Rotating strings and particles in AdS: Holography at weak gauge coupling and without conformal symmetry

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- Tomotaka Kitamura (Rikkyo U.), Shoichiro Miyashita (Natl. Dong Hwa U.), YS, *PTEP* 2022(2022) 4, 043B03.
- YS, *PTEP* 2020 (2020) 2, 021B01.

Outline

- The question:
  What is the gauge/gravity correspondence at weak gauge (‘t Hooft) coupling?
  (We consider $N \to \infty$, and zero temperature.)

- Our “testing ground” (next slide):
  Example of gauge/gravity correspondence with maximal SUSY, but <u>without</u> conformal symmetry.

- Result:
  Reproduce the free-field correlators of gauge theory from string theory (based on “string bit” picture).

Gauge/gravity correspondence for Dp-branes

- Conjecture: [Itzhaki, Maldacena, Sonnenschein, Yankielowicz, '98]

Superstring on the near-horizon limit of Dp-brane solution

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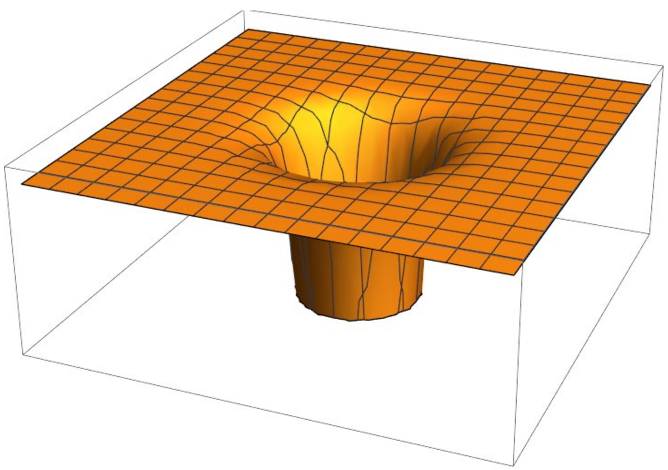
$SU(N)$ Super Yang-Mills in $(p+1)$ dimensions with 16 supercharges



$$S = \frac{1}{g_{\rm YM}^2}\int dx^{p+1}\left\{\frac{1}{4}F_{MN}F^{MN} + \frac{i}{2}\Psi\Gamma^M D_M\Psi\right\}$$

(Dimensional reduction of (9+1)D Super Yang-Mills)

(We consider $0 \leq p \leq 4$)

$p = 3$: Conformally invariant ($\mathrm{AdS}_5/\mathrm{CFT}_4$)

$p \neq 3$: Non-conformal (coupling const. $g_{\rm YM}$ has dimension)

Background geometry

- Near-horizon Dp-brane solution in string frame

$$ds^2 = H^{-1/2}\left(-dt^2 + dx_a^2\right) + H^{1/2}\left(dr^2 + r^2 d\Omega_{8-p}^2\right)$$

$$e^{\phi} = g_s H^{\frac{3-p}{4}}, \quad H = \frac{q}{r^{7-p}} \qquad q \propto (g_s N)\ell_s^{7-p}$$

  - String coupling e^{ϕ} depends on the position (for $p \neq 3$).
- Metric is Weyl equivalent to $\mathrm{AdS}_{p+2} \times S^{8-p}$:

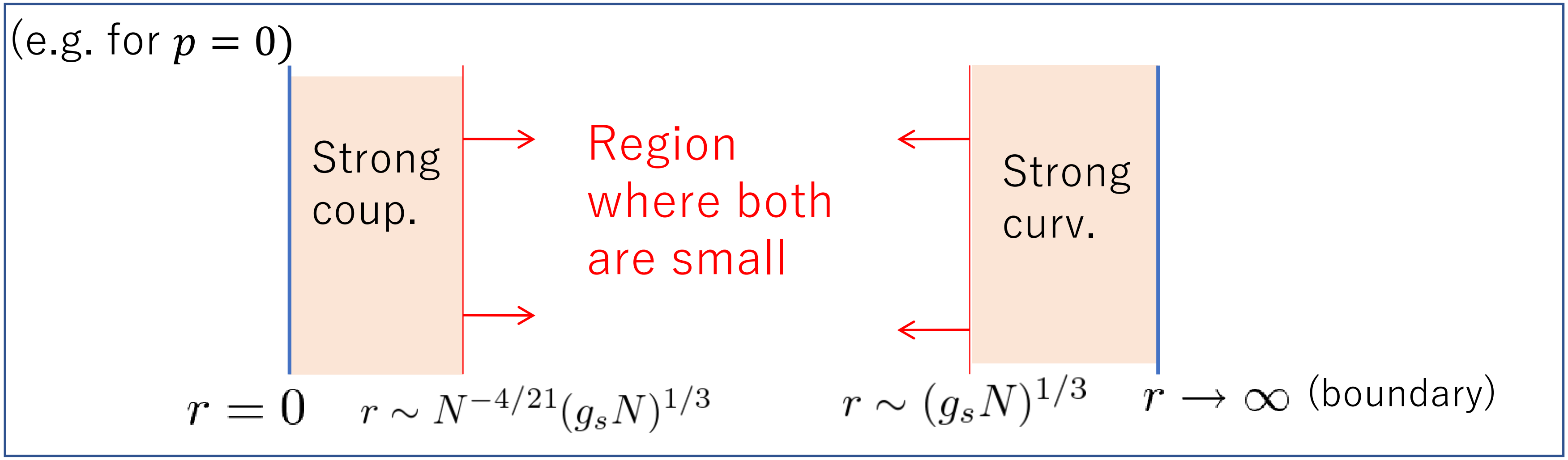
$$ds^2 = H^{1/2} r^2 \left[\left(\frac{2}{5-p}\right)^2 \left(\frac{dt^2 + dx_a^2 + dz^2}{z^2}\right) + d\Omega_{8-p}^2\right]$$

$$z = \frac{2}{5-p}(g_s N)^{1/2} l_s^{(7-p)/2} r^{-(5-p)/2}$$

  - No AdS isometry for $p \neq 3$.
- Note: for $p \neq 3$, we cannot rely on the identification (Energy in global AdS)=(scaling dimension of CFT)

Review of strong ‘t Hooft coupling (1)

- If we want to use weakly coupled SUGRA, we need a large region where e^{ϕ} and curvature are both weak.



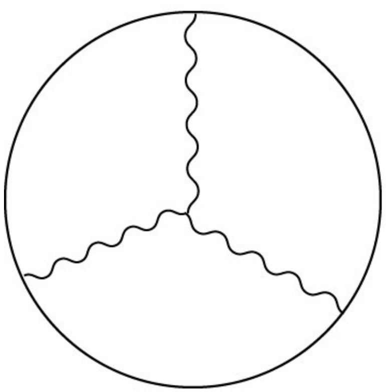
- This is satisfied if one takes $N \to \infty$ with $g_s N$ fixed and large. (i.e., SUGRA corresponds to ‘t Hooft limit with large ‘t Hooft coupling) [e.g., Sekino-Yoneya, ’99].

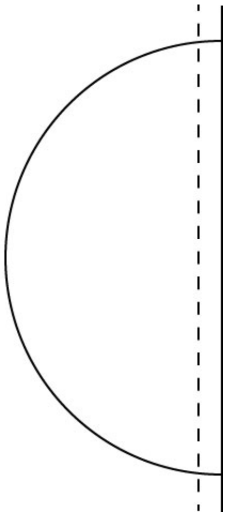
Review of strong ‘t Hooft coupling (2)

- Gubser-Klebanov-Polyakov-Witten (GKPW) prescription:

$$Z[\phi_0] = \langle e^{\int d^{p+1}x\, \phi_0(x) O(x)} \rangle$$

  - Evaluate the classical SUGRA action as a function of the b.c., and obtain generating functional of gauge theory correlators.





  - Boundary condition $\phi(x,r) = \phi_0(x)$ at the (regulated) boundary, and decaying $\phi(x,r) \to 0$ near the center.
  - Remark: Due to this b.c., the singularity at r =0 is no problem.

Review of strong 't Hooft coupling (3)

- From GKPW, correlators corresponding to SUGRA modes obey power law (even though there is no conformal sym.)

[YS, Yoneya, '99, YS, '00, Kanitscheider, Skenderis, Taylor, '08]

  - e.g., for "BMN operator" $O = \mathrm{Tr}\,(Z^J)$, $\quad (Z \equiv X_8 + iX_9)$

$$\langle \mathcal{O}(x')\mathcal{O}(x)\rangle = \frac{\delta}{\delta\phi_0(x')}\frac{\delta}{\delta\phi_0(x)} e^{-S_{\mathrm{SG}}[\phi_0]} \sim \frac{1}{|x-x'|^{\frac{4J}{5-p}+c_p}}.$$

$$\left(\; c_p = -\frac{(3-p)^2}{5-p}. \; \right)$$

- The powers (fractional numbers in general) are different from the free-field values (for $p \neq 3$).
  - Not understood in gauge theory, analytically.
  - For $p = 0$, confirmed by Monte Carlo simulation.

[Hanada, Nishimura, YS, Yoneya, '09, '11]

- Not known how to obtain free-field behavior from the bulk.

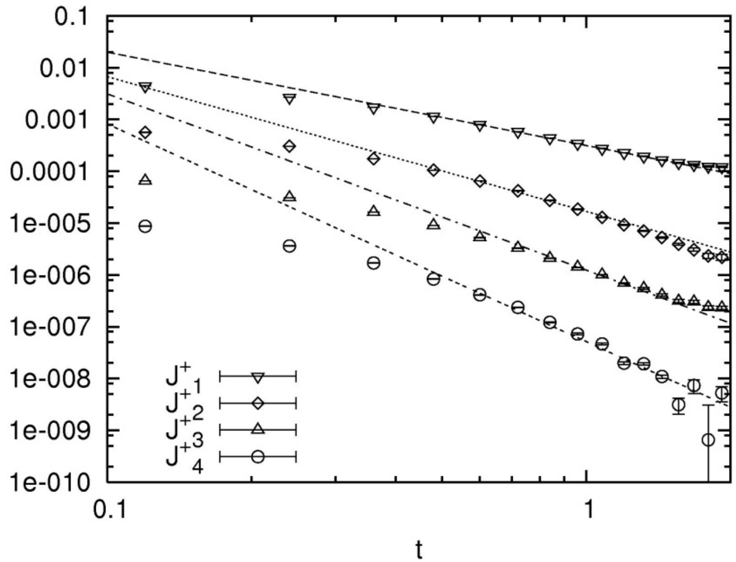
From Hanada, Nishimura, Sekino, Yoneya, '11

Results (supergravity modes)

- $$J^{+ij}_{l,i_1,\cdots,i_l} = \frac{1}{N}\mathrm{Str}\left(F_{ij}X_{i_1}\cdots X_{i_l}\right) \qquad (l = 1,2,3,4)$$

  Predicted power law $\nu = 2l/5$

  seen at $0.5 \lesssim t \lesssim 1.5$ (in unit of $(g_{YM}^2 N)^{-1/3}$)



$N = 3,\ \beta = 4,\ \Lambda = 16$

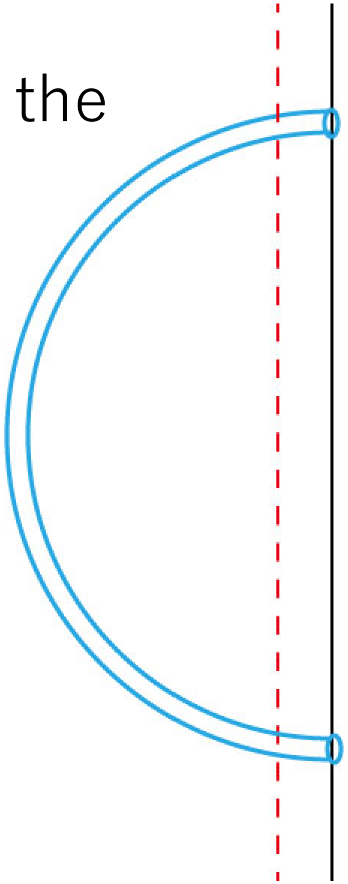
Worldsheet analysis

The need of string theory:

- To understand weak ‘t Hooft coupling, we need to study highly curved background (in string unit).
- In this limit, string excited modes are lighter than the curvature scale ⇒ SUGRA analysis is insufficient.

The string that we first consider

- A point-like configuration of closed string moving along a geodesic which connects two points on the boundary.
- We include quantum fluctuations around it. (Using technical results from ’02-’03.)
- We will consider states with angular momentum along S^{8-p}, and along AdS_{p+2}



State with angular momentum along S^{8-p}

- "BMN Operator" [Berenstein, Maldacena, Nastase, '02] $O = \mathrm{Tr}\,(Z^J) \quad (Z \equiv X_8 + iX_9)$ corresponds to a SUGRA mode with angular momentum J along S^{8-p}.

- SUGRA mode (lowest mode of string) is massless in 10D.
  - By KK reduction, it is massive in AdS_{p+2} with mass
$$m = \frac{2}{5-p} J$$
  - As long as we consider on-shell massless mode in 10D, Weyl factor plays no role (can be absorbed by worldline metric.)

- We will consider AdS_{p+2} with Euclidean signature (for which GKPW is formulated).

Review of the formalism (1)

- Geodesic in Euclidean AdS:

$$z = \frac{\ell}{\cosh \tau}, \qquad x = \ell \tanh \tau$$

  - Half-circle $x^2 + z^2 = \ell^2$
  - Connects two points on boundary; coordinate distance $|x_f - x_i| = 2\ell$

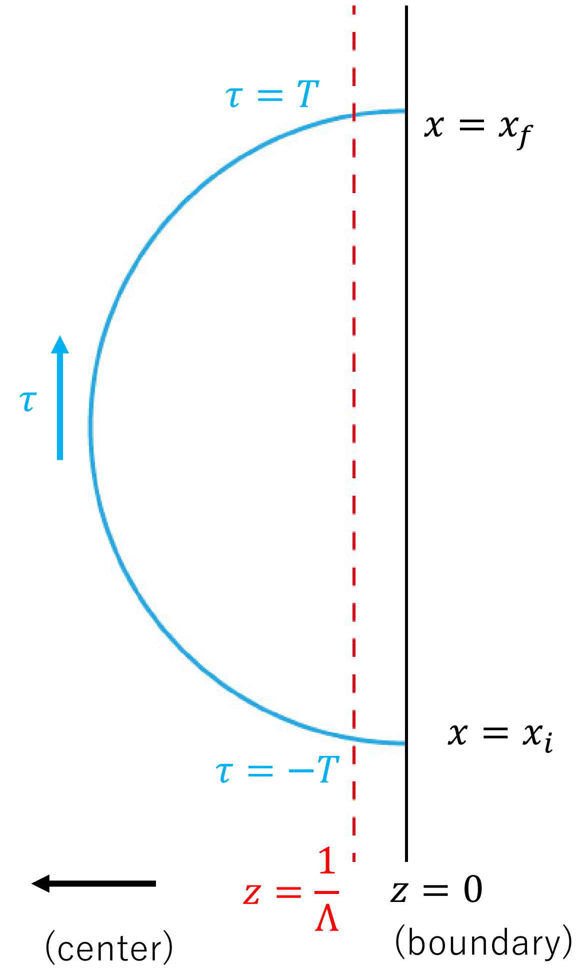
- Cutoffs:

$$z \geq \frac{1}{\Lambda} \quad \text{and} \quad -T \leq \tau \leq T$$

  - Related by $e^T \sim |x_f - x_i|\Lambda$

- Geodesic approximation (Classical contribution from Routh function in 10D)

$$\langle \mathcal{O}(x_f)\mathcal{O}(x_i)\rangle = e^{-m\int_{-T}^{T} d\tau} = e^{-\frac{4}{5-p}JT} = \frac{1}{(\Lambda|x_f - x_i|)^{\frac{4J}{5-p}}}$$



Consistent with GKPW. Leading exponent in J is obtained.

Review of the formalism (2)

- We study fluctuations around this geodesic, by expand world-sheet fields around it:

$$x^{\mu}_{\mathrm{full}}(\tau,\sigma) = x^{\mu}_{\mathrm{cl}}(\tau) + x^{\mu}(\tau,\sigma)$$

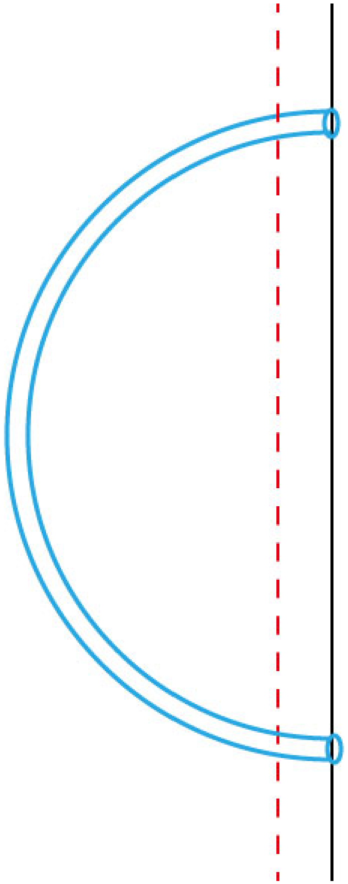
- Perform "double Wick rotation"

[Dobashi-Shimada-Yoneya, '02]

  - Time (in AdS and worldsheet) is Euclidean.
  - To keep angular momentum J real, we take an angle in S^{8-p} effectively imaginary:

$$J = P_{\psi} \sim \int d\sigma \, \frac{\partial\psi}{\partial\tau}$$

  - Under this prescription,
    (Gauge-theory correlator)
      = (Euclidean amplitude of closed string)



Review of the formalism (3)

[Asano-Sekino-Yoneya, '03, Asano-Sekino, '04]

($p=3$ case: BMN, '02 (Lorentzian), DSY, '02 (Euclidean))

- Worldsheet action at quadratic order
  - 8 bosons and 8 fermions. Bosonic part:

$$S^{(2)} = \frac{1}{4\pi\ell_s^2}\int d\tau \int_0^{2\pi\tilde{\alpha}} d\sigma \Big\{ \dot{x}_a^2 + \tilde{r}^{p-3}(\tau) x_a'^{\,2} + m_x^2 x_a^2 + \dot{y}_i^2 + \tilde{r}^{p-3}(\tau) y_i'^2 + m_y^2 y_i^2 \Big\}$$

  - Fluctuations are massive (These are the mass of SUGRA modes, due to background curvature)

$m_x = 1$ $(p+1)$ fields along AdS_{p+2}

$m_y = \dfrac{2}{5-p}$ $(7-p)$ fields along S^{8-p}

$m_f = \dfrac{7-p}{2(5-p)}$ 8 fermionic fields

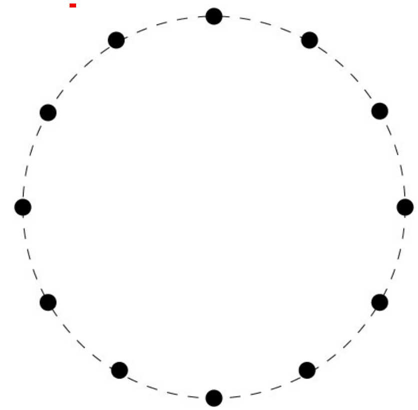
  - String excited modes have time-dependent mass, from the factor $\bar{r}^{p-3}(\tau)$ (due to Weyl factor) for the σ-derivative.

String bit picture

[YS, ‘20; See also BMN, ‘02, Verlinde, ’02, etc.]

Assumptions:

- The string corresponding to $\mathbf{Tr}\,(Z^J)$ has the σ direction discretized into J “bits.”
- One bit has a unit angular momentum along S^{8-p}.



- Reason 1: (In gauge theory,) string states are constructed by inserting “impurities” into $\mathbf{Tr}\,(Z^J)$. Can only represent J sites. e.g.,

$$a_{i,n}^{\dagger}|0\rangle \leftrightarrow \sum_{k=0}^{J} e^{ikn/J}\mathrm{Tr}\left(Z^k X_i Z^{J-k}\right)$$

- Reason 2: (In string theory,) we may imagine angular momentum J to be realized by acting a creation operator in position representation (in terms of the worldsheet σ) instead of the usual momentum rep.

BMN operator at zero ‘t Hooft coupling

[YS, 2020]

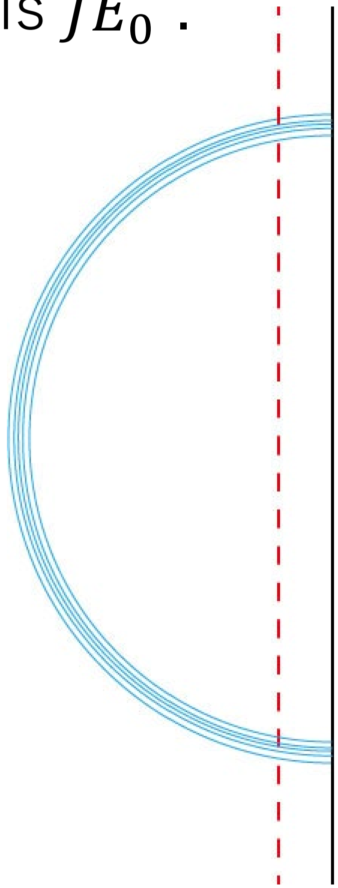
- String tension is zero at zero ‘t Hooft coupling. The string can be regard as a collection of J free bits.
    - Then, zero-point energy for the ground state is JE_0 . (E_0 for each bit, from 8 bosons and fermions)

$$E_0 = \frac{1}{2}\left((p+1)m_x + (7-p)m_y - 8m_f\right) = -\frac{(3-p)^2}{2(5-p)}.$$

- Amplitude (correlator for $O = \mathrm{Tr}\,(Z^J)$):

$$\langle \mathcal{O}(x_f)\mathcal{O}(x_i)\rangle = e^{-2(m+JE_0)T} = \frac{1}{(\Lambda|x_f - x_i|)^{(p-1)J}}.$$

    - Agrees with the free-field result. (Dimensional analysis: one scalar field in $(p+1)$D has dimension $(p-1)/2$.)



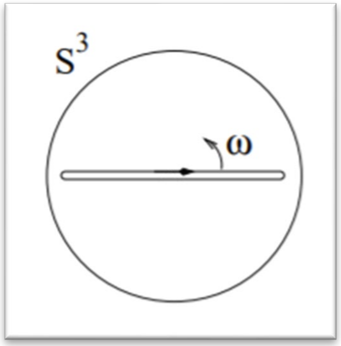
Operators with angular momentum along $\mathbf{AdS}_{p+2}$

[Kitamura, Miyashita, YS, 2022]

- "GKP operator" [Gubser, Klebanov, Polyakov, '02]

$$\mathrm{Tr}\left(Z^k D^S Z^{J-k}\right) \qquad (D \equiv D_1 + iD_2)$$

- For $p = 3$, at strong 't Hooft coupling: Interpreted as a folded string spinning in Lorentzian $\mathbf{AdS}_5$. (By identifying AdS energy with scaling dimension.)



(Figure: GKP, '02)

- For general p (without symmetry):
  - Consider Euclidean $\mathbf{AdS}_{p+2}$.
  - Generalizing Dobashi-Shimada-Yoneya, take the angles imaginary: along $\mathbf{AdS}_{p+2}$, $\psi \to i\psi$, and along S^{8-p}, $\tilde{\psi} \to i\tilde{\psi}$, to keep angular momenta real.

(In the following, we briefly explain the general formalism, and state the result for zero 't Hooft coupling.)

The general formalism (1)

<u>The "Spinning" string solution:</u>

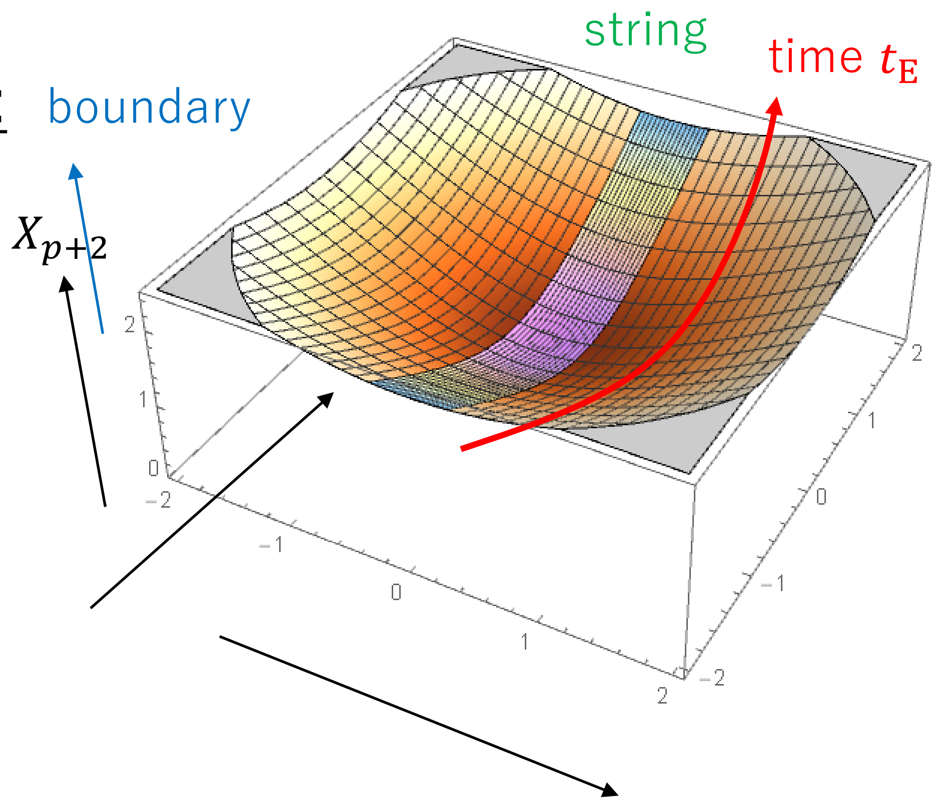
- Euclidean AdS_{p+2} in $R^{p+2,1}$:

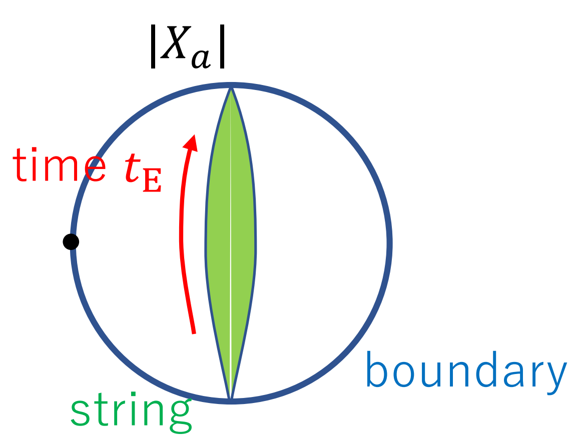
$$-X_{p+2}^2 + X_0^2 + \sum_{a=1}^{p+1} X_a^2 = -L^2$$

- Global coordinates:

$$X_{p+1} = \cosh\rho \cosh t_E$$
$$X_0 = \cosh\rho \sinh t_E$$
$$X_a = \sinh\rho\, \Omega_a$$

- We have taken the rotating angle (not shown in the figure) imaginary.
- The string moves from boundary to boundary.





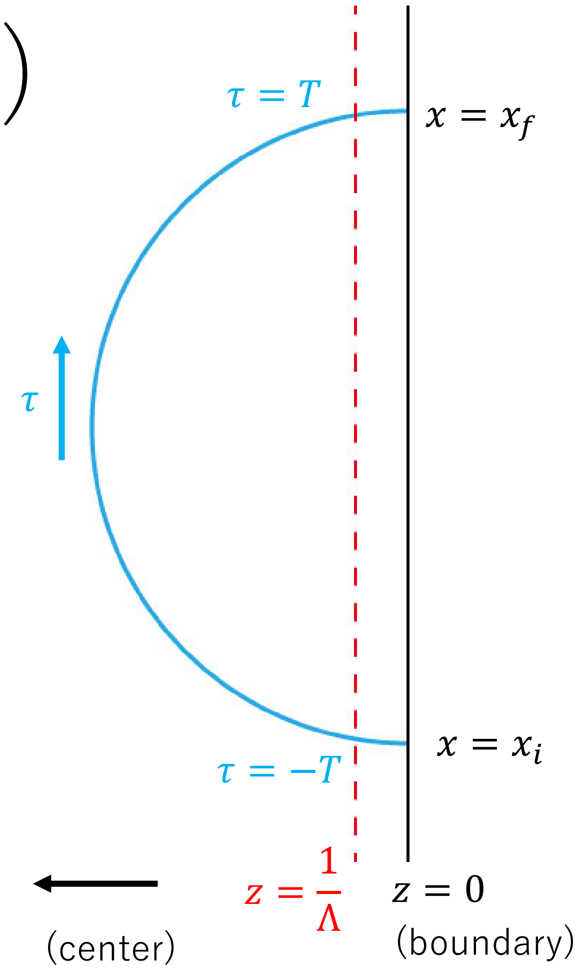
The general formalism (2)

- Center of string ($x_i = 0$) follows the same trajectory as a massive particle.
- In the Poincare coordinates,

$$t_{\rm P} = \tilde{\ell} \tanh \tau_{\rm E},$$

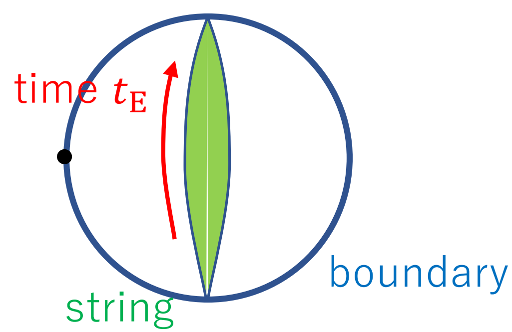
$$z = \frac{\tilde{\ell}}{\cosh \rho \cosh \tau_{\rm E}},$$

$$x_i^2 = z^2 \sinh^2 \rho = \frac{\tilde{\ell}^2 \tanh^2 \rho}{\cosh^2 \tau_{\rm E}},$$



- String reduces to a "single point" at boundary ($x_i^2 \to 0$ as $z \to 0$):
  - In comfort with the fact that this corresponds to a local operator.
- (Correlator) = (Amplitude).
  - For $p = 3$,

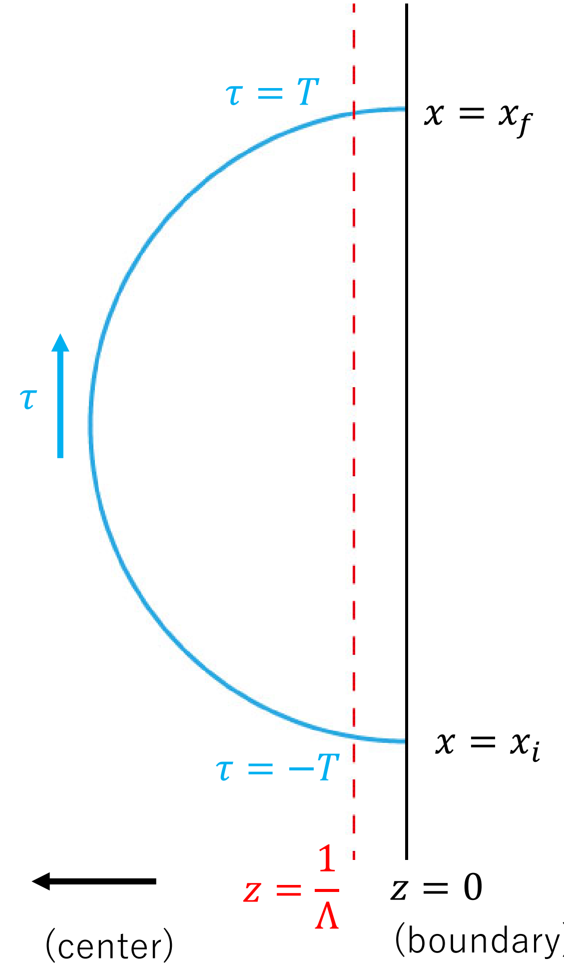
$$e^{-\int_{-T}^{T} d\tau_{\rm E} H(\tau_{\rm E})} = e^{-2HT} = \frac{1}{(\Lambda |t_{\rm f} - t_{\rm i}|)^{2H}}.$$



Zero ‘t Hooft coupling (general $\boldsymbol{p}$)

- Assumption (as before):
  - One bit has a unit angular momentum along S^{8-p}.
- At zero ‘t Hooft coupling, can ignore interaction among bits.
  - To keep S real, take the corresponding angle imaginary.
    - R: radius of rotation, Θ: angle
  - Consider motion of one bit with S units of angular momentum along AdS_{p+2}.
    - Follows trajectory similar to $S=0$ case.

$$I=\int_{-T}^{T} d\tau\, \frac{\tilde{\alpha}}{2} L^2 \left[\left(\frac{2}{5-p}\right)^2 \frac{1}{z^2}\left\{\dot{t}^2+\dot{z}^2+\dot{R}^2-R^2\dot{\Theta}^2+\cdots\right\} + \left(\dot{\theta}^2-\cos^2\theta\,\dot{\psi}^2+\cdots\right)\right]$$



- Amplitude of J free bits with total angular momentum S along AdS_{p+2} (by including the zero-point energy for each bit):

$$\langle t_f, J, S | t_i, J, S \rangle \simeq e^{-(I + J(\psi_f - \psi_i) + S(\Theta_f - \Theta_i))} Z^{(2)}_{J-bit}$$
$$= e^{-((p-1)J + 2S)T}$$
$$= \left(\frac{2J + S(5-p)}{2J} \right)^{\frac{p-1}{2}J + S} \frac{1}{\Lambda^{(p-1)J+2S}} \frac{1}{|t_f - t_i|^{(p-1)J+2S}}$$

  - Agrees with the free-field result in gauge theory. (A covariant derivative is just a partial derivative at zero coupling, and contributes S to the dimension.)

  - Remark: The spectrum of the fluctuations around this geodesic is different from the previous one (with $S = 0$), but the zero-point energy is the same value.

Summary

- Consider the gauge/gravity correspondence between superstring on near-horizon limit of Dp-brane solution, and SYM in $(p+1)$ dim.

- Studied the case of zero ‘t Hooft coupling.
  - Taking the string-bit picture.

- Reproduced the free-field result of gauge theory from string theory.
  - Confirmed this for general operators.

Issues to be clarified

1. Are there α'-corrections to the near-horizon Dp-brane background (for general p)?
   - We are assuming there it not.
   - The $p = 3$ case $(\mathrm{AdS}_5 \times S^5)$ is believed to be α'-exact, but for general p, not known.

2. We got the free-field result by taking into account fluctuations around geodesic, only up to quadratic order. No higher-order corrections?
   - There is no small parameter; everything is order 1.
   - This could be exact because of the high symmetry of the $\mathrm{AdS}_{p+2} \times S^{8-p}$ background. (This is calculation of a single bit (particle), so Weyl factor does not play a role.)

Future directions

1. Study of non-zero gauge coupling:
    - Include interactions among bits perturbatively, and see if it reproduces the perturbation in gauge theory.
    - If successful, this can be regarded as a proof for gauge/gravity correspondence.

2. Classical solution of rotating string for $p \neq 3$
    - Will be useful for the study of strong ‘t Hooft coupling.
    - Technically challenging, in the absence of AdS isometry.

3. Formulation of finite temperature case
4. Application to other backgrounds