Friedrich-Alexander-Universität
Naturwissenschaftliche Fakultät

FAU

QCNN as a Phase Detection Circuit on the Toric Code

Leon Sander, Nathan McMahon, Petr Zapletal
and Michael J. Hartmann

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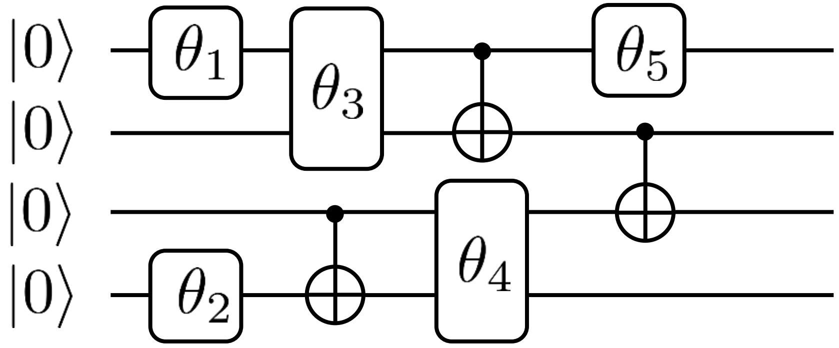
Quantum Neural Networks

Context and Motivation

- Quantum Neural Network:
  - Parameterized quantum circuit on layers of qubits
  - Subset of Variational Quantum Algorithms (VQAs) with network-like structure
- VQAs as promising tool during NISQ era
  - Design quantum circuit with parameterized qubit rotation
  - Measure qubits and compute cost function
  - Update circuit parameters
- Problem of Barren plateaus: Tradeoff between trainability and expressivity
  M. Ragone et al. arXiv.2309.09342 (2023).
- Interestingly, for Quantum Convolutional Neural Networks:
  - No Barren plateaus - A. Pesah et al. Phys. Rev. X **11**(4), 041011 (2021)
  - Variance of gradient vanishes no faster than polynomially → guarantee for trainability

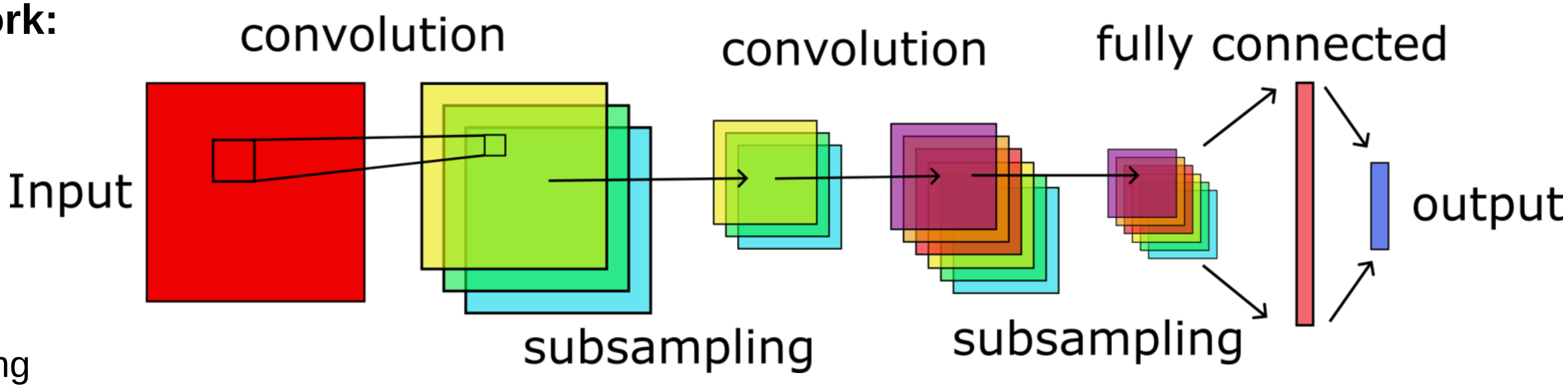


Quantum Convolutional Neural Network

A (parameterized) quantum circuit inspired by MERA

Classical Convolutional Neural Network:

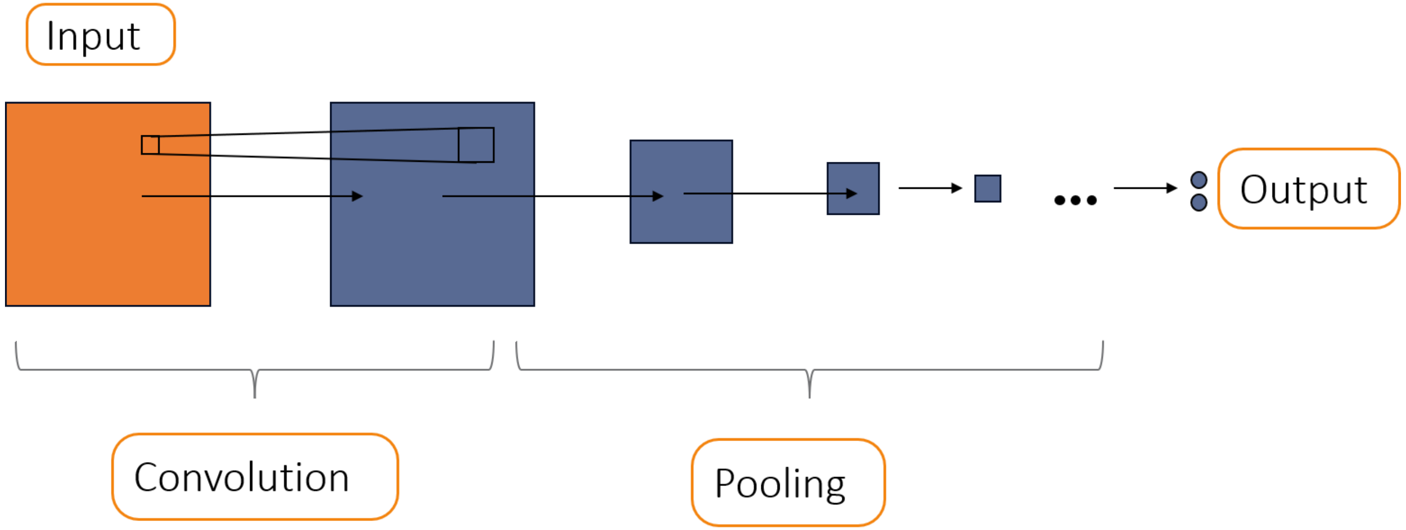
- Characterization of translationally invariant data (images)
- Sequential application of convolutions with kernels (filters) and pooling
- Reduction of number of neurons → mapping of most import information to output neurons



Quantum Convolutional Neural Network:

- Qubits replace neurons
- Convolution is Clifford circuit
- Pooling consists of controlled Pauli-gates

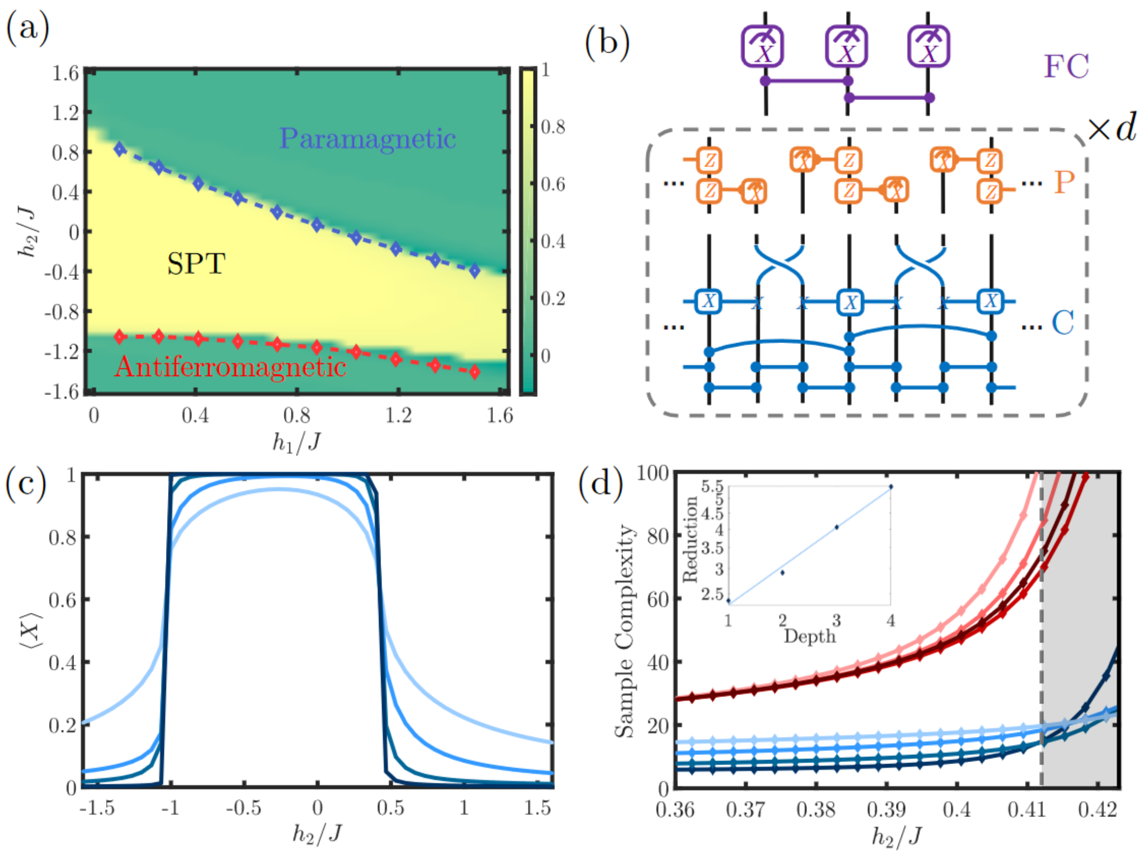
$$U_{\mathrm{QCNN}} = U_{\mathrm{FC}} \cdot \prod_{j=1}^{d} U_{\mathrm{CP}}^{(j)}$$



Quantum Convolutional Neural Network

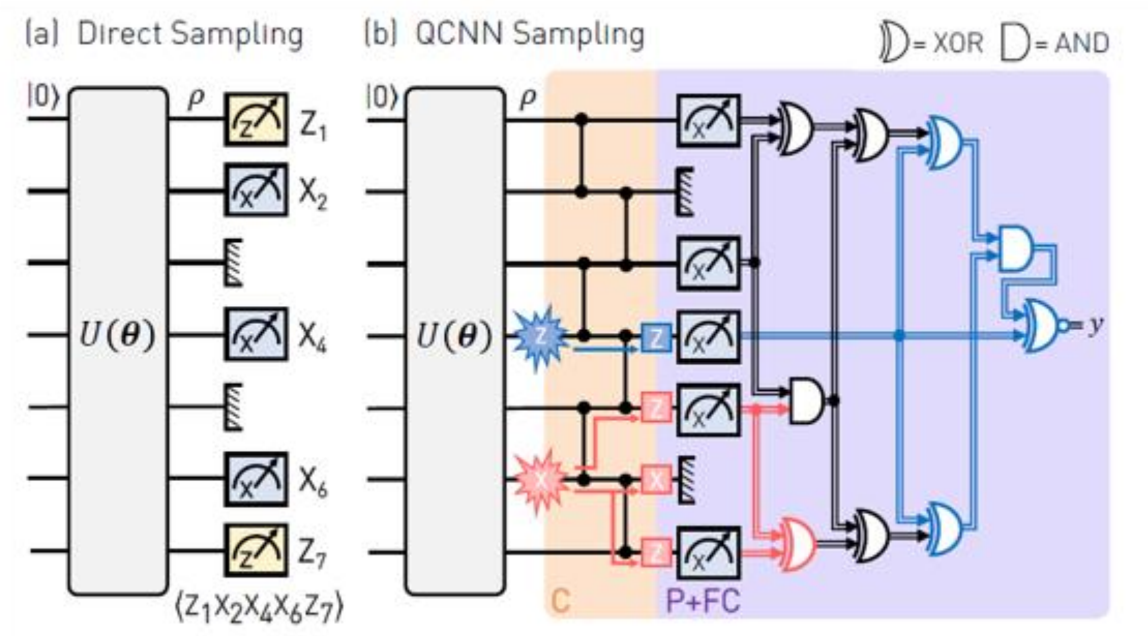
A (parameterized) quantum circuit inspired by MERA

Cong et al. 2019: Quantum Convolutional Neural Network



Cong, I., Choi, S. & Lukin, M. D. *Nat. Phys.* **15**, 1273–1278 (2019).

Herrmann et al. 2022: Realizing Quantum Convolutional Neural Networks on a Superconducting Quantum Processor to Recognize Quantum Phases



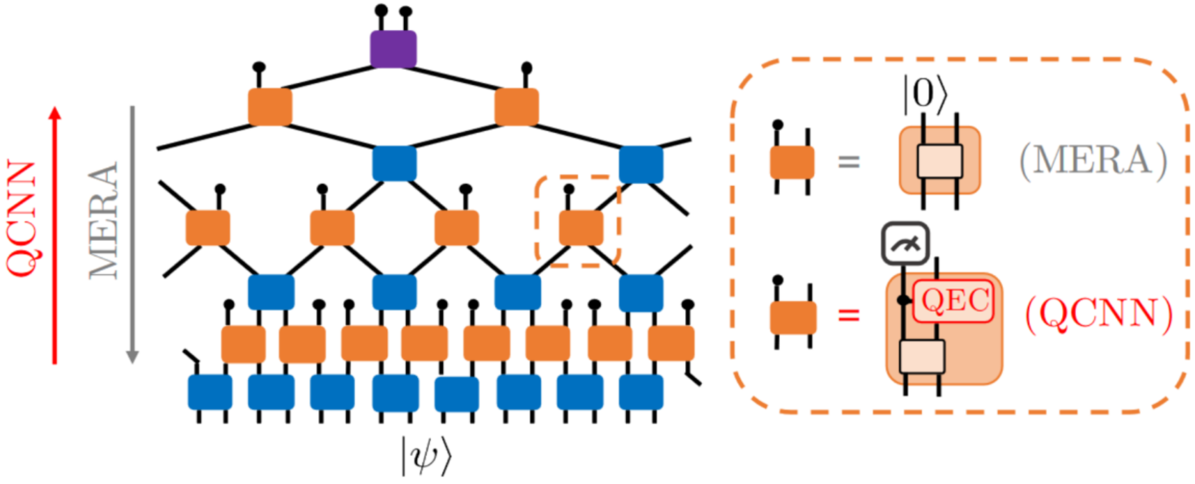
Herrmann, J. et al. *Nat Commun* **13**, 4144 (2022).

Construction of the QCNN

Inspiration by MERA and RG flow

Multi-scale Entanglement Renormalization Ansatz (MERA)

- Tensor network representation of quantum many-body state on D-dimensional lattice
- Circuit consisting of isometric and disentangling operations
- Can be optimized to approximate ground state of local Hamiltonian



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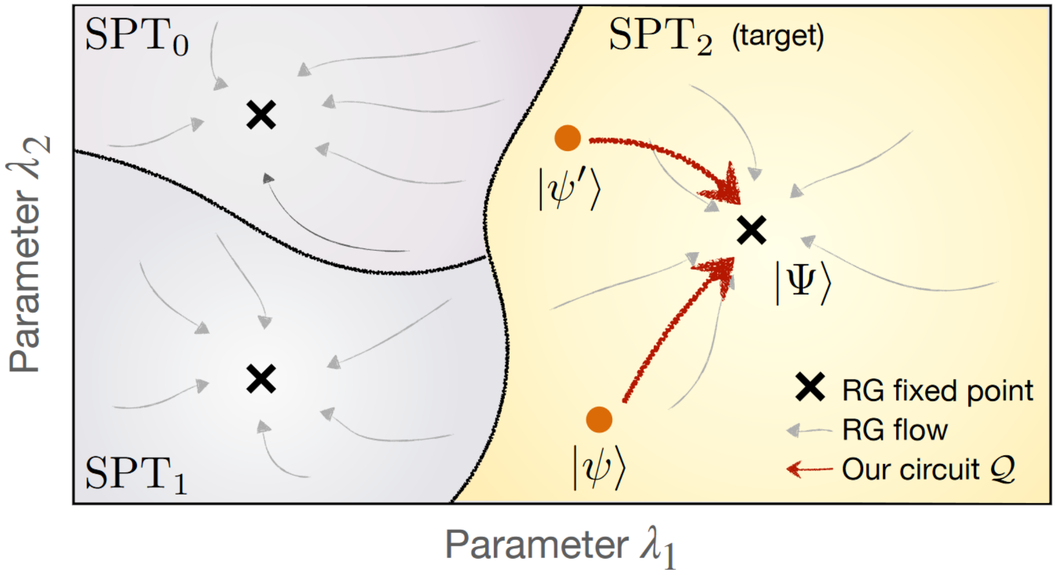
Renormalization Group flow:

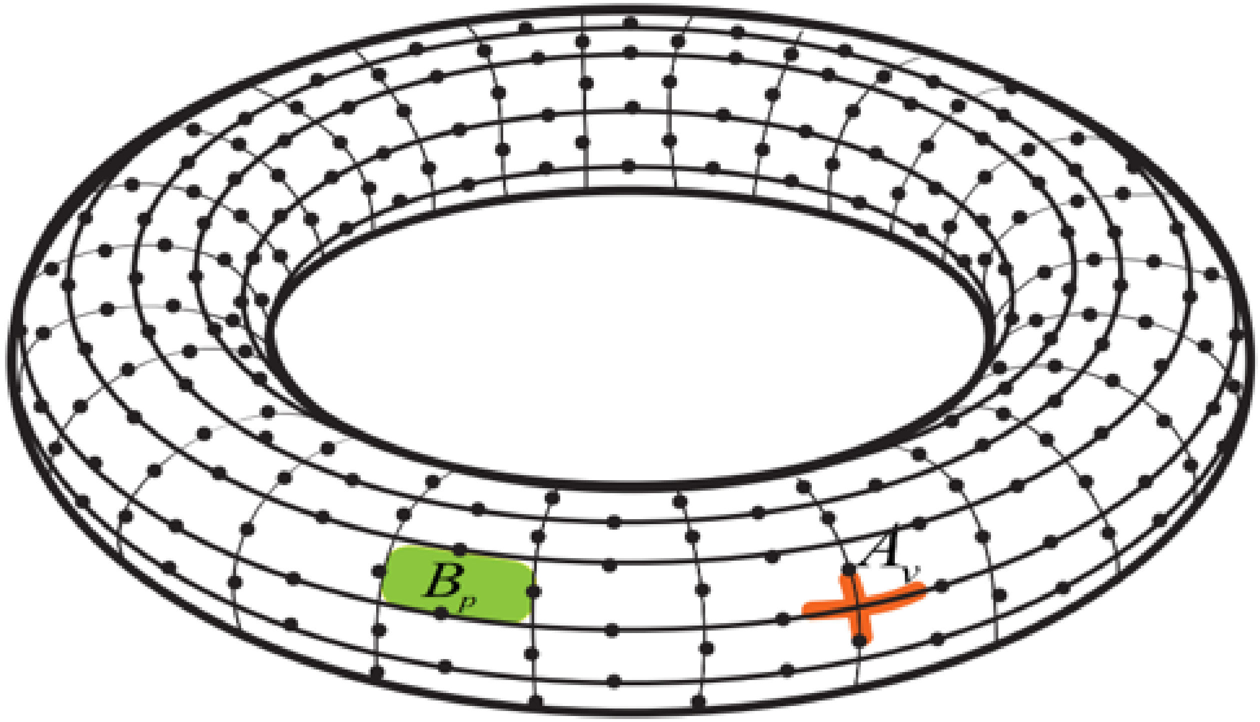
- Define fixed point states for different phases
- Other states of the same phase correspond to fixed point states + local unitary perturbations
- Apply error correction to recover fixed point state



Lake, E., Balasubramanian, S. & Choi, S. arXiv.2211.09803 (2022).

Toric Code

2D spin lattice model



Zarei, M. H. *Phys. Rev. B* **100**, 125159 (2019).

Toric code Hamiltonian:

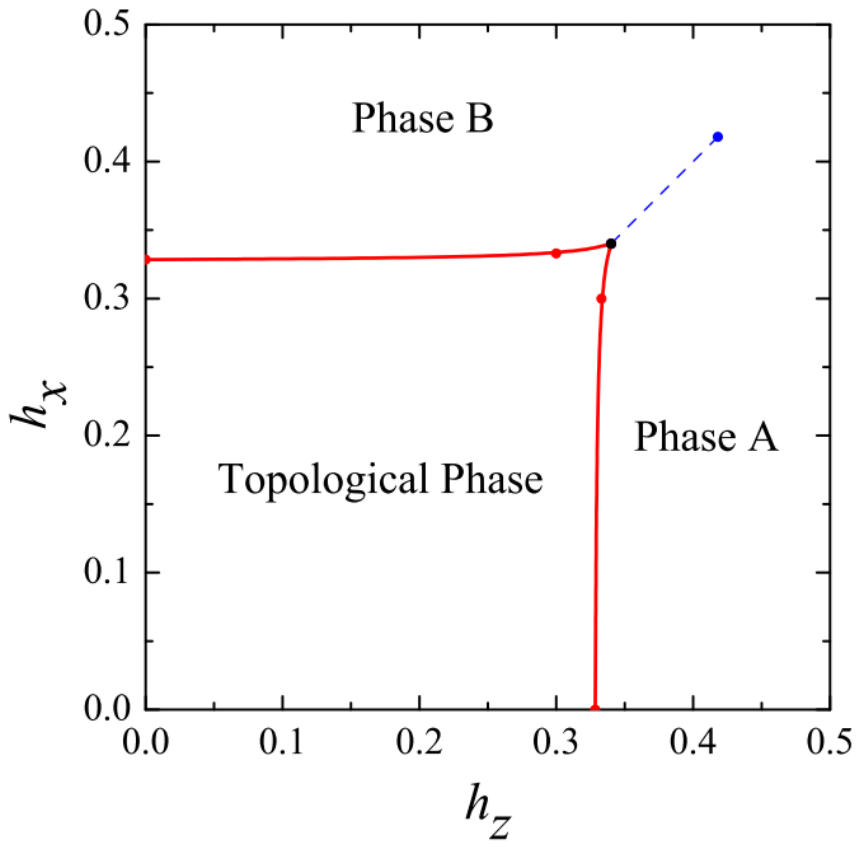
$$H = -\sum_i A_i - \sum_i B_i$$

$$A_i = \prod_{j \in \mathrm{vertex}} \sigma_{z,j}$$

$$B_i = \prod_{j \in \mathrm{plaq}} \sigma_{x,j}$$

Toric Code

2-dimensional spin lattice model



Wu, F., Deng, Y. & Prokof'ev, N. *Phys. Rev. B* **85**, 195104 (2012).

Toric code Hamiltonian in a parallel magnetic field:

$$H = -\sum_i A_i - \sum_i B_i - h \sum_i \sigma_{z,i}$$

$$A_i = \prod_{j \in \text{vertex}} \sigma_{z,j}$$

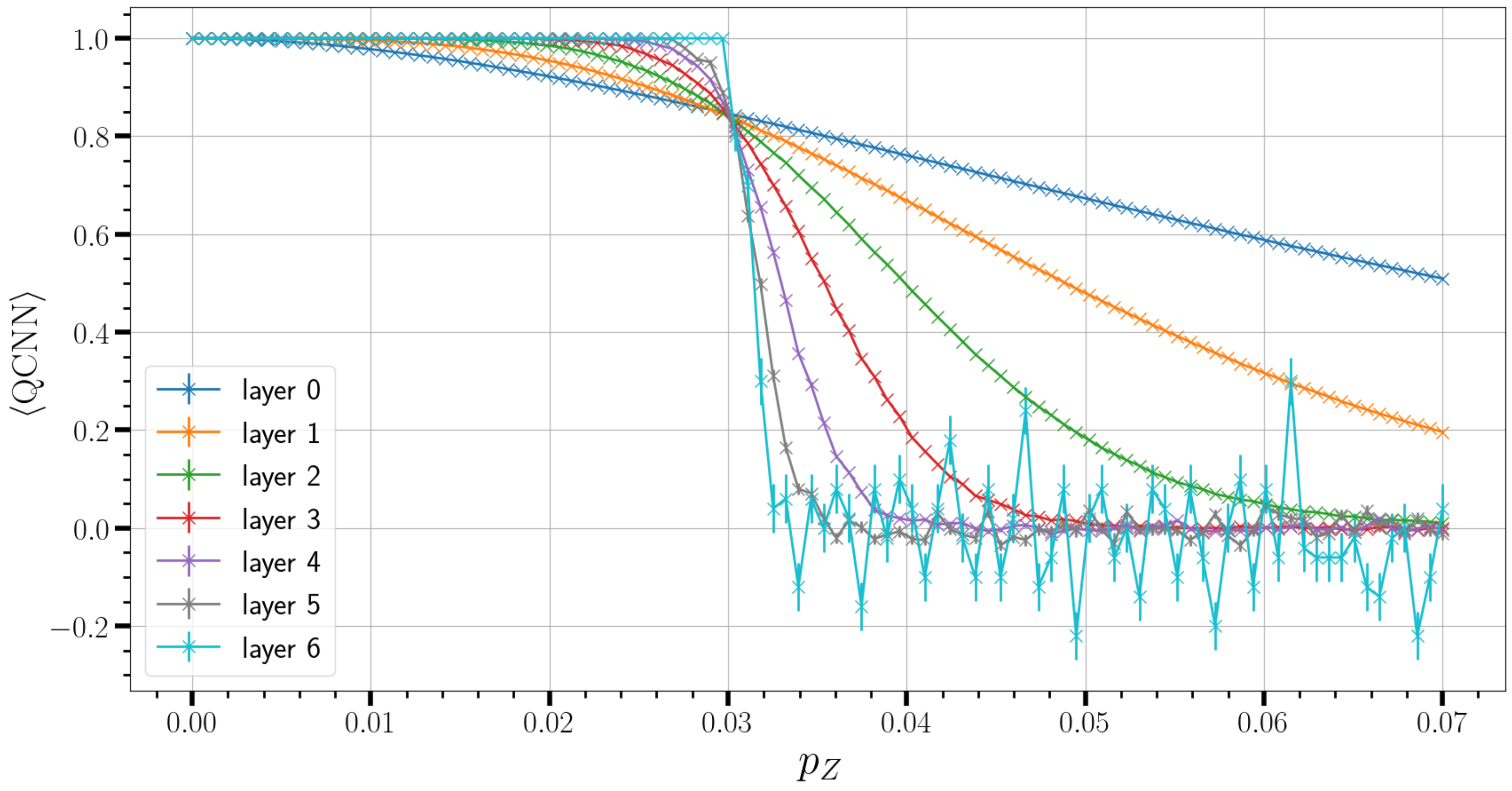
$$B_i = \prod_{j \in \text{plaq}} \sigma_{x,j}$$

Simulations for random Pauli noise

Numerical Results – incoherent errors

$$\rho \rightarrow (1 - p_Z)\, \mathbb{1}\rho\mathbb{1} + p_Z\, Z\rho Z$$

- QCNN output $\langle \mathrm{QCNN} \rangle$ over Pauli error rate p_Z
- $\langle \mathrm{QCNN} \rangle = 1$ corresponds to topologically ordered phase
- $\langle \mathrm{QCNN} \rangle = 0$ corresponds to disordered phase (random outcome between output -1 and 1)
- Subsequent layers of the QCNN show increased sharpness at transition

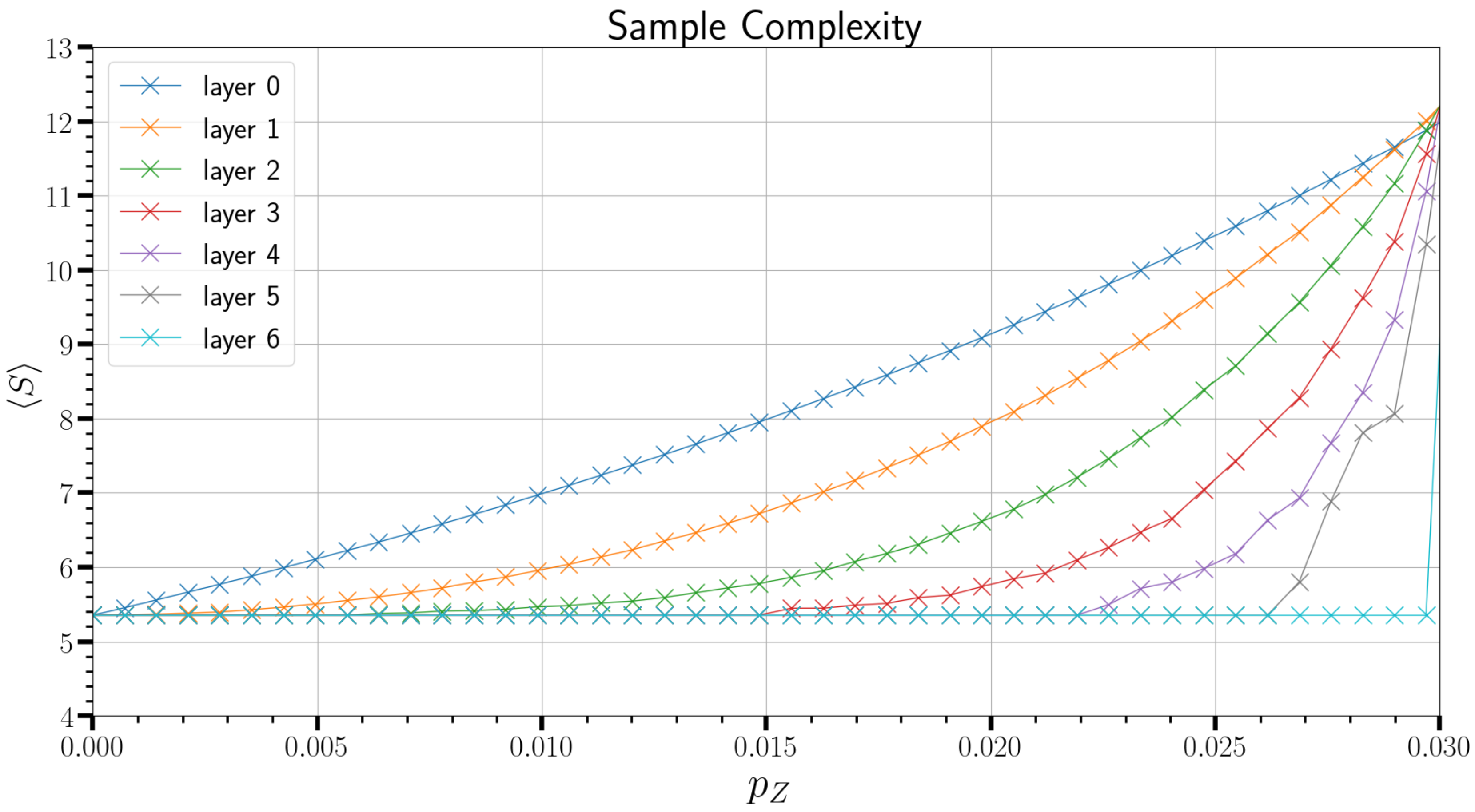


Simulations for random Pauli noise

Numerical Results – incoherent errors

$$\rho \to (1 - p_Z)\, \mathbb{1}\rho\mathbb{1} + p_Z\, Z\rho Z$$

- Sample complexity: Required number of samples for determining the state to be in the topological phase with 95% confidence
- Greater number of QCNN layers leads to reduction in sample complexity for increasing error rates

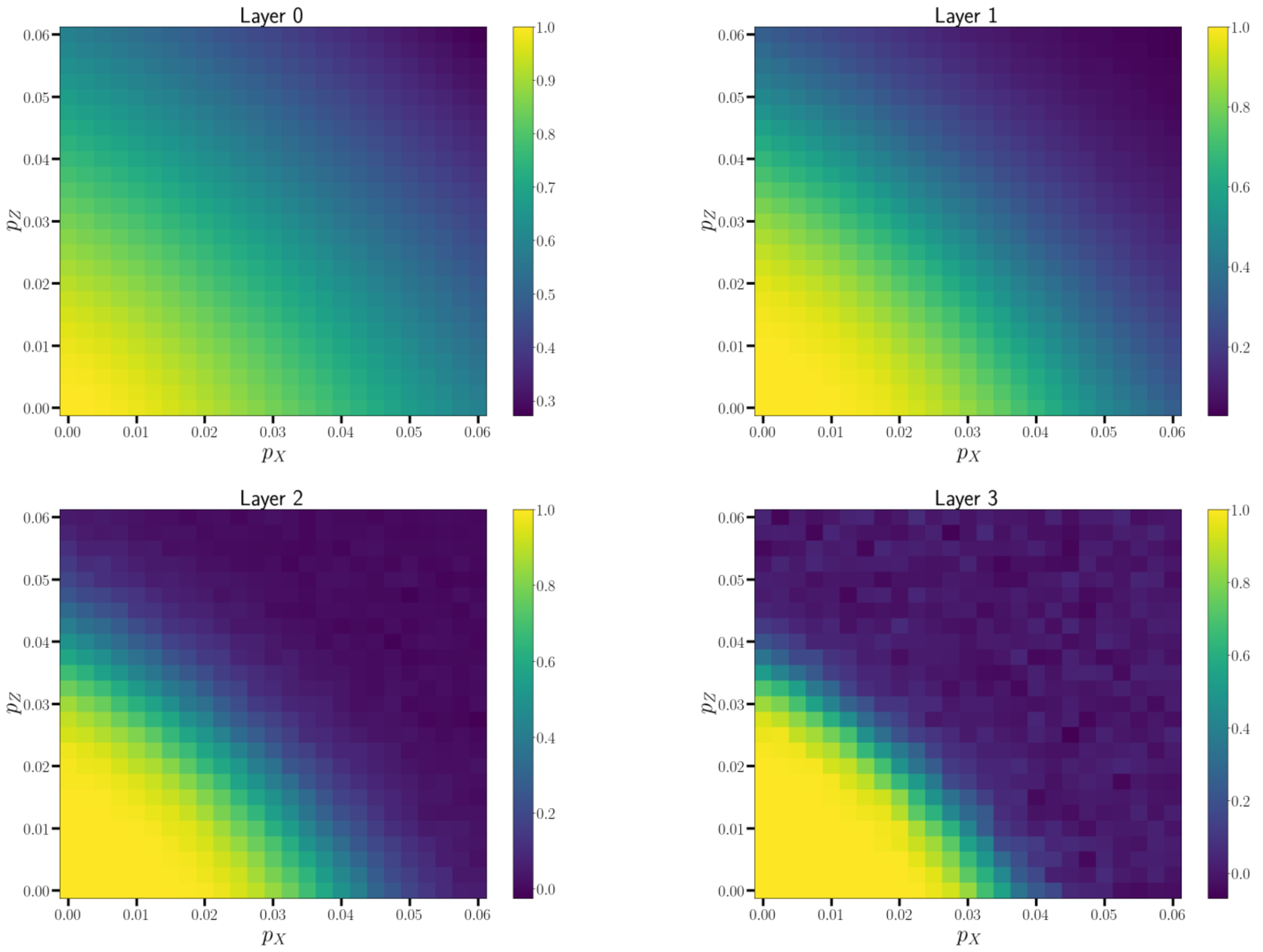


Simulations for random Pauli noise

Numerical Results – incoherent errors

$$\rho \rightarrow (1-p_Z)(1-p_X)\,\mathbb{1}\rho\mathbb{1} + p_Z\, Z\rho Z + p_X\, X\rho X + p_X p_Z\, Y\rho Y$$

- Output of a 4-layer QCNN for simultaneous Pauli-X and Pauli-Z noise
- Sharpness of transition as well as as topological characterized area increases with successive layers
- Errors of the different basis interact on the lattice (diagonal line for error threshold)

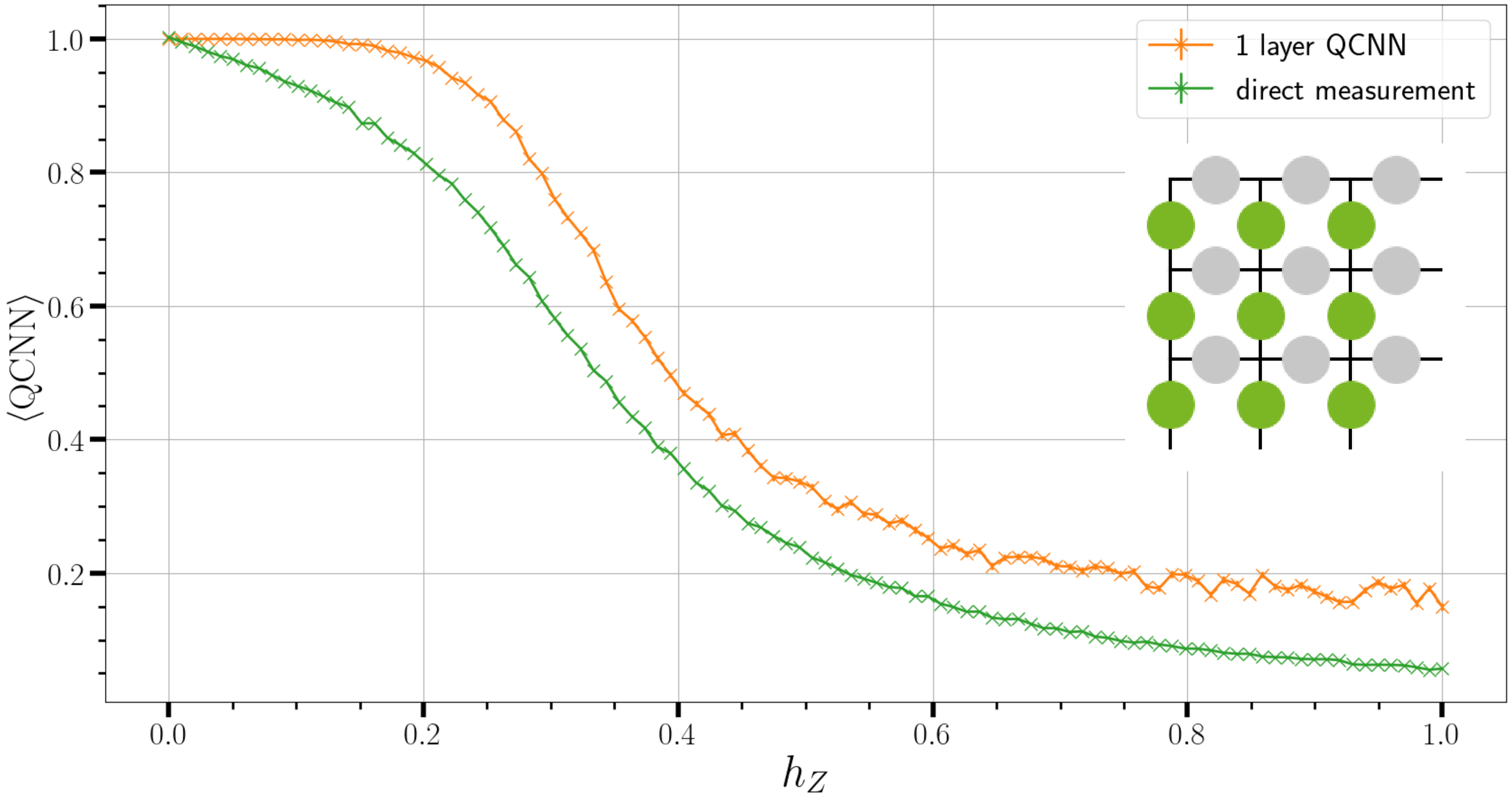


Simulations for exact ground state

Numerical Results – external magnetic field

$$H = -\sum_i A_i - \sum_i B_i - h \sum_i \sigma_{z,i}$$

- QCNN output for 1 layer on sampling from 18 qubit toric code ground state
- $\langle \mathrm{QCNN} \rangle = 1$ corresponds to topologically ordered phase
- $\langle \mathrm{QCNN} \rangle = 0$ corresponds to disordered phase (random outcome between output -1 and 1)
- Comparison between direct measurement of all qubits, measurement after convolution and output after pooling

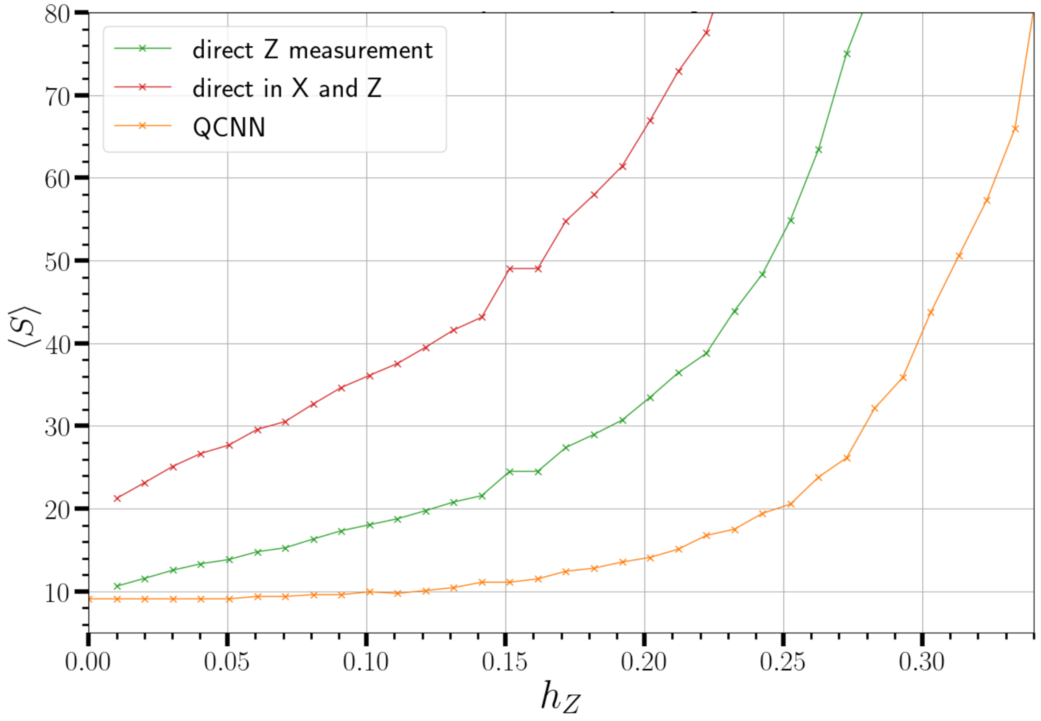


Simulations for exact ground state

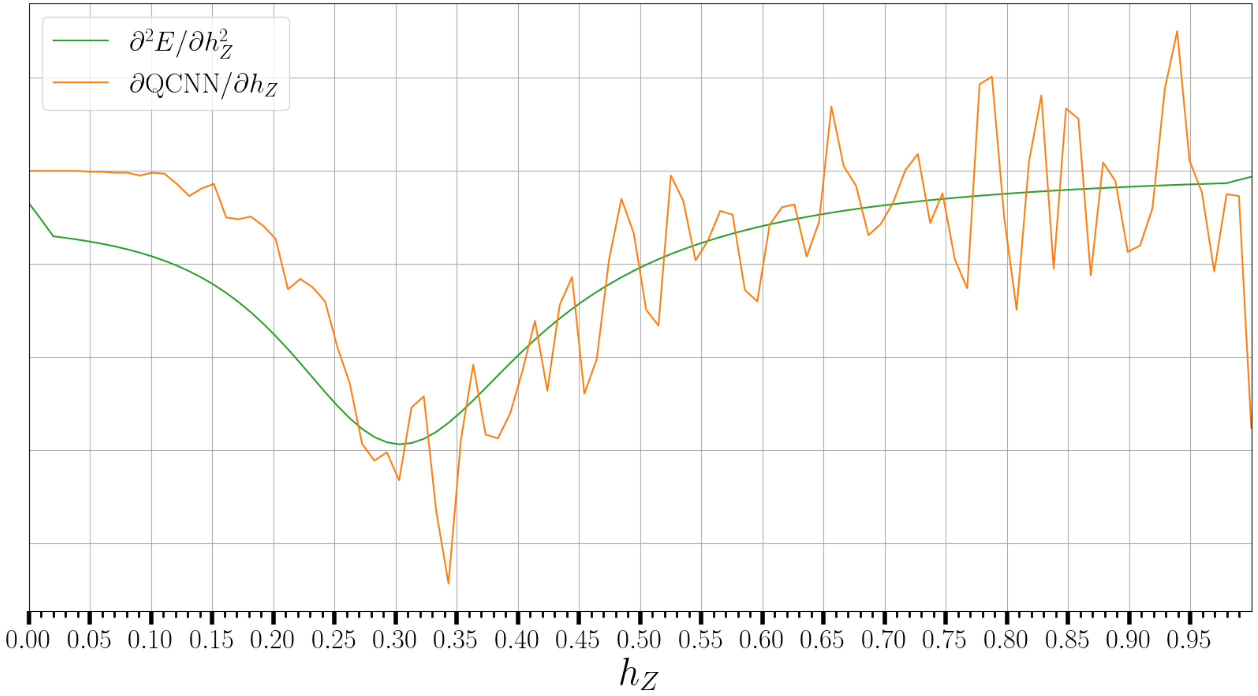
Numerical Results – external magnetic field

$$H = -\sum_i A_i - \sum_i B_i - h\sum_i \sigma_{z,i}$$

Reduction in sample complexity



Does the QCNN show the correct transition?



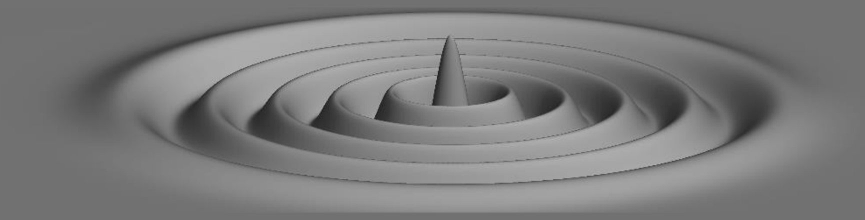
Conclusion

- **Summary:**
  - Successful construction of QCNN for 2D toric code with phase identification
  - Application for lattice perturbation by random Pauli noise and parallel magnetic field
  - QCNN identifies phase transition by design, no training required
- **Outlook**:
  - Improvements for QCNN pooling?
    - Increase error threshold for random Pauli noise?
    - Optimal threshold of $p_{\mathrm{MWPM}} \approx 11\%$ using Minimum Weight Perfect Matching algorithm
    - Improvements by training of parameterized circuit?
  - Implementation on Hardware for small system?
  - Further exploration of the phase space:
    - Test data generated by translationally invariant Clifford circuits
      - hard to sample for large systems
      - explore phase space with coherent errors for large systems
    - Use 2D tensor network to generate test data
      - Apply magnetic field mapped to imaginary time evolution
      - Difficult to contract tensor network for large system

Thank you for your attention!

References

1. M. Ragone, B.N. Bakalov, F. Sauvage, A.F. Kemper, C.O. Marrero, M. Larocca, and M. Cerezo, “A Unified Theory of Barren Plateaus for Deep Parametrized Quantum Circuits,”. Preprint at https://doi.org/10.48550/arXiv.2309.09342 (2023).
2. A. Pesah, M. Cerezo, S. Wang, T. Volkoff, A.T. Sornborger, and P.J. Coles, “Absence of Barren Plateaus in Quantum Convolutional Neural Networks,” Phys. Rev. X **11**(4), 041011 (2021).
3. Cong, I., Choi, S. & Lukin, M. D. Quantum Convolutional Neural Networks. *Nat. Phys.* **15**, 1273–1278 (2019).
4. Herrmann, J. *et al.* Realizing Quantum Convolutional Neural Networks on a Superconducting Quantum Processor to Recognize Quantum Phases. *Nat Commun* **13**, 4144 (2022).
5. Zarei, M. H. Ising order parameter and topological phase transitions: Toric code in a uniform magnetic field. *Phys. Rev. B* **100**, 125159 (2019).
6. Wu, F., Deng, Y. & Prokof’ev, N. Phase diagram of the toric code model in a parallel magnetic field. *Phys. Rev. B* **85**, 195104 (2012).
7. Lake, E., Balasubramanian, S. & Choi, S. Exact Quantum Algorithms for Quantum Phase Recognition: Renormalization Group and Error Correction. Preprint at https://doi.org/10.48550/arXiv.2211.09803 (2022).



Leon Sander

Institute for Theoretical Physics II
Chair for Quantum Technologies Theory

Friedrich-Alexander-Universität (FAU) Erlangen-Nürnberg
Staudtstraße 7
91058 Erlangen

www.quantumtheory.nat.fau.eu

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