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Born rule extension for non-replicable systems and its consequences for Unruh-DeWitt detectors

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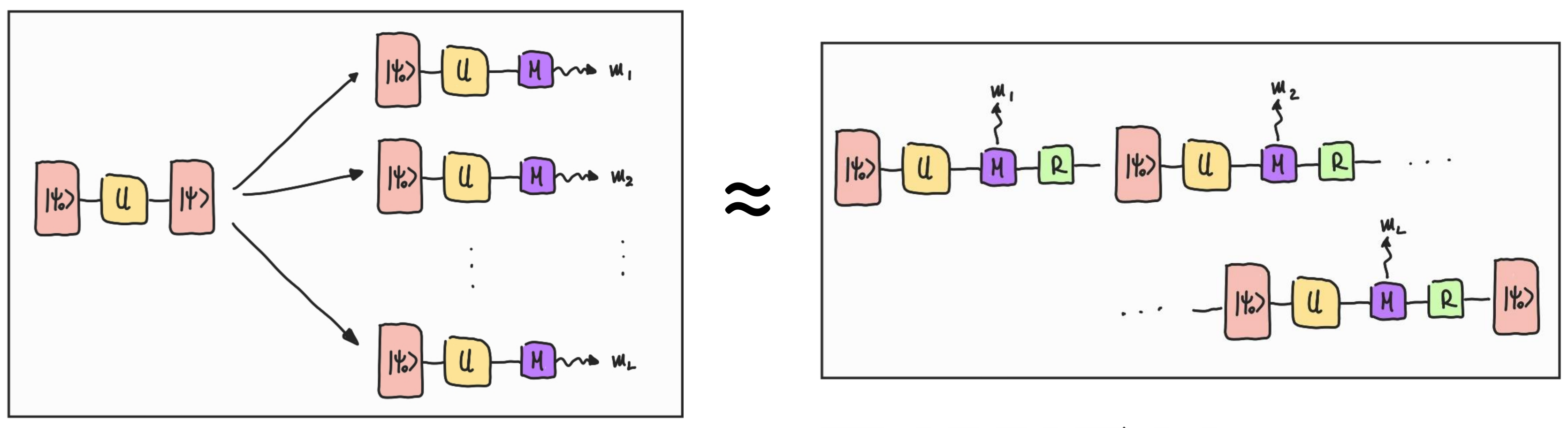
Abstract

The Born rule describes the probability of obtaining a specific outcome when measuring an observable of a quantum system. As it can only be tested by measuring many copies of the system under consideration, it cannot hold strictly for non-replicable systems. For these systems, we give a procedure to predict the future statistics of measurement outcomes through Repeated Measurements (RM). We prove that if the statistics of the results acquired via RM is sufficiently similar to that obtained by the Born rule, the latter can be used effectively. We apply our framework to a repeatedly measured Unruh-DeWitt detector interacting with a massless scalar quantum field, which is an example of a system (detector) interacting with an uncontrollable environment (field) for which using RM is necessary. Analysing what an observer learns from the RM outcomes, we find a regime where history-dependent RM probabilities are close to the Born ones. Consequently, the latter can be used for all practical purposes. Finally, we study numerically inertial and accelerated detectors showing that an observer can test the Unruh effect via RM.

Born rule

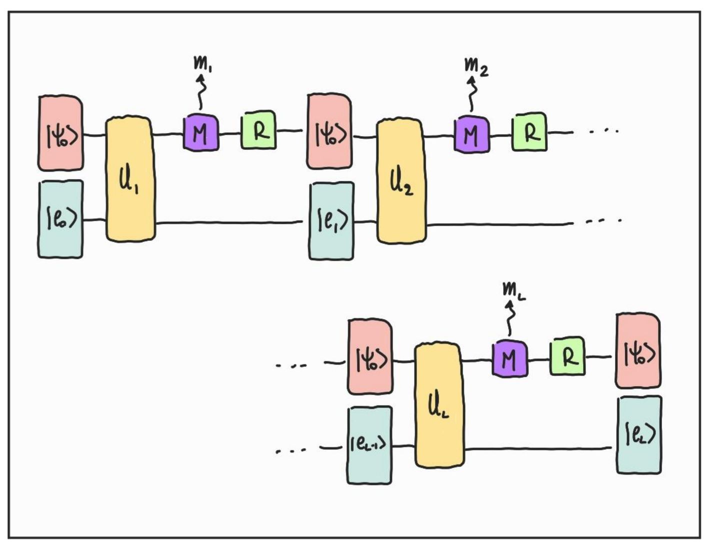
To test the Born rule we either need:
1) replicas of the target system[1] (below right); or,
2) a way to reset the system's state after a measurement (below left).



Doing so, we can prove the Born rule: $p(m_j) = \langle\psi|\hat{M}_j|\psi\rangle$

We call **non-replicable** a systems for which none of the above options is available.

Repeated Measurements (RM)



We model non-replicable systems by an open system (measured system + uncontrollable environment $\mathcal{E}$) which we prepare and measure multiple times (left). Each repetition modifies the state of the environment, making later measurements **not *i.i.d.***

Assuming

$$\hat{U}_k = \hat{U}\otimes\mathbb{I}_{\mathcal{E}} + \epsilon\sum \hat{A}_l\otimes\hat{B}_l(k) + O(\epsilon^2)$$

(i.e. unitaries weakly entangle the system and the environment) the outcome probabilities are

$$\boxed{p_k(m_j)[f] = p(m_j) + \epsilon Q_k^{(1)}(m_j)[f] + O(\epsilon^2)}$$

where $|f\rangle$ is the history-dependent state of $\mathcal{E}$ before the k-th measurement. The RM probability of getting a string of results is

$$p_{\mathrm{RM}}(m_1,\dots,m_L) = p_{\mathrm{B}}(m_1,\dots,m_L) + \epsilon\Delta p(m_1,\dots,m_L)$$

where p_{B} is the *i.i.d.* outcome probability related to the bit-string (Born).

Whenever

$$\boxed{\frac{\Delta p(m_1,\dots,m_L)}{p_{\mathrm{B}}(m_1,\dots,m_L)} \ll 1}$$

we can replace the 'real' RM rule with the standard Born one, for all practical purposes.

Unruh-DeWitt detectors

We study a bipartite system composed of a two-level system (called Unruh-DeWitt detector) with free Hamiltonian $\hat{H}_D = \omega|1\rangle\langle 1|$, and a real massless scalar quantum field in Minkowski spacetime[2,3]. Given the detector's worldline $X(\tau) = (t(\tau), \mathbf{x}(\tau))$, the detector and field interact via

$$\hat{H}_{\text{int}}(\tau) = \lambda\chi(\tau)\hat{m}(\tau)\otimes\hat{\phi}(X(\tau))$$

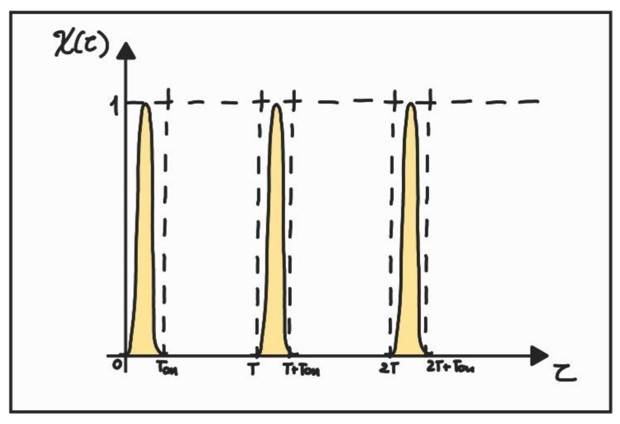
where $\chi(\tau)$ is a switching function describing the shape of the detector-field interaction, and $\hat{m}(\tau)$ is the detector's dipole momentum operator in the interaction picture. Then, the system evolves according to

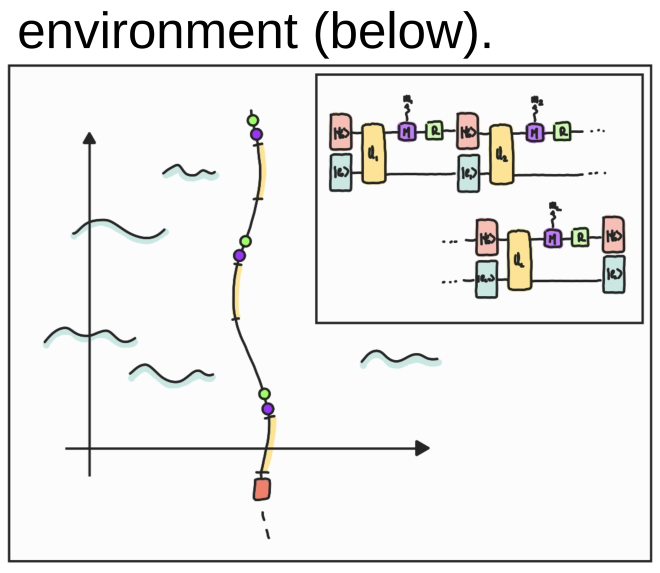
$$|0\rangle\otimes|0_M\rangle \mapsto |0\rangle\otimes|0_M\rangle - i\lambda\int_{\text{Supp}(\chi)} d\tau\chi(\tau)e^{-i\omega\tau}|1\rangle\otimes\hat{\phi}(X(\tau))|0_M\rangle + O(\lambda^2)$$

meaning that the interaction entangles the detector and the field. Hence, **measuring the detector also collapses the state of the field**.

RM on Unruh-DeWitt detectors

We choose the switching function to be a collection of identical consecutive peaks (left), measure the detector after each peak, and reset its state after each measurement. This reproduces the RM scheme on the UDW detector, with the field playing the role of the environment (below).





After each measurement, (1) the state of the field collapses following a contextual Lüders rule in the future light-cone of the measurement, and (2) it does not collapse outside of it[4]. Defining

$$\int_J = 2\int_{TJ}^{TJ+T_{\text{on}}} du_i \int_0^{u_i-TJ} ds_i \chi(u_i)\chi(u_i-s_i)\cos(\omega s_i)$$

we obtain the **history-dependent probabilities** of getting the outcome 1 as

$$\boxed{\mathcal{P}_n = \frac{\lambda^{2n}}{\prod_{j<1}\mathcal{P}_j}\iint_{\text{intervals where we found 1}} \mathcal{W}_{2n}(X_n, X_n', \dots, X_1, X_1')}$$

where

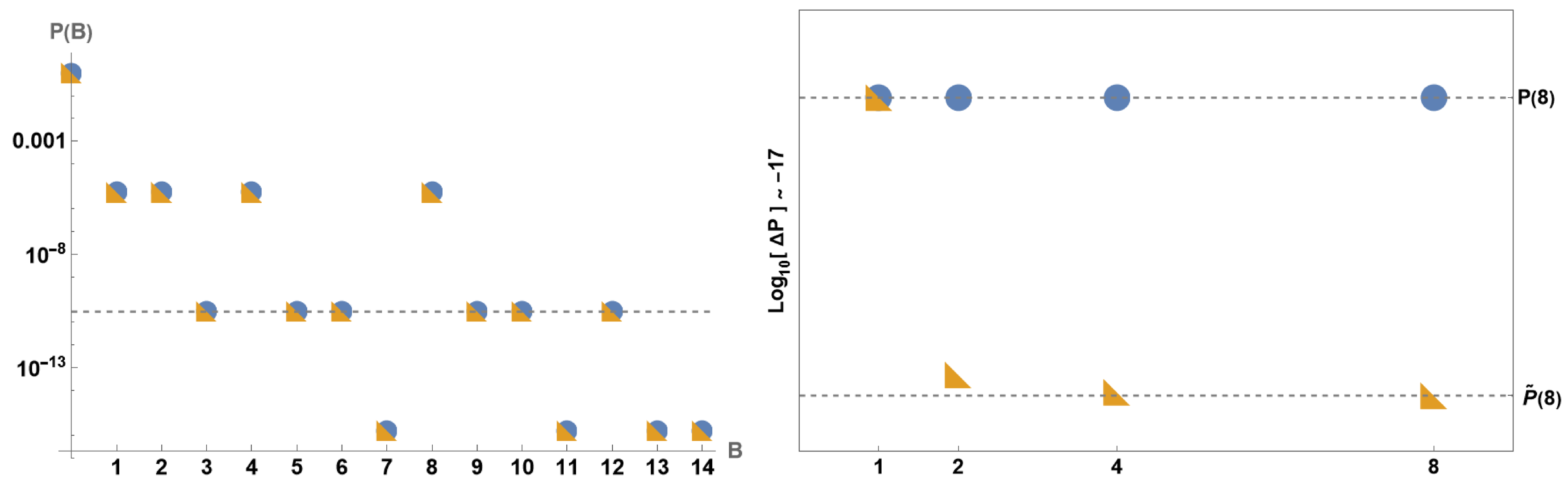
$$\mathcal{W}_{2n}(X_n, X_n', \dots, X_1, X_1') = \langle 0|\hat{\phi}(X(u_n))\hat{\phi}(X(u_n-s_n))\dots\hat{\phi}(X(u_1))\hat{\phi}(X(u_1-s_1))|0\rangle$$

is the Wightman 2n-point function, which can be analytically reduced to sum of products of easier 2-point functions.

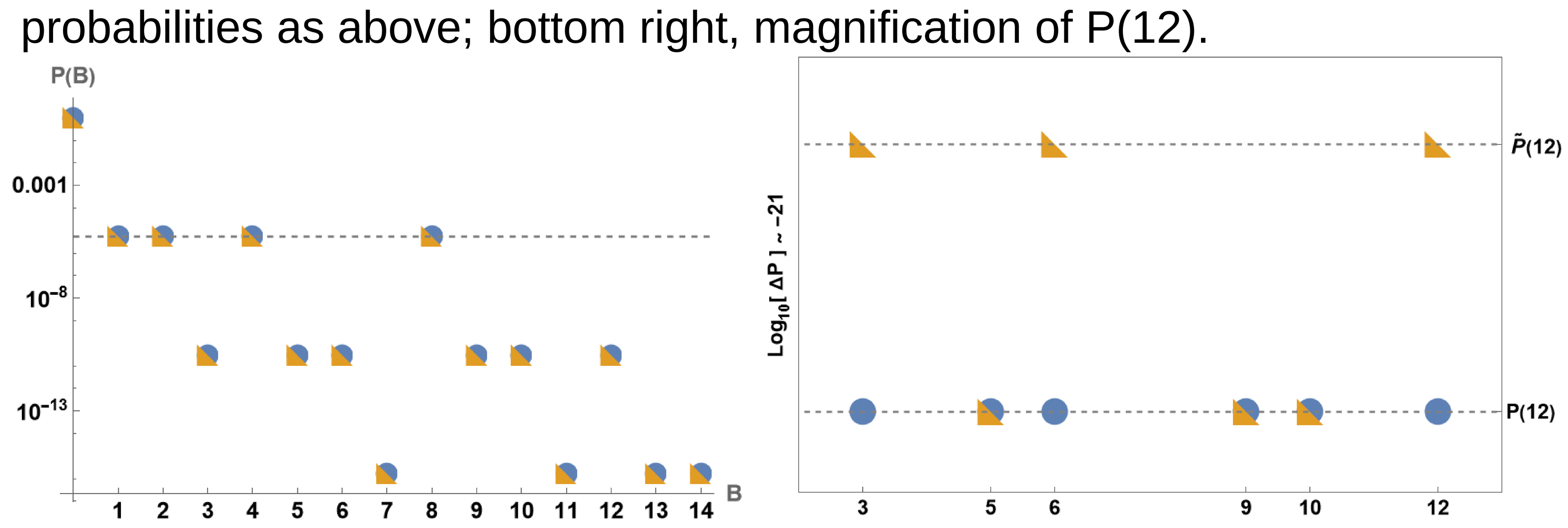
Numerical results

By selecting the switching function to be a collection of gaussians[5], we numerically obtain and compare the Born and RM probabilities.

Inertial detector: bottom left, Born (blue) and RM (orange) probabilities for all 4 digits bit-strings, in base 10; bottom right, magnification of P(8).



Accelerated detector: bottom left, Born (blue) and RM (orange) probabilities as above; bottom right, magnification of P(12).



Since the differences between Born and RM probabilities are so small, **we can test QFT in non-inertial spacetimes via RM**; e.g. study the Unruh effect (accelerated observers see an inertial observer's vacuum as a thermal bath of particles[2,3]) via RM on one detector.

Main references

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