Physics of Information and Quantum Technologies Group

The Quantum Information of Virtual Particles

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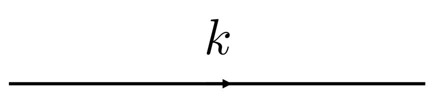
- **Quantum entanglement** is a fundamental property of quantum states which fuels the vast majority of **quantum information applications**.
- The **fundamental interactions** in High Energy Physics (HEP) **can entangle quantum degrees of freedom** of particles in an initial separable quantum state. These interactions are mediated by **virtual particles**.
- We show that virtual particles, despite being unobservable, can be described by **qubit operators** which can be interpreted under certain conditions as valid qubit quantum states. For spin ½ virtual fermions, we show they are related to **mixed 2-qubit thermal operators**, while for spin 1 bosons we have mixed 4-qubit operators.
- We demonstrate how every process in HEP can be described by a **quantum circuit** by using the example of **fermion pair creation**.

1 - Qubit operator of a virtual fermion

A virtual fermion is embodied by the Feynman propagator



$$D_F(k) = \frac{1}{\not{k} - m} = \frac{\not{k} + m}{k^2 - m}$$

We can express the Feynman propagator in the eigenbasis as

$$D_F(k) = \frac{1}{M_k - m}\sum_s \check{u}^s(k)\bar{\check{u}}^s(k) + \frac{1}{M_k + m}\sum_s \check{v}^s(k)\bar{\check{v}}^s(k)$$

Off-shell mass parameter

$$M_k = \sqrt{k_0^2 - k_1^2 - k_2^2 - k_3^2}$$

where we have

$$\check{u}^1 = W_k(1,0,0,0)^T = W_k\,|00\rangle$$
$$\check{u}^2 = W_k(0,1,0,0)^T = W_k\,|01\rangle$$
$$\check{v}^1 = W_k(0,0,1,0)^T = W_k\,|10\rangle$$
$$\check{v}^2 = W_k(0,0,0,1)^T = W_k\,|11\rangle$$

Spinors of virtual (or real) particles

2-qbit states

$$W_k\,|\lambda_E \lambda_s\rangle \equiv |\lambda_E \lambda_s; k\rangle$$

Charge qubit Spin qubit

$$W_k = \frac{1}{\sqrt{2M_k(k_0+M_k)}}\begin{pmatrix} k_0+M_k & 0 & k_3 & k_1 - ik_2 \\ 0 & k_0+m_k & k_1+ik_2 & -k_3 \\ k_3 & k_1-ik_2 & k_0+M_k & 0 \\ k_1+ik_2 & -k_3 & 0 & k_0+M_k \end{pmatrix}$$

The propagator is not Hermitian, so it can't be interpreted as a density matrix. However, a simple modification fixes this:

$$\check{\rho}(k) = \left(\frac{k^2-m^2}{4k_0}\right) D_F(k)\gamma_0$$
$$= \sum_{\lambda_E,\lambda_s} \frac{1}{4}\left(\frac{M_k}{m} + (-1)^{\lambda_E}\right)|\lambda_E,\lambda_s;k\rangle\langle\lambda_E,\lambda_s;k|$$

Eigenvalues: $\lambda_\pm = \frac{1}{4}(1 \pm r_k)$ with $r_k = \frac{\sqrt{k_1^2+k_2^2+k_3^2+m^2}}{k_0}$

For $|r_k| \leq 1$, i.e. for $|M_k| \geq m$, the quantity $\check{\rho}(k)$ can be interpreted as a **2-qubit mixed density matrix**.

One can show that $\check{\rho}(k)$ is a thermal ensemble:

$$\check{\rho}(k) = \frac{e^{-\beta H}}{\mathrm{Tr}[e^{-\beta H}]}$$

Hamiltonian

$$H = -\frac{1}{|r_k|}(m\gamma^0 - k_i\gamma^i\gamma^0)$$

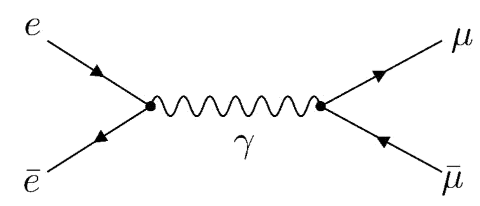
Inverse temperature

$$\beta = \frac{1}{2k_0}\log\left(\frac{1+|r_k|}{1-|r_k|}\right)$$

By performing a PPT test, one can show that $\check{\rho}(k)$ is a separable density matrix, in the regime where it is well-define as a quantum state.

2 - Qubit operator of a virtual boson

The case of a virtual spin 1 boson (photon) is less straightforward since the propagator only contains classical spacetime indexes. To find the qubit operator one can write a simple scattering amplitude involving virtual photons and isolate the spinors:



$$\mathcal{M}_{e^+e^-\to\mu^+\mu^-} = -\frac{ie^2}{p^2}[v^{\dagger s_2}(p_2)\gamma^0\gamma^\mu u^{s_1}(p_1)]\left(-g_{\mu\nu} + (1-\xi)\frac{p_\mu p_\nu}{p^2}\right)[u^{\dagger s_3}\gamma^0(p_3)\gamma^\mu v^{s_4}(p_4)]$$

$$\mathcal{M}_{e^+e^-\to\mu^+\mu^-} = -\frac{ie^2}{p^2}\mathrm{Tr}\left[(I\otimes v^{s_4}(p_4)u^{\dagger s_3}(p_3))D_\gamma(p)(u^{s_1}(p_1)v^{\dagger s_2}(p_2)\otimes I)\right]$$

The action of the virtual photon is thus condensed into the object

$$D_\gamma(p) = \sum_i \Xi^i(p)\otimes\Xi^{i\dagger}(p) \qquad \text{with} \qquad \Xi^i(p) = \varepsilon_\mu^{(i)}(p)\gamma^\mu\gamma^0$$

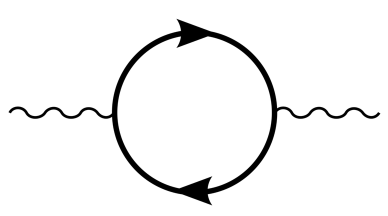
The quantity $\Xi^i(k)$ is a particular case of the fermionic propagator and thus can be decomposed into a qubit form in the same manner, resulting in

$$\Xi^i(k) = M_{\varepsilon^{(i)}(k)}\sum_{i_1,i_2}|i_1,i_2;\varepsilon^{(i)}(k)\rangle\langle i_1,i_2;\varepsilon^{(i)}(k)|$$

This is a 4-qubit operator which can be interpreted as a 4-qubit density matrix, although only for certain gauges.

3 - Qubit picture of fermionic pair creation

With the qubit interpretation of virtual fermions and bosons, one can deconstruct any process in HEP as a quantum computation. For a pair of virtual fermions, for example, we have



$$\mathcal{M}_{1-\mathrm{loop}} = -e^2|\varepsilon_0(p)|^2\langle\check{\rho}_{2F}(p)\rangle_\varepsilon$$

$$\langle\check{\rho}_{2F}(p)\rangle_\varepsilon = \frac{\sum_J\langle J,\varepsilon|\,\check{\rho}_{2F}(p)\,|J,\varepsilon\rangle}{\sum_J\langle J,\varepsilon|J,\varepsilon\rangle}$$

$$|J,\varepsilon\rangle \equiv |i_1,i_2;\varepsilon(p)\rangle\otimes|j_1,j_2;\varepsilon^*(p)\rangle$$

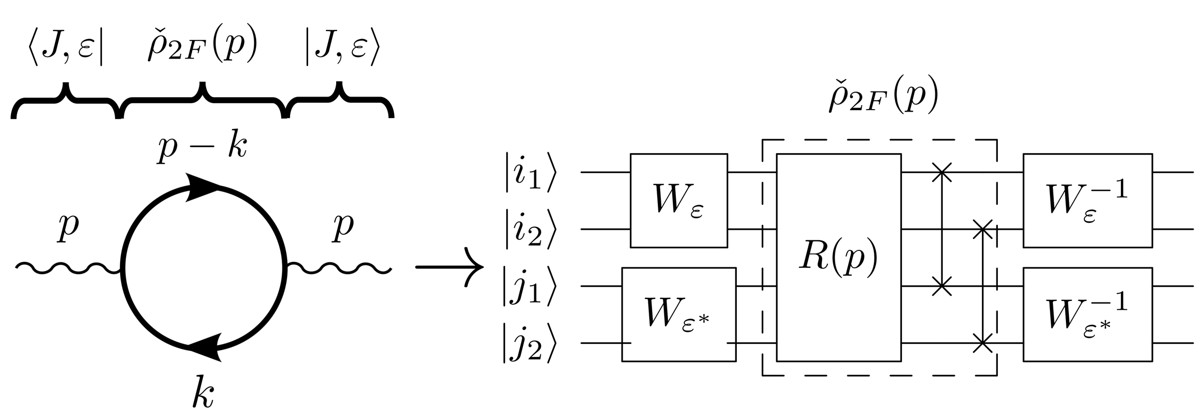
Entangled 4-qubit operator

$$\check{\rho}_{2F}(p) = \int\frac{d^4k}{(2\pi)^4}[D_F(p-k)\otimes D_F(k)]S$$

SWAP gate

$$S = \sum_{\substack{\lambda_E,\lambda_s \\ \lambda_{E'},\lambda_{s'}}}|\lambda_{E'},\lambda_{s'}\rangle|\lambda_E,\lambda_s\rangle\langle\lambda_E,\lambda_s|\langle\lambda_{E'},\lambda_{s'}|$$

The corresponding quantum circuit immediately follows:



5 - Conclusions

- We have identified qubit operators to virtual fermions and bosons, thereby providing a new quantum information picture of HEP processes;
- Although these operators can only sometimes be associated to valid density matrices in momentum with a well-defined temperature, they always provide an analog quantum circuit to every HEP process.

References

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