Nonlocality of deep thermalization

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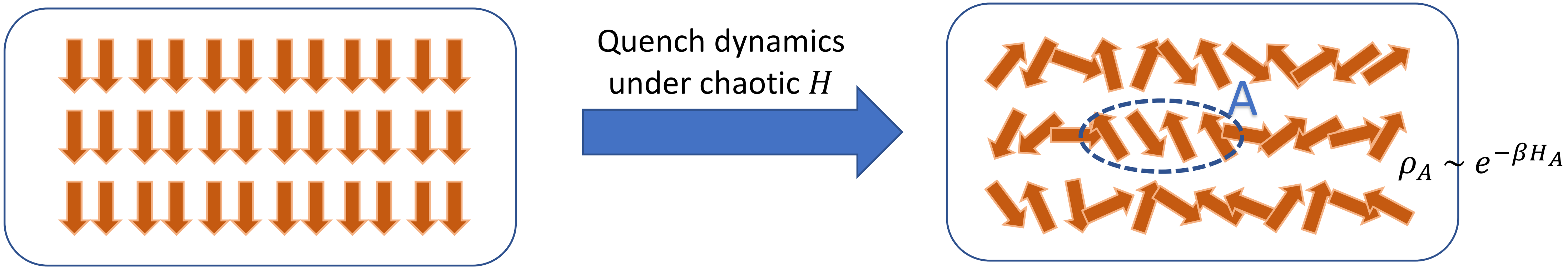
Setting the stage

- Thermalization: Quantum chaotic many body systems locally relax to maximally entropic thermal states constrained only by global conservation laws



Many body system in simple initial state

Highly entangled state

Mechanism for emergence of thermalization

Consider closed many-body quantum systems

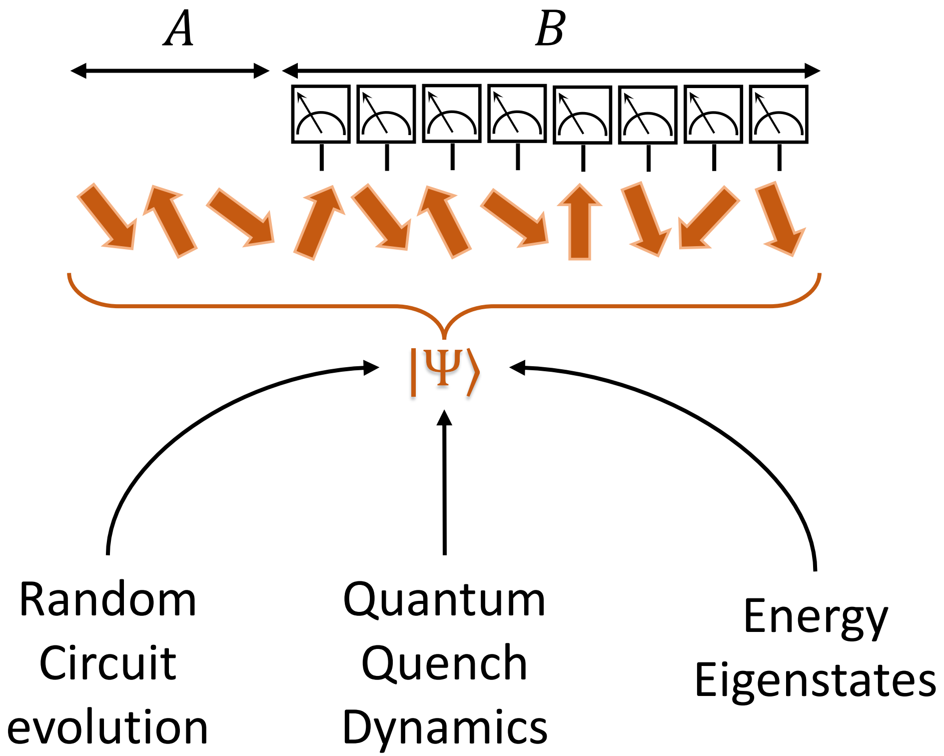
Eigenstate Thermalization Hypothesis: RMT based approach which allows to study equilibrium values for local observables (only makes prediction about average state of the subsystem)

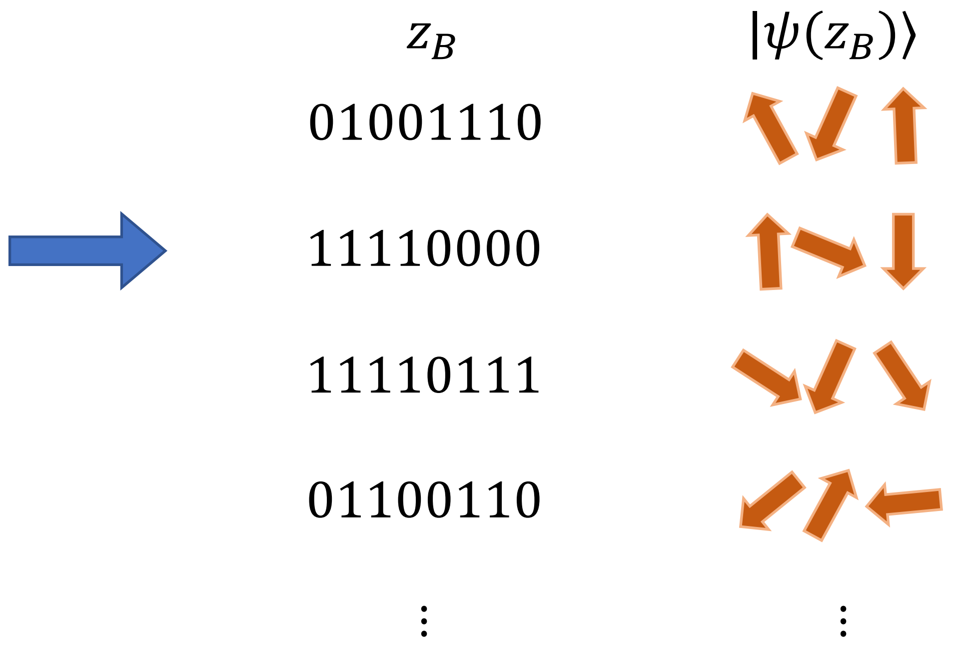
Delocalization of information due to growth of entanglement in the global state of the system

These approaches ignore the information contained within the bath

Projected ensembles: new approach that takes into account information from the bath

Projected ensembles*





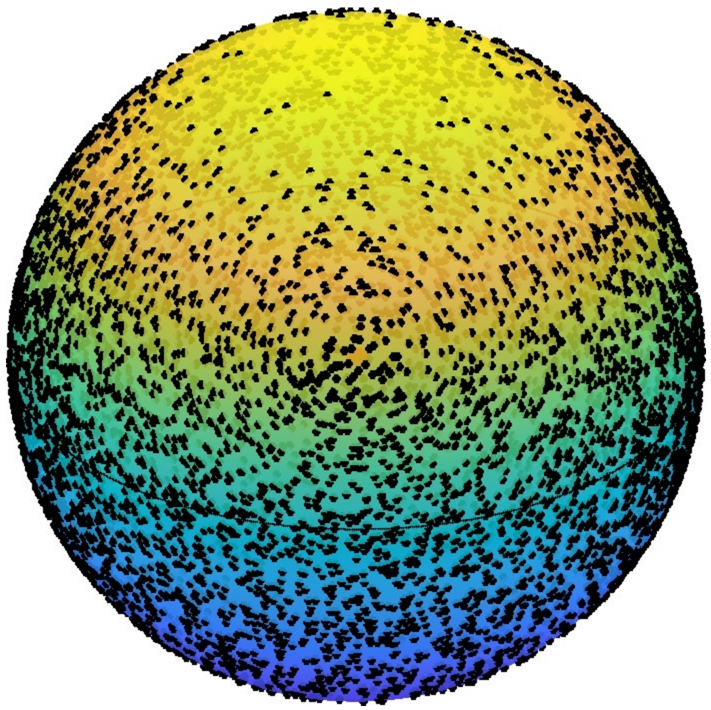
Projected ensemble:

$$\mathcal{E} := \{p(z_B), |\psi(z_B)\rangle_A\}$$

*Cotler, Jordan S., et al; *PRX Quantum* 4.1 (2023): 010311
Choi, Joonhee, et al; *Nature* 613.7944 (2023): 468-473

Visualizing Projected Ensembles

Consider $\mathcal{E} := \{p(z_B), |\psi(z_B)\rangle_A\}$ with $N_A = 1$



Thermalization vs Deep Thermalization

- Thermalization concerned with the density matrix of subsystem (maximize entropy subject to global conservation laws)

$$\rho_A \sim e^{-\beta H_A}$$

- In the case of Projected ensemble: maximize entropy of the *distribution* of states $|\psi(z_B)\rangle_A$ over the Hilbert space

$$|\psi(z_B)\rangle_A \sim |\psi_{Haar}\rangle_A$$ (if no conservation laws) "Deep Thermalization"

- Probing the emergence of such *universality* via quench dynamics is within reach of experimental platforms (cold atoms, trapped ions, superconducting qubits etc.)

Characterizing the Projected Ensemble

- Construct moments of the distribution as

$$\rho^{(1)} = \sum_{z_B} p(z_B)|\psi(z_B\ )\rangle\langle\psi(z_B)| = \rho_A$$

Reduced density matrix over A recovered

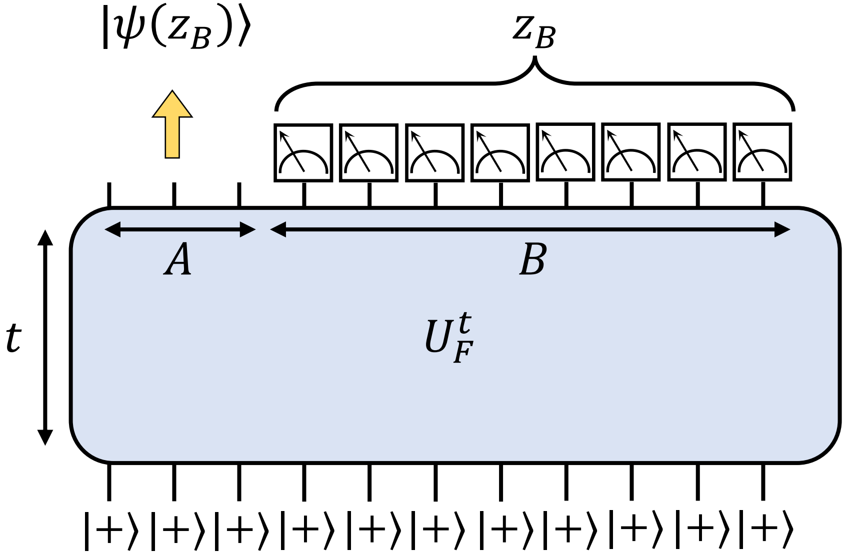
⋮

Quantum State k-design: Any weighted ensemble of pure q. states $\{p_i, |\psi_i\rangle\}$ such that it duplicates any statistical property up to k^{th} moment obtained from Haar-randomly distributed states

$$\sum_i p_i|\psi_i\rangle\langle\psi_i|^{\otimes k} = \int_{\psi\sim Haar} d\psi(|\psi\rangle\langle\psi|)^{\otimes k}$$

Exact emergent state designs*

- Model: 1D periodically kicked Ising model



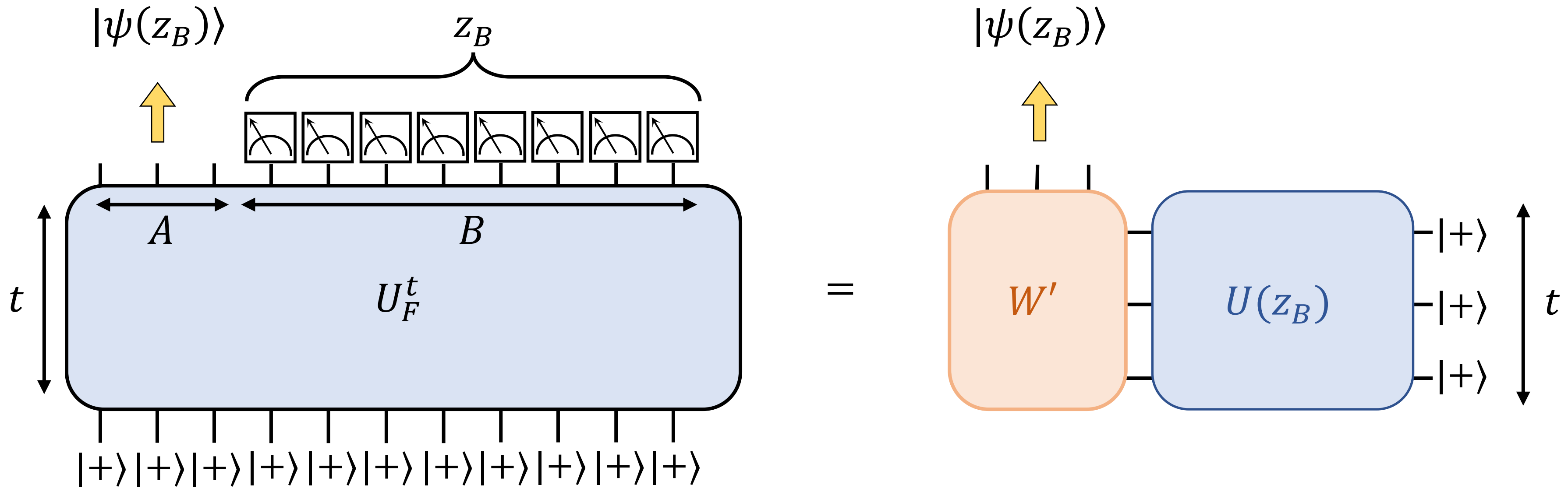
$$\mathcal{E}_{KIM} = \{p(z_B), |\psi(z_B)\rangle_A\}$$

Result: For KIM tuned to self-dual point and $t \geq N_A$, projected ensemble $\mathcal{E}_{KIM}$ forms an exact state design as $N_B \to \infty$

*Ho, W. W., & Choi, S. (2022); *Physical Review Letters*, *128*(6), 060601.

Exact emergent state designs: Proof intuition

- Key property of underlying quantum circuit: Dual Unitarity of model



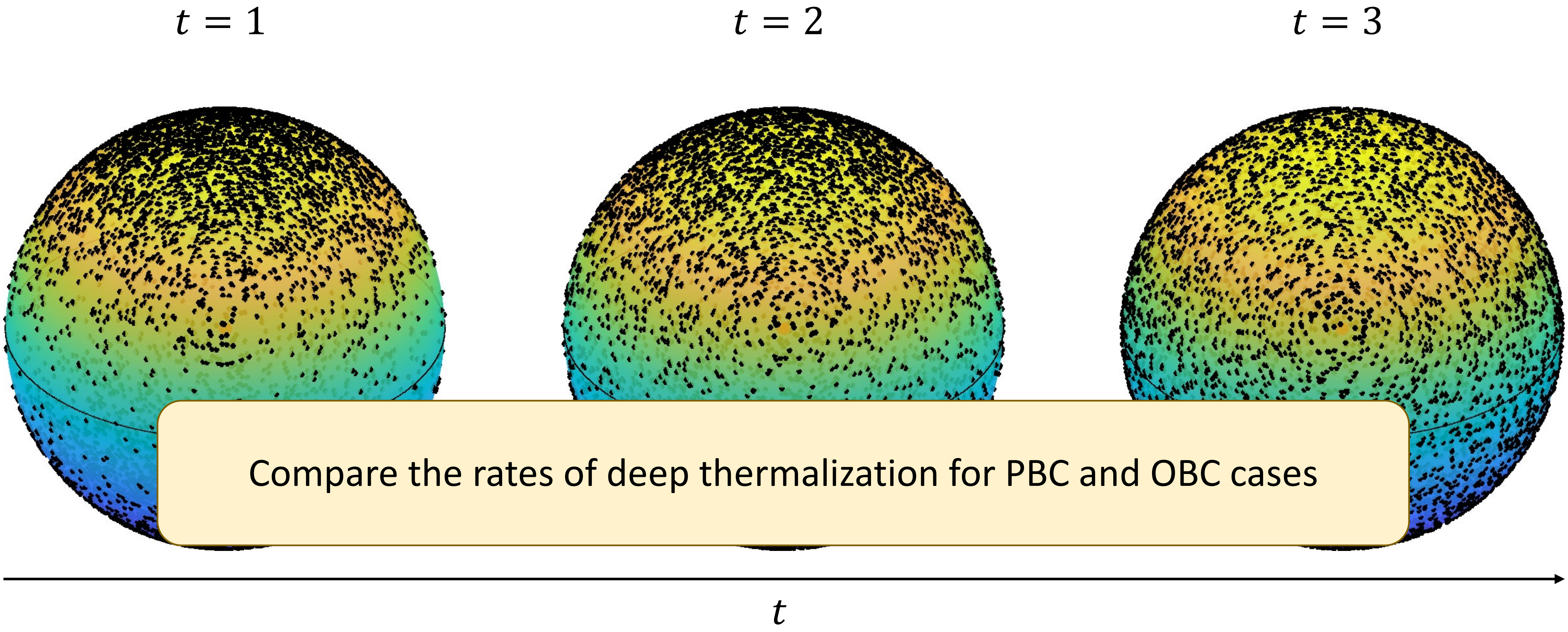
Claim 1: In the limit $N_B \to \infty$, quantum circuits $U(z_B)$ are indistinguishable from Haar random unitaries

Do boundary conditions play a role?

- LR bounds dictate the speed at which correlations grow in a many body systems
- Correlations created by local Hamiltonians vanish exponentially outside the effective light cone
- PBC vs OBC: Expect ρ_A to become mixed at same rate since correlations grow at same rate

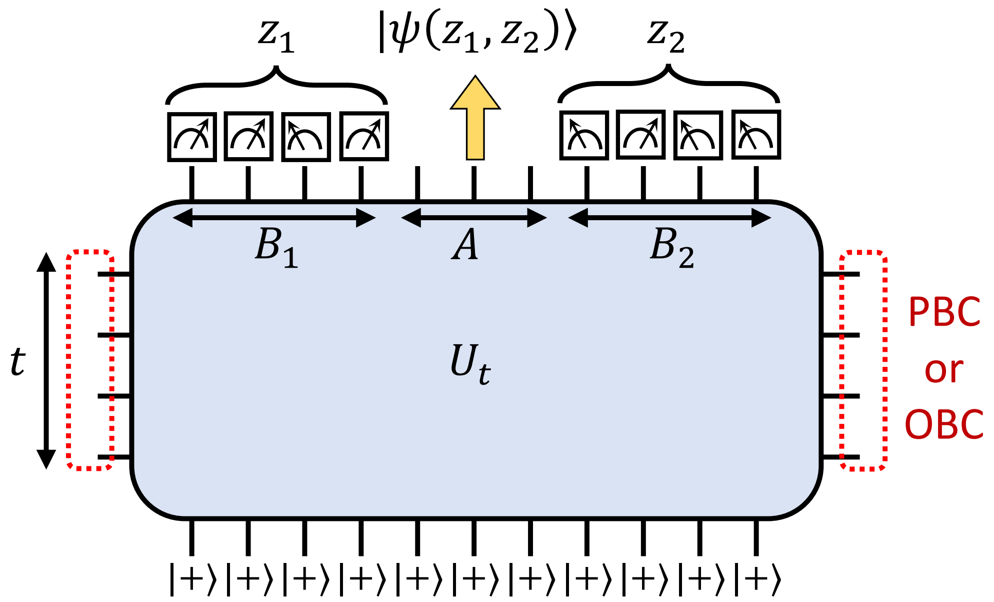
Our result: Boundary conditions can govern the rate of emergence of universal randomness

Rate of Deep Thermalization



Compare the rates of deep thermalization for PBC and OBC cases

Projected ensemble in our scenario

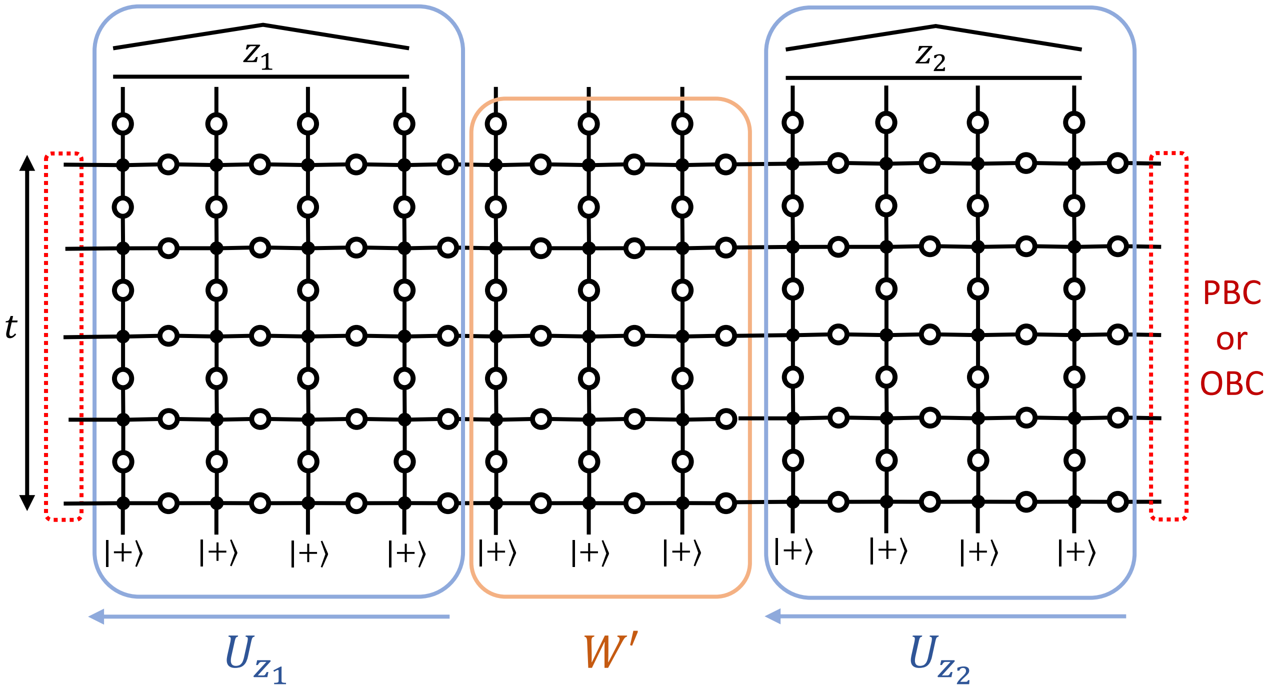


$$U_t = \left(U_h e^{-iH_{ising}\tau}\right)^t$$

$$\mathcal{E} = \{p(z_1, z_2), |\psi(z_1, z_2)\rangle_A\}$$

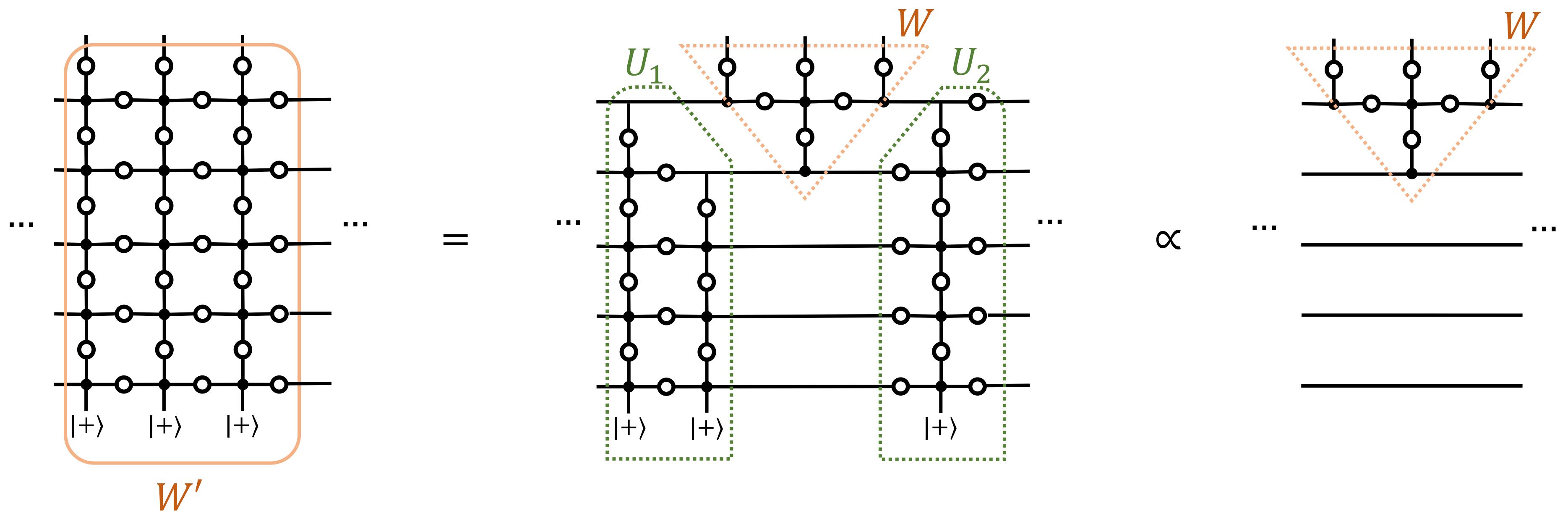
Quantum circuit representation

$\left|\tilde{\psi}(z_1, z_2)\right\rangle \propto$

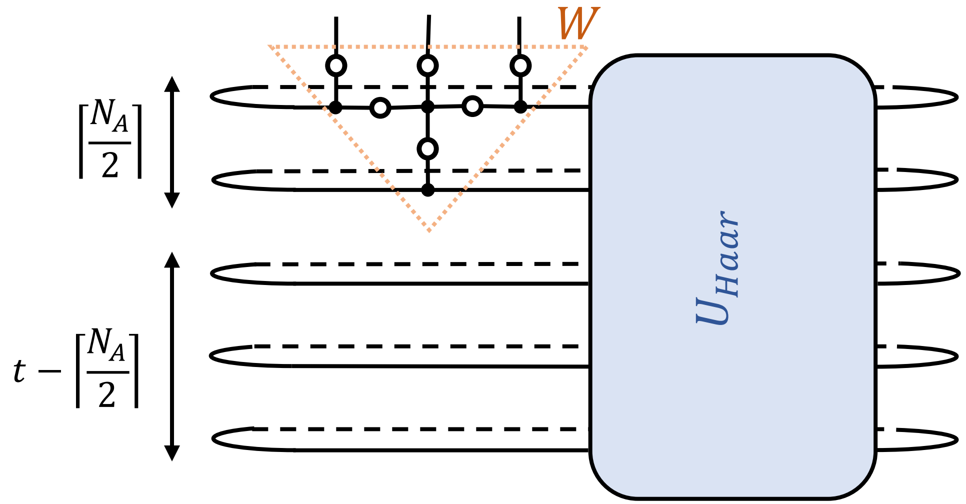


Using Claim 1: $U_{z_1} \to U_{Haar}$ and $U_{z_2} \to U'_{Haar}$ as $N_{B_1}, N_{B_2} \to \infty$

Simplifications

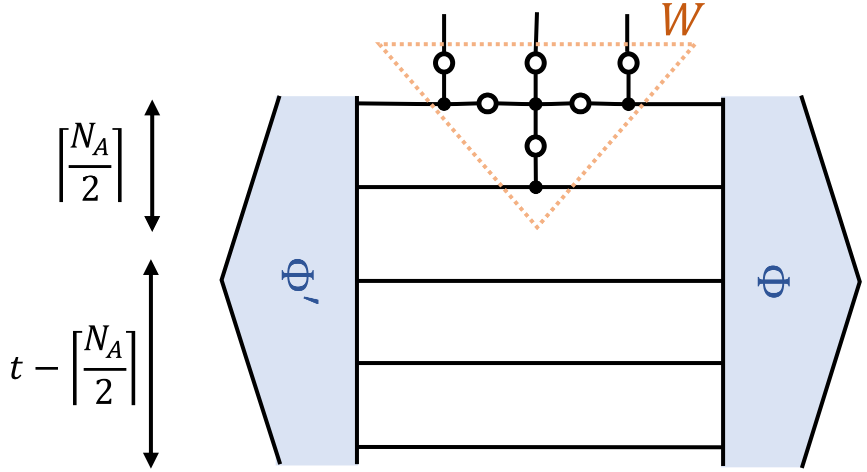


PBC case



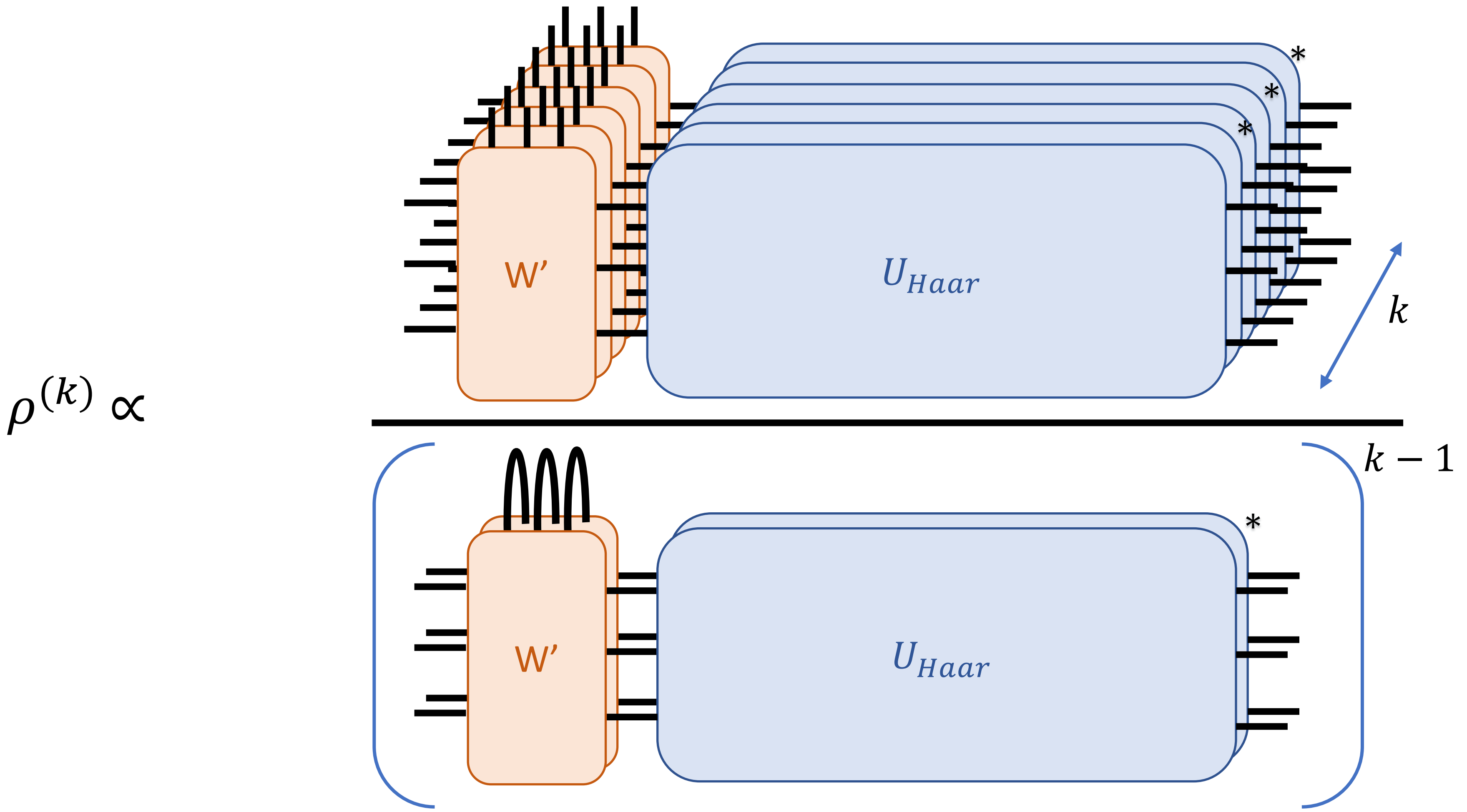
W
$\left\lceil \frac{N_A}{2} \right\rceil$
$t - \left\lceil \frac{N_A}{2} \right\rceil$
U_{Haar}

OBC case



W
$\left\lceil \frac{N_A}{2} \right\rceil$
$t - \left\lceil \frac{N_A}{2} \right\rceil$
Φ'
Φ

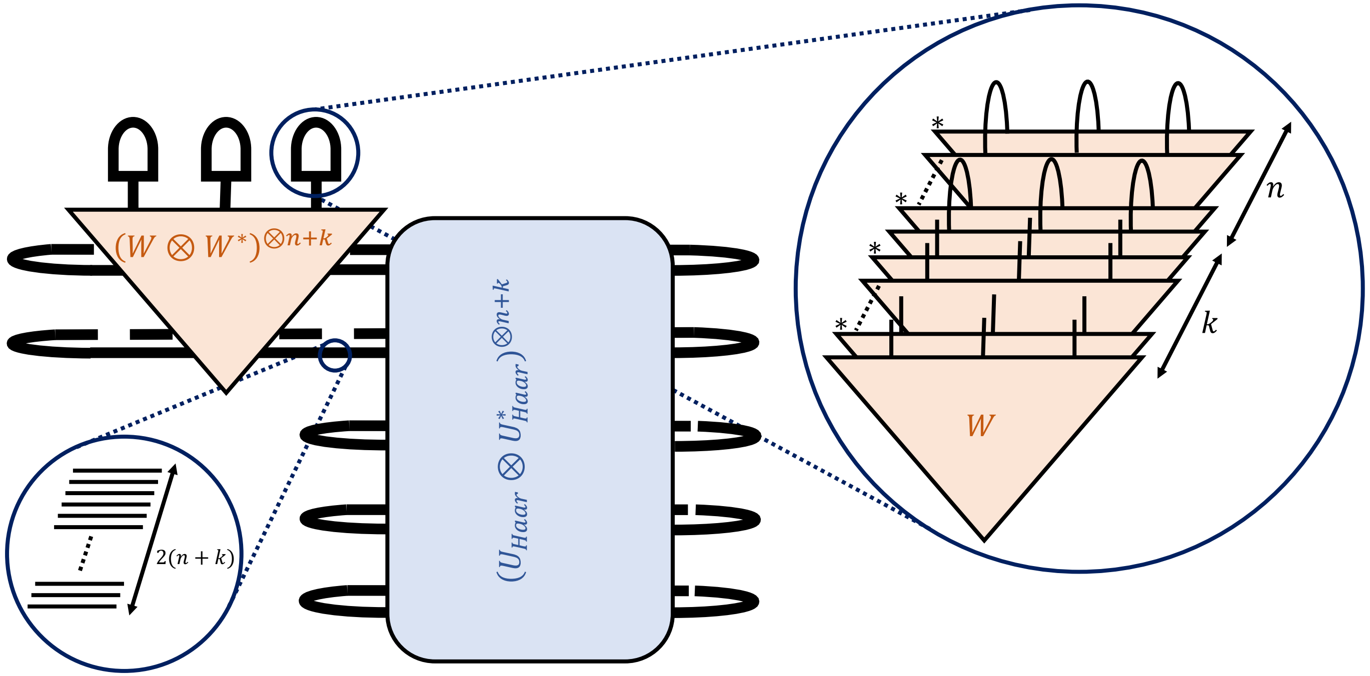
Setup k^{th} moment of state as $\rho^{(k)} = \sum_{z_B} \langle \tilde{\psi}_{z_B} | \tilde{\psi}_{z_B} \rangle^{1-k} |\tilde{\psi}_{z_B}\rangle\langle\tilde{\psi}_{z_B}|^{\otimes k}$



How do we estimate a rational function of Haar random unitaries?

Replica trick

$\rho^{(k,n)} \propto$



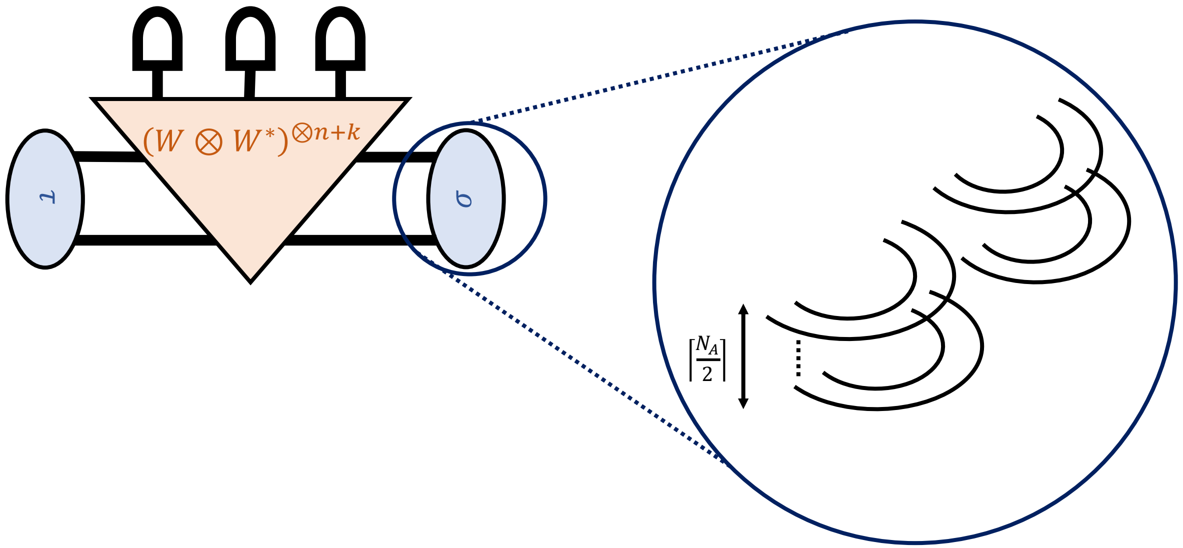
$$\rho^{(k)} = \lim_{n \to 1-k} \rho^{(k,n)}$$

Main Results

- Haar random ensemble emerges in both PBC and OBC cases in the limit of large times
- (Nonlocal nature of deep thermalization) Separation in the rate at which Haar randomness emerges in the 2 cases

Result 1: Intuition

$$\sum_{\sigma,\tau\in S_{k+n}} f(\sigma,\tau)$$



$$f(\sigma,\tau) \begin{cases} Wg(\sigma\tau^{-1})\left(2^{t-\lceil \frac{N_A}{2} \rceil}\right)^{\#(\sigma^{-1}\tau)} & \text{PBC} \\ \left(2^{t-\lceil \frac{N_A}{2} \rceil}\right)^{\#(\sigma^{-1}\tau)} & \text{OBC} \end{cases}$$

- Diagonal terms $(\sigma = \tau) = \rho_{Haar}$
- $\|Diag\ terms\| \geq 2^t \|Off\ diag\ terms\|$

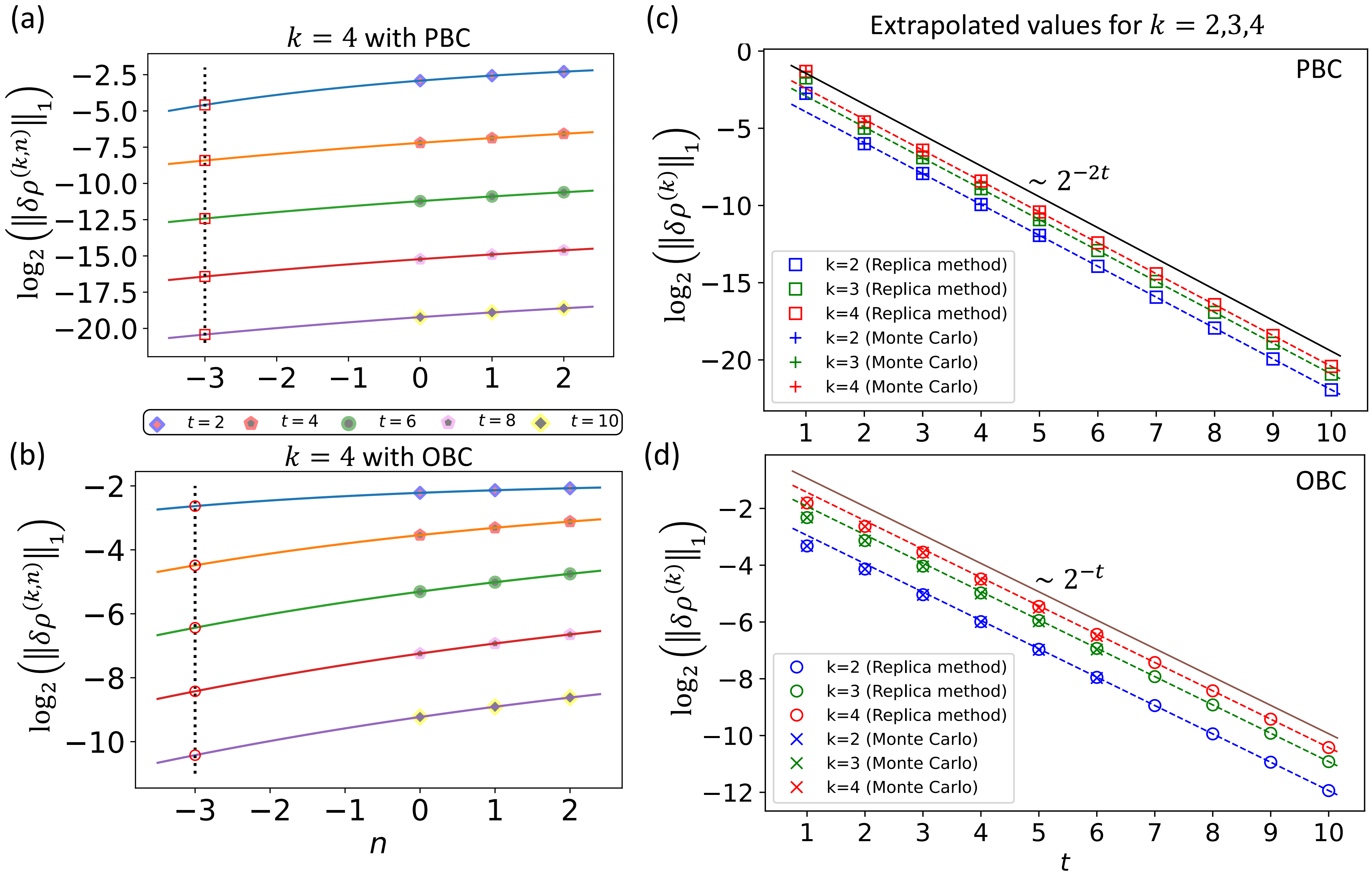
Result 2: Numerical evidence

Separation in convergence rate for PBC and OBC cases supported by numerics in 2 ways:

- $\left\|\delta\rho^{(k,n)}\right\|_1$ vs ‘t’ plots for k=2,3,4 from Replica trick, where

$$\delta\rho^{(k,n)} = \rho^{(k,n)} - \rho_{Haar}^{(k)}$$

- Monte Carlo sampling



(a)
$k = 4$ with PBC
$\log_2\left(\|\delta\rho^{(k,n)}\|_1\right)$
$t=2$ $t=4$ $t=6$ $t=8$ $t=10$
(b)
$k = 4$ with OBC
$\log_2\left(\|\delta\rho^{(k,n)}\|_1\right)$
n
(c)
Extrapolated values for $k = 2,3,4$
PBC
$\sim 2^{-2t}$
$\log_2\left(\|\delta\rho^{(k)}\|_1\right)$
k=2 (Replica method)
k=3 (Replica method)
k=4 (Replica method)
k=2 (Monte Carlo)
k=3 (Monte Carlo)
k=4 (Monte Carlo)
(d)
OBC
$\sim 2^{-t}$
$\log_2\left(\|\delta\rho^{(k)}\|_1\right)$
k=2 (Replica method)
k=3 (Replica method)
k=4 (Replica method)
k=2 (Monte Carlo)
k=3 (Monte Carlo)
k=4 (Monte Carlo)
t

Outlook

- We studied **Deep Thermalization** using the framework of **Projected Ensembles**
- Deep Thermalization requires emergence of a uniform ensemble of states at the level of subsystem
- Boundary conditions can affect the rate of Deep Thermalization
- Future directions:
  - Entanglement structure for PBC vs OBC
  - Scrambling rates for PBC vs OBC

Thanks for listening!