}\right|^2\right] + \lim_{n\to 1}\frac{1}{1-n}\log\left\langle \sigma_n\left(w^{\mathrm{New}}_{\mathrm{Y}_1},\overline{w}^{\mathrm{New}}_{\mathrm{Y}_1}\right)\overline{\sigma}_n\left(w^{\mathrm{New}}_{\mathrm{Y}_2},\overline{w}^{\mathrm{New}}_{\mathrm{Y}_2}\right)\right\rangle_{2\epsilon}$$

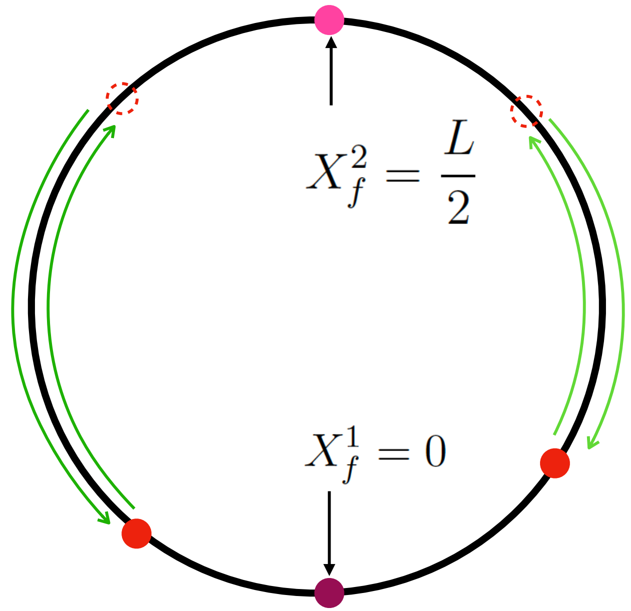


$$\mathcal{O}(1) \ll \mathcal{O}(1/\epsilon)$$

Define the spatial position as $X_X^{\mathrm{New}} = \dfrac{w_X^{\mathrm{New}} - \overline{w}_X^{\mathrm{New}}}{2i}$.



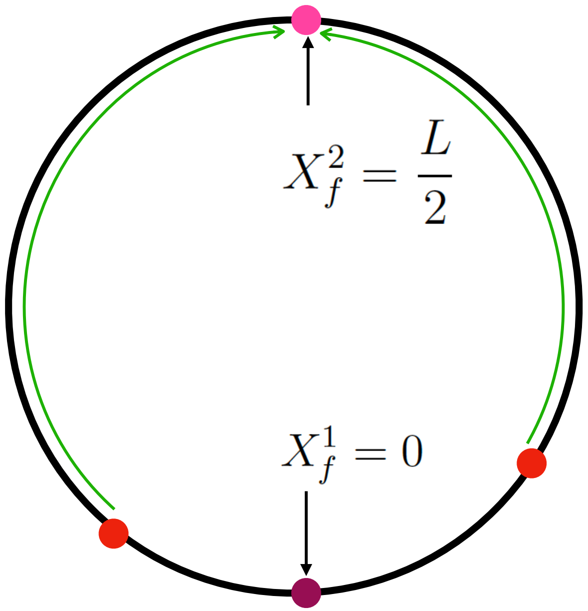
(a) The SSD time evolution



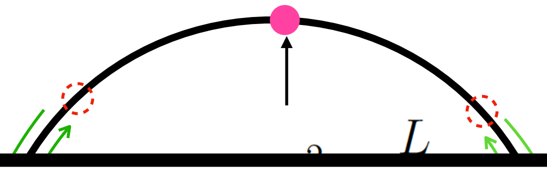
(b) The Möbius time evolution

During the SSD evolution, ***the operators move to*** $X = X_f^2 = \dfrac{L}{2}$.

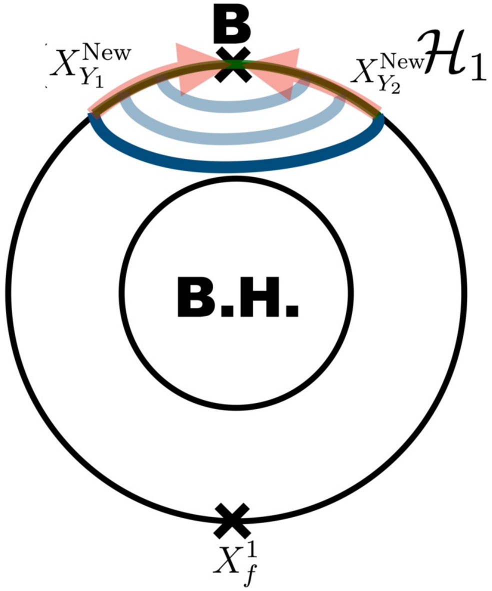
Define the spatial position as $X_X^{\mathrm{New}} = \frac{w_X^{\mathrm{New}} - \overline{w}_X^{\mathrm{New}}}{2i}$.



(a) The SSD time evolution



The (effective) size of B monotonically decrease with times.

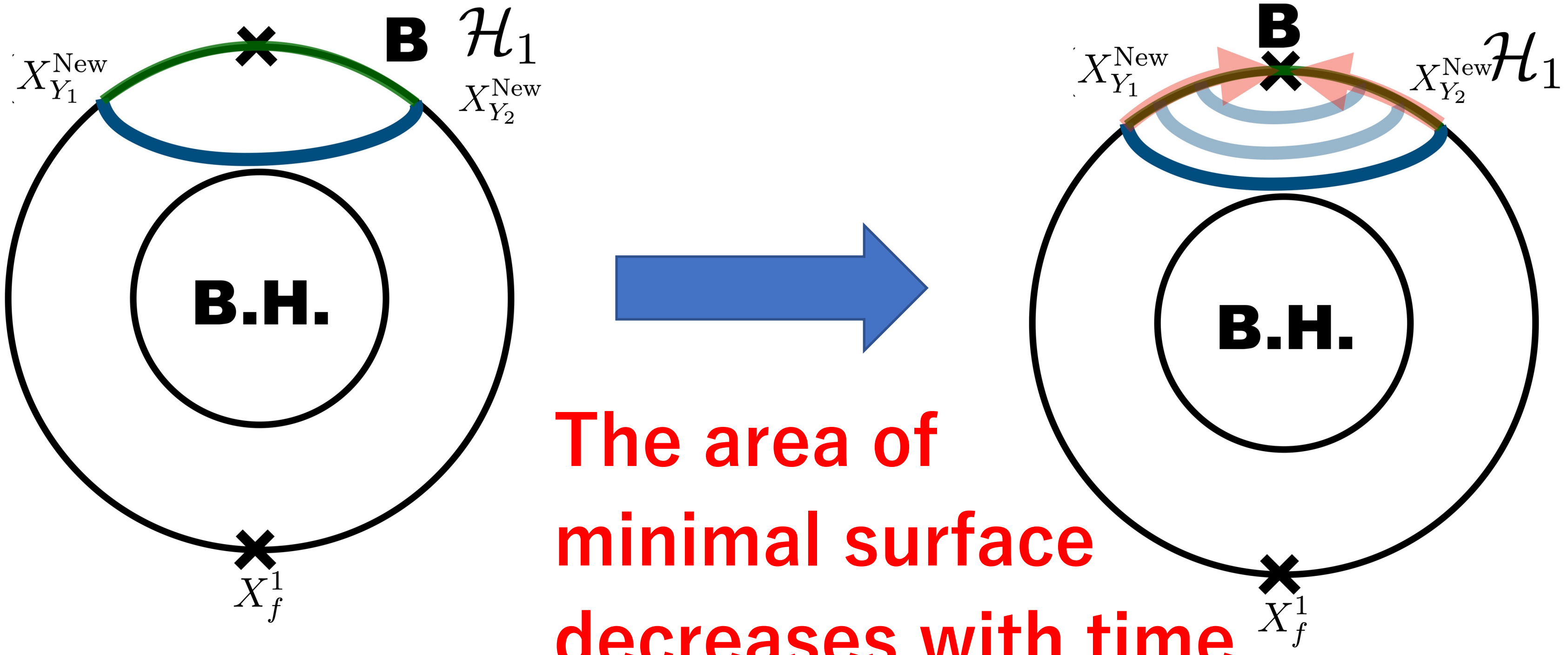


ıtion

Under the SSD evolution, ***the o***

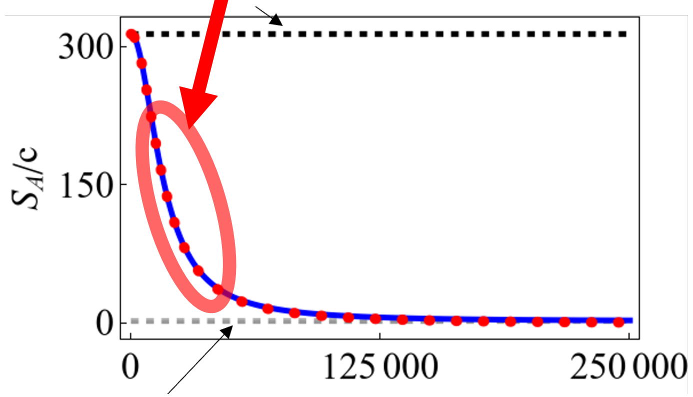
Vacuum entanglement entropy

$$S_B = -\frac{c}{12}\log\left[\prod_{i=1,2}\left|\frac{dw^{\mathrm{New}}_{Y_i}}{dw_{Y_i}}\right|^2\right] + \lim_{n\to1}\frac{1}{1-n}\log\left\langle\sigma_n\left(w^{\mathrm{New}}_{Y_1},\overline{w}^{\mathrm{New}}_{Y_1}\right)\overline{\sigma}_n\left(w^{\mathrm{New}}_{Y_2},\overline{w}^{\mathrm{New}}_{Y_2}\right)\right\rangle_{2\epsilon}$$



The area of minimal surface decreases with time.

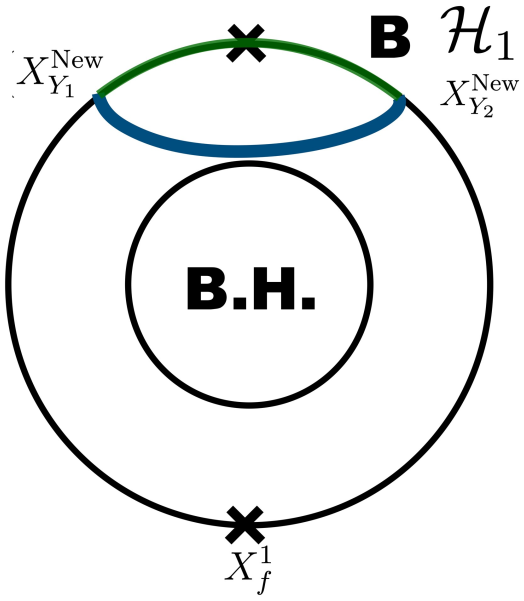
EE decreases with time.

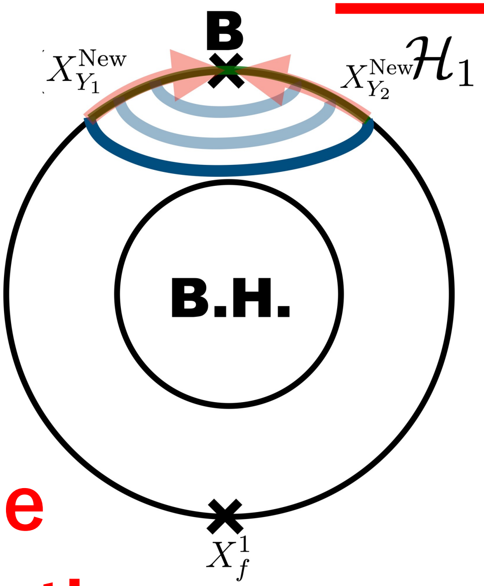


Vacuum entanglement entropy

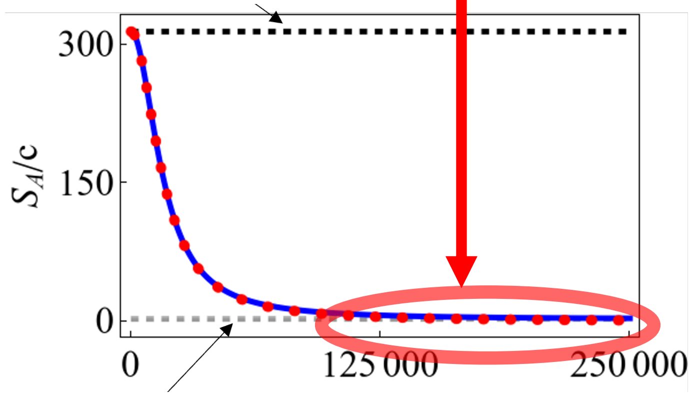
$$S_B = \underline{-\frac{c}{12}\log\left[\prod_{i=1,2}\left|\frac{dw^{\mathrm{New}}_{Y_i}}{dw_{Y_i}}\right|^2\right]} + \underline{\lim_{n\to 1}\frac{1}{1-n}\log\left\langle \sigma_n\left(w^{\mathrm{New}}_{Y_1},\overline{w}^{\mathrm{New}}_{Y_1}\right)\overline{\sigma}_n\left(w^{\mathrm{New}}_{Y_2},\overline{w}^{\mathrm{New}}_{Y_2}\right)\right\rangle_{2\epsilon}}$$

$$\mathcal{O}(1) + \mathcal{O}(1) \approx S_A^{\mathrm{Vacuum}} = \underline{\frac{c}{3}\log\left[\frac{L}{\pi}\sin\left[\frac{\pi(Y_1-Y_2)}{L}\right]\right]}$$





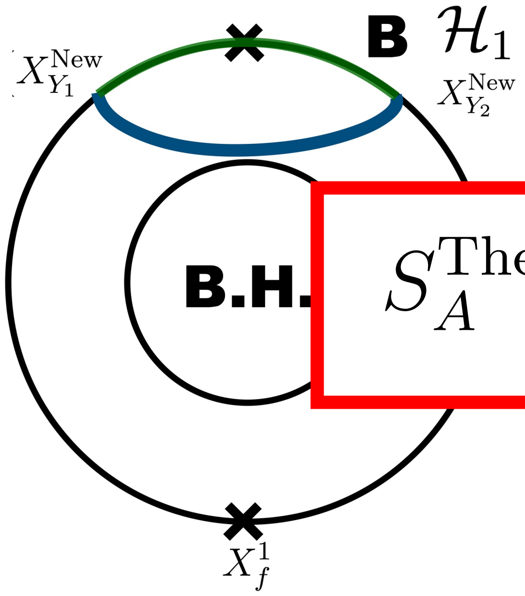
The area of minimal surface decreases with time.



Vacuum entanglement entropy

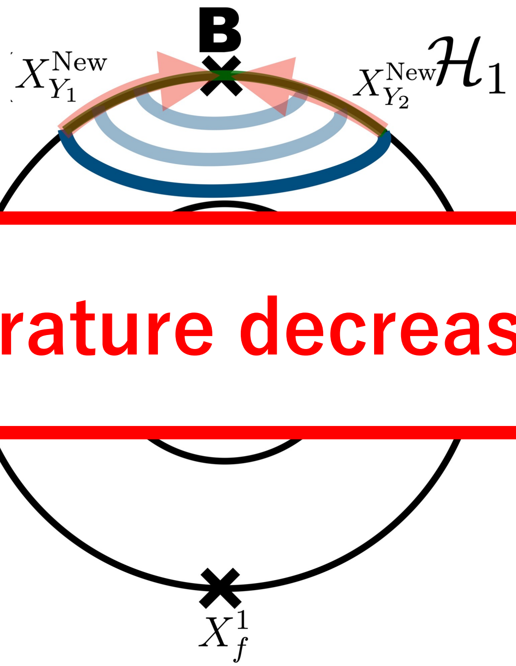
$$S_B = \underbrace{-\frac{c}{12}\log\left[\prod_{i=1,2}\left|\frac{dw^{\text{New}}_{Y_i}}{dw_{Y_i}}\right|^2\right]} + \underbrace{\lim_{n\to1}\frac{1}{1-n}\log\left\langle\sigma_n\left(w^{\text{New}}_{Y_1},\overline{w}^{\text{New}}_{Y_1}\right)\overline{\sigma}_n\left(w^{\text{New}}_{Y_2},\overline{w}^{\text{New}}_{Y_2}\right)\right\rangle_{2\epsilon}}$$

$$\mathcal{O}(1) + \mathcal{O}(1) \approx \underline{S_A^{\text{Vacuum}} = \frac{c}{3}\log\left[\frac{L}{\pi}\sin\left[\frac{\pi(Y_1-Y_2)}{L}\right]\right]}$$



$$S_A^{\text{Thermal}} \Rightarrow S_A^{\text{Vacuum}}$$ **:Local temperature decreases.**

The area of minimal surface decreases with time.

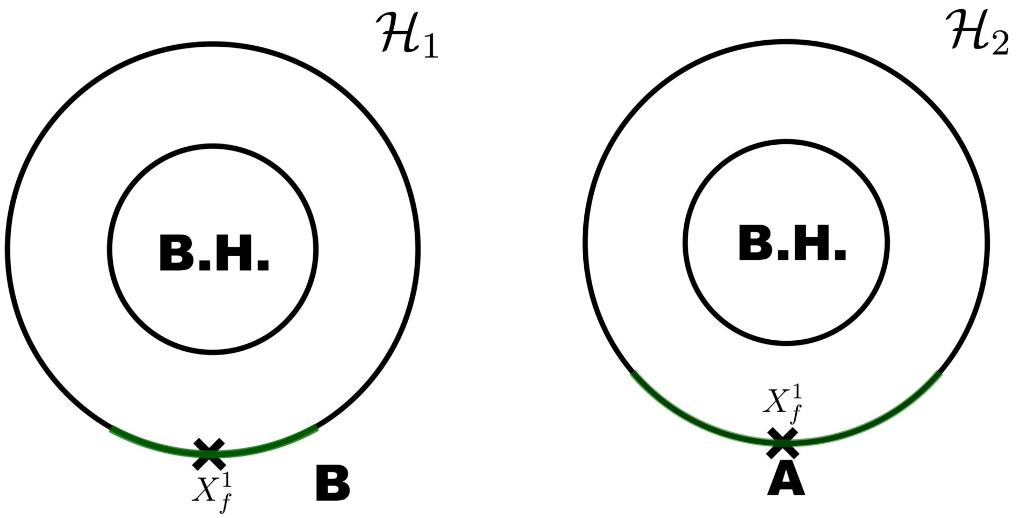


Mutual information of thermofield double state

As a simple example,

$$|\phi(t_1)\rangle = \left(e^{-it_1 H^1_{\mathrm{SSD}}} \otimes \mathbf{1}_2\right) |\mathrm{TFD}\rangle$$

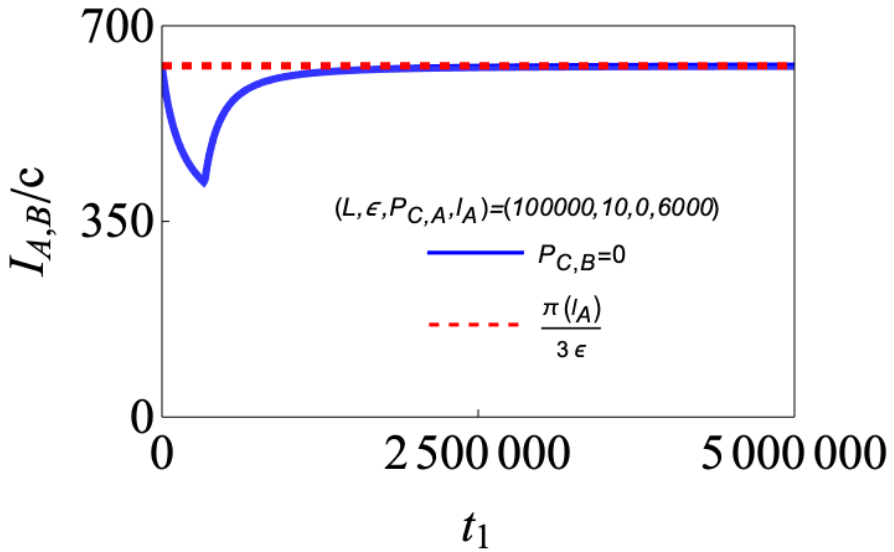
let us report the time-dependence of $I_{A,B}$.



Let A and B denote the subsystems of $\mathcal{H}_2$ and $\mathcal{H}_1$,respectively.

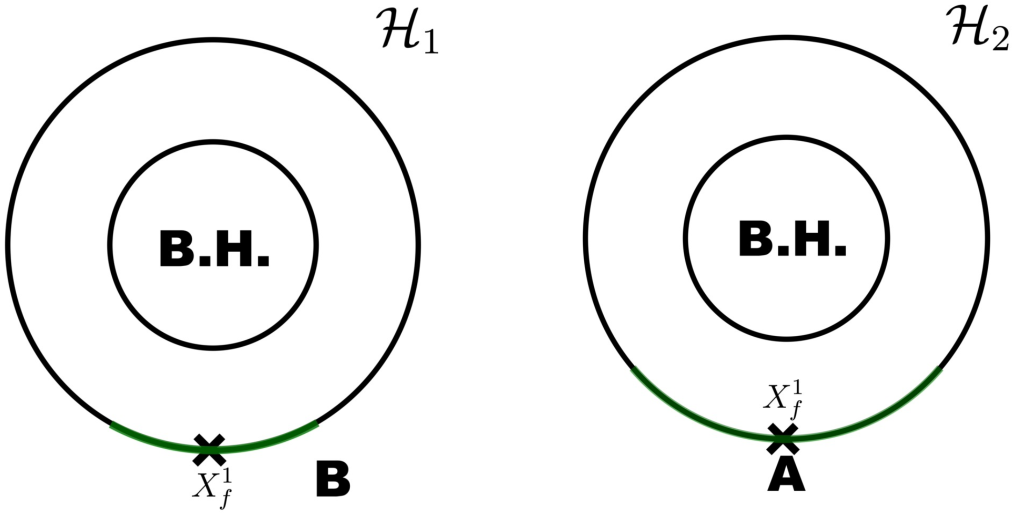
B **includes** X^1_f .

Here, $l_{\mathcal{V}=A,B}$, $P_{C,\mathcal{V}=A,B}$ denote the subsystem sizes and centers of A and B, respectively.



Mutual information of thermofield double state

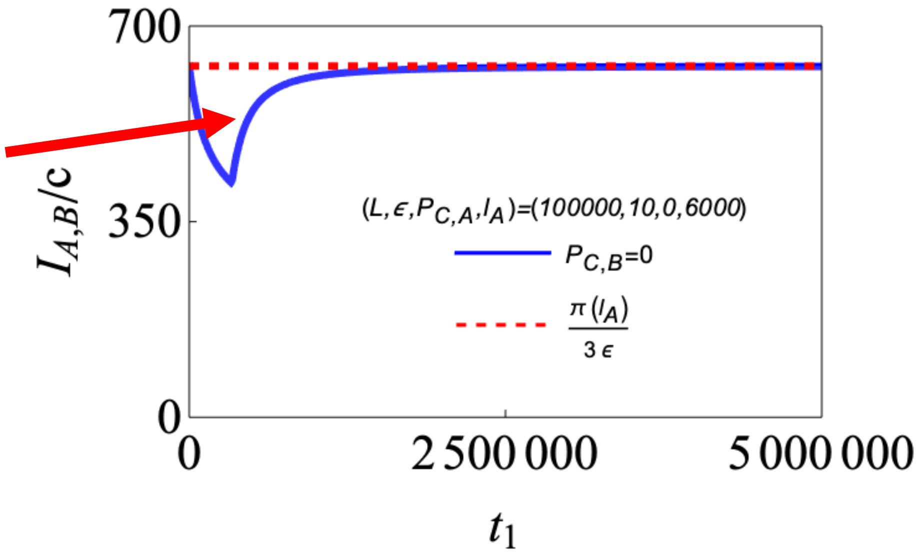
As a simple example,
$|\phi(t_1)\rangle = \left(e^{-it_1 H^1_{\text{SSD}}} \otimes \mathbf{1}_2\right)|\text{TFD}\rangle$
let us report the time-dependence of $I_{A,B}$.



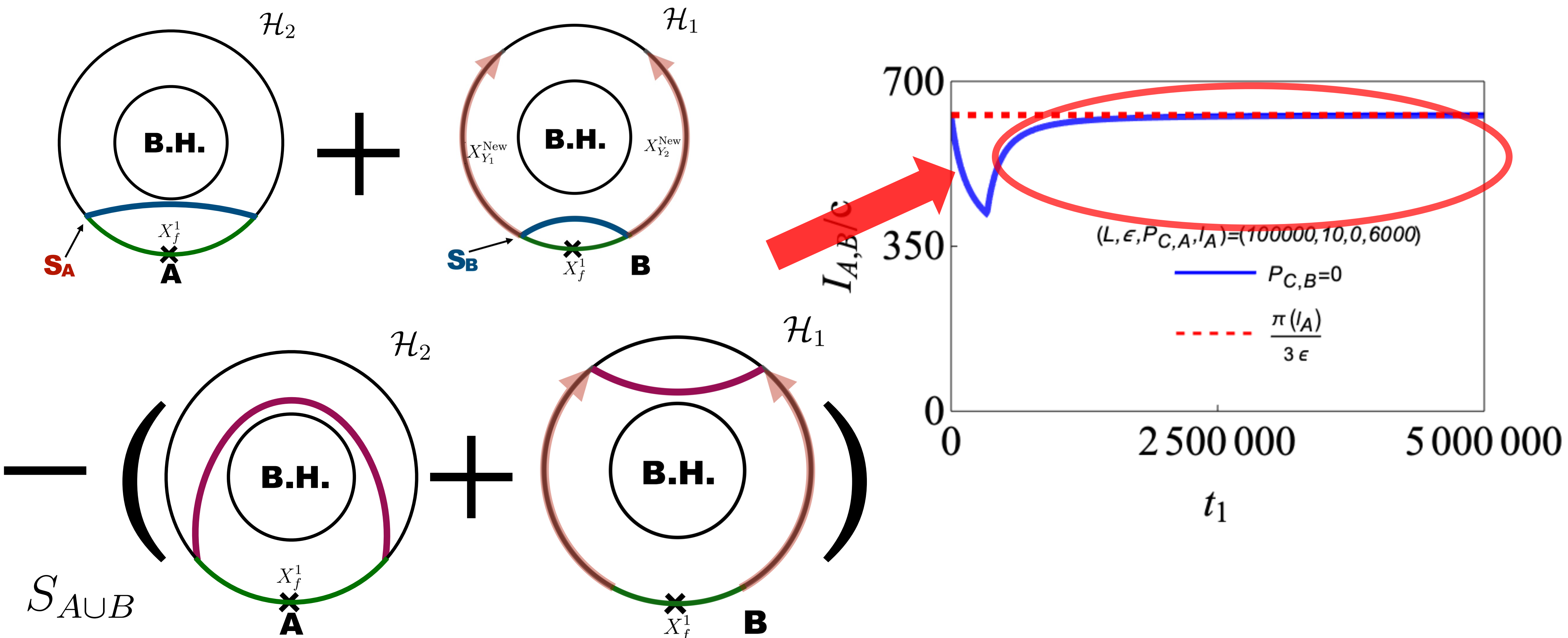
Let A and B denote the subsystems of

Under the SSD-time evolution, mutual information between A and B returns to the initial value.

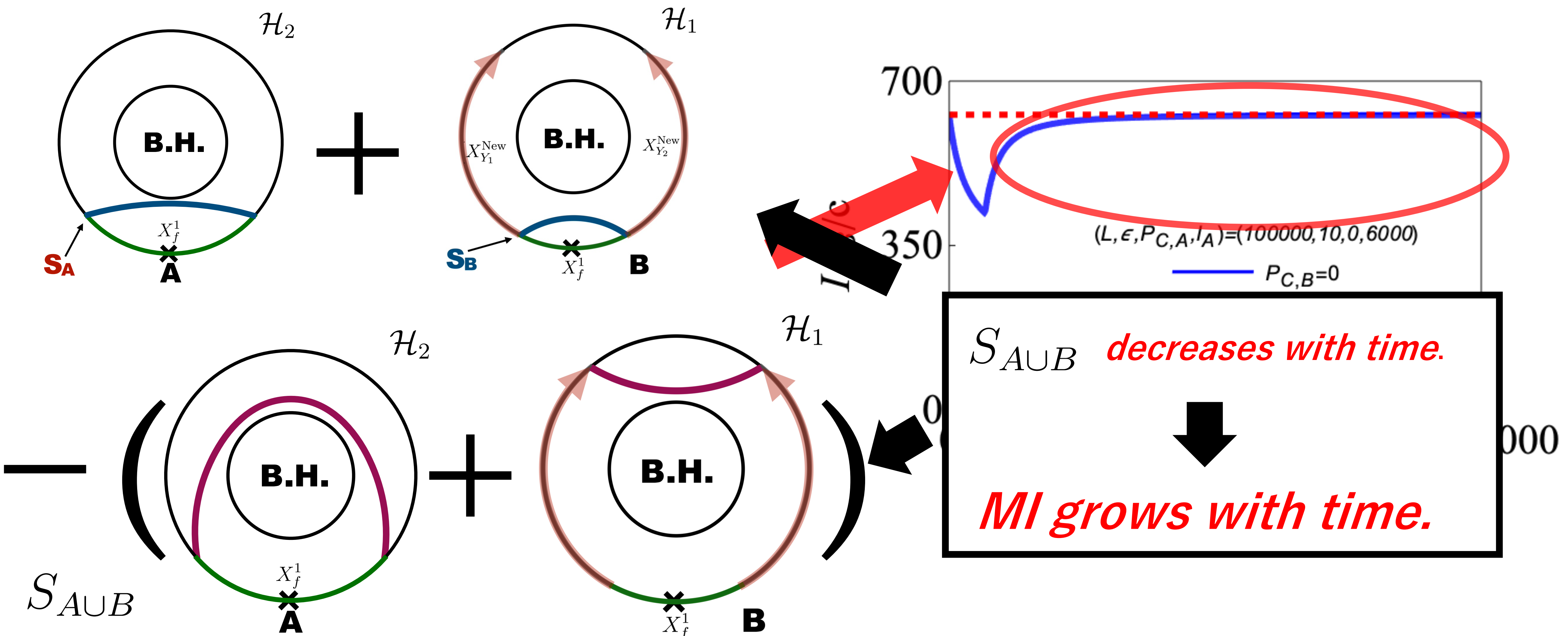
subsystem sizes and centers of A and B, respectively.



Why does mutual information revive?



Why does mutual information revive?

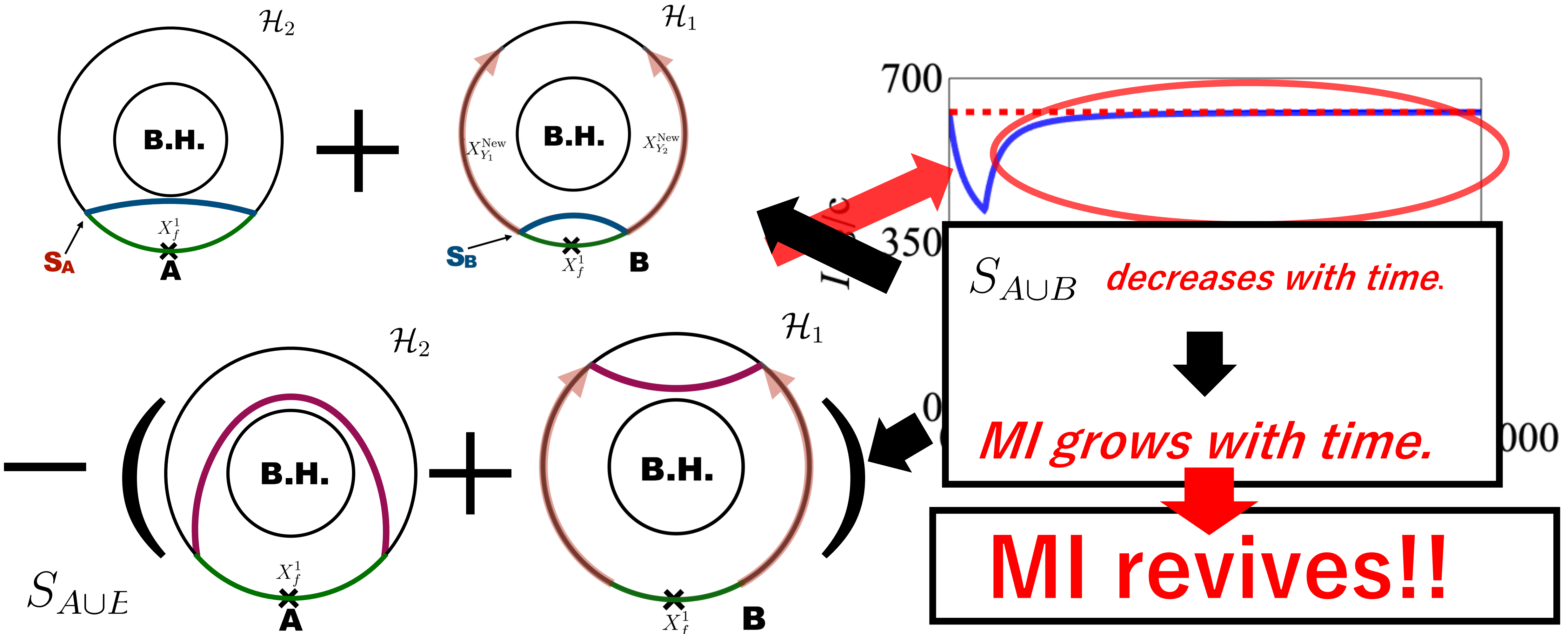


$S_{A\cup B}$ *decreases with time.*

↓

MI grows with time.

Why does mutual information revive?

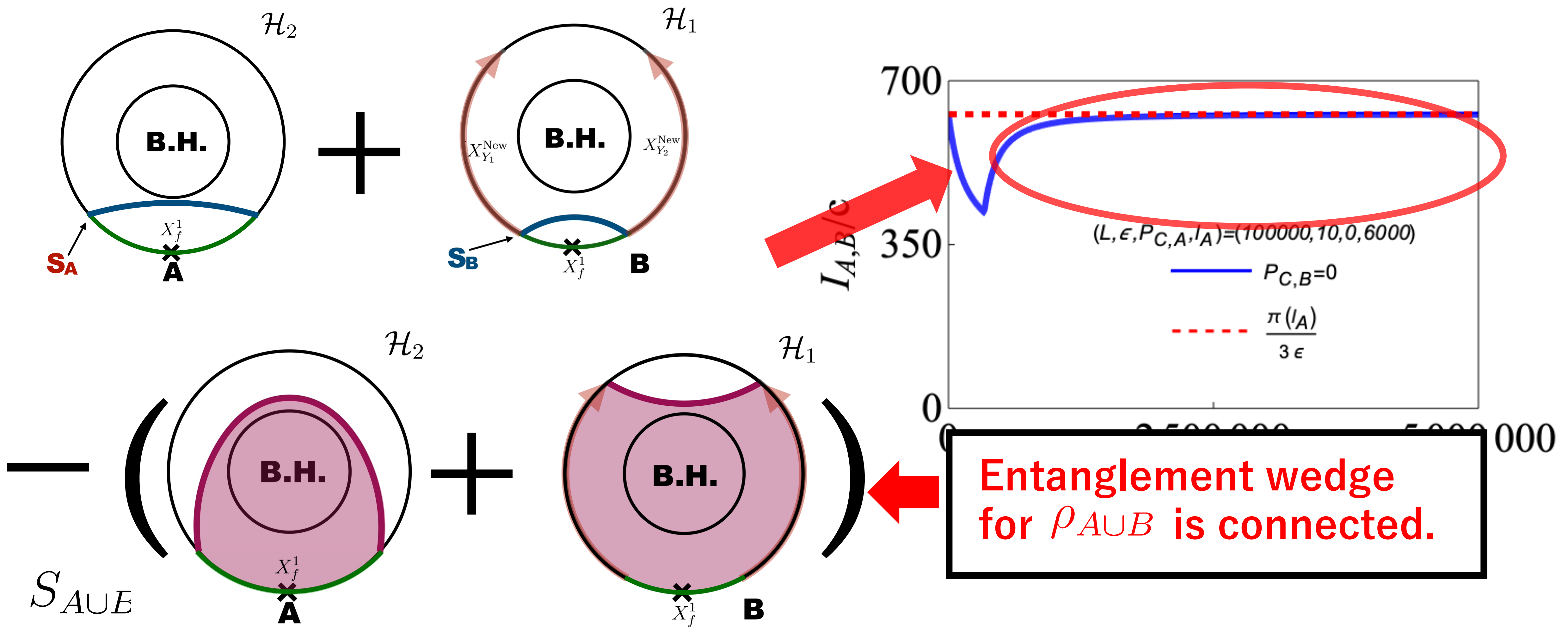


$S_{A\cup B}$ *decreases with time.*

MI grows with time.

MI revives!!

Why does mutual information revive?



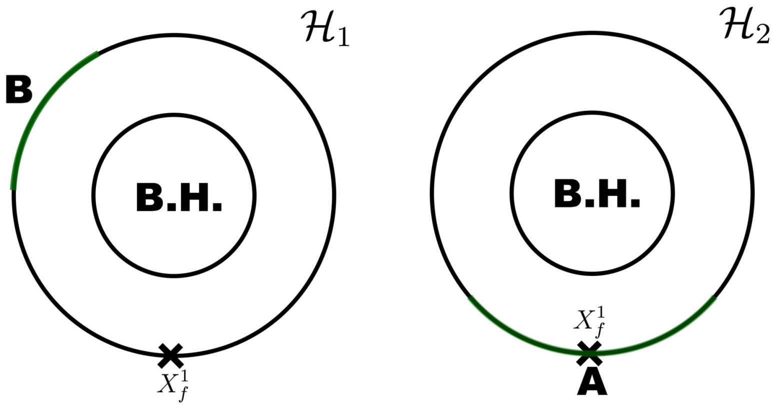
Entanglement wedge for $\rho_{A\cup B}$ is connected.

Mutual information of thermofield double state

We start from thermofield double state,
then evolve the system with the 2d holographic Hamiltonian,

$$|\Psi(t_0)\rangle = \left(e^{-iH_0^1 t_0} \otimes \mathbf{1}_2 \right) |\mathrm{TFD}\rangle$$

For the time-regime, $t_0 \gg \mathcal{O}(L)$,
$I_{A,B}$ ***should be completely destroyed*.**



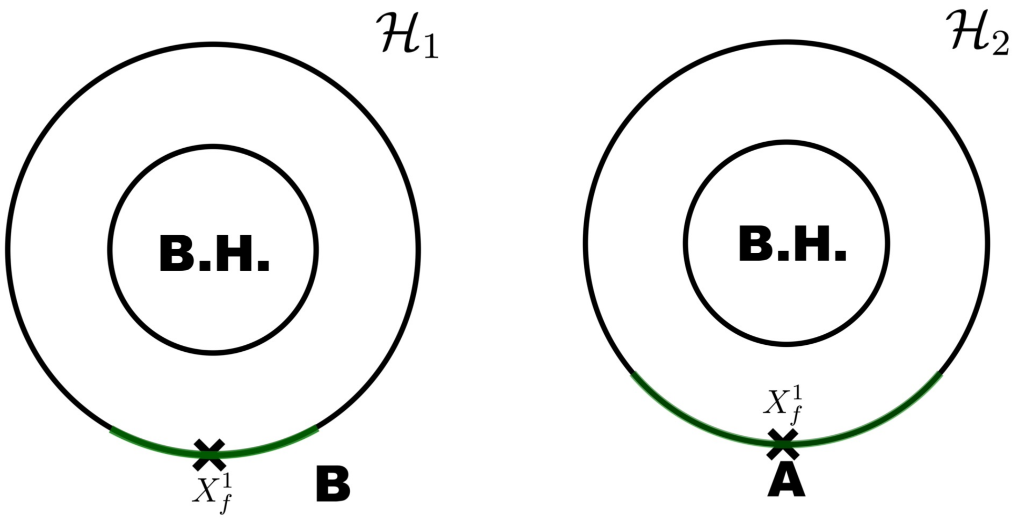
Mutual information of thermofield double state

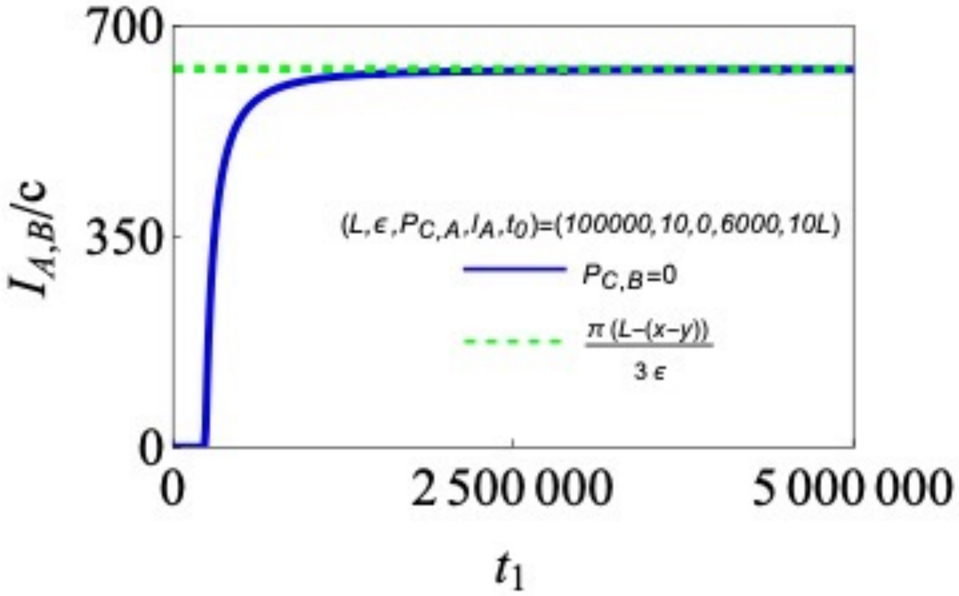
For the time-regime, $t_0 \gg \mathcal{O}(L)$,

$I_{A,B}$ ***should be completely destroyed*.**

Then, we evolve this state with the SSD Hamiltonian,

$$|\Psi(t_1, t_0)\rangle = \left(e^{-iH^1_{\text{SSD}}t_1} \otimes \mathbf{1}_2\right)|\Psi(t_0)\rangle \,.$$





Mutual information of thermofield double state

For the time-regime, $t_0 \gg \mathcal{O}(L)$,

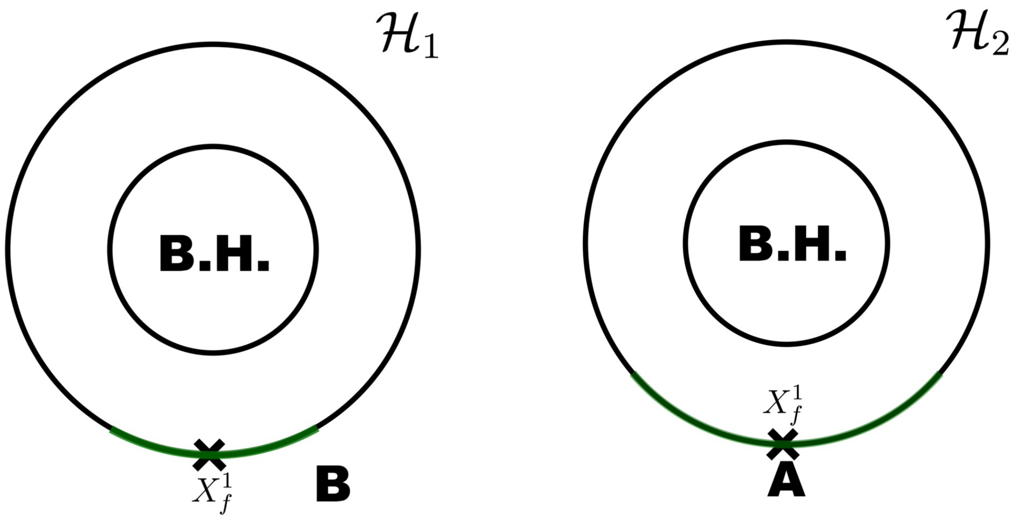
$I_{A,B}$ ***should be completely destroyed***.

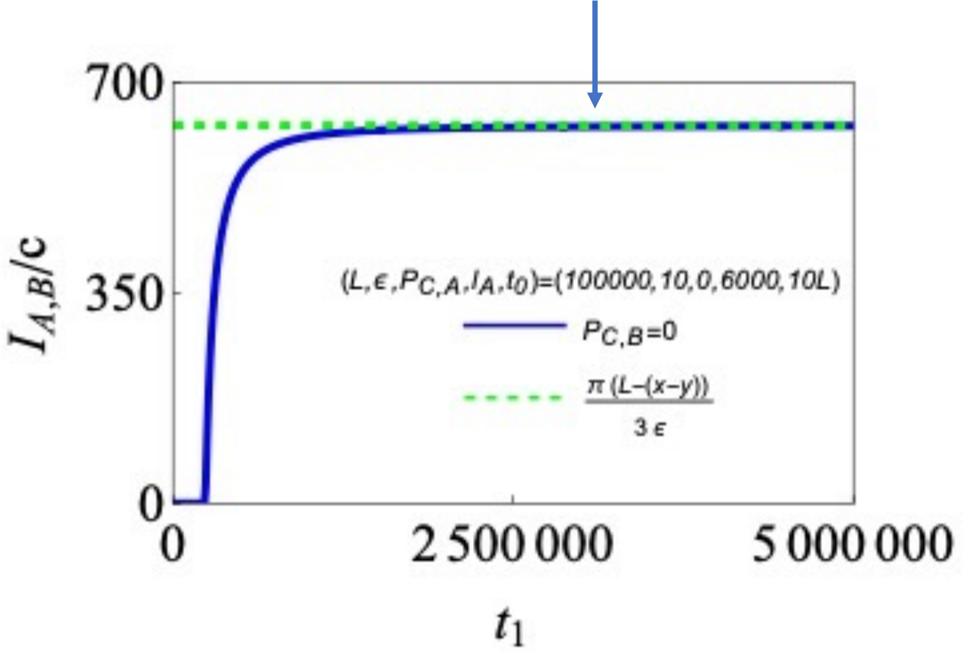
Then, we evolve this state with the SSD Hamiltonian,

$$|\Psi(t_1,t_0)\rangle = \left(e^{-iH^1_{\mathrm{SSD}}t_1} \otimes \mathbf{1}_2\right)|\Psi(t_0)\rangle \, .$$

Correlation revives.

(Entanglement wedge: disconnected → connected)





Mutual information of thermofield double state

For the time-regime, $t_0 \gg \mathcal{O}(L)$,

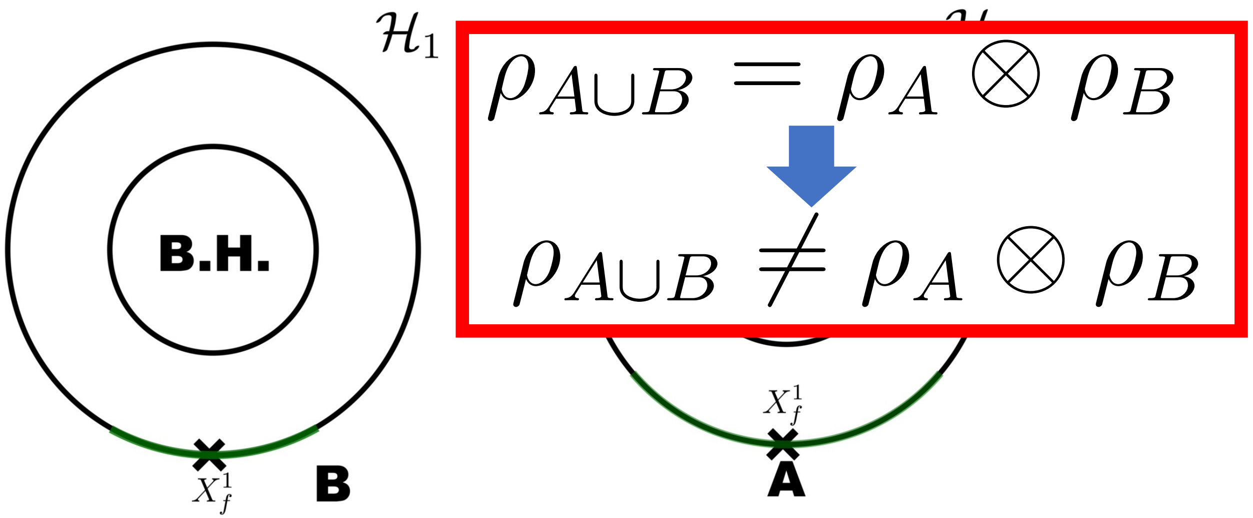
$I_{A,B}$ ***should be completely destroyed*.**

Then, we evolve this state with the SSD Hamiltonian,

$$|\Psi(t_1, t_0)\rangle = \left(e^{-iH^1_{\text{SSD}}t_1} \otimes \mathbf{1}_2\right)|\Psi(t_0)\rangle \,.$$

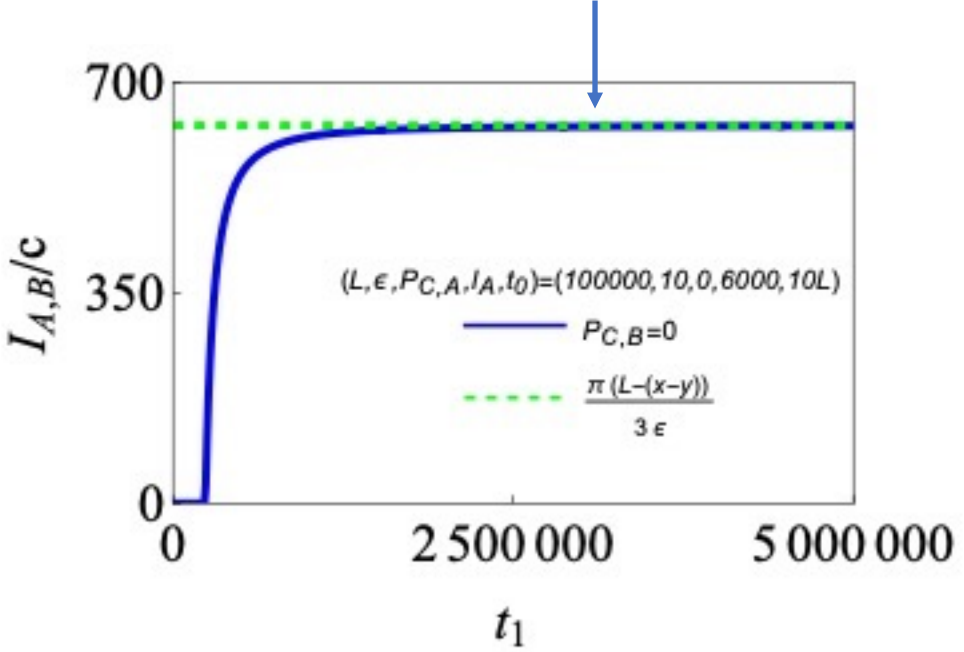
Correlation revives.

(Entanglement wedge: disconnected
➡ *connected)*



$$\rho_{A\cup B} = \rho_A \otimes \rho_B$$

$$\rho_{A\cup B} \neq \rho_A \otimes \rho_B$$



Results in Summary 4

The thermodynamic on the curved spacetime

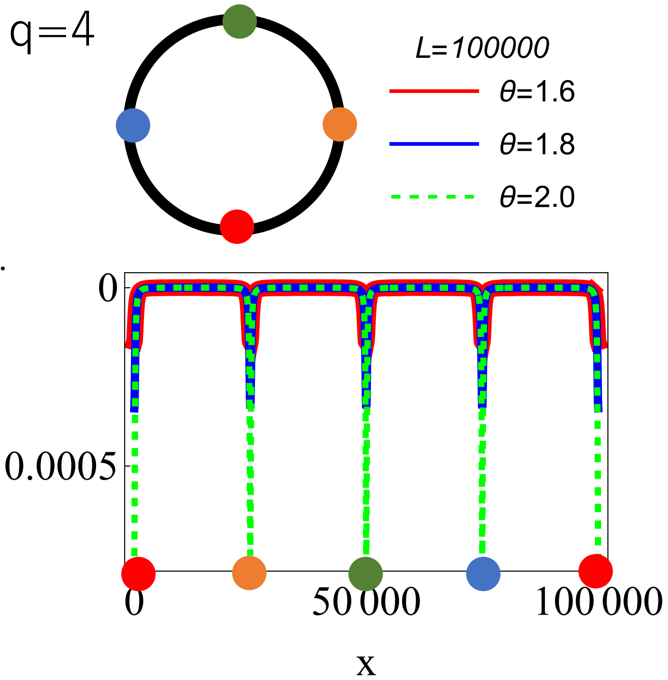
Our thermal state: $\rho = \dfrac{e^{-\beta H_{q-\text{Möbius}}}}{\text{tr}e^{-\beta H_{q-\text{Möbius}}}}$

Hamiltonian: $H_{q-\text{Möbius}} = \int_0^L dx \left[1 - \tanh 2\theta \left(1 - 2\sin^2\left(\frac{q\pi x}{L}\right)\right)\right] (T(x) + \overline{T}(x))$

(q is an integer.) $= \int_0^L dx \sqrt{-\det g}(T(x) + \overline{T}(x))$

Geometry: $ds^2 = -f^2(x,\theta)dt^2 + dx^2, \quad f(x,\theta) = 1 - \tanh 2\theta \left(1 - 2\sin^2\left(\frac{q\pi x}{L}\right)\right).$

Curvature: $R(\theta, x) = -\dfrac{2\partial_x^2 f(\theta,x)}{f(\theta,x)} = \dfrac{8\pi^2 q^2 \tanh(2\theta)\cos\left(\frac{2\pi q x}{L}\right)}{L^2\left(\tanh(2\theta)\cos\left(\frac{2\pi q x}{L}\right) - 1\right)}$



The thermodynamic on the curved spacetime

Our thermal state: $\rho = \dfrac{e^{-\beta H_{q-\text{Möbius}}}}{\mathrm{tr} e^{-\beta H_{q-\text{Möbius}}}}$

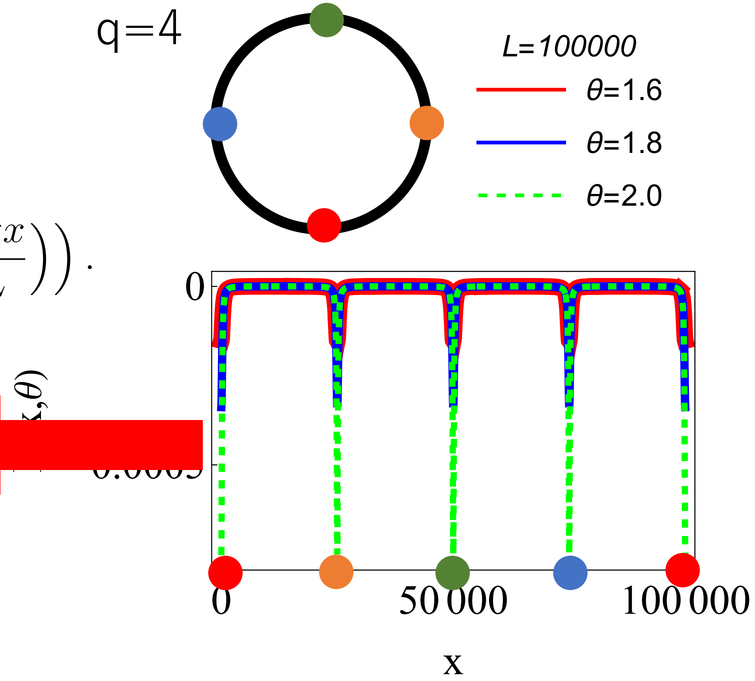
Hamiltonian: $H_{q-\text{Möbius}} = \displaystyle\int_0^L dx \left[1 - \tanh 2\theta \left(1 - 2\sin^2\left(\frac{q\pi x}{L}\right)\right)\right] (T(x) + \overline{T}(x))$

(q is an integer.) $\displaystyle = \int_0^L dx \sqrt{-\det g} (T(x) + \overline{T}(x))$

Geometry: $ds^2 = -f^2(x,\theta)dt^2 + dx^2, \;\; f(x,\theta) = 1 - \tanh 2\theta \left(1 - 2\sin^2\left(\frac{q\pi x}{L}\right)\right).$

Curvat

Curvature around x=0,L/4,L/2,3L/4 negatively grows with θ.



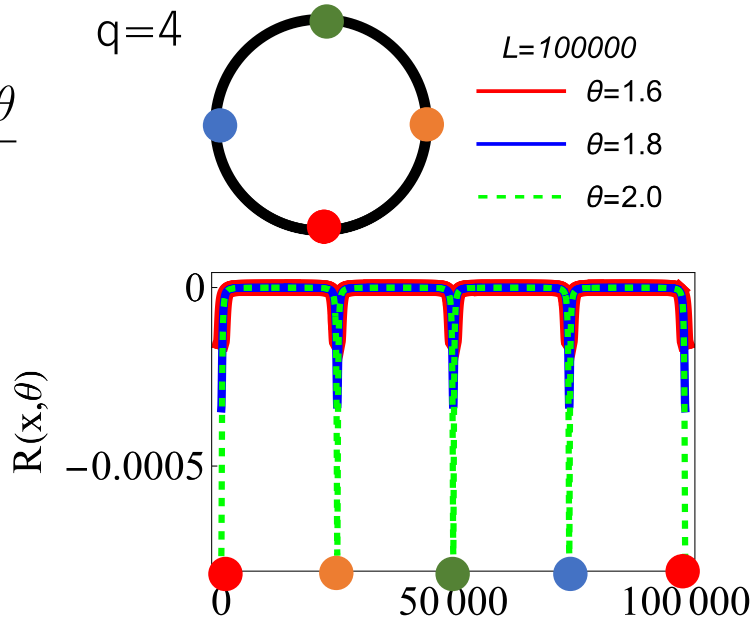
The thermodynamic on the curved spacetime

Our thermal state: $\rho = \frac{e^{-\beta H_{q-\text{Möbius}}}}{\mathrm{tr} e^{-\beta H_{q-\text{Möbius}}}}$

We assume $L/\beta < 1$ (low temp.).

For $\theta = 0$ (flat), entropy is $S \approx \mathcal{O}(1)$.

The moduli parameter of this torus is $\tau = \frac{L \cosh 2\theta}{\beta}$



The thermodynamic on the curved spacetime

Our thermal state: $\rho = \frac{e^{-\beta H_{q-\text{Möbius}}}}{\mathrm{tr} e^{-\beta H_{q-\text{Möbius}}}}$

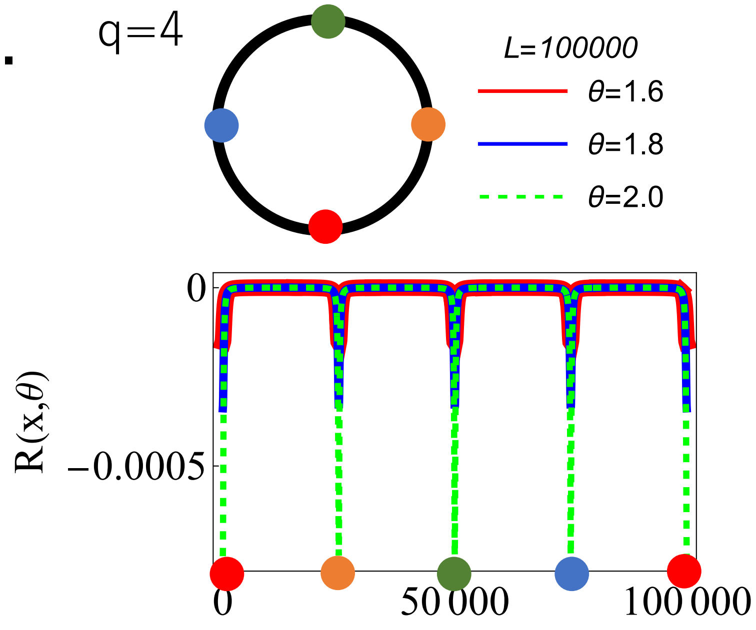
We assume $L/\beta < 1$ (low temp.).

For $\theta = 0$ (flat), entropy is $S \approx \mathcal{O}(1)$.

The moduli parameter of this torus is $\tau = \frac{L \cosh 2\theta}{\beta}$.

Therefore, if θ increases, then

$$S \approx \begin{cases} \mathcal{O}(1) & L \cosh 2\theta / \beta < 1 \\ \frac{c\pi L \cosh 2\theta}{6\beta} \underset{\theta \gg 1}{\propto} \frac{Le^{2\theta}}{\epsilon} & L \cosh 2\theta / \beta > 1 \end{cases}$$



The thermodynamic on the curved spacetime

Our thermal state: $\rho = \frac{e^{-\beta H_{q-\text{Möbius}}}}{\text{tr} e^{-\beta H_{q-\text{Möbius}}}}$

We assume $L/\beta < 1$ (low temp.).

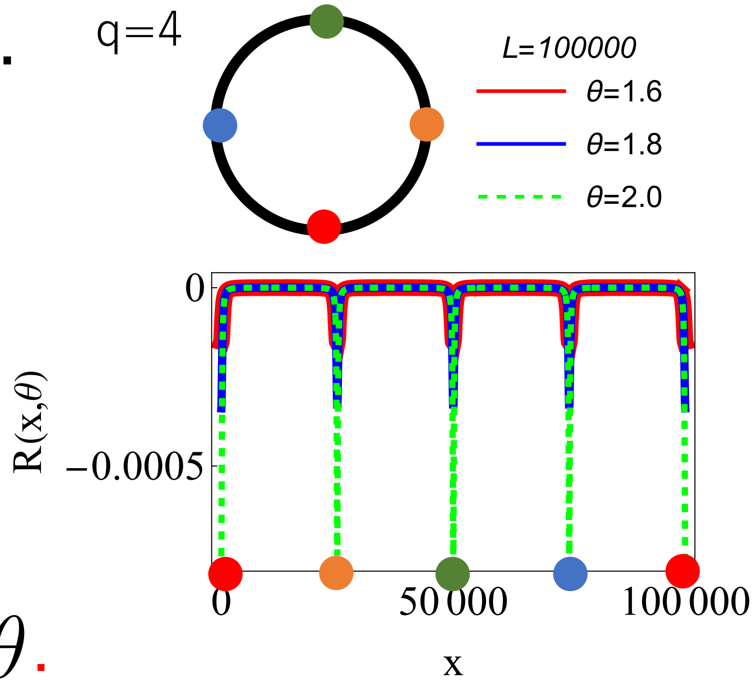
For $\theta = 0$ (flat), entropy is $S \approx \mathcal{O}(1)$.

The moduli parameter of this torus is $\tau = \frac{L \cosh 2\theta}{\beta}$.

Therefore, if θ increases, then

$$S \approx \begin{cases} \mathcal{O}(1) & L\cosh 2\theta/\beta < 1 \\ \frac{c\pi L\cosh 2\theta}{6\beta} \underset{\theta\gg 1}{\propto} \frac{Le^{2\theta}}{\epsilon} & L\cosh 2\theta/\beta > 1 \end{cases}$$

Hawking-Page transition is induced by θ.



The thermodynamic on the curved spacetime

Our thermal state: $\rho = \frac{e^{-\beta H_{q-\text{Möbius}}}}{\mathrm{tr} e^{-\beta H_{q-\text{Möbius}}}}$

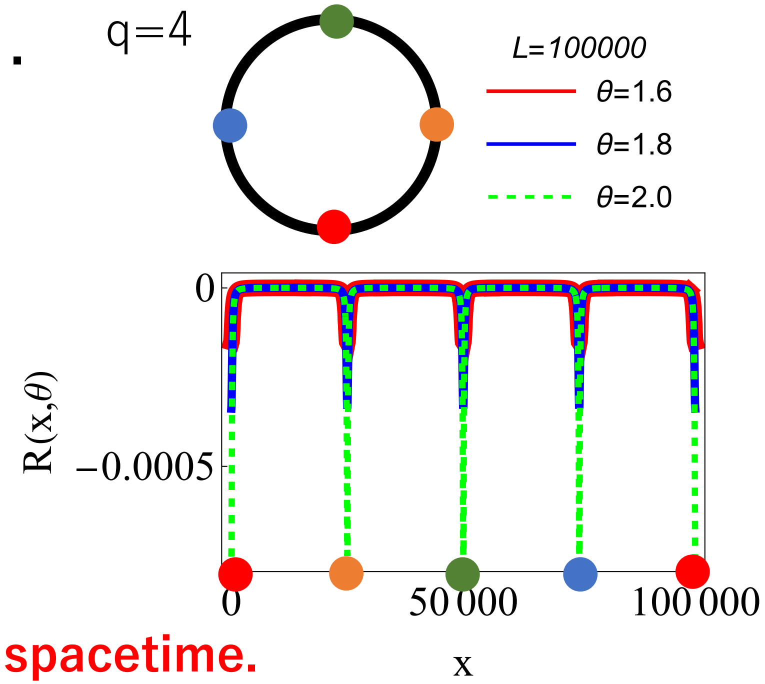
We assume $L/\beta < 1$ (low temp.).

For $\theta = 0$ (flat), entropy is $S \approx \mathcal{O}(1)$.

The moduli parameter of this torus is $\tau = \frac{L \cosh 2\theta}{\beta}$.

Therefore, if θ increases, then

$$S \approx \begin{cases} \mathcal{O}(1) & L \cosh 2\theta / \beta < 1 \\ \frac{c\pi L \cosh 2\theta}{6\beta} \underset{\theta \gg 1}{\propto} \frac{L e^{2\theta}}{\epsilon} & L \cosh 2\theta / \beta > 1 \end{cases}$$



Hawking-Page transition is induced by spacetime.

The thermodynamic on the curved spacetime

Our thermal state: $\rho = \frac{e^{-\beta H_{q-\text{Möbius}}}}{\text{tr} e^{-\beta H_{q-\text{Möbius}}}}$

We assume $L/\beta < 1$ (low temp.).

Here, **A** doesn't include x=0,L/4,L/2,3L/4.

B includes x=L/2.

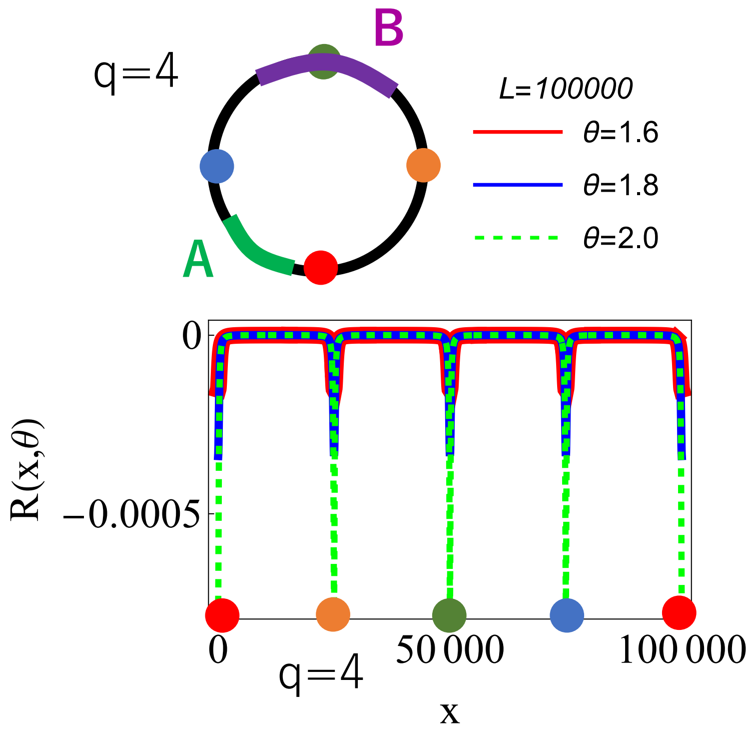
In the large θ limit, the behavior of entanglement entropy is

$$S_A \approx \frac{c}{3}\log\left[\frac{L}{4\pi}\sin\left(\frac{4\pi l_A}{L}\right)\right]$$

➡ **Independent of θ.**

Similar to Vacuum EE on the interval of L/4

$$S_B \approx \frac{c \cdot C_{\text{cof.}} L e^{2\theta}}{\beta}$$



The thermodynamic on the curved spacetime

Our thermal state: $\rho = \dfrac{e^{-\beta H_{q-\text{Möbius}}}}{\text{tr}e^{-\beta H_{q-\text{Möbius}}}}$

We assume $L/\beta < 1$ (low temp.).

Here, **A** doesn’t include x=0,L/4,L/2,3L/4.

B includes x=L/2.

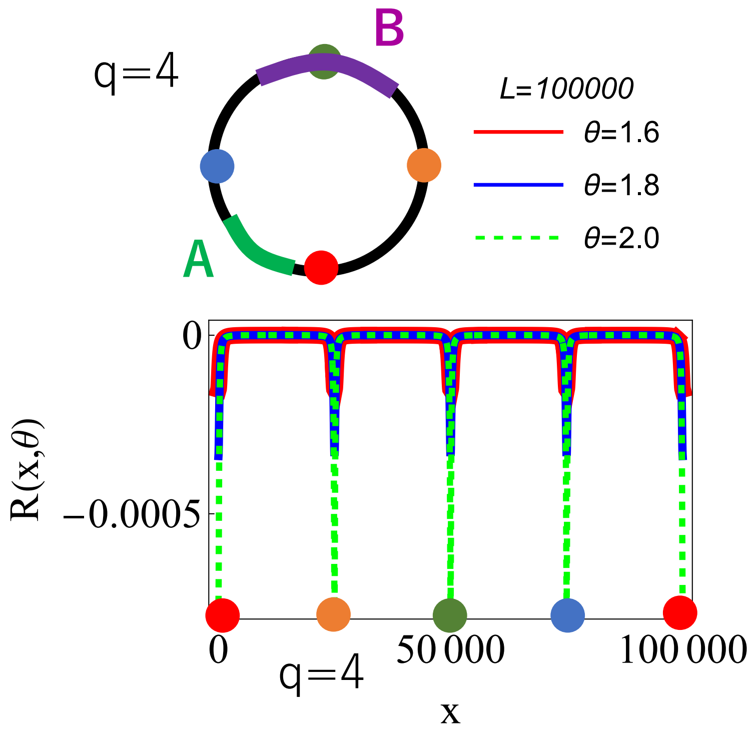
In the large θ limit, the behavior of entanglement entropy is

$$S_A \approx \frac{c}{3}\log\left[\frac{L}{4\pi}\sin\left(\frac{4\pi l_A}{L}\right)\right]$$

➡ **In dependent of θ. Similar to Vacuum EE on the interval of L/4.**

$$S_B \approx \frac{c \cdot C_{\text{cof.}} L e^{2\theta}}{\beta}$$

➡ **Exponential growth with θ.**



The thermodynamic on the curved spacetime

Our thermal state: $\rho = \dfrac{e^{-\beta H_{q-\text{Möbius}}}}{\text{tr} e^{-\beta H_{q-\text{Möbius}}}}$

We assume $L/\beta < 1$ (low temp.).

When the subsystem includes the region where curvature is negatively large, entanglement entropy grows with θ.

3L/4.

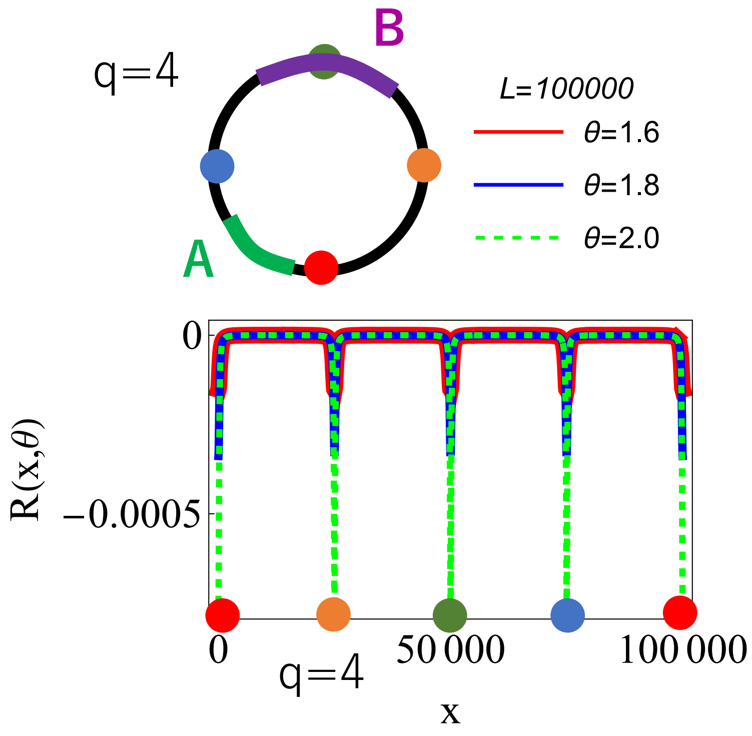
entanglement entropy is

$$S_A \approx \frac{c}{3} \log \left[\frac{L}{4\pi} \sin\left(\frac{4\pi l_A}{L} \right) \right]$$

In dependent of θ. Similar to Vacuum EE on the interval of L/4.

$$S_B \approx \frac{c \cdot C_{\text{cof.}} L e^{2\theta}}{\beta}$$

Exponential growth with .



The thermodynamic on the curved spacetime

Our thermal state: $\rho = \frac{e^{-\beta H_{q-\text{Möbius}}}}{\text{tr} e^{-\beta H_{q-\text{Möbius}}}}$

We assume $L/\beta < 1$ (low temp.).

2,3

When the subsystem includes the region where curvature is negatively large, entanglement entropy grows with θ.

Phase transition may be induced by the entanglement phase transition in the region where curvature is negatively large.

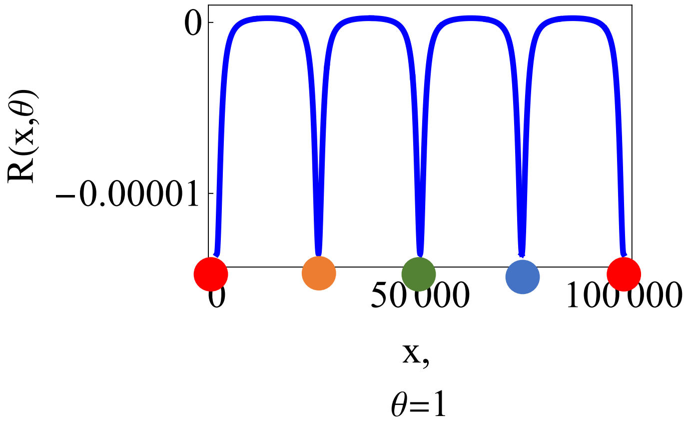
entanglement entropy is

$$S_A \approx \frac{c}{3} \log \left[\frac{L}{4\pi} \sin \left(\frac{4\pi l_A}{L} \right) \right]$$

In dependent of θ.
Similar to Vacuum EE on the interval of L/4.

$$S_B \approx \frac{c \cdot C_{\text{cof.}} L e^{2\theta}}{\beta}$$

Exponential growth with θ.



The thermodynamic on the curved spacetime

Our thermal state: $\rho = \frac{e^{-\beta H_{q-\text{Möbius}}}}{\text{tr}e^{-\beta H_{q-\text{Möbius}}}}$

We assume $L/\beta < 1$ (low temp.).

2,3

When the subsystem includes the region where curvature is negatively large, entanglement entropy grows with θ.

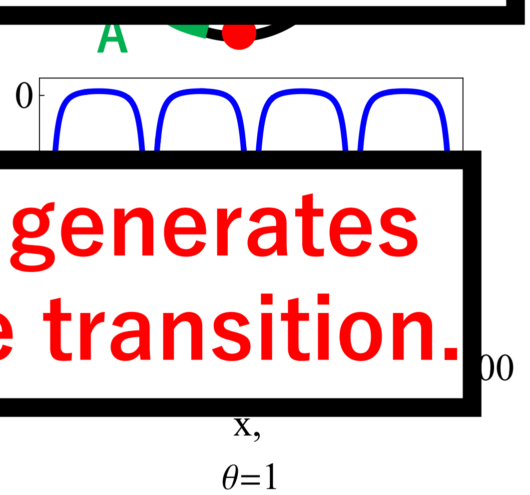
Phase transition may be induced by the entanglement phase transition in the region where curvature is negatively large.

entanglement entropy is

In dependent of θ.

Spacetime (maybe, curvature? or geometry?) generates entanglement or induces entanglement phase transition.

β



Summaries

- We studied the time dependence and thermodynamic properties of entanglement entropy and mutual information in two-dimensional inhomogeneous conformal field.

- During the time evolution from the TFD, **mutual information can revive**(non-local correlation retrieval).

- In holographic CFT on the curved spacetime, **the phase transition (entanglement phase transition) related to curvature may occurs.**

Future directions

- Quantum many body-scars
- Measurement-induced phase transition
- ETH and thermalization on the curved spacetime
- Quantum simulation
- Cosmology
- Experiments

Thank organizers for this great long-term workshop!!

- Quantum Information: Kohtaro Kato(Nagoya), Isaac Kim (UC Davis), Yuki Takeuchi(NTT), **Tomoyuki Morimae** (YITP, Kyoto, Cochair), Yoshifumi Nakata(YITP, Kyoto)

- String Theory: Michal Heller(Ghent), Norihiro Iizuka (Osaka), Tatsuma Nishioka (Osaka), **Tadashi Takayanagi** (YITP, Kyoto, Cochair), Tomonori Ugajin (Rikkyo)

- Cosmology and Relativity: Roberto Emparan(Barcelona), **Akihiro Ishibashi**(Kindai, Cochair), Keiju Murata (Nihon),Tetsuya Shiromizu (Nagoya), Norihiro Tanahashi(Chuo)

- Condensed Matter: Chisa Hotta (Tokyo), Tomotoshi Nishino (Kobe), **Kouichi Okunishi** (Niigata, Cochair), Frank Pollmann(TUM), Masaki Tezuka (Kyoto)