Chaos in Driven CFTs and their Bulk Dual

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Abstract

The Out-of-Time-Order correlators (OTOCs) are known to provide a diagnostic for early-time chaos (scrambling that precedes thermalization) in typical non-integrable quantum systems and have mostly been studied in thermal equilibrium cases. We study OTOCs for a particular non-equilibrium setup in 1+1 D conformal field theories (CFTs) in which the evolution Hamiltonian is subjected to a drive protocol. In [1] we show that, for large c CFTs, OTOCs in different dynamical phases i.e. heating phase, non-heating phase, and on the phase boundary show exponential, oscillatory, and power law behaviour respectively. We find in [2] that the holographic dual geometry of these phases corresponds to AdS_3 metrics with different AdS_2 slicing.

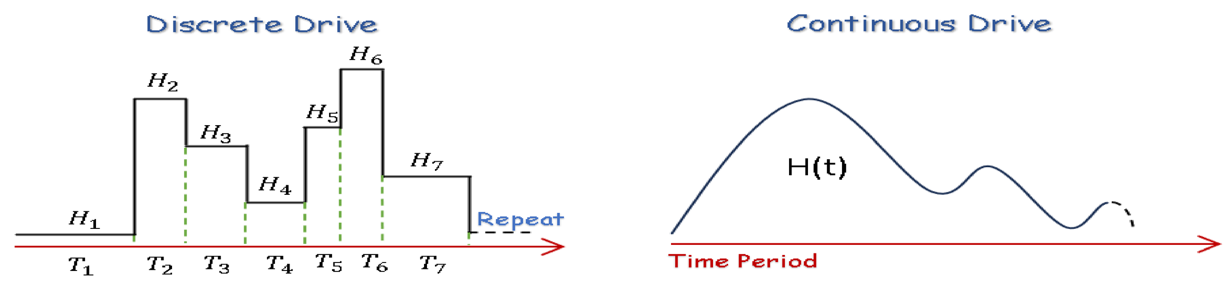
The Driven CFT Protocol

A (periodically) driven CFT protocol simply means to evolve a CFT system with different (non-commutative) Hamiltonians by changing some control parameters of the Hamiltonian each time within a stroboscopic time period T and analyze the system at time t after repeating the evolution process for integer 'n' times i.e. $t = nT$.

We focus on the periodically driven CFTs on a ring of circumference L where the drive Hamiltonian

$$H = \frac{1}{2\pi}\int_0^L (f(x)T_{00} + g(x)T_{01})dx$$

has the control functions $f(x)$ and $g(x)$ that can be changed continuously or in discrete steps.

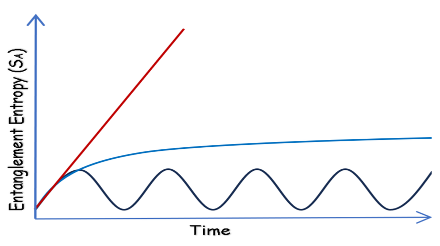


- Main advantage: Since such deformed Hamiltonian can be written in terms of generators of virasoro algebra, in Heisenberg picture, the time evolution of a primary operator (with conformal weights $h, \bar{h}$) gets translated into the dynamics of the operator under conformal transformation:

$$U^\dagger(t,0)O(z,\bar{z})U(t,0) = \left(\frac{\partial z_n}{\partial z}\right)^h \left(\frac{\partial \bar{z}_n}{\partial \bar{z}}\right)^{\bar{h}} O(z_n, \bar{z}_n)$$

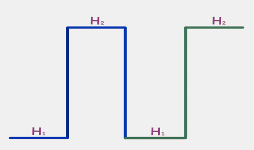
where, $t = nT$ and $U(t,0) = \prod_{j=1}^{n} U_j(T,0) = e^{-iH_F nT}$, with effective Hamiltonian H_F generating time evolution after 'n' stroboscopic time.

- Such driven systems can be classified into different dynamical phases: heating phase, non-heating along with a phase boundary by tuning the control parameters.
- The entanglement entropy and energy density show different behavior in these different phases.



SL_2 Discrete Drive

We consider Hamiltonians made out of only the global generators $L_0,\ L_1,\ L_{-1}$. A simple example is the following two-step discrete drive protocol, where the Hamiltonian H is changed twice in each time period $T = T_1 + T_2$, and is given by:



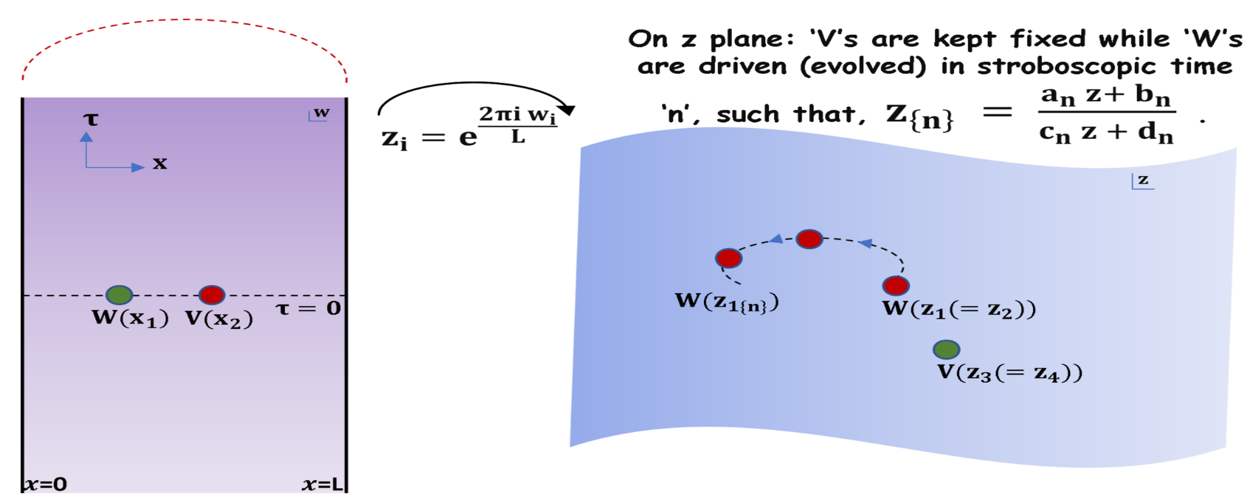
$$H_\phi = \int_0^L dx T_{00}\left(1 - \tanh(2\phi)\cos\left(\frac{2\pi x}{L}\right)\right)$$

$$H_1 = H_{\phi=0}\,, \quad H_2 = H_{\phi\neq 0} \to \text{deformed CFT hamiltonian}$$

- Since, Hamiltonians are made out of generators of $SL(2,\mathbb{R})$ algebra, time evolution operator: $U_i = \begin{pmatrix} a_i & b_i \\ c_i & d_i \end{pmatrix} \in SL(2,\mathbb{R})$ generates Möbius transformation in the complex plane $z \to z' = \frac{a_i z + b_i}{c_i z + d_i}$.
- Note that these generators keep the vacuum state invariant, and therefore only unequal time correlators can capture the dynamics of the drive. One such interesting correlator, which is not fixed only by symmetries, is the 4-point OTOC.

OTOC set-up in Driven CFTs

- One begins with a normalized 4 point Euclidean correlator $\langle W(z_1,\bar{z}_1)W(z_2,\bar{z}_2)V(z_3,\bar{z}_3)V(z_4,\bar{z}_4)\rangle$ computed in the vacuum state of CFT. Such 4- pt correlators are in general undetermined functions of the cross ratios $F(\eta,\bar{\eta})$.



On this cylinder: Two 'V's are placed at x_1 and two 'W's at x_2.

- Analytic continuation of $\langle W(z_{1n},\bar{z}_{1n})W(z_{2n},\bar{z}_{2n})V(z_3,\bar{z}_3)V(z_4,\bar{z}_4)\rangle$ with $n_i = n_i + i\epsilon_i$, where $\epsilon_i \to 0$, corresponds to different Lorentzian correlators depending upon different orders in which $\epsilon_i \to 0$ is taken. For $\epsilon_1 > \epsilon_3 > \epsilon_2 > \epsilon_4 \to 0$ one gets the OTOC $\langle V(0)W(t)V(0)W(t)\rangle$

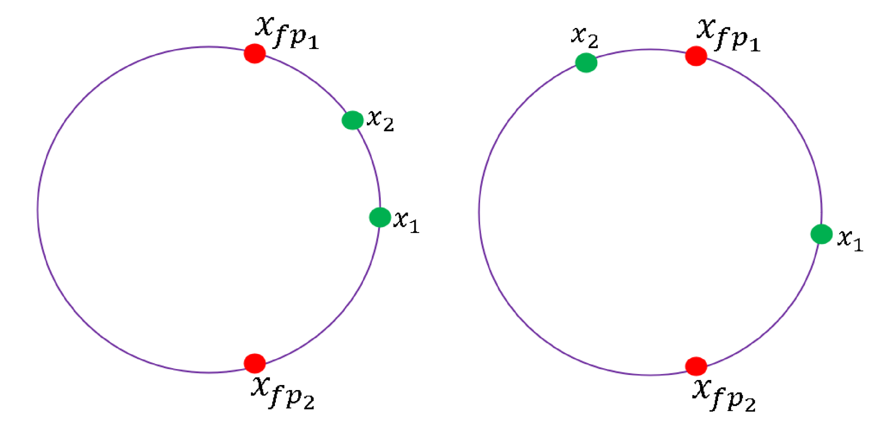
OTOCs at finite temperatures have been proposed as a diagnostic of chaos. In the case of large c CFTs, the normalized OTOC in the thermal state is known to show exponential behaviour, $\sim 1 - ge^{\lambda(t-t_s)}$ for $t < t_s$ where t_s is the scrambling time.

- Key difference in driven CFTs:

OTOCs in large-c driven CFTs show exponential behaviour in a particular phase i.e. heating phase even in the vacuum state.

Results for Driven CFTs

- For large c 2D CFTs, with two heavy operators W and light operators V, a closed form expression for $F(\eta,\bar{\eta})$ is well known.
- We find that in the heating phase of such large-c CFTs, the cross-ratio η and therefore, the OTOC shows an exponential behaviour, for sufficiently large time, smaller than the scrambling time t_s.
- However, unlike in large-c thermal CFT, this exponential behaviour of OTOCs, in case of driven CFTs, depends crucially on the initial operator location $x_{1,2}$, the range of which is controlled by the two fixed points $z_n = z$ of $SL(2,\mathbb{R})$ evolution.
- For two-step discrete drive setup, this range of x_1 and x_2 is: $-1 \leq \frac{-\sin(\frac{\pi(x_1-x_2)}{L})}{\sin(\frac{\pi(x_1+x_2)}{L})} \leq 1$.



OTOC can show exponential behaviour for both operators on the same side

No exponential behaviour for operators on different sides

- One can show that the butterfly velocity is position-dependent. This is a consequence of the lack of translation invariance in these systems. For fixed x_1, in discrete drive, $v_B = \lambda_L \frac{L}{2\pi}\sin\left(\frac{2\pi x_2}{L}\right)$ with Lyapunov exponent $\lambda_L = \frac{4\phi}{T_1+T_2}$.
- In the non-heating phase, there is no exponential growth for any value of x. For driven Ising CFT, on the other hand, the OTOC does not show exponential behavior even in the heating phase.

Geometries in the Bulk dual

- Since $SL(2,\mathbb{R})$ transformations do not change the vacuum, the holographic dual geometry of CFT vacuum under $SL(2,\mathbb{R})$ drive remains AdS spacetime. However, one expects to distinguish between the different phases and also to calculate the exponential behavior of OTOC from the bulk.
- To compute the dual bulk metric in a $SL(2,\mathbb{R})$ driven CFT, we start with a boundary effective Hamiltonian $H_F = \alpha(L_0 + \bar{L}_0) + \beta(L_1 + \bar{L}_1) + \gamma(L_{-1} + \bar{L}_{-1})$ and lift it into the bulk, by replacing the global Virasoro generators by its AdS_3 representations to write down $H_b = \alpha(L_{b,0} + \bar{L}_{b,0}) + \beta(L_{b,1} + \bar{L}_{b,1}) + \gamma(L_{b,-1} + \bar{L}_{b,-1})$. The parameters $\alpha,\ \beta,\ \gamma$ depend on drive parameters, i.e. $T_1,\ T_2, \phi$ in each period.
- Then we rewrite AdS-Poincaré metric in terms of tangent curves generated by H_b to get different bulk metrics for different phases.

Interestingly, we find that different phases of the drive (characterized by $d = \frac{\alpha^2}{4\beta^2} - \frac{\gamma}{\beta}$) correspond to AdS_3 metrics foliated by different bulk AdS_2 slices [2].

- For non-heating phase ($d > 0$):

$$ds^2 = \frac{d\phi^2}{\sin^2\phi} + \frac{1}{\sin^2\phi}\underbrace{\frac{4\beta^2(-dt^2 + d\theta^2)}{sin^2(2\beta\sqrt{d}\theta)}}_{\text{Global } AdS_2 \text{ patch}}$$

Here, 'θ' and 't' are the boundary coordinates and 'ϕ' is the bulk coordinate.

- For heating phase ($d < 0$):

$$ds^2 = \frac{d\phi^2}{\sin^2\phi} + \frac{1}{\sin^2\phi}\underbrace{\frac{4\beta^2(-dt^2 + d\theta^2)}{sinh^2(2\beta\sqrt{d}\theta)}}_{AdS_2 \text{ Black hole}}$$

- On transition line ($d = 0$):

$$ds^2 = \frac{d\phi^2}{\sin^2\phi} + \frac{1}{\sin^2\phi}\underbrace{\frac{(-dt^2 + d\theta^2)}{\theta^2}}_{AdS_2 \text{ Poincaré patch}}$$

- The presence of a horizon in the heating phase is reminiscent of the bulk dual picture corresponding to the un-driven thermal CFT. Hence, it is expected to see the chaotic behavior from bulk OTOC calculations as well. In our recent work, we have been able to match the boundary two-point correlators and the Lyapunov exponents from the bulk calculations.

References

[1] S. Das, B. Ezhuthachan, A. Kundu, S. Porey, B. Roy and K. Sengupta, JHEP 08, 221 (2022) doi:10.1007/JHEP08(2022)221 [arXiv:2202.12815 [hep-th]].

[2] S. Das, B. Ezhuthachan, A. Kundu, S. Porey, B. Roy and K. Sengupta, [arXiv:2212.04201 [hep-th]].