# Holographic duality beyond AdS/CFT via $T\bar{T}$ deformation

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# How to use holography and quantum information to understand gravity in the real world?

AdS/CFT → 

Astrophysics and cosmology

-Minkowski

-de Sitter

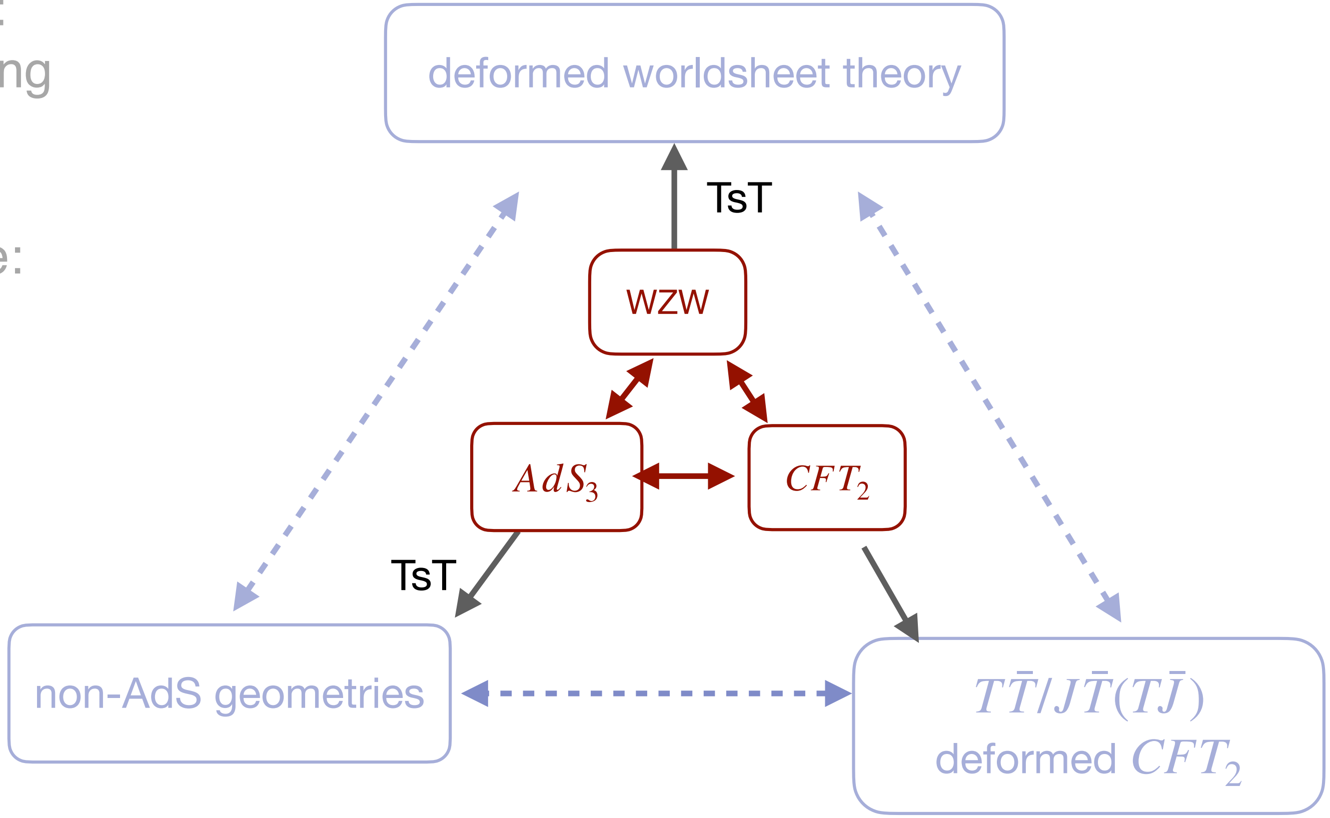
-Kerr

# A top down approach of non-AdS holography: The TsT/$T\bar{T}$ correspondence

- the inner triangle: $AdS_3/CFT_2$ in string theory

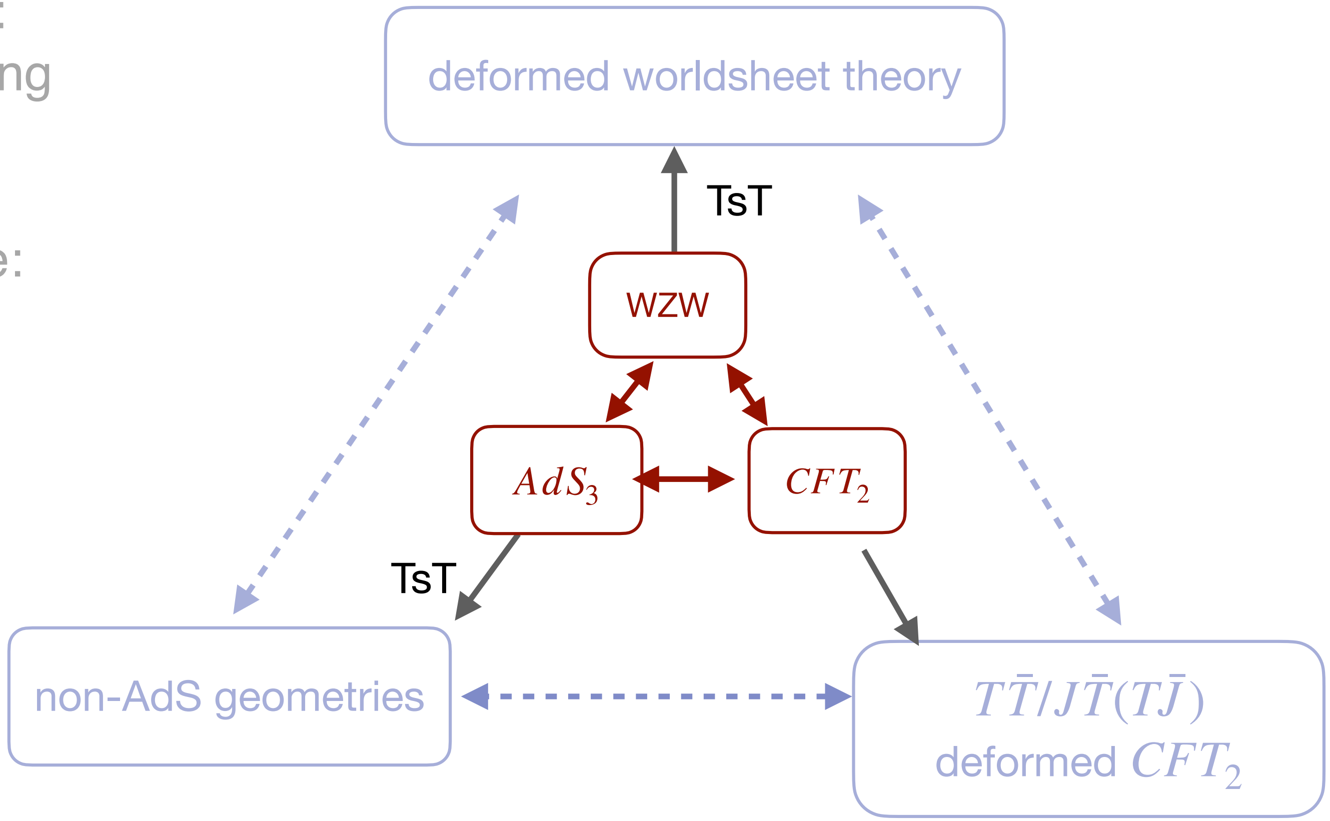
- the outter triangle: the TsT / $T\bar{T}$

- the evidence



# A top down approach of non-AdS holography: The TsT/$T\bar{T}$ correspondence

- the inner triangle: $AdS_3/CFT_2$ in string theory
- the outter triangle: the TsT / $T\bar{T}$
- the evidence




Based on work with Luis Apolo, Wei Cui, Stephane Detournay, Pengxiang Hao, Wenxin Lai, Hongfei Shu, Juntao Wang, and Boyang Yu
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# I. The inner triangle: $AdS_3/CFT_2$ in string theory

# I. The inner triangle: AdS$_3$

The bulk theory: II B string theory on $AdS_3 \times \mathcal{N}$ with NS-NS flux.

$$d\tilde{s}^2 = k(d\rho^2 + e^{2\rho} d\tilde{\gamma} d\bar{\tilde{\gamma}}),$$

$$\tilde{B} = -\frac{k}{2} e^{2\rho} d\tilde{\gamma} \wedge d\bar{\tilde{\gamma}}\,.$$

$$e^{2\tilde{\Phi}} = \frac{k}{N},$$

→ # of NS5 branes, magnetic charge

→ # of NS1 branes, electric charge

The isometry group is SL(2,R) x SL(2,R), with translational symmetries generated by $\partial_{\tilde{\gamma}}, \quad \partial_{\bar{\tilde{\gamma}}}$

# I. The inner triangle: WZW

The bulk theory: II B string theory on $AdS_3 \times \mathcal{N}$ NS-NS flux allows a weakly coupled worldsheet theory. The AdS3 part is a $SL(2,R)_k$ WZW model.

- Spacetime isometry becomes group symmetries on the worldsheet.

i.g. translational symmetries $\partial_{\tilde{\gamma}} \leftrightarrow j^-$

- The Hilbert space contains short strings $D_j^{\pm,w}$ and long strings $C_{j,\alpha}^w$ *[Maldacena-Ooguri]*

$w$ is a spectral flow parameter, can be interpreted as winding around the boundary circle for global $AdS_3$.

- Classical solution $t^{(w)} = t + w\tau, \phi^{(w)} = \phi + w\sigma$

- Classical solutions in Poincare coordinates,

$$\rho^{(w)} = \tilde{\rho} - \frac{w}{2}\log z\bar{z}, \quad \gamma^{(w)} = z^w \tilde{\gamma}, \quad \bar{\gamma}^{(w)} = \bar{z}^w \tilde{\bar{\gamma}}$$

- Physical states satisfy the Virasoro constraint

$$-\frac{j(j-1)}{k-2} - w(h + w\frac{k}{4}) + \Delta_{rest} = 0$$

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*[Argurio-Giveon-Shomer]*
*[Eberhardt-Gaberdiel]*
*[Eberhardt]*

## I. The inner triangle: $\mathrm{CFT}_2$

The boundary theory is a $\mathrm{CFT}_2$ , the full theory of which is not yet clear for generic value of k.

Superstring theory at k=1

The long string sector at generic k

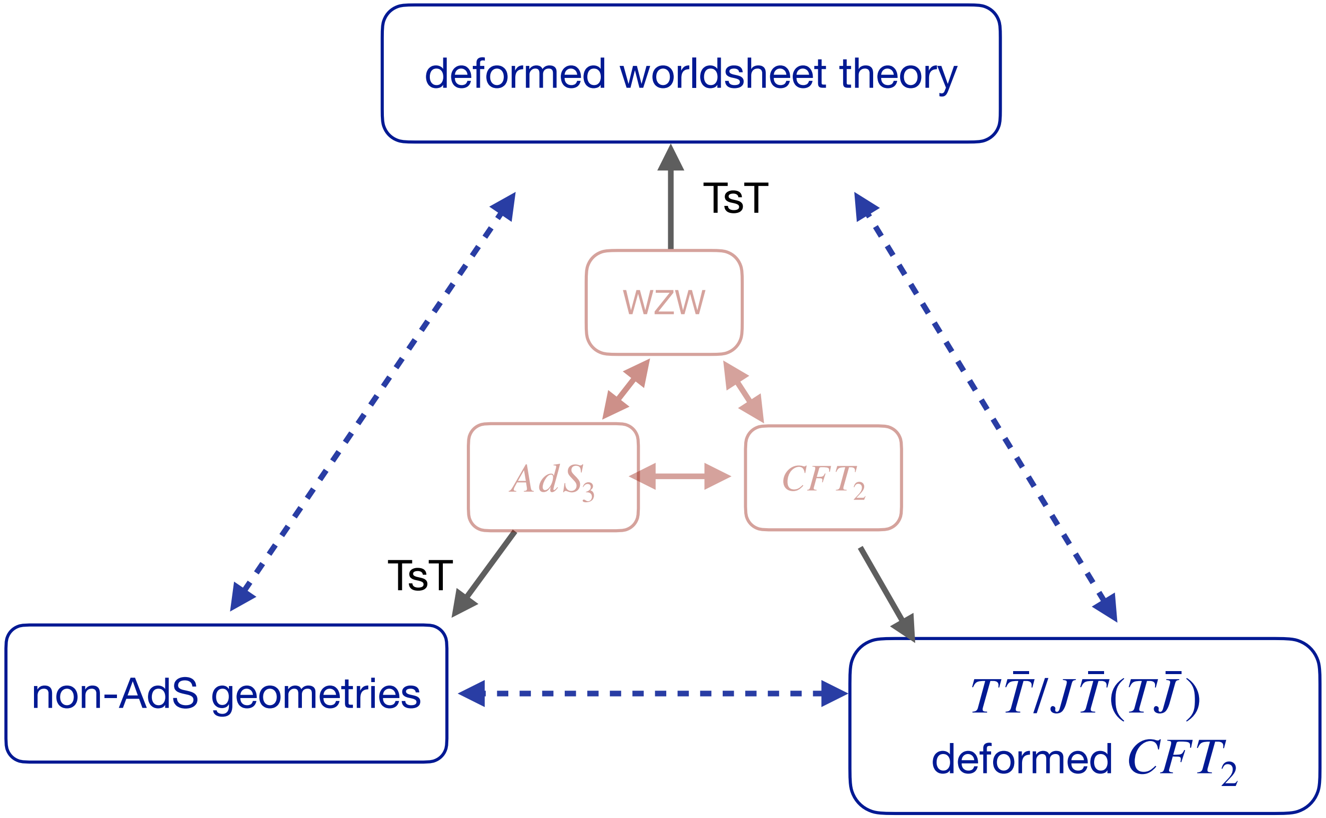
$\longleftrightarrow$ $\mathcal{M}^N / S_N$, $c_M = 6k$

*We focus on the long string sector in this talk*

- null coordinates $\tilde{\gamma} \leftrightarrow x,$
- Neother current $j^- \leftrightarrow T = \sum_i^N T_i$ stress tensor
- eigenvalue of $j_0^3 \leftrightarrow$ conformal weight $h$
- spectral-flowed sector $w \leftrightarrow n$ twisted sector

# II. The outer triangle: the TsT/$T\bar{T}$ correspondence

—a class of toy models for non-AdS holography



# Deformation in the bulk theory: TsT transformation

## II. The outer triangle: the TsT/$T\bar{T}$ correspondence

TsT(T-duality, shift, T-duality) transformations are solution-generating techniques.

Starting from a background with two $U(1)$ isometries along $X^m, X^n$,

$$TsT_{(X^m,X^n,\frac{\lambda}{k})} : T_{X^m} \to shift(X^n \to X^n - \frac{2\lambda}{k}X^m) \to T_{X^m}$$

- TsT transformations change the local geometry
- depends on the two $U(1)$ directions and a deformation parameter

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# II. The outer triangle: the TsT/$T\bar{T}$ correspondence

Poincare AdS

$$d\tilde{s}^2 = k(d\rho^2 + e^{2\rho} d\tilde{\gamma} d\tilde{\bar{\gamma}}), \quad \tilde{B} = -\frac{k}{2} e^{2\rho} d\tilde{\gamma} \wedge d\tilde{\bar{\gamma}} .$$

$$e^{2\tilde{\Phi}} = \frac{k}{N},$$

TsT along two U(1)s in $AdS_3$

$$ds^2 = k(d\rho^2 + \frac{e^{2\rho}}{1 + 2\lambda e^{2\rho}} d\gamma d\tilde{\bar{\gamma}}), \quad B = -\frac{k e^{2\rho} d\gamma \wedge d\bar{\gamma}}{2(1 + 2\lambda e^{2\rho})} .$$

$$e^{2\Phi} = \frac{k}{N}(1 + 2\lambda e^{2\rho})^{-1},$$

- $\lambda$ is the deformation parameter
- The resulting geometry interpolates between

IR: locally AdS

UV: asymptotically flat spacetime in the string frame, with a linear dilaton

A two parameter family of locally AdS3 solutions with cylindrical boundary

$$d\tilde{s}_3^2 = \ell^2 \left\{ \frac{dr^2}{4\left(r^2 - 4T_u^2 T_v^2\right)} + r du dv + T_u^2 du^2 + T_v^2 dv^2 \right\},$$

$$(u, v) \sim (u + 2\pi, v + 2\pi)$$

$$e^{2\tilde{\Phi}} = \frac{k}{N}$$

- $T_u^2 \geq 0, T_v^2 \geq 0$ : BTZ black holes
- $T_u = T_v = 0$ : massless BTZ
- $T_u^2 < 0, T_v^2 < 0$: conical defect
- $T_u^2 = T_v^2 = -\frac{1}{4}$: global $AdS_3$

TsT along two U(1)s in $AdS_3$

↓

$$\frac{ds_3^2}{\ell^2} = \frac{dr^2}{4\left(r^2 - 4T_u^2 T_v^2\right)} + \frac{r du dv + T_u^2 du^2 + T_v^2 dv^2}{1 + 2\lambda r + 4\lambda^2 T_u^2 T_v^2}$$

$$e^{2\Phi} = \frac{k}{N}\left(\frac{1 - 4\lambda^2 T_u^2 T_v^2}{1 + 2\lambda r + 4\lambda^2 T_u^2 T_v^2}\right) e^{-2\phi_0}$$

- $T_u^2 \geq 0, T_v^2 \geq 0$, BHs
- smooth solution
- $\lambda = 1/2$ : Horne-Horowitz 91'

- TsT transformation generates a class of asymptotically flat solutions in string frame, with a linear dilaton

## Deformation in the boundary theory: the single-trace $T\bar{T}$ deformation

## (double-trace ) $T\bar{T}$ deformations

[Zamolodchikov;Smirnov, Zamolodchikov; Cavaglia, Negro, Szecsenyi, Tateo; Cardy; Dubovsky, Flauger, Gorbenko; Dubovsky, Gorbenko, Mirbabayi; Conti, Iannella, Negro, Tateo; Frolov; …]

$$\frac{\partial S_\mu}{\partial \mu} = \int dx^2 \det T^\mu{}_\nu = \int dx^2 \left(T_{xx}T_{\bar{x}\bar{x}} - T_{x\bar{x}}T_{\bar{x}x}\right)$$

$x = \phi + t,\ \bar{x} = \phi - t$

$T_{\mu\nu}$ : *stress tensor of the deformed theory at* $\mu$

- solvable irrelevant deformation
- spectrum on a cylinder with $(x, \bar{x}) \sim (x + 2\pi R, \bar{x} + 2\pi R)$

$$E(\mu) = -\frac{R}{2\mu}\left(1 - \sqrt{1 + \frac{4\mu E}{R} + \frac{4\mu^2 J^2}{R^2}}\right), \quad J(\mu) = J$$

- modular invariance, connections to random metric, Nambu-Goto action, JT gravity, string theory, CDD factors…*[Review: Jiang]*

For $CFT_2$, modular invariance+large c+ sparseness condition
$\Longrightarrow$ universal torus partition function *[Hartman-Keller-Stocia]*

$T\bar{T}$ deformed CFTs are shown to be modular invariant *[Datta-Jiang]*
Assuming large $c$ and sparseness condition, the partition function

*[Apolo-WS-Yu]*

$$\log Z_{T\bar{T}}(\mu) \approx \begin{cases} -\frac{1}{2}(\beta_L+\beta_R)\, RE_{\text{vac}}(\mu), & \beta_L\beta_R > 1, \\ -2\pi^2\left(\frac{1}{\beta_L}+\frac{1}{\beta_R}\right) RE_{\text{vac}}\left(\frac{4\pi^2}{\beta_L\beta_R}\mu\right), & \beta_L\beta_R < 1, \end{cases}$$

The entropy in the microscopic ensemble is

$$S_{T\bar{T}} = 2\pi[\sqrt{\frac{c}{6}E_L(\mu)\left[1+2\mu E_R(\mu)\right]} + \sqrt{\frac{c}{6}E_R(\mu)\left[1+2\mu E_L(\mu)\right]}], \quad E_{L/R} = \frac{1}{2R}(E \pm J)$$

$$S_{T\bar{T}}(E_L(\mu), E_R(\mu)) = S_{Cardy}(E_L, E_R)$$

The spectrum on a cylinder with
$(x, \bar{x}) \sim (x + 2\pi R, \bar{x} + 2\pi R)$

$$E(\mu) = -\frac{R}{2\mu}\left(1 - \sqrt{1 + \frac{4\mu E}{R} + \frac{4\mu^2 J^2}{R^2}}\right), \quad J(\mu) = J, \qquad \mu < 0$$

- real energy for the ground state
- complex spectrum at very high energy

The spectrum on a cylinder with
$(x, \bar{x}) \sim (x + 2\pi R, \bar{x} + 2\pi R)$

$$E(\mu) = -\frac{R}{2\mu}\left(1 - \sqrt{1 + \frac{4\mu E}{R} + \frac{4\mu^2 J^2}{R^2}}\right), \quad J(\mu) = J, \qquad \mu < 0$$

- real energy for the ground state
- complex spectrum at very high energy
- Proposed holographic dual for $\mu < 0$:
  cutoff $AdS_3$ in Einstein gravity *[McGough-Mezei-Verlinde]*
  Mixed boundary condition *[Guica-Monten]*

The spectrum on a cylinder with $(x, \bar{x}) \sim (x + 2\pi R, \bar{x} + 2\pi R)$

$$E(\mu) = -\frac{R}{2\mu}\left(1 - \sqrt{1 + \frac{4\mu E}{R} + \frac{4\mu^2 J^2}{R^2}}\right), \quad J(\mu) = J, \qquad \mu > 0$$

- ground state $E^{vac}(\mu) = -\frac{1}{2\mu}(1 - \sqrt{1 - \frac{c\mu}{3R^2}})$, complex if $\lambda \equiv \frac{c\mu}{6R^2} > \frac{1}{2}$

*critical value:* $\lambda_c = 1/2$

- Hagedorn growth at very high energy $E(\mu) \gg \frac{1}{\mu}$, $\quad S_{T\bar{T}} \sim 2\pi\sqrt{\frac{c\mu}{3}E(\mu)}$
- temperatures $T_{L/R} \equiv \left(\partial S_{T\bar{T}}/\partial E_{L/R}\right)^{-1}$, have an upper bound

$$T_L T_R \leq T^2_{Hagedorn} = \frac{3}{4\pi^2 c\mu}.$$

[Giveon-Itzhaki-Kutasov, Apolo-Detournay-WS]

## $T\bar{T}$ with $\mu > 0$

The spectrum on a cylinder with $(x, \bar{x}) \sim (x + 2\pi R, \bar{x} + 2\pi R)$

$$E(\mu) = -\frac{R}{2\mu}\left(1 - \sqrt{1 + \frac{4\mu E}{R} + \frac{4\mu^2 J^2}{R^2}}\right), \quad J(\mu) = J, \qquad \mu > 0$$

- ground state $E^{vac}(\mu) = -\frac{1}{2\mu}(1 - \sqrt{1 - \frac{c\mu}{3R^2}})$, complex if $\lambda \equiv \frac{c\mu}{6R^2} > \frac{1}{2}$

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- temperatures $T_{L/R} \equiv \left(\partial S_{T\bar{T}}/\partial E_{L/R}\right)^{-1}$, have an upper bound

$T_L T_R \leq T^2_{Hagedorn} = \frac{3}{4\pi^2 c\mu}$. *[Giveon-Itzhaki-Kutasov, Apolo-Detournay-WS]*

- Proposed holographic dual: Mixed boundary condition *[Guica-Monten]*

*Glue-on* $AdS_3$ *[Apolo-Hao-Lai-WS]*

*Wenxin Lai's poster later today!*

# The single-trace $T\bar{T}$ deformation

## A single trace version of $T\bar{T}$ deformation

A single trace version of $T\bar{T}$ deformation can be defined for as a symmetric product $(\mathcal{M}_\mu)^N/S_N$ , where the seed theory is a (double trace) $T\bar{T}$ deformed CFT$_2$.

- The spectrum in the untwisted sector with n=1 agrees with the double trace case *[Giveon-Itzhaki-Kutasov]*
- The spectrum in the twisted sector is given by

$$E_L^{(n)}(0) = E_L^{(n)}(\mu) + \frac{2\mu}{nR} R E_L^{(n)}(\mu) E_R^{(n)}(\mu)$$

*[Apolo-WS-Yu]*

- The entropy is

$$S_{T\bar{T}}^{single\ trace}\left(E_L, E_R\right) = 2\pi\left[\sqrt{\frac{c}{6} R E_L(\mu)\left[1 + \frac{2\mu}{RN} E_R(\mu)\right]} + \sqrt{\frac{c}{6} R E_R(\mu)\left[1 + \frac{2\mu}{RN} E_L(\mu)\right]}\right]$$

- Torus partition function is universal at large $N$, without assuming sparseness condition. The density of states saturates the sparseness bound.

# TsT $\leftrightarrow T\bar{T}/J\bar{T}(T\bar{J})/J\bar{J}$

A conjecture *[Apolo-Detournay-WS]*

*[Giveon-Itzhaki-Kutasov, Giribet]*
*[Apolo-WS]*
*[Araujo, Colgáin, Sakatani, Sheikh-Jabbari,Yavartanoo]*

Starting from IIB string theory on locally $AdS_3 \times \mathcal{N}$ with NSNS background flux,

$$\mathrm{TsT}_{(X^m, X^{\bar{m}}; \lambda/k)} \Longleftrightarrow \frac{\partial S_{\mathcal{M}_\mu}}{\partial \mu} = -4 \int J_{(m)} \wedge J_{(\bar{m})}$$

Examples:

a toy model for flat holography/ a toy model for Kerr/CFT

TsT with two $U(1)$s both in $AdS_3$ / one in $AdS_3$ and the other in $\mathcal{N}$ / both in $\mathcal{N}$

*[Apolo-Detournay-WS]* *[Chakraborty-Giveon-Kutasov; Apolo-WS]*

$\Updownarrow$

single trace $T\bar{T}$ / $J\bar{T}(T\bar{J})$ / $J\bar{J}$ deformations

*[Giveon-Itzhaki-Kutasov, Giribet]* *[Guica]*

## Stationary solutions with two parameters with cylindrical boundary

$$\frac{ds_3^2}{\ell^2} = \frac{dr^2}{4\left(r^2 - 4T_u^2 T_v^2\right)} + \frac{rdudv + T_u^2 du^2 + T_v^2 dv^2}{1 + 2\lambda r + 4\lambda^2 T_u^2 T_v^2} \qquad (u, v) \sim (u + 2\pi, v + 2\pi)$$

$$e^{2\Phi} = \frac{k}{p}\left(\frac{1 - 4\lambda^2 T_u^2 T_v^2}{1 + 2\lambda r + 4\lambda^2 T_u^2 T_v^2}\right) e^{-2\phi_0}$$

- $T_u = T_v = 0 \leftrightarrow \text{Ramond vacuum}$ *[Giveon-Itzhaki-Kutasov]*
- $T_u = T_v = \frac{i}{2\lambda}(1 - \sqrt{1 - 2\lambda}) \leftrightarrow \text{ground state, NS vacuum}$ *[Apolo-Detournay-WS]*

  *critical value:* $\lambda_c = 1/2$
- Range of parameters from the bulk:
- $0 < \lambda < \frac{1}{2}$, smooth and real solution,
- $\lambda < 0$, CTC and curvature singularities
- upper bound for the temperatures by requiring real dilaton

## Stationary solutions with two parameters

$$\frac{ds_3^2}{\ell^2} = \frac{dr^2}{4\left(r^2 - 4T_u^2T_v^2\right)} + \frac{rdudv + T_u^2du^2 + T_v^2dv^2}{1 + 2\lambda r + 4\lambda^2T_u^2T_v^2} \qquad (u, v) \sim (u + 2\pi, v + 2\pi)$$

$$e^{2\Phi} = \frac{k}{p}\left(\frac{1 - 4\lambda^2T_u^2T_v^2}{1 + 2\lambda r + 4\lambda^2T_u^2T_v^2}\right)e^{-2\phi_0}$$

- $T_u = T_v = 0 \leftrightarrow$ Ramond vacuum *[Giveon-Itzhaki-Kutasov]*
- $T_u = T_v = \frac{i}{2\lambda}(1 - \sqrt{1 - 2\lambda}) \leftrightarrow$ ground state, NS vacuum *[Apolo-Detournay-WS]*
- Range of parameters from the bulk:
  - $0 < \lambda < \frac{1}{2}$, smooth and real solution
  - $\lambda < 0$, CTC and curvature singularities
  - upper bound for the temperatures by requiring real dilaton
- $T_u, T_v \geq 0$ black holes
  Bekenstein Hawking entropy $\leftrightarrow$entropy of single trace $T\bar{T}$
  $S_{TsT} = S_{T\bar{T}}^{single\ trace}\left(E_L, E_R\right)$

*[Apolo-Detournay-WS]*
*[Apolo-WS-Yu]*

## The dictionary

# TsT solution space/single-trace $T\bar{T}$ deformed CFT

$$\ell_s^2 \lambda \equiv \ell^2 \hat{\mu} \Longleftrightarrow \mu$$

| | | |
|---|---|---|
| TsT of Poincare AdS | $\Longleftrightarrow$ | vacuum on the plane |
| TsT of Massless BTZ | $\Longleftrightarrow$ | RR vacuum on the cylinder |
| smooth solution from TsT | $\Longleftrightarrow$ | NSNS vacuum on the cylinder |
| TsT of BTZ black hole | $\Longleftrightarrow$ | thermal states on the cylinder |
| Bekenstein Hawking entropy | $\Longleftrightarrow$ | microscopic entropy of single trace $T\bar{T}$ |

# more evidence

✓classical solution space & black hole thermodynamics

➡the action and partition functions

- the deformed spectrum
- correlation functions

# The action

The string worldsheet action $S_{WS} = -\ell_s^{-2} \int d^2 z\, M_{\mu\nu} \partial X^\mu \bar{\partial} X^\nu ,$

$$M_{\mu\nu} \equiv G_{\mu\nu} + B_{\mu\nu}$$

*$G_{\mu\nu}$: Target space metric*
*$B_{\mu\nu}$: NS-NS potential*

TsT along $TsT_{X^m, X^{\bar{m}}; \hat{\mu}}$:

$$M = \tilde{M}\left(I + 2\hat{\mu}\Gamma\tilde{M}\right)^{-1}, \; \Gamma_{\mu\nu} = \delta_\mu^m \delta_\nu^{\overline{m}} - \delta_\mu^{\overline{m}} \delta_\nu^m$$

$\boldsymbol{j}_{(m)}$, $\boldsymbol{j}_{(\overline{n})}$ *are* ***worldsheet*** *Noether 1-forms associated to $\partial_{X^m}$, and $\partial_{X^{\bar{m}}}$*

TsT on string worldsheet : $\dfrac{\partial S_{WS}}{\partial \hat{\mu}} = -4 \int \boldsymbol{j}_{(m)} \wedge \boldsymbol{j}_{(\overline{n})}$

↕

*marginal deformation on the worldsheet*

*irrelavent deformation on the dual theory*

$T\bar{T}/J\bar{T}$ on the dual field theory $(\mathscr{M}_\mu)^p / S_p$ : $\dfrac{\partial S_{\mathscr{M}_\mu}}{\partial \mu} = -4 \int J_{(m)} \wedge J_{(\overline{m})}$

$J_{(m)}$, $J_{(\overline{m})}$ *are the* ***boundary spacetime*** *Noether 1-forms associated to $\partial_{X^m}$, and $\partial_{X^{\bar{m}}}$*

The torus partition has been calculated from string theory and the symmetric product theory *[Apolo-WS-Yu]* *[Hashimoto-Kutasov]*

see also [*Benjamin-Collier-Kruthoff-Verlinde-Zhang*]

- The torus partition function is modular invariant

$$Z_N(\tau,\bar{\tau};\mu) = \sum_{\{k_1,\ldots,k_N\}} \frac{1}{\prod_{n=1}^{N} n^{k_n} k_n!} \prod_{n=1}^{N} \left(T'_n Z\right)(\tau,\bar{\tau};\mu)^{k_n}$$

- The generating function

$$\mathscr{Z}(\tau,\bar{\tau};\mu;p) := \sum_{N=0}^{\infty} p^N Z_N(\tau,\bar{\tau};\mu) = \exp\left(\sum_{n=1}^{\infty} \frac{p^n}{n} \left(T'_n Z\right)(\tau,\bar{\tau};\mu)\right)$$

# more evidence

- ✓classical solution space & black hole thermodynamics
- ✓the action and partition functions
- ➡the deformed spectrum
- correlation functions

## III. Evidence for TsT/$T\bar{T}$ : the spectrum on the cylinder

After the TsT, string spectrum on a cylinder can be obtained by two observations:

1. Before the TsT, string spectrum on a cylinder with winding $w$ can be obtained from zero winding by "spectral flow" with parameter $w$ [Maldacena,Ooguri]

2. TsT on the WS⇔ field redefinition: [Alday]
string solutions on new background with periodic b.c.
$\Longleftrightarrow$ strings on the old background with twisted boundary conditions depending on the momentum $p_{(1)}, p_{(\bar{2})}$.
*assuming $j_{(1)}/j_{(\bar{2})}$ to be chiral/antichiral up to total derivative terms (satisfied for the WZW model)*
$\Longleftrightarrow$ momentum dependent "spectral flow"

The final string spectrum on a cylinder after TsT can hence be obtained from string spectrum on a cylinder before TsT with a momentum dependent "spectral flow". Comparing the Virasoro constraints can give us the relation between the spectrum before and after the TsT. This relation is just the $T\bar{T}/J\bar{T}(T\bar{J})$ spectrum [Apolo, WS; Apolo, WS; Apolo,Stephane,WS;]

## III. Evidence for TsT/$T\bar{T}$ : the spectrum on the cylinder

- The Virasoro zero mode on AdS$_3$ background with winding $w$ along the circle
  $\phi(\sigma+2\pi) = \phi(\sigma+2\pi) + 2\pi w, u = t + \phi, v = \phi - t$

is related to those of on AdS$_3$ background without winding

$$\hat{L}_0 = \quad \tilde{L}_0 \quad + wRp$$

*Relation?*

---

- The Virasoro mode on the TsT background with winding w

is related to those of on AdS$_3$ background by spectral flow transformations

$$\hat{L}_0 = \quad \tilde{L}_0 \quad + wRp(\hat{\mu}) \quad + 2\hat{\mu}p(\hat{\mu})\bar{p}(\hat{\mu})$$

# III. Evidence for TsT/$T\bar{T}$ : the spectrum on the cylinder

Relation between string spectra with winding $w$

before and after the TsT transformations

$$\hat{L}_0 = \tilde{L}_0 - wRp(\hat{\mu}) + 2\hat{\mu}p(\hat{\mu})\bar{p}(\hat{\mu})$$

$\tilde{L}_0$: keep fixed

$= -wRp(0)$

The dictionary $\mu = \ell^2\hat{\mu}$

- $p \leftrightarrow \ell E_L, \quad \bar{p} \leftrightarrow -\ell E_R$
- w=1: untwisted sector
- w>1: twisted sector

String spectrum matches that of the single trace $T\bar{T}$ deformation

$$E_L(0) = E_L(\mu) + \frac{2\mu}{w\ell}E_L(\mu)E_R(\mu),$$

w=1: agrees with the spectrum of double trace $T\bar{T}$

w>1: agrees with the derivation of $(\mathcal{M}_\mu)^p/S_p$ [Apolo-WS-Yu]

# more evidence

- ✓ classical solution space & black hole thermodynamics
- ✓ the action  and partition functions
- ✓ the deformed spectrum
- ➡ correlation functions

To consider the correlation functions, we write the Virasoro constraint on the plane, and do the same trick

$$\hat{L}_0 = \tilde{L}_0 - w(h_{\tilde{\lambda}} + \frac{k}{4}w) + 2\tilde{\lambda}p\bar{p}$$

keep fixed

$$= - w(h + kw/4)$$

*[Cui-Shu-WS-Wang]*

The weights of the single trace $T\bar{T}$ deformation is momentum dependent

$$h_{\tilde{\lambda}} = h + 2\frac{\tilde{\lambda}}{w}p\bar{p}, \quad \bar{h}_{\tilde{\lambda}} = \bar{h} + 2\frac{\tilde{\lambda}}{w}p\bar{p}\,.$$

## The correlation functions from TsT side using string theory feature momentum-dependent weights

*[Cui-Shu-WS-Wang]*

$$h_{\tilde{\lambda}} = h + 2\frac{\tilde{\lambda}}{w} p\bar{p}, \quad \bar{h}_{\tilde{\lambda}} = \bar{h} + 2\frac{\tilde{\lambda}}{w} p\bar{p}\,.$$

Two point functions in momentum space

$$G_2^{(\lambda)}(p) = \frac{\pi 2^{2-4(h+\frac{2\tilde{\lambda}p\bar{p}}{\omega})}\Gamma\left(1-2(h+\frac{2\tilde{\lambda}p\bar{p}}{\omega})\right)}{\Gamma(2(h+\frac{2\tilde{\lambda}p\bar{p}}{\omega}))}(p\bar{p})^{2h-1+\frac{4\tilde{\lambda}p\bar{p}}{\omega}}\,.$$

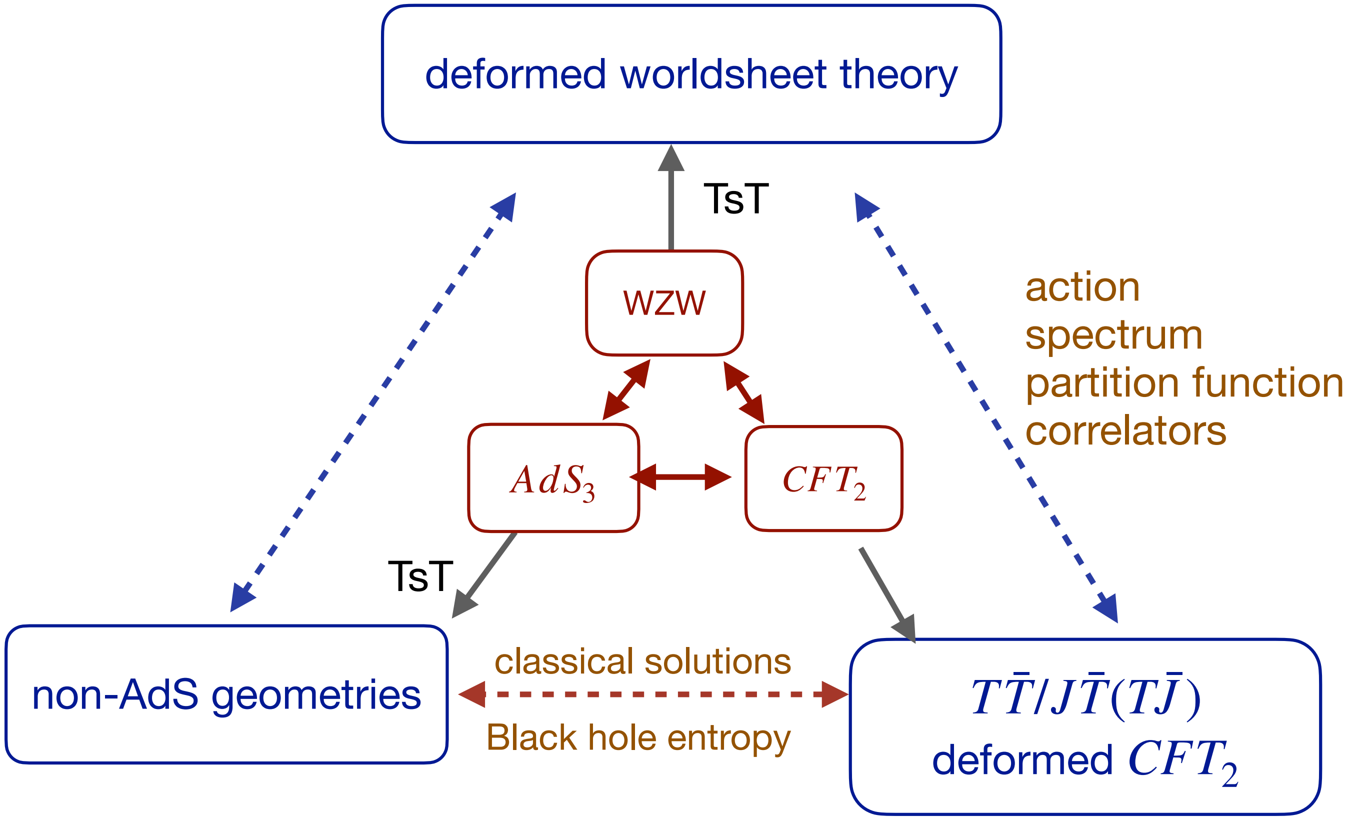
consistent with

- conformal perturbation on the worldsheet

Correlation functions from the $T\bar{T}$ side

- Perturbation on the dual CFT *[Kraus-Liu-Marolf, He-Shu, He-Sun]*
- Callan-Symanzik equation *[Cardy, Hirano-Nakajima-Shigemori]]*
- non-perturbative calculation using JT gravity *[Aharony-Barel]*
- symmetry argument *[Chakraborty-Georgescu-Guica]*

# A class of tractable toy models for non-AdS holography




Thank you!