Horizons Decohere Quantum Superpositions
"Horizons are Watching You"

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Princeton University

YITP Workshop: Holography, Gravity and Quantum Information

D. Danielson, G.S., & R.M. Wald Phys. Rev. D 105, 086001 (2022) [arXiv:2112.10798]

D. Danielson, G.S., & R.M. Wald Int. J. Mod. Phys. D 2241003 (2022) [arXiv:2205.06279]
Gravity Research Foundation Essay: 3rd Prize

D. Danielson, G.S. & R.M. Wald (2023) [arXiv:2301.00026]

D. Danielson, J. Kudler-Flam, G.S. (to appear)

September 27, 2023

A Gedankenexperiment in Flat Spacetime

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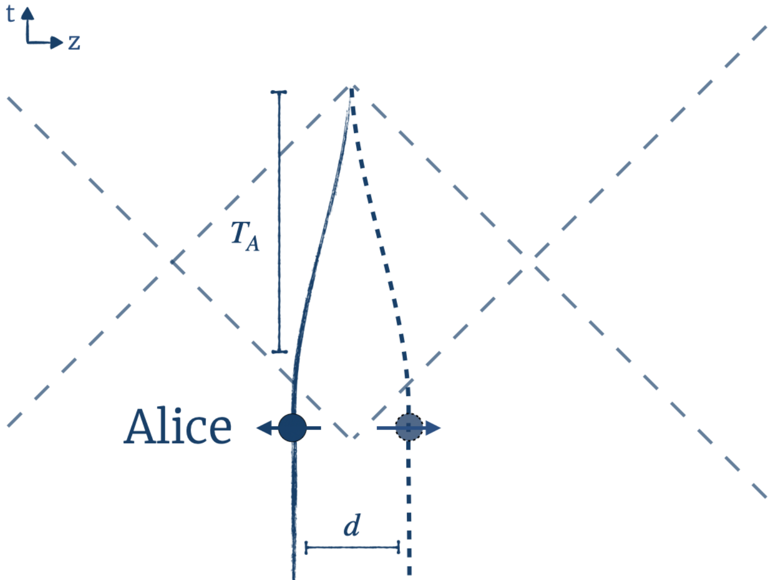

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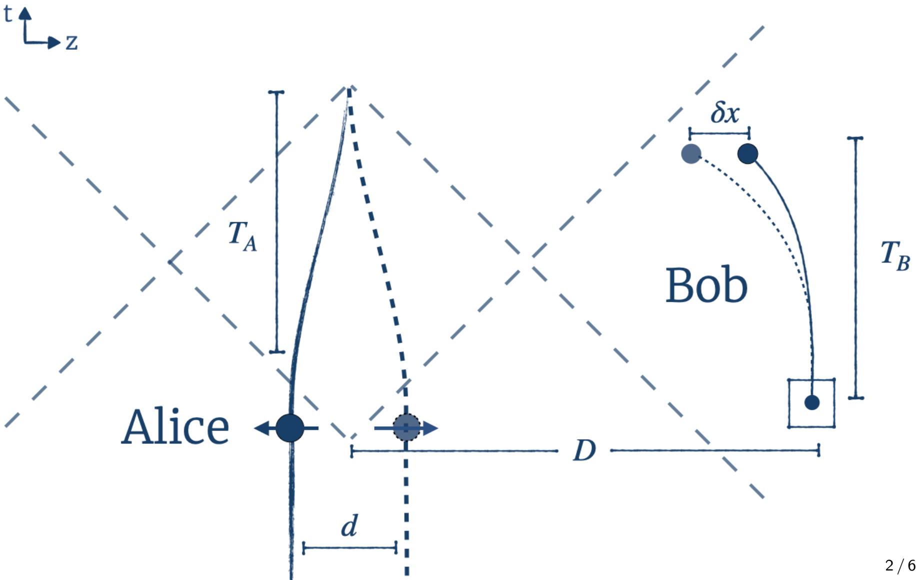

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- If $|\Psi_1\rangle$ and $|\Psi_2\rangle$ correspond to the electromagnetic radiation states along each path then amount of decoherence due to radiation is

$$\mathscr{D} = 1 - |\langle\Psi_1|\Psi_2\rangle| = 1 - e^{-\frac{1}{2}\langle N\rangle_{\Psi_1 - \Psi_2}} \qquad \text{where } \langle N\rangle \sim (q_A d/T_A)^2$$

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- <u>Lesson:</u> *Alice must decohere herself (by emitting entangling radiation) at least as much as any Bob(s) could decohere her.* If Alice recombines sufficiently adiabatically, in flat spacetime, (i.e. $T_A \gg q_A d$) then she can maintain coherence and, similarly, any Bobs cannot obtain "which-path" information

A Gedankenexperiment Outside of a Black Hole

- Suppose a black hole is present and Bob(s) are *inside* the black hole. Alice is performing her experiment in the exterior of the black hole and can recombine adiabatically as she likes.

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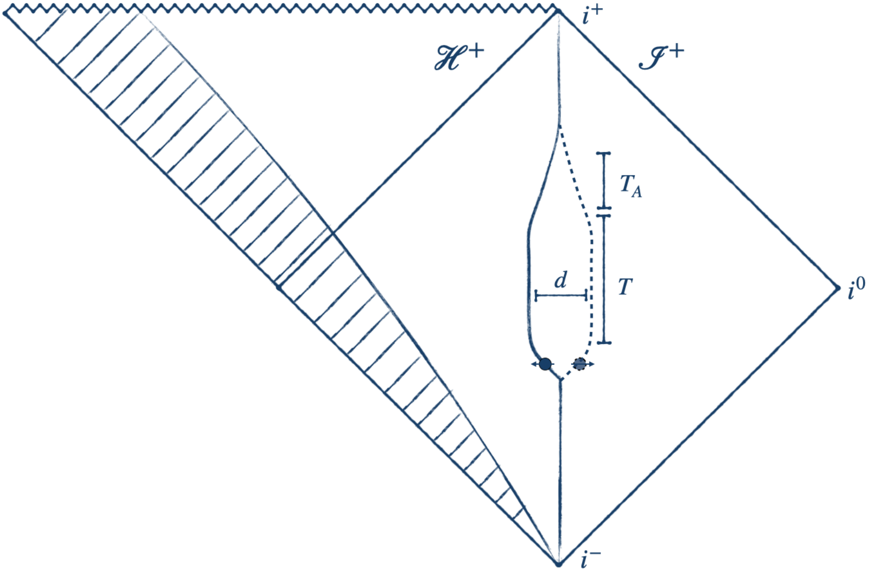

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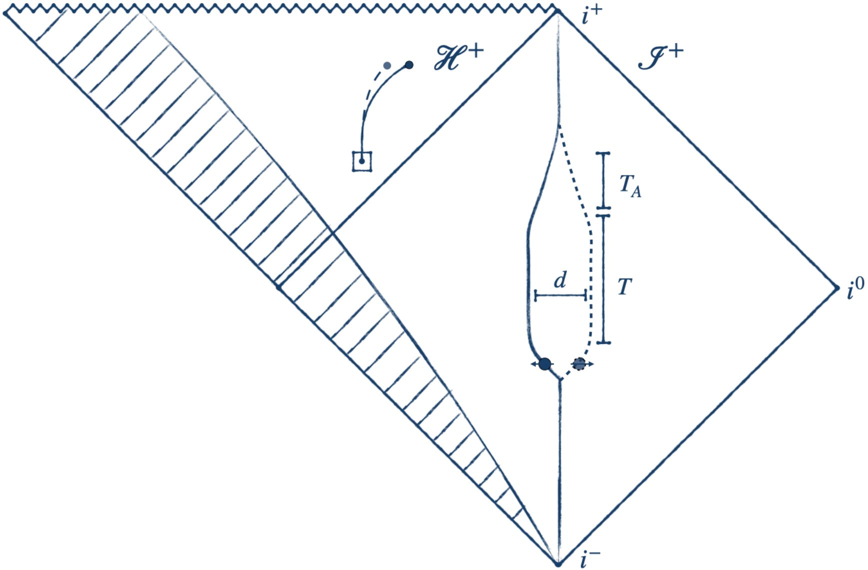

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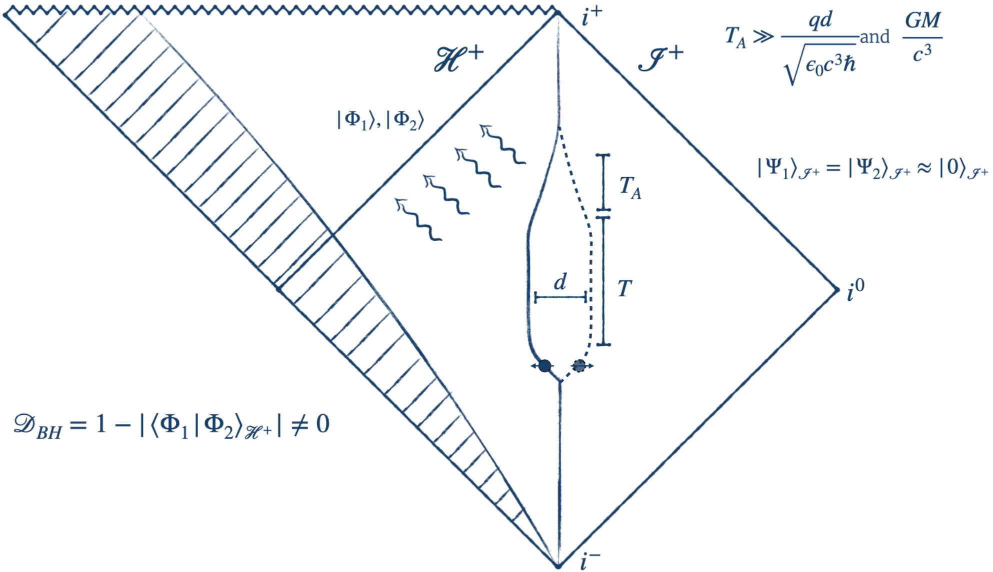

$$T_A \gg \frac{qd}{\sqrt{\epsilon_0 c^3 \hbar}} \text{ and } \frac{GM}{c^3}$$

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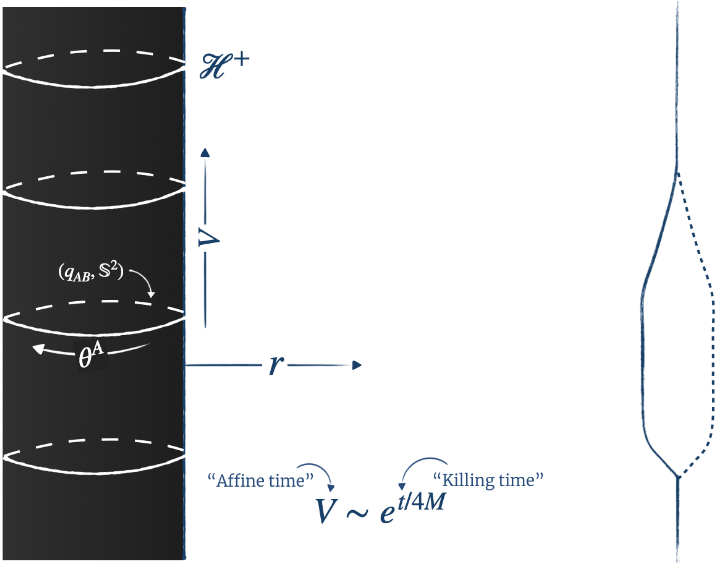

Warm-Up: Displacing a Classical Charge Outside a Black Hole

- Let's consider the simpler warm-up problem of the radiation produced by a charged body initially at a position $r = D$ and then is displaced to a position $r = D + d$ where $d \ll D$.

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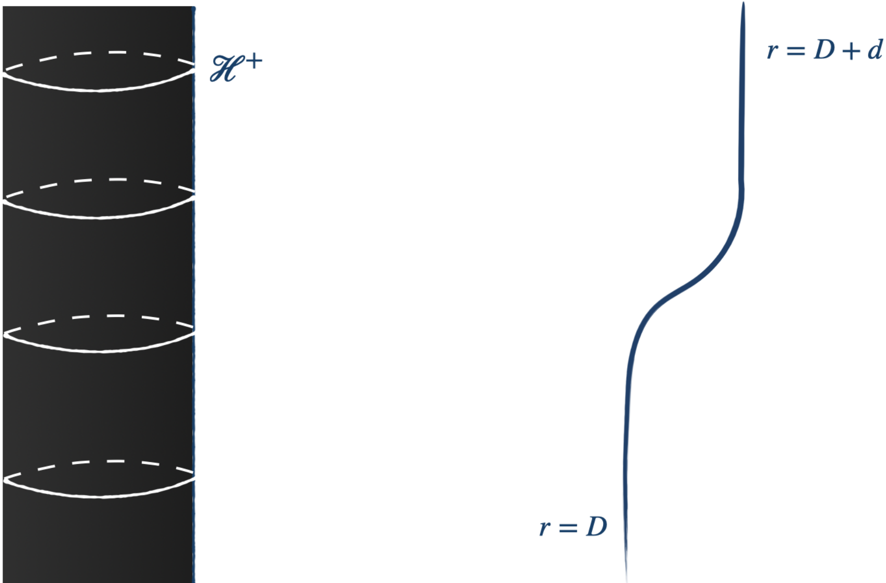

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- To analyze the radiation on the horizon we need Maxwell's equation on $\mathscr{H}^+$. Maxwell's equation relates changes in the "Coulombic field" E_r on the horizon to "horizon radiation" E_A which propagates into the horizon.

$$\mathcal{D}^A E_A = -\partial_V E_r$$

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- For a static test charge at $r = D$, E_A *vanishes* and $|E_r| \sim q/D^2$. Throughout this process, E_A is very small however, due to the displacement,

$$|\Delta E_r| \sim \frac{qd}{D^3} \implies \int dV \; E_A \neq 0$$

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- In terms of the vector potential on the horizon $E_A = \partial_V A_A$, this implies that A_A must suffer a permanent change between early and late times.

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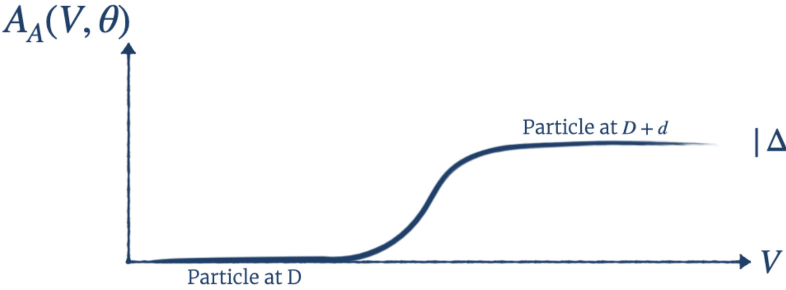


$$|\Delta A_A| \propto \frac{qd}{D^3}$$

$$D^A E_A = -\partial_V E_r \qquad E_A = \partial_V A_A$$

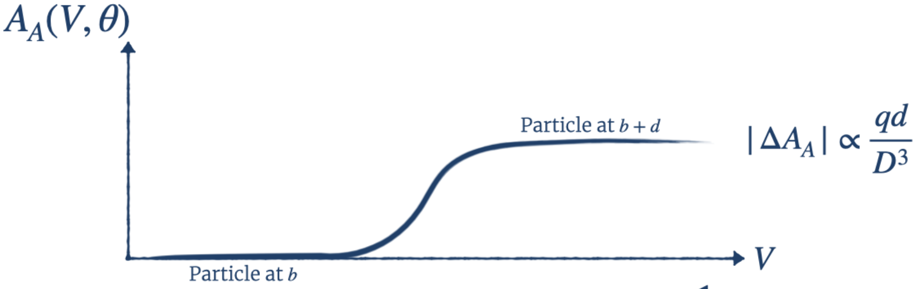
$$\Delta E_r \neq 0 \implies \Delta A_A \neq 0$$

Number of Horizon Photons

- The quantum state of a classical solution A_a is a coherent state $|\Phi\rangle$ "above" the Unruh vacuum. The expected number of horizon photons is

$$\langle N \rangle_\Phi = ||A||^2 = \int_{\mathbb{S}^2} d\Omega \int_0^\infty \omega d\omega \, |\tilde{A}_A(\omega, \theta^A)|^2$$

which is the norm in the corresponding one-particle Hilbert space.



$$\Delta A_A \neq 0 \implies \tilde{A}_A(\omega,\theta) \sim \frac{1}{\omega}$$

$$\implies \langle N \rangle = ||A||^2 \sim \int_0^\infty \frac{d\omega}{\omega}$$

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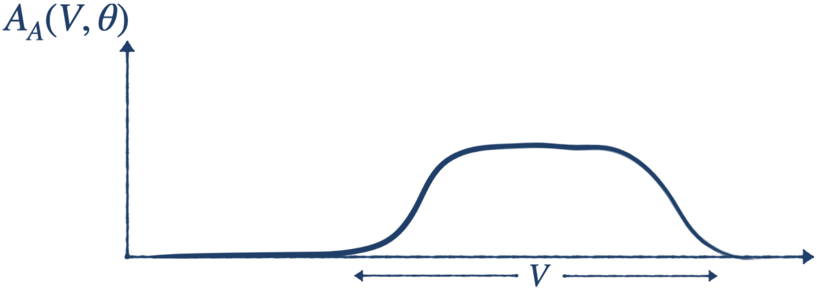
- Displacing a charged body outside of a black hole and keeping there *forever* results in the emission of an infinite number of soft horizon photons.

- If it is displaced back after a time T then $\langle N \rangle_\Phi$ is *finite* but large for large T

Number of Horizon Photons



$$V = e^{T/4M}$$

$$\langle N \rangle = ||A||^2 \sim \frac{G^4 M^4 q^2 d^2}{\hbar c^9 D^6} \ln V = \frac{G^3 M^3 q^2 d^2}{\hbar c^5 D^6} T$$

Black Holes Decohere Quantum Superpositions

- Both branches of Alice's superposition will be forced to radiate soft photons through the horizon over a proper time T such that number of entangling soft photons is given by

$$\langle N \rangle_{\Phi_1 - \Phi_2} \propto T$$

- The decoherence entirely due to the presence of the black hole is given by

$$\mathscr{D}_{\mathrm{BH}} = 1 - |\langle \Phi_1 | \Phi_2 \rangle_{\mathscr{H}^+}| = 1 - e^{-\langle N \rangle_{\Phi_1 - \Phi_2}}$$

  and so after the emission of an $O(1)$ number of entangling soft photons the superposition completely decohered.

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- A charged/massive spatial superposition held outside of a black hole is totally decohered in a time

$$T_{\mathrm{D}}^{\mathrm{EM}} \sim \frac{D^6}{M^3 q^2 d^2} \qquad T_{\mathrm{D}}^{\mathrm{GR}} \sim \frac{D^{10}}{M^5 m^2 d^4}$$

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- If performed sufficiently adiabatically, the gravitational field of an ordinary massive body ($r_{\mathrm{body}} \gg r_{\mathrm{S}}$) will not decohere Alice's particle [**G.S.**, S. Carot-Huot, (in prep.)].

Summary and Conclusions

- If one creates a quantum spatial superposition in the presence of a Black hole horizon, the long range fields of the quantum superposition register on the horizon. This results in a constant rate of production of "entangling radiation" into the horizon.
- This analysis immediatley generalizes for any stationary superposition in the presence of a Killing horizon. Eventually, the horizon will decohere any quantum superposition. [Danielson, **G.S.**, Wald (2023)]
- This effect is *not* due to thermal radiation or the local acceleration of Alice's lab.
- We give a more precise description of Alice's protocol in terms of quantum channels and relate the decoherence of the channel due to "optimal" measurements made in the black hole interior. [Danielson, Kudler-Flam, **G.S.** (to appear)]

We believe the fact that black holes (and cosmological horizons) will eventually decohere any quantum superposition in their vicinity may be of fundamental significance to our understanding of the nature of such structures in a quantum theory of gravity

Rindler Horizons Decohere Quantum Superpositions

- A similar analysis holds for any stationary superposition in the presence of a (Killing) horizon (i.e. a Rindler horizon in flat spacetime, cosmological horizon in de Sitter . . .) [Danielson, **G.S.**, Wald, 2023]. In the Rindler case, one can analyze this effect from both the co-accelerating and inertial perspectives.

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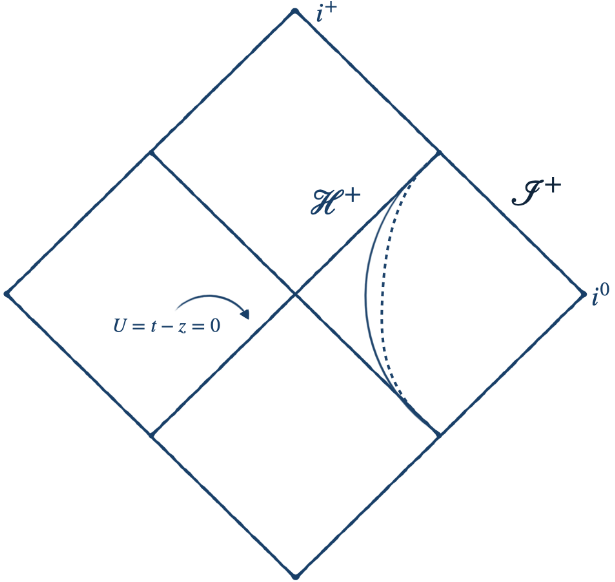

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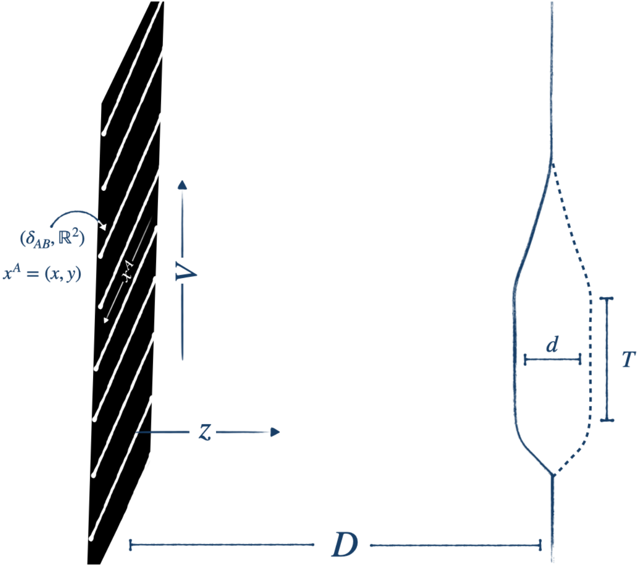

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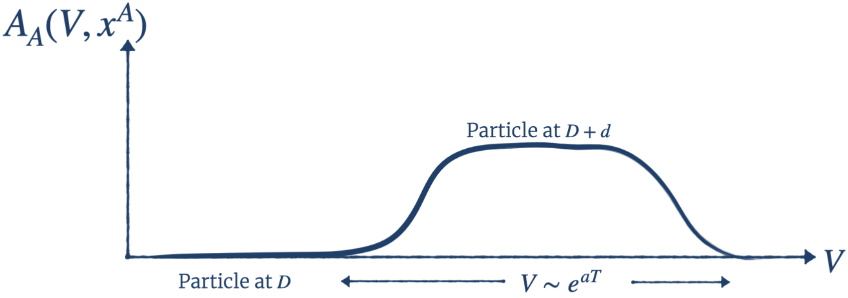

$$D^A E_A = -\partial_V E_z \qquad E_A = \partial_V A_A$$

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- A superposition held stationary in a lab uniformly accelerating with acceleration a in flat spacetime is forced to radiate soft Rindler photons with (Rindler) frequencies $\omega_{\mathrm{Rind.}} \sim ae^{-aT}$. The superposition is completely decohered in a time

$$T_{\mathrm{D}}^{\mathrm{EM}} \sim \frac{\epsilon_0 \hbar c^6}{a^3 q^2 d^2}, \qquad T_{\mathrm{D}}^{\mathrm{GR}} \sim \frac{\hbar c^{10}}{G m^2 d^4 a^5}$$

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- The decoherence can also be calculated (with considerably more effort!) from the more conventional inertial perspective and one can show that the flux of entangling photons at $\mathscr{I}^+$ yields $\langle N \rangle_{\mathscr{I}} \propto T$. Due to the relative blue shift between the inertial observer and the accelerating charge, the entangling radiation is "hard" ($\omega_{\mathrm{inert.}} \sim ae^{aT}$).

Cosmological Horizons Decohere Quantum Superposition

- In the cosmological case, this decoherence occurs even if Alice's lab is inertial at at the "center" of the de Sitter universe.

- In de Sitter spacetime with horizon radius R_{H} an inertial superposition will be completely decohered due to the emission of soft photons/gravitons in a time

$$T_{\mathrm{D}}^{\mathrm{EM}} \sim \frac{\hbar \epsilon_0 R_{\mathrm{H}}^3}{q^2 d^2} \quad \text{and} \quad T_{\mathrm{D}}^{\mathrm{GR}} \sim \frac{\hbar R_{\mathrm{H}}^5}{G m^2 d^4}.$$

- Since $d \ll R_{\mathrm{H}}$ the decoherence time will be much larger than the Hubble time R_{H}/c unless $q \gg q_P \sim 10\mathrm{e}$ or $m \gg m_P \sim 10\mu\mathrm{g}$.

What About Thermal Radiation?

- Killing horizons thermally radiate [Kay & Wald, '93] and so, in all cases, a thermal bath is present in Alice's lab. Is this effect simply due to collisions with thermal radiation?
- For example, the decoherence times for an accelerating charged superposition in flat spacetime due to collisions with the Unruh bath as compared to our effect are given by

$$T^{\mathrm{EM}}_{\mathrm{therm.}} \sim \frac{m^2}{q^4 d^2 a^5} \quad \text{and} \quad T^{\mathrm{EM}}_{\mathrm{rad.}} \sim \frac{1}{q^2 d^2 a^3}$$

  so the effects are distinct.
- Furthermore, using the fact that $d > q/m$, it is straightforward to show that $T^{\mathrm{EM}}_{\mathrm{rad.}} \ll T^{\mathrm{EM}}_{\mathrm{therm.}}$ as long as $d \ll \lambda_{\mathrm{therm.}} \sim 1/a$.

More generally, the decoherence rate due to the emission of "soft photons/gravitons" through any (Killing) horizon is always much larger decoherence due to the thermal radiation if the size of the superposition is smaller than the wavelength of the thermal radiation.

Quantum Information Inside the Black Hole

- What about the decoherence due to Bob(s) inside the black hole? Let $\mathscr{B}_t$ be a bounded region inside the black hole, and let Ψ_1^{EM} and Ψ_2^{EM} be the two possible (mixed) states of the EM field restricted to $\mathscr{B}_t$.

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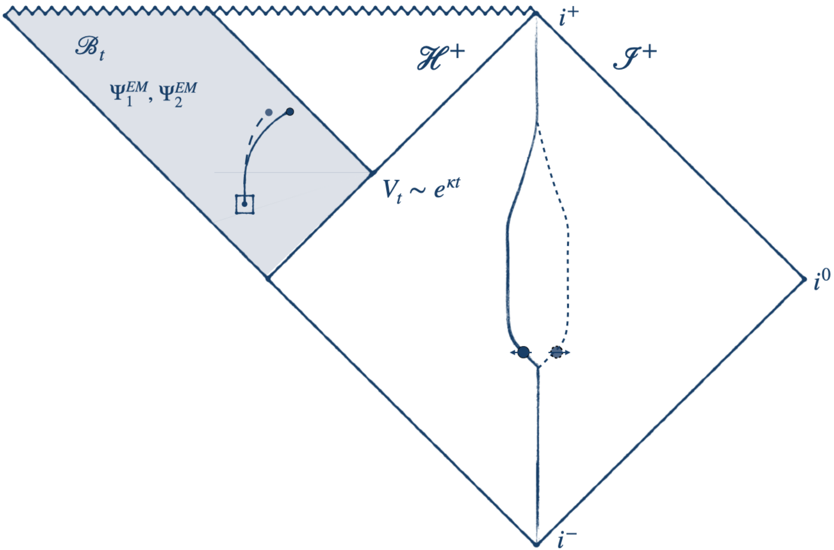

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- Decoherence due to Bob(s) "optimal" measurement is determined by "fidelity"
  - Fidelity - The error in Bob's "optimal" measurement in distinguishing states Ψ_1 and Ψ_2 in $\mathscr{B}_t$ and is denoted by $0 \leq F_t(\Psi_1^{\rm EM}, \Psi_2^{\rm EM}) \leq 1$

In quantum mechanics, the fidelity of two density matrices is $F_t^2 = \mathrm{Tr}[\sqrt{\rho_1}\sqrt{\rho_2}]$.
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- The fidelity is monotonic as the region $\mathscr{B}_t$ increases. However we prove that
  - $F_t(\Psi_1^{\rm EM}, \Psi_2^{\rm EM})$ is a strictly decreasing function of t
  - $F_t(\Psi_1^{\rm EM}, \Psi_2^{\rm EM}) > \exp(-\frac{1}{2}\langle N \rangle_t)$ where $\langle N \rangle_t \sim (q^2 d^2 M^3/D^6) \cdot t$
  - $\lim_{t\to\infty} F_t(\Psi_1^{\rm EM}, \Psi_2^{\rm EM}) = \exp(-\frac{1}{2}\langle N \rangle_T)$

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The decoherence due to the black hole — or any horizon — is equivalent to the decoherence due to the optimal "which-path" measurement made in its interior

Number of Photons

- The number of photons in a coherent state relative to the Hartle Hawking vacuum — which is equivalent to Unruh vacuum at low frequencies — is given by

$$\langle N \rangle = \frac{2c}{\hbar} \int\limits_{\mathbb{S}^2} d\Omega \int\limits_0^\infty \omega d\omega \, |\tilde{A}_A(\omega, \theta^A)|^2$$

  where $\tilde{A}_A$ is the Fourier transform of A_A.

- Since $\langle N \rangle$ is expressed in terms of the Fourier transform $\tilde{A}_A$, the number of photons emitted into the black hole seems to depend on the early and late time behavior of the horizon (i.e. during the collapse/evaporation where the spacetime is non-stationary). However, the above formula can equivalently be expressed as a local integral

$$\langle N \rangle = -\frac{2c}{\hbar} \int\limits_{\mathbb{R}^2 \times \mathbb{S}^2} dV_1 dV_2 d\Omega \, \frac{q_{AB} A^A(V, \theta^C) A^B(V, \theta^C)}{(V_1 - V_2 - i0^+)^2}$$

  where A_A only has support on the horizon during the time when the horizon is stationary.