Measurement-based quantum simulation of Abelian lattice gauge theories

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SciPost 14 (5), 129 (arXiv: 2210.10908), and works in prep.

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Introduction

- Quantum simulation of lattice gauge theories
- Gate-based quantum computers, quantum simulators.
- **Measurement-Based Quantum Computation (MBQC)** is also a model capable of (universal) quantum computation.
- In this vein, we formulated an MBQC scheme for simulating a class of spin models that includes gauge theories.
- **Measurement-based quantum simulation (MBQS)**
- *Achieve a quantum simulation by measuring a tailor-made entangled state.*
- *What's the properties of the entangled state?*

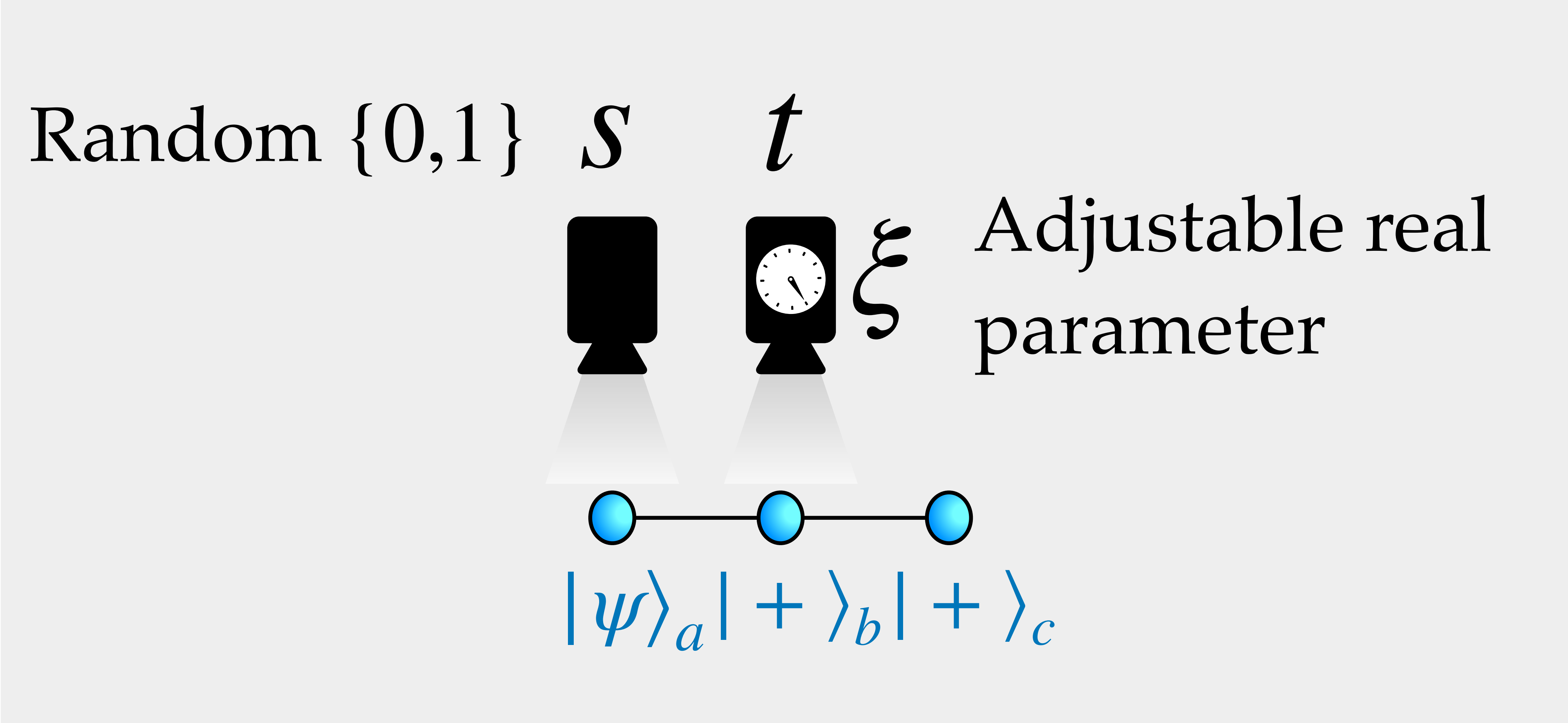
Plan

- MBQC (MBQS) in (0+1)dimensions
- Wegner’s generalized Ising models
- MBQS for Wegner’s models
- Higher-form symmetries, an SPT order, and a holographic interplay.
- Generalizations

A simple MBQC (MBQS) in (0+1)dimensions

MBQC: (0+1)dimensions

- A one-qubit state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ as an *a*-qubit. Couple it to *b*- and *c*-qubits.
- Measure *a*-qubit and ***then*** *b*-qubit.



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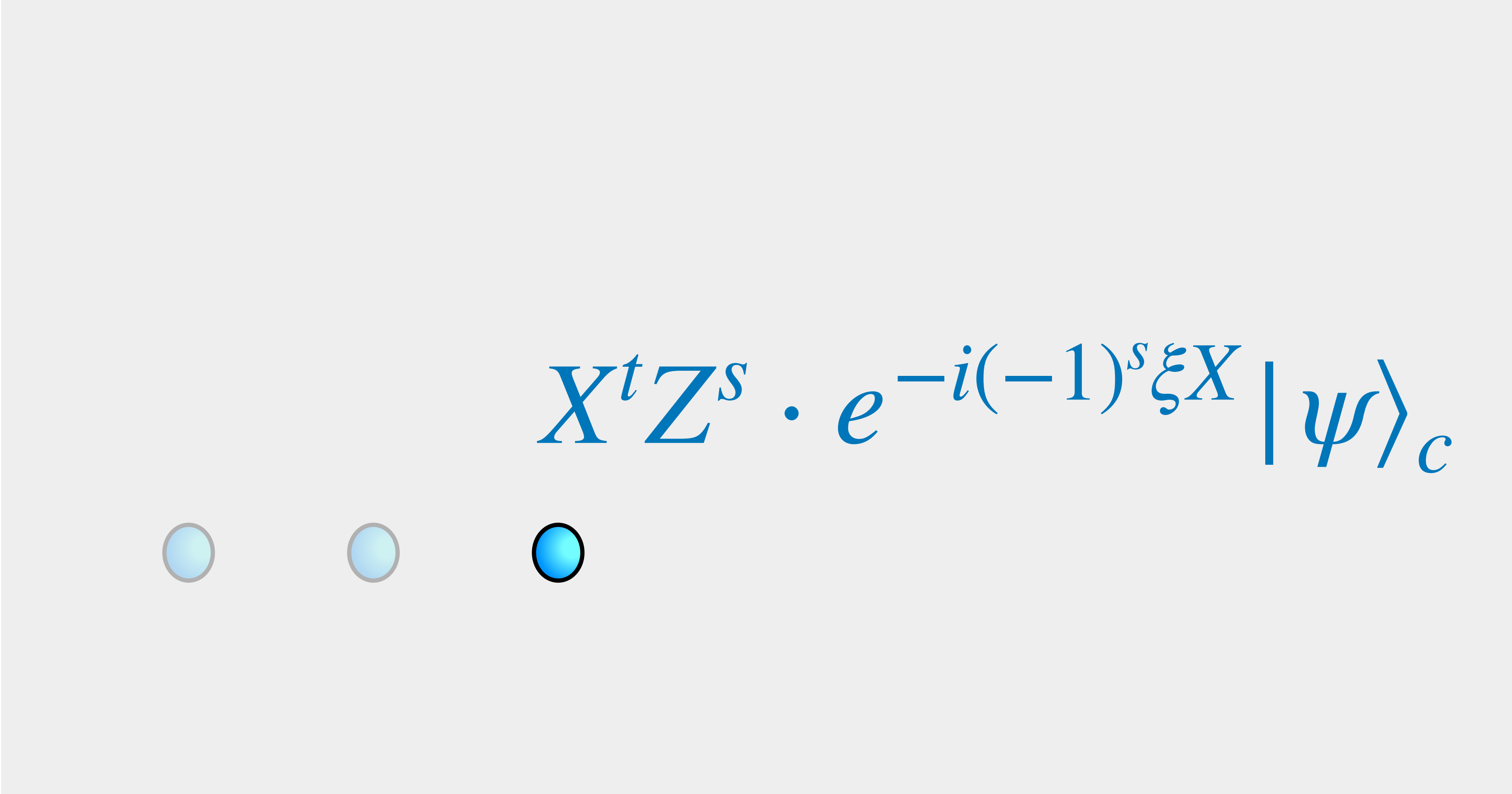
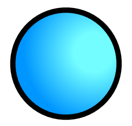


$X|+\rangle = |+\rangle$ — $CZ_{a,b} := |0\rangle\langle 0|_a \otimes I_b + |1\rangle\langle 1|_a \otimes Z_b = CZ_{b,a}$

- *Gate teleportation*.
- Choose $\xi = (-1)^s \alpha$ in the second measurement to counter the randomness of the first measurement and realize $e^{-i\alpha X}$ *deterministically*.

MBQC: (0+1)dimensions

1-qubit state



$$|\psi\rangle$$

MBQC: (0+1)dimensions

$|\psi\rangle$

1d entangled state (resource state) $|\psi_{\text{resource}}\rangle$

E.g. AKLT state, cluster state, …

MBQC: (0+1)dimensions

$|\psi\rangle$

MBQC: (0+1)dimensions

$$X^{\#}Z^{\#} \cdot U_1 |\psi\rangle$$

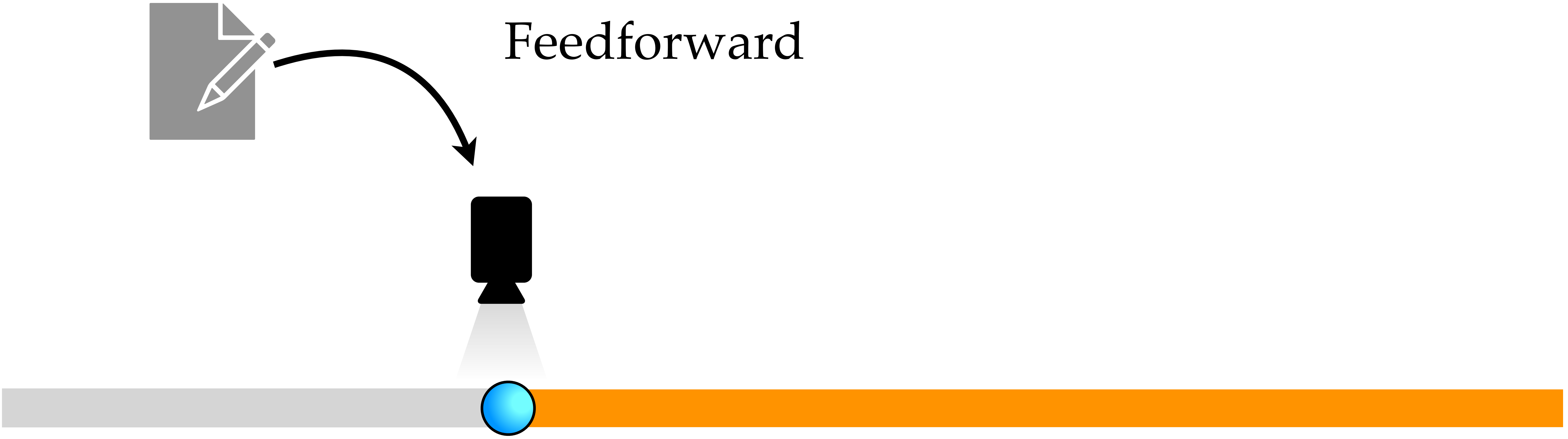
MBQC: (0+1)dimensions

$$X^{\#}Z^{\#} \cdot U_1 \,|\psi\rangle$$

MBQC: (0+1)dimensions

$$X^{\#}Z^{\#} \cdot U_2 U_1 |\psi\rangle$$

MBQC: (0+1)dimensions



$$X^{\#}Z^{\#} \cdot U_2 U_1 |\psi\rangle$$

MBQC: (0+1)dimensions

$$X^{\#}Z^{\#}\cdot\cdots U_2 U_1 |\psi\rangle$$

MBQC: (0+1)dimensions

Post-measurement product state

$$X^{\#}Z^{\#} \cdot U_N \cdots U_2 U_1 |\psi\rangle$$

Simulated state

MBQC: (0+1)dimensions

Post-measurement product state

$U_N \cdots U_2 U_1 |\psi\rangle$

Simulated state
(Cleaned up)

MBQC

What we have just shown is a simple example of MBQC.

MBQC (measurement-based quantum computation)

(Universal) quantum computation can be achieved by
(1) **preparing a resource state**
(2) **measuring the resource state in a certain adaptive pattern**.
(3) **post-processing (unwanted) byproduct operators**

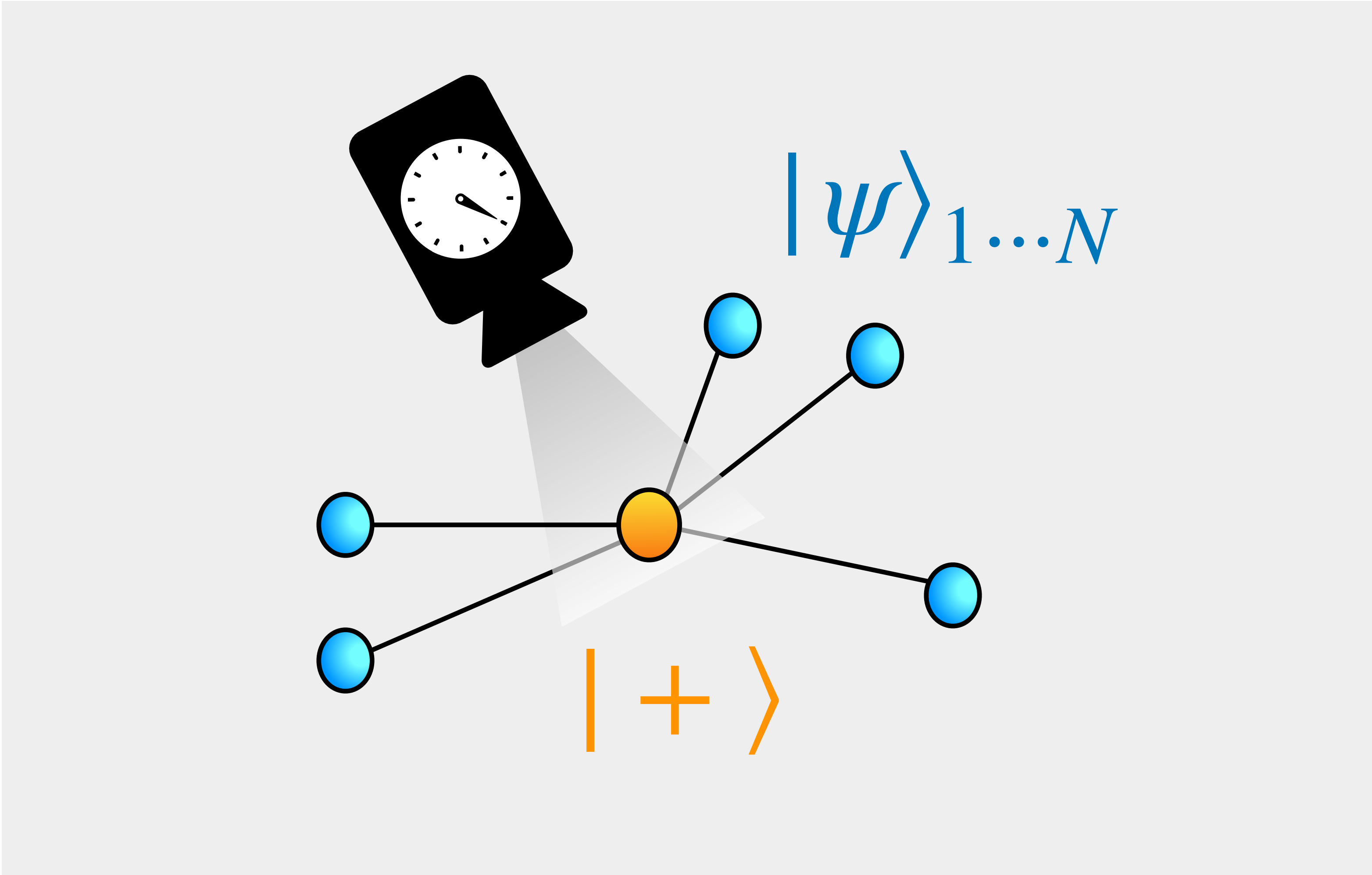
[Raussendorf, Briegel, Browne, Nielsen…]
Review article: e.g. [T.-C. Wei (2023)]

However, our goal below is not the universal quantum computation, but a quantum simulation of Wegner's Ising models.

MBQC: multi-body interaction term

More with CZ and measurement:

- Consider a general state $|\psi\rangle_{1\ldots N}$



X-basis

$$\frac{1+(-1)^s Z_1\cdots Z_N}{2}|\psi\rangle_{1\ldots N}$$

$\rightarrow$ Parity check

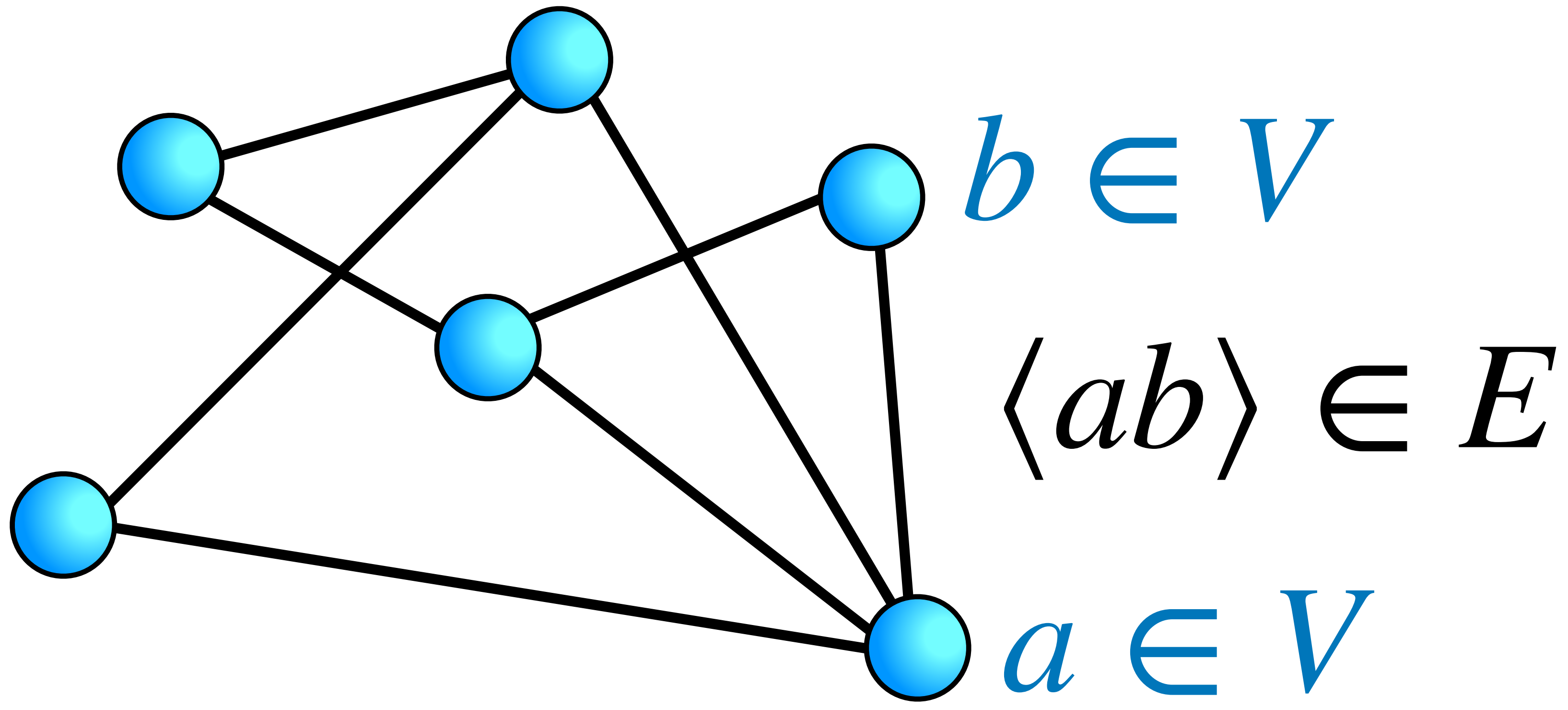
ξ-rotated basis

$$(Z_1\cdots Z_N)^s e^{-i\xi Z_1\cdots Z_N}|\psi\rangle_{1\ldots N}$$

$\rightarrow$ Multi-qubit rotation

Graph state

There is a class of states generated by $|+\rangle$ and CZ, which are called *graph states*.



- Graph = $\{V, E\}$
- V: vertices $\leftrightarrow$ qubits $|+\rangle^{\otimes V}$ are placed
- E: edges $\leftrightarrow$ $CZ_{a,b}$ is applied on $\langle ab \rangle \in E$ $(a, b \in V)$
- Graph state $\subset$ Stabilizer state
- Translationally invariant graph states are called *cluster states*.

Graph state

- In terms of state vectors,

$$|\psi_{\mathscr{C}}\rangle = \prod_{\langle vv'\rangle \in E} CZ_{v,v'} |+\rangle^{\otimes V}$$

- In terms of stabilizers (i.e., $K_v |\psi_{\mathscr{C}}\rangle = |\psi_{\mathscr{C}}\rangle$),

$$|+\rangle^{\otimes V} \quad \longleftrightarrow \quad \left\{ X_v \;\middle|\; v \in V \right\}$$

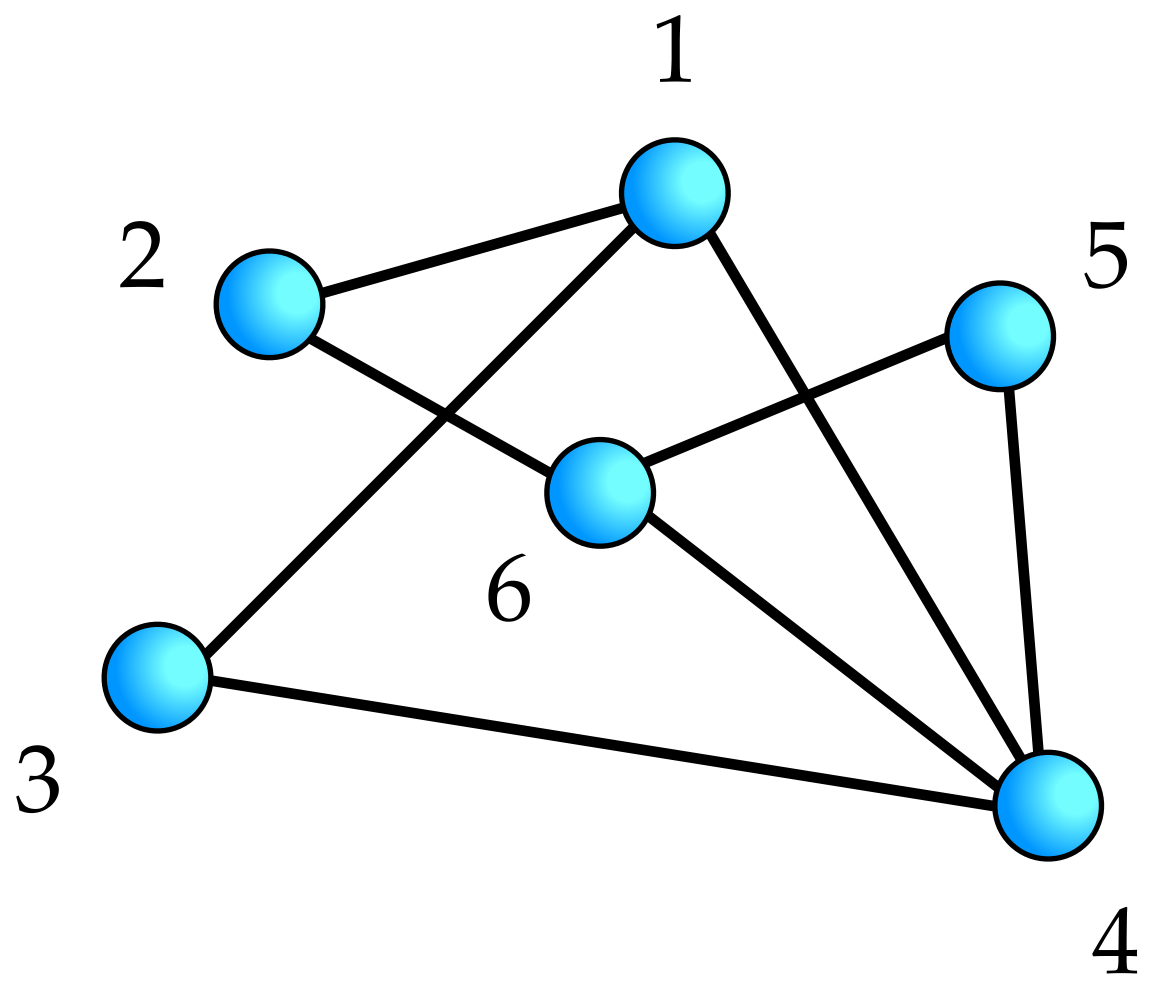
$$|\psi_{\mathscr{C}}\rangle \quad \longleftrightarrow \quad \left\{ K_v \;\middle|\; v \in V \right\}$$

$$K_v = \left(\prod_{\langle vv'\rangle \in E} CZ_{v,v'} \right) \cdot X_v \cdot \left(\prod_{\langle vv'\rangle \in E} CZ_{v,v'} \right) = X_v \prod_{\langle vv'\rangle \in E} Z_{v'}$$

Graph state

$$K_1 = X_1 Z_2 Z_3 Z_4$$

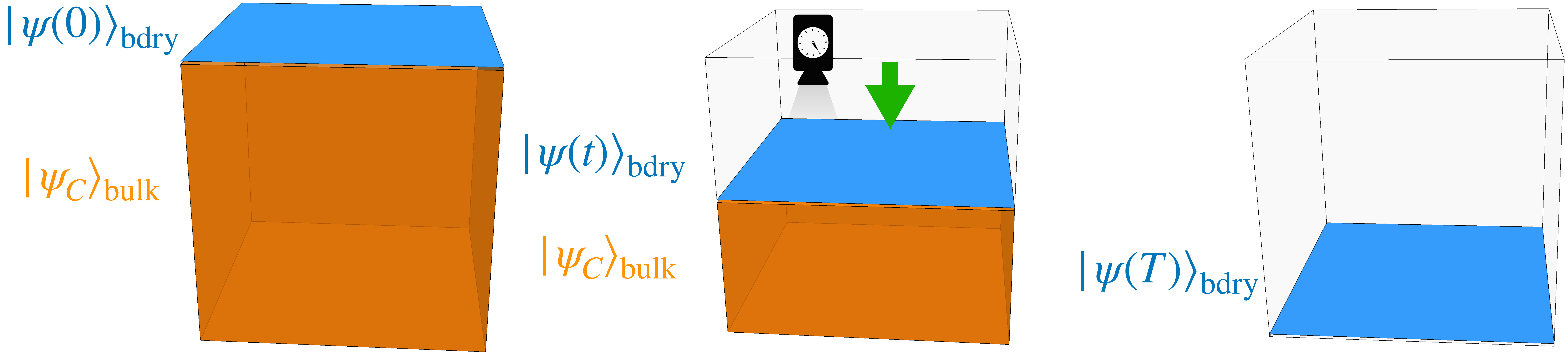
$$K_2 = X_2 Z_1 Z_6$$



$$K_4 = X_3 Z_1 Z_4$$

etc.

Our idea



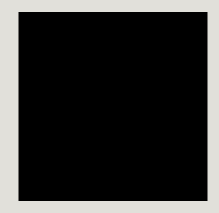
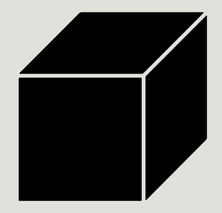
$|\psi(t)\rangle_{\text{bdry}}$: **simulated state of Wegner's model $M_{(d,n)}$** with the Trotterized time evolution $T(t)$,

$$|\psi(t)\rangle_{\text{bdry}} = T(t)\,|\psi(0)\rangle\ .$$

$|\psi_C\rangle_{\text{bulk}}$: **resource state** to be measured — **generalized cluster state (gCS)**.

Spin model to be simulated
—Wegner's generalized Ising models—

Cell simplex σ_i

σ_0	σ_1	σ_2	σ_3
			

$\breve{\sigma}_i$: cell simplices in d dimensional hypercube lattice

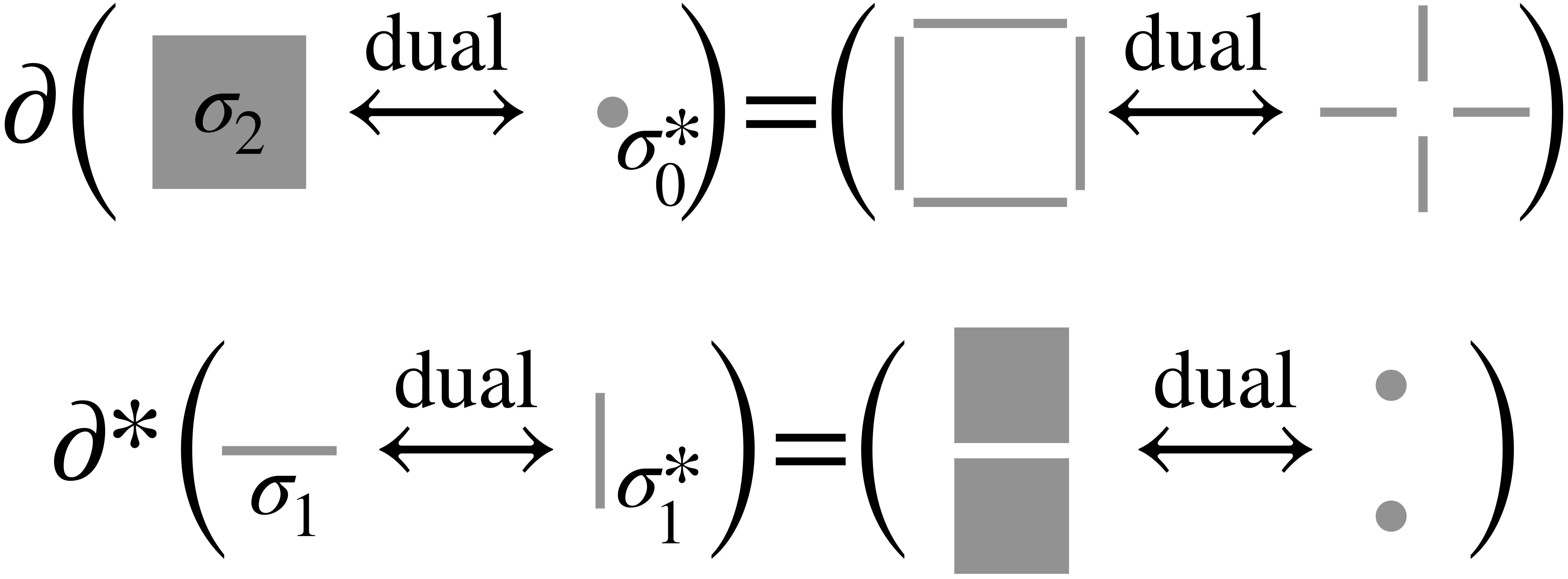
σ_i : cell simplices in $d-1$ dimensional hypercube lattice

$$\breve{\sigma}_i = \sigma_i \times \{j\} \text{ or } \breve{\sigma}_{i+1} = \sigma_i \times [j, j+1]$$

Point Interval x_d coordinate

Similarly, we have cell simplices in the dual lattice with $\sigma_i \simeq \sigma^*_{d-i}$.

We have $\partial^2 = 0$ (and $(\partial^*)^2 = 0$) and a chain complex.



Wegner's generalized Ising model

Model $M_{(d,n)}$:
Classical spin variables $S_{\breve{\sigma}_{n-1}} \in \{+1, -1\}$ living on $(n-1)$-cells in the d-dimensional hybercubic lattice. [Wegner (1971)]

Euclidean action (classical Hamiltonian) I :

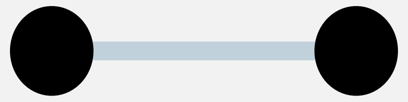
$$I = -J \sum_{\breve{\sigma}_n} \left(\prod_{\breve{\sigma}_{n-1} \subset \partial \breve{\sigma}_n} S_{\breve{\sigma}_{n-1}} \right).$$

Via the transfer matrix formalism, we obtain a quantum Hamiltonian in $(d-1)$ dimensions with the continuous time. [Kogut (1979) etc.]

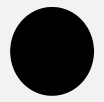
$$H_{(d,n)} = -\sum_{\sigma_{n-1}} X(\sigma_{n-1}) - \lambda \sum_{\sigma_n} Z(\partial \sigma_n) .$$

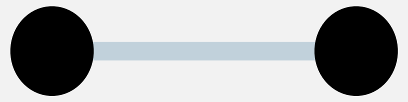
Wegner's generalized Ising model

	Classical Ising model	Transverse field Ising model
$M_{(d,1)}$	$I = -J \sum_{\text{edge}} S(\partial \breve{\sigma}_1)$	$H_{(d,1)} = -\sum_{\sigma_0} X(\sigma_0) - \lambda \sum_{\sigma_1} Z(\partial \sigma_1)$

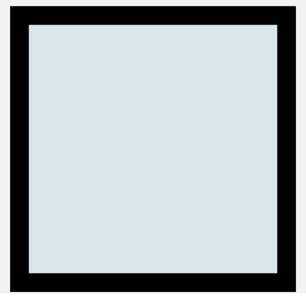


site variable

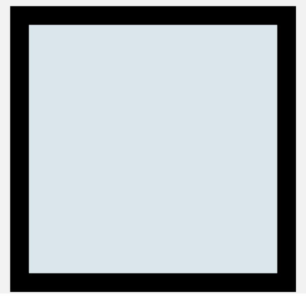




	Gauge theory (Wilson's plaquette action for $G = \mathbb{Z}_2$)	Quantum pure gauge theory
$M_{(d,2)}$	$I = -J \sum_{\text{plaquette}} S(\partial \breve{\sigma}_2)$	$H_{(d,2)} = -\sum_{\sigma_1} X(\sigma_1) - \lambda \sum_{\sigma_2} Z(\partial \sigma_2)$



link variable



Wegner's generalized Ising model

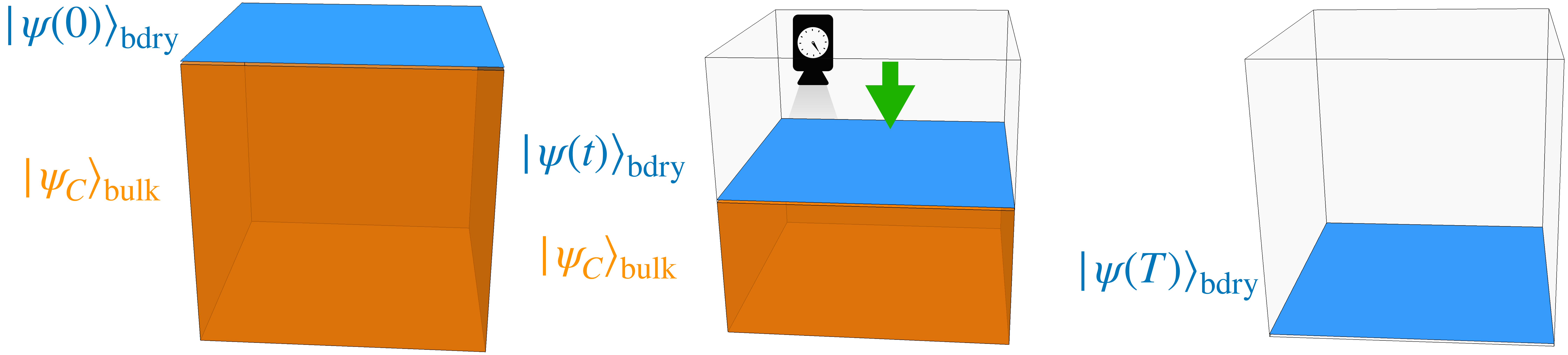
We wish to simulate a Trotterized (real) time evolution:

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle$$

with

$$T(t = j\Delta t) = \left(\prod_{\sigma_{n-1}} e^{i\Delta t X(\sigma_{n-1})} \prod_{\sigma_n} e^{i\Delta t \lambda Z(\partial\sigma_n)}\right)^j .$$

MBQS



$|\psi(t)\rangle_{\rm bdry}$: **simulated state of $M_{(d,n)}$** with the Trotterized time evolution $T(t)$,

$$|\psi(t)\rangle_{\rm bdry} = T(t)|\psi(0)\rangle \ .$$

$|\psi_C\rangle_{\rm bulk}$: **resource state** to be measured — **generalized cluster state (gCS)**.

Resource state and MBQS

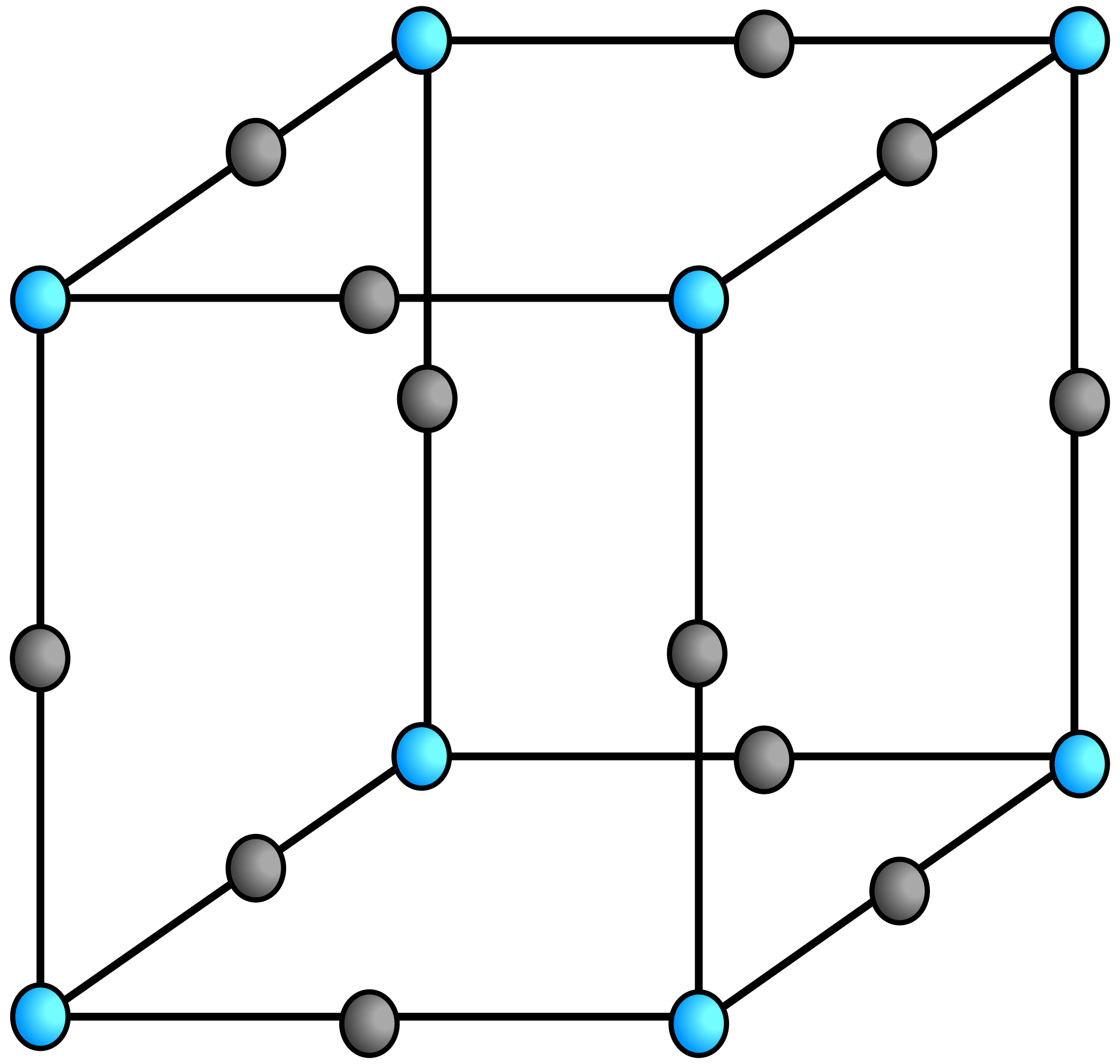
MBQS

Entanglement in our resource state, $gCS_{(d,n)}$ (generalized cluster state), is **tailored** to reflect the space-time structure of the model $M_{(d,n)}$:

$$|gCS_{(d,n)}\rangle := \mathcal{U}_{CZ} |+\rangle^{\breve{\Delta}_n} |+\rangle^{\breve{\Delta}_{n-1}}$$

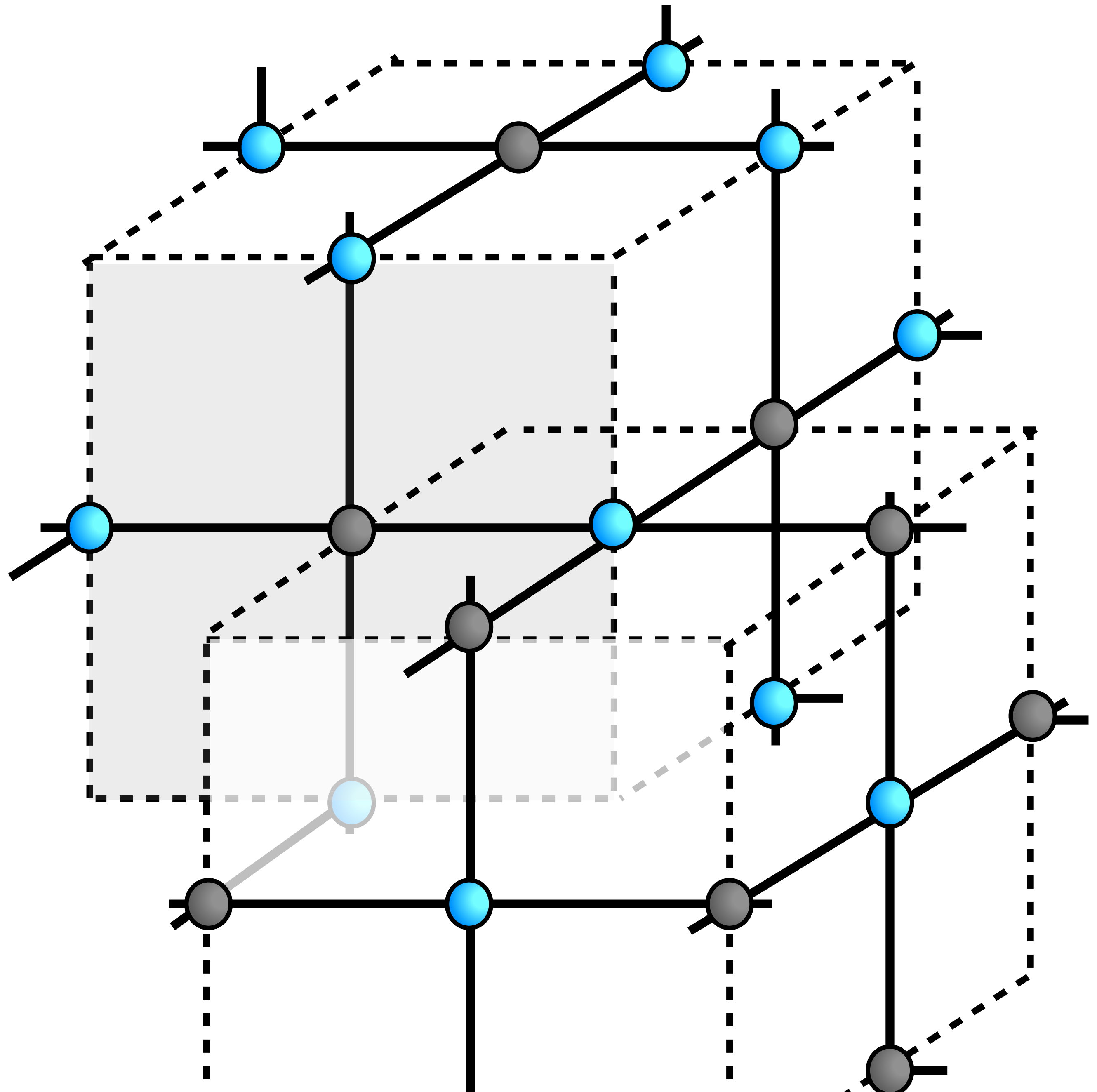
$$\mathcal{U}_{CZ} = \prod_{\breve{\sigma}_n \in \breve{\Delta}_n} \left(\prod_{\breve{\sigma}_{n-1} \subset \partial \breve{\sigma}_n} CZ_{\breve{\sigma}_{n-1}, \breve{\sigma}_n} \right).$$

$(d,n) = (3,1)$

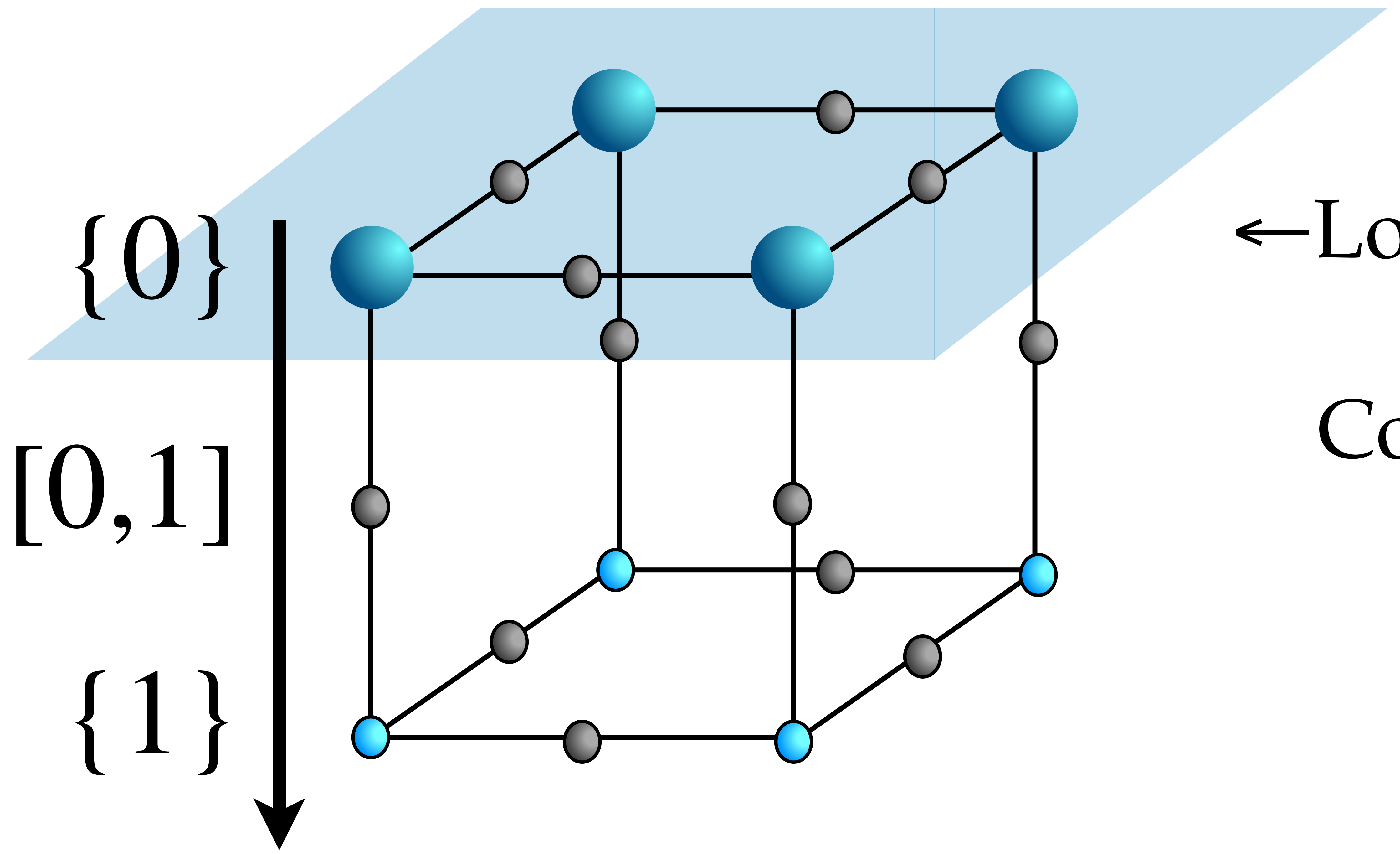


$(d,n) = (3,2)$

[Raussendorf Bravyi Harrington (2007)]



MBQS: simulating $M_{(3,1)}$ on $gCS_{(3,1)}$

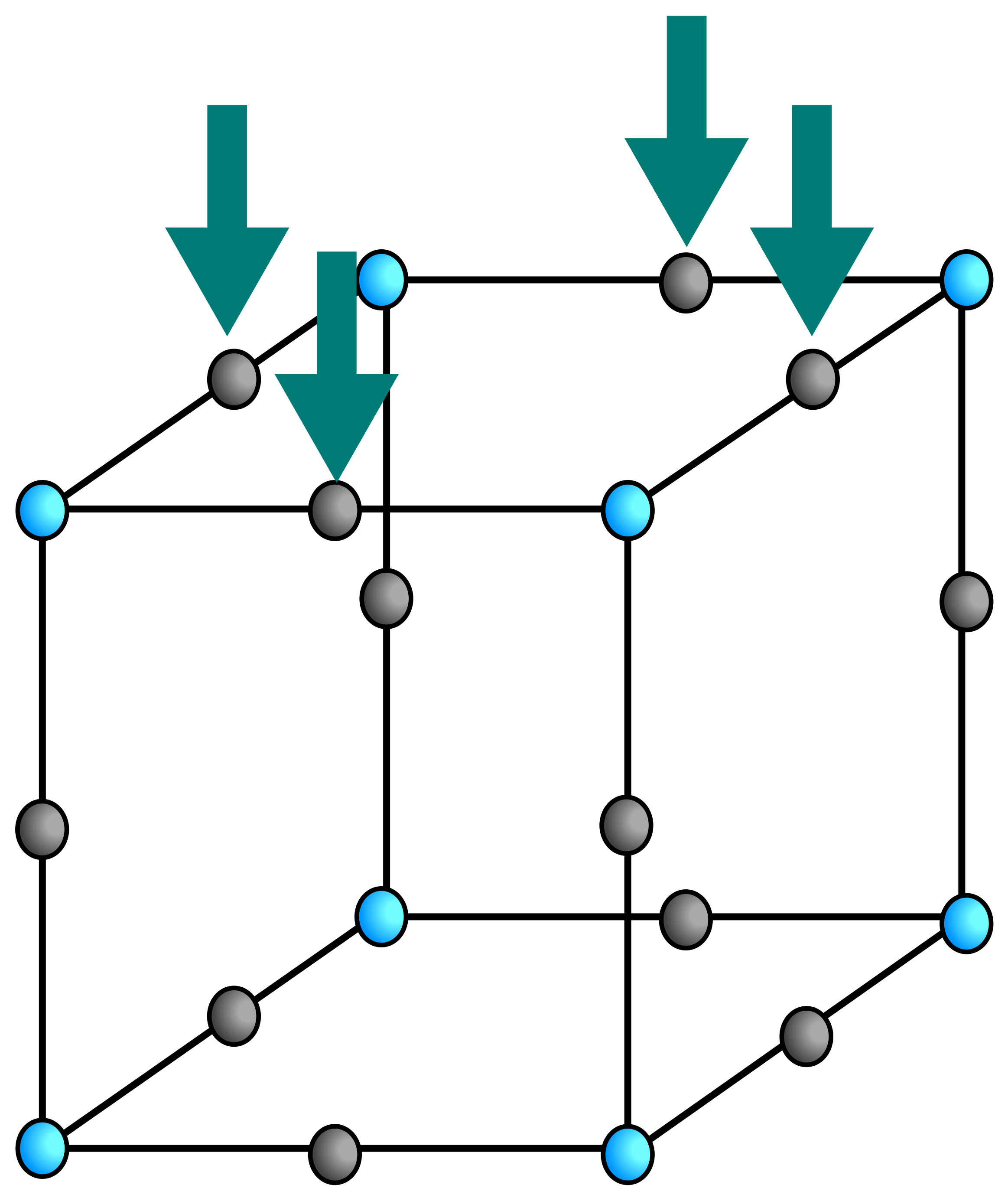


x_3-direction
="time" in the simulated world

←Load a 2d initial state $|\psi(0)\rangle_{\text{bdry}}$ at $x_3 = 0$.

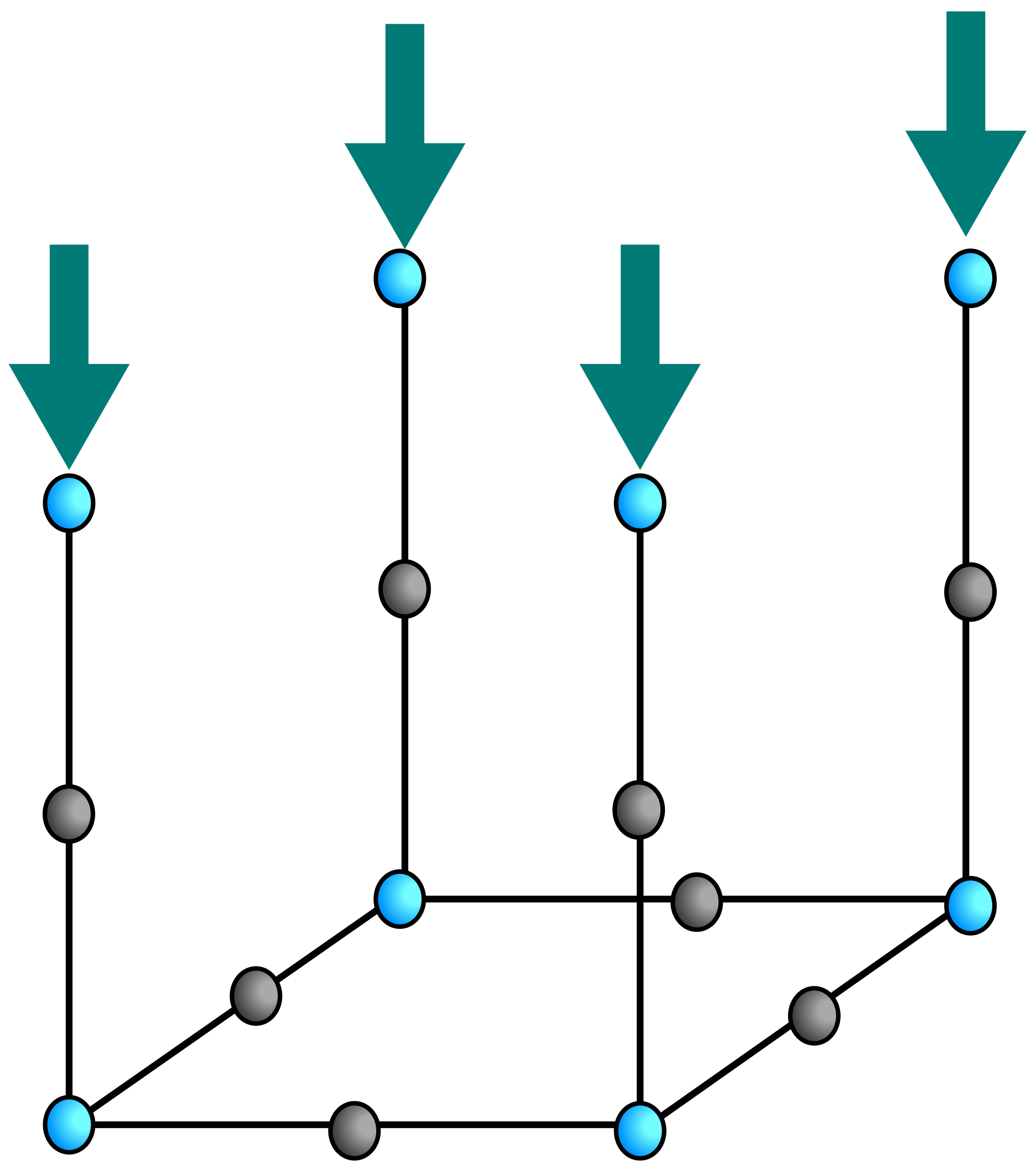
Couple it to the rest of the resource state.

MBQS: simulating $M_{(3,1)}$ on $\mathrm{gCS}_{(3,1)}$



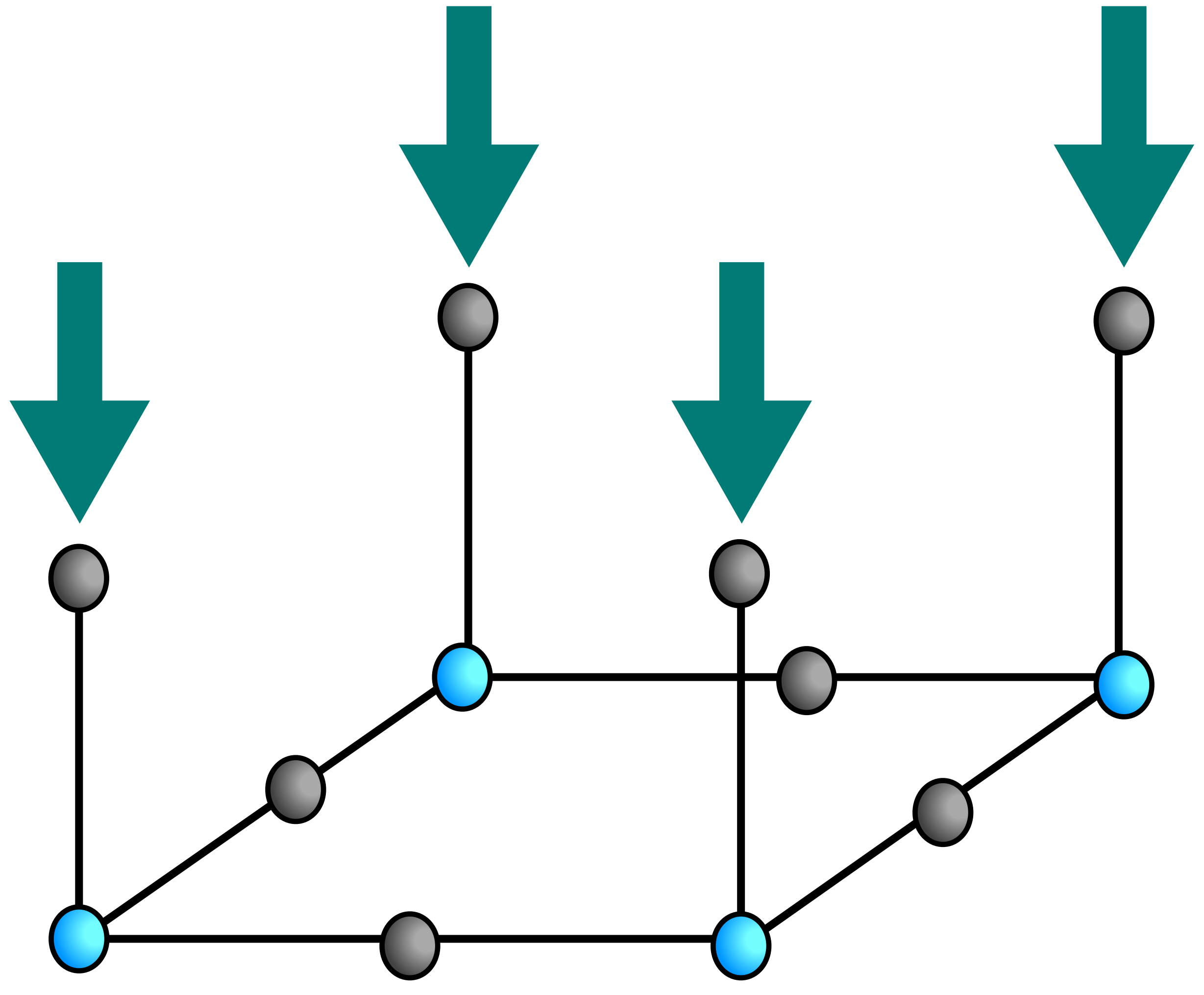
$\breve{\sigma}_1 = \sigma_1 \times \{j\}$

$$\prod_{\sigma_1} e^{-i\xi_1 Z(\partial \sigma_1)}$$



$\breve{\sigma}_0 = \sigma_0 \times \{j\}$

teleported to $[j, j+1]$

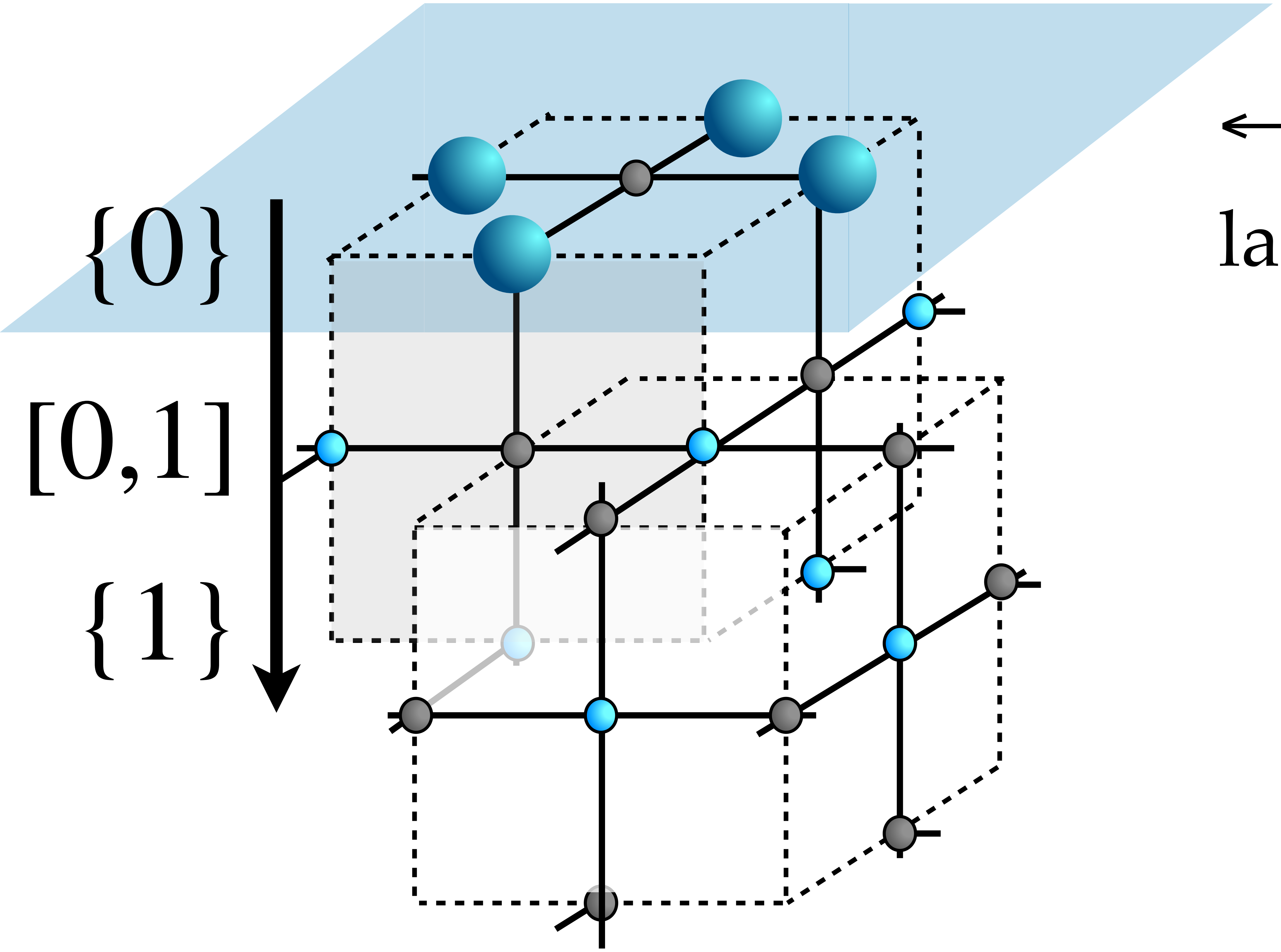


$\breve{\sigma}_1 = \sigma_0 \times [j, j+1]$

$$\prod_{\sigma_0} e^{-i\xi_3 X(\sigma_0)}$$

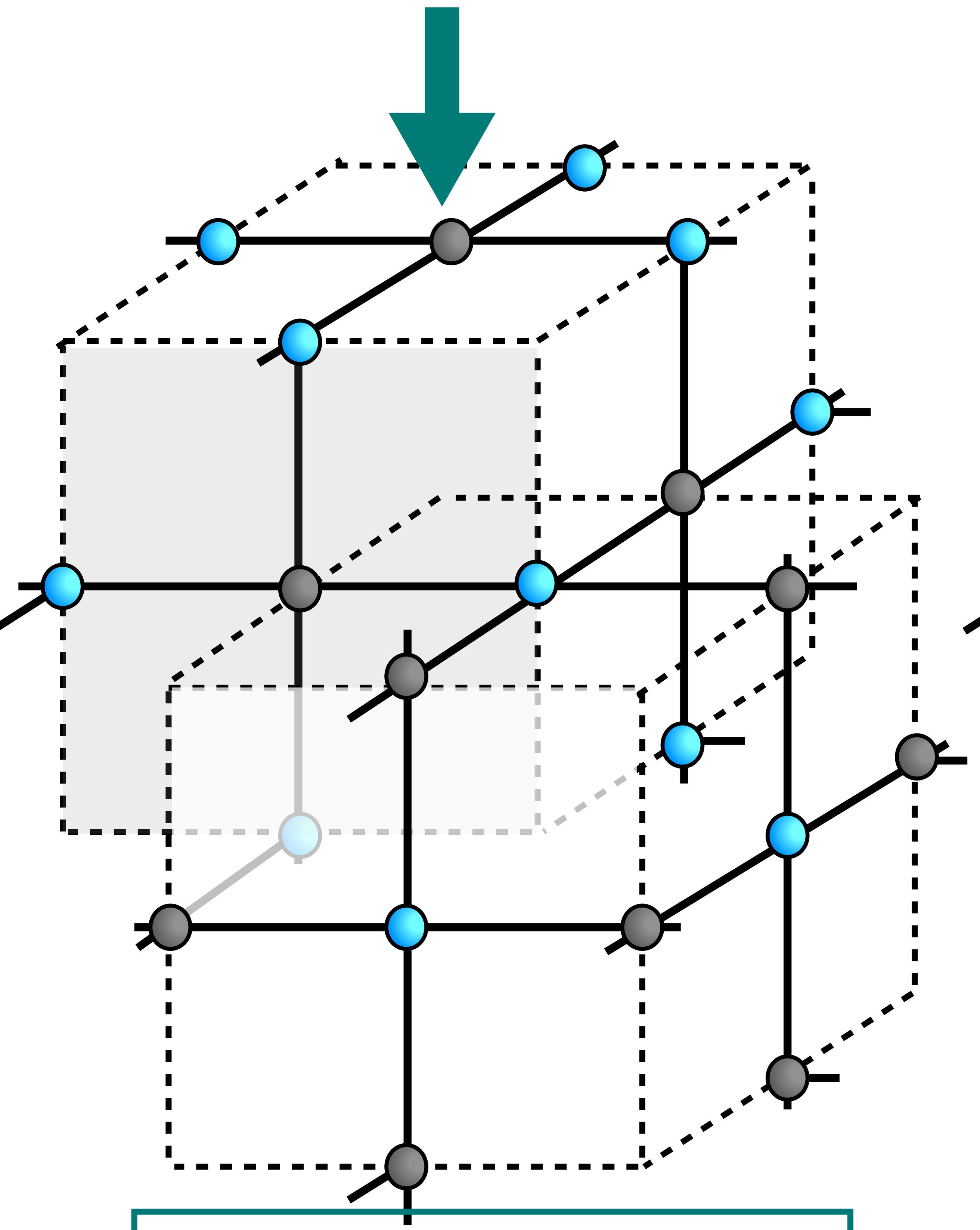
teleported to $\{j+1\}$

MBQS: simulating $M_{(3,2)}$ on gCS$_{(3,2)}$



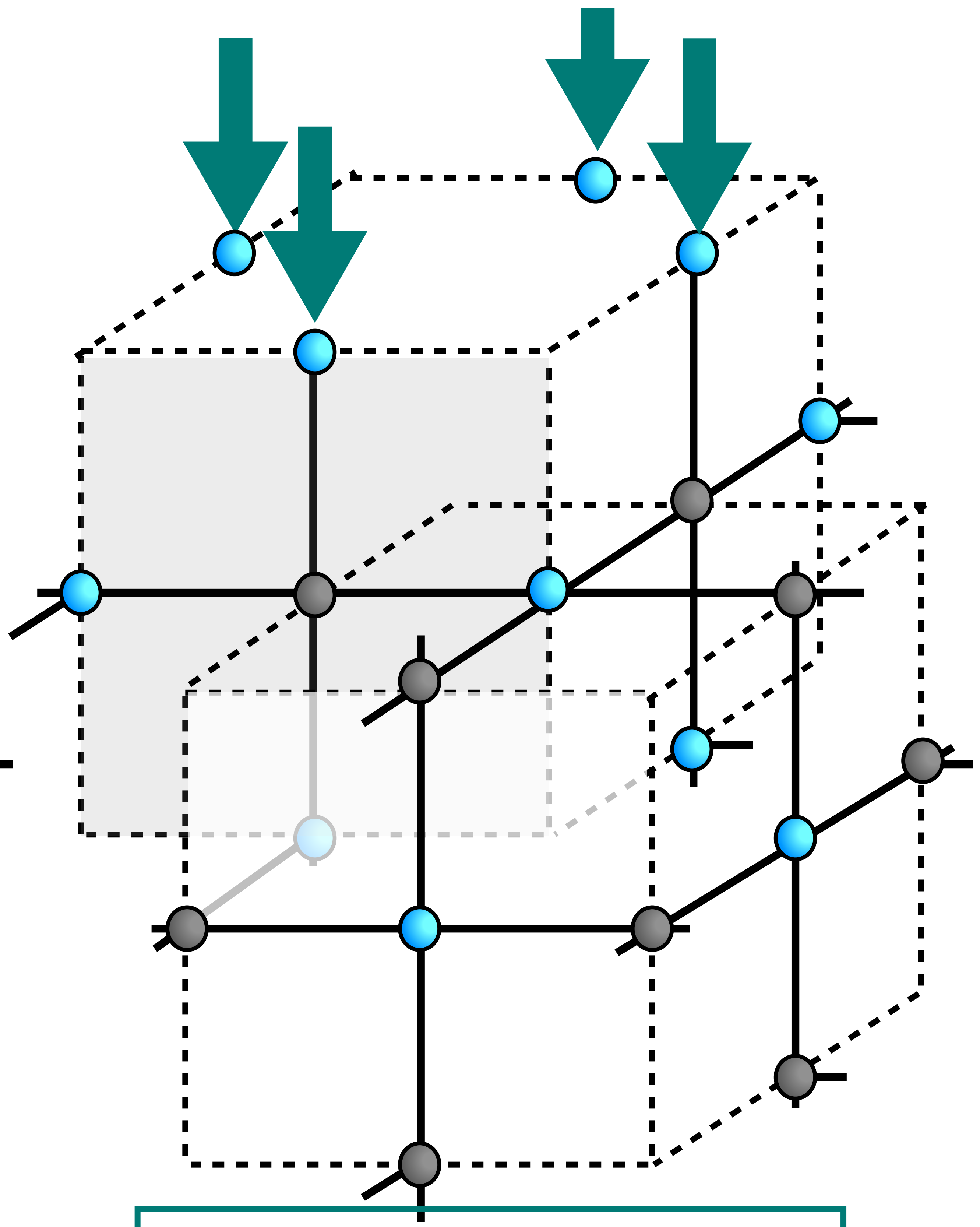
←Load a 2d initial state $|\psi(0)\rangle_{\mathrm{bdry}}$ of the (2 + 1)d lattice $\mathbb{Z}_2$ gauge theory

MBQS: simulating $M_{(3,2)}$ on $\mathrm{gCS}_{(3,2)}$



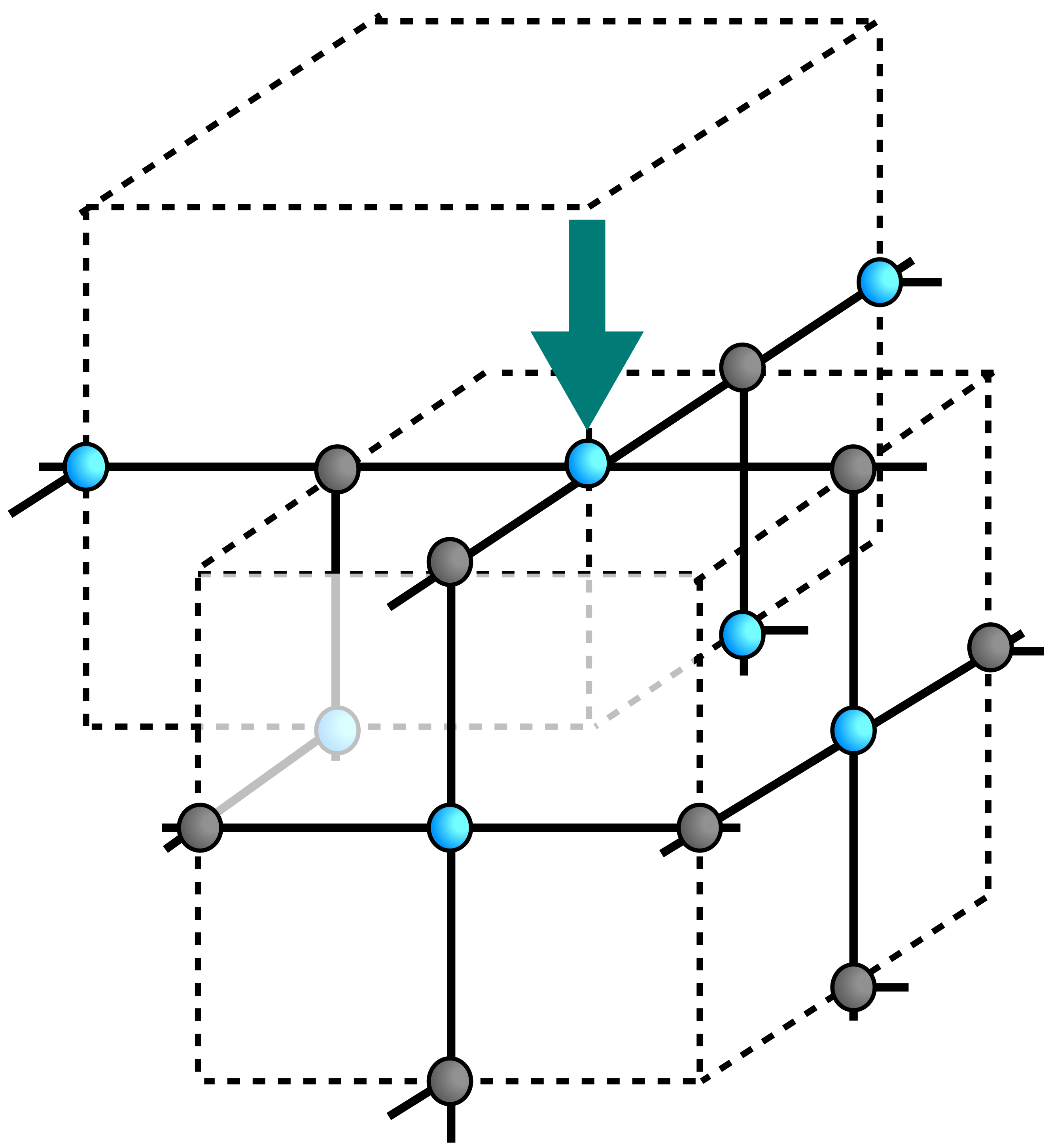
$\breve{\sigma}_2 = \sigma_2 \times \{j\}$

$$\prod_{\sigma_2} e^{-i\xi_1 Z(\partial\sigma_2)}$$



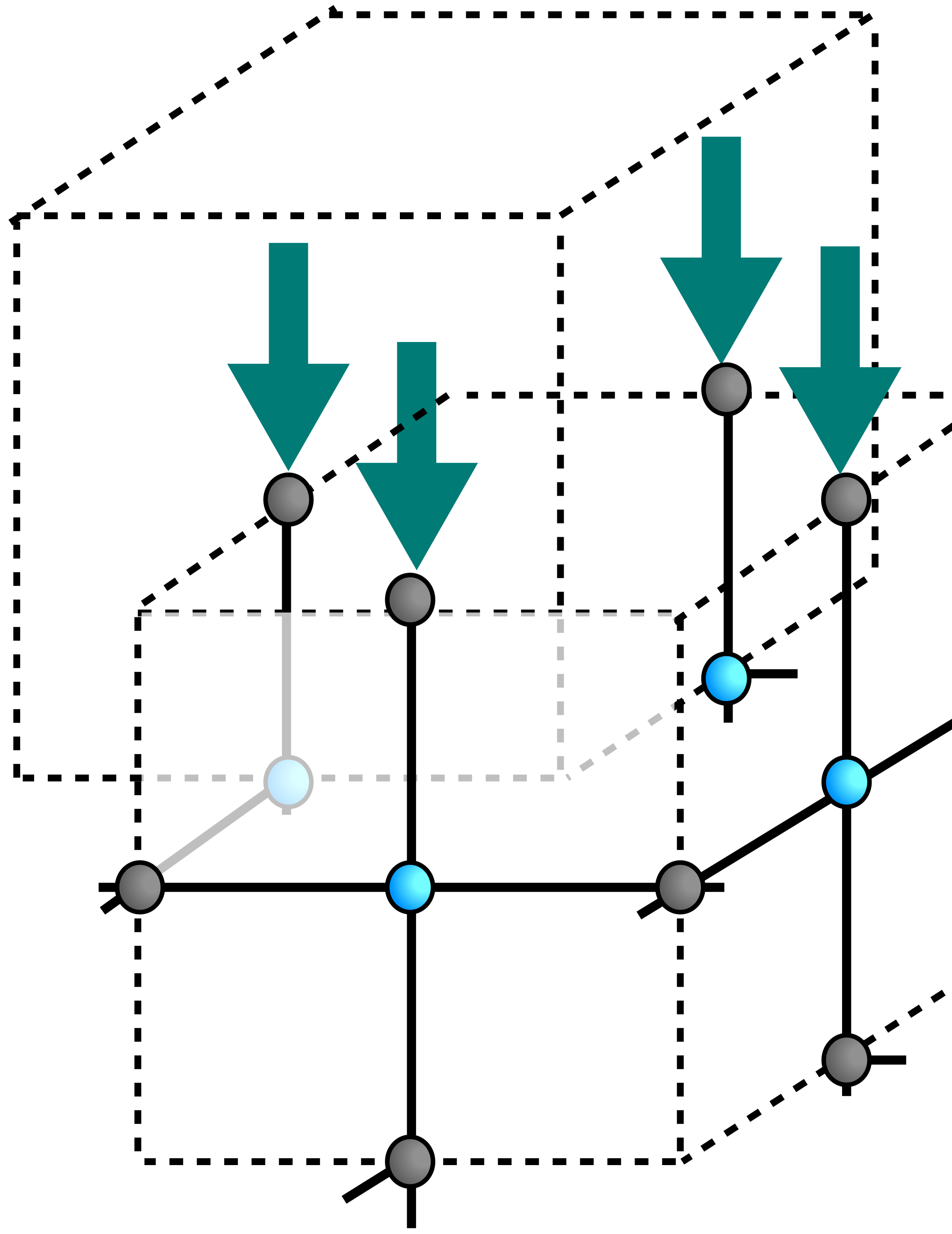
$\breve{\sigma}_1 = \sigma_1 \times \{j\}$

teleported to $[j, j+1]$



$\breve{\sigma}_1 = \sigma_0 \times [j, j+1]$

Parity check for
Gauss law

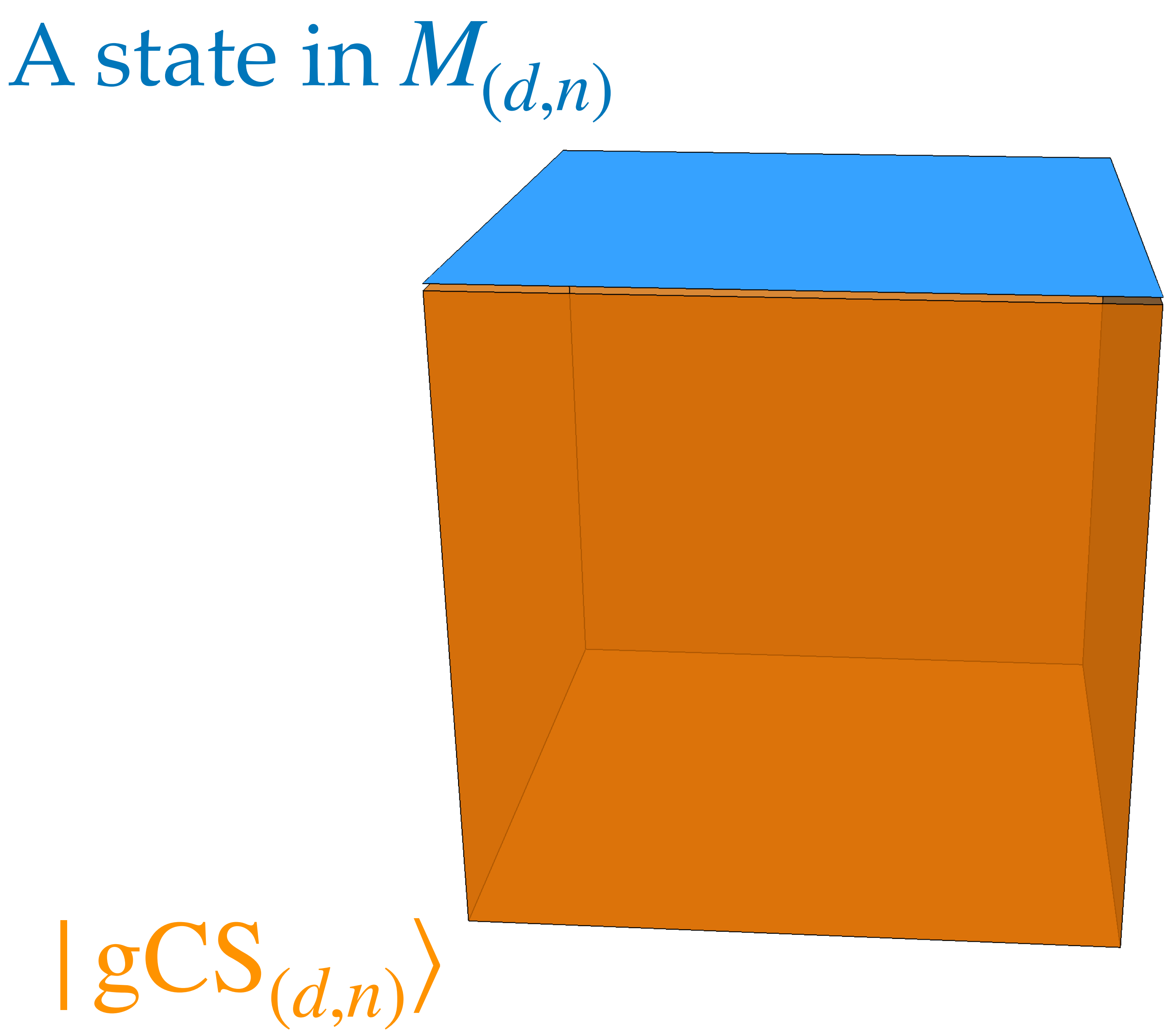


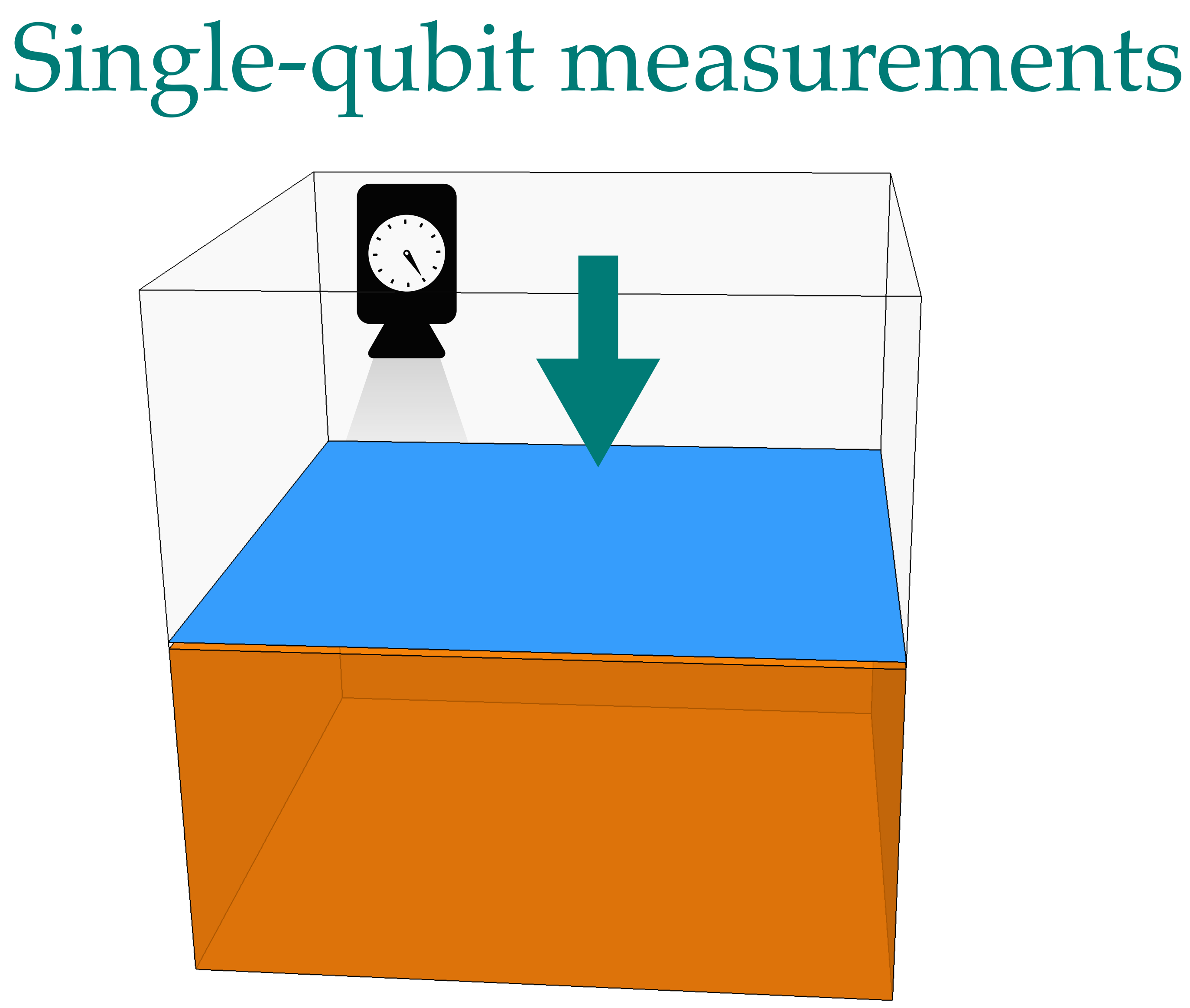
$\breve{\sigma}_2 = \sigma_1 \times [j, j+1]$

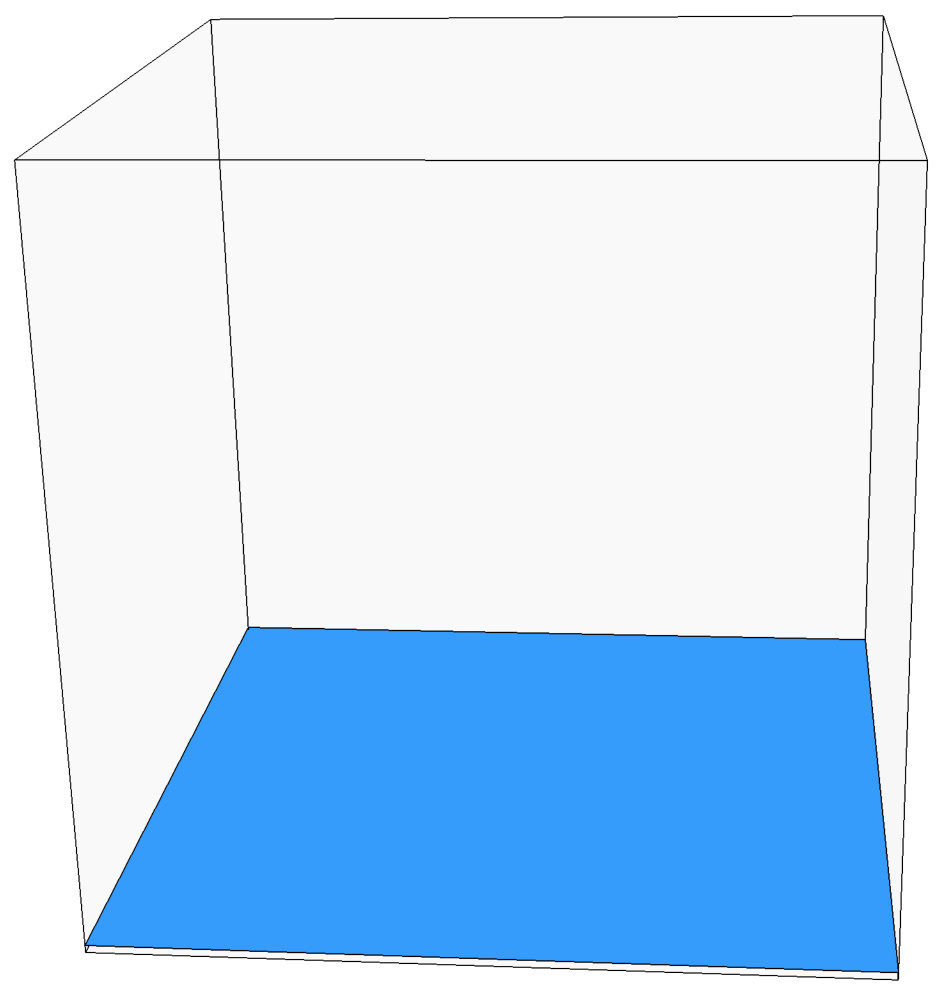
$$\prod_{\sigma_1} e^{-i\xi_4 X(\sigma_1)}$$

teleported to $\{j+1\}$

MBQS: simulating $M_{(d,n)}$ on $\mathrm{gCS}_{(d,n)}$





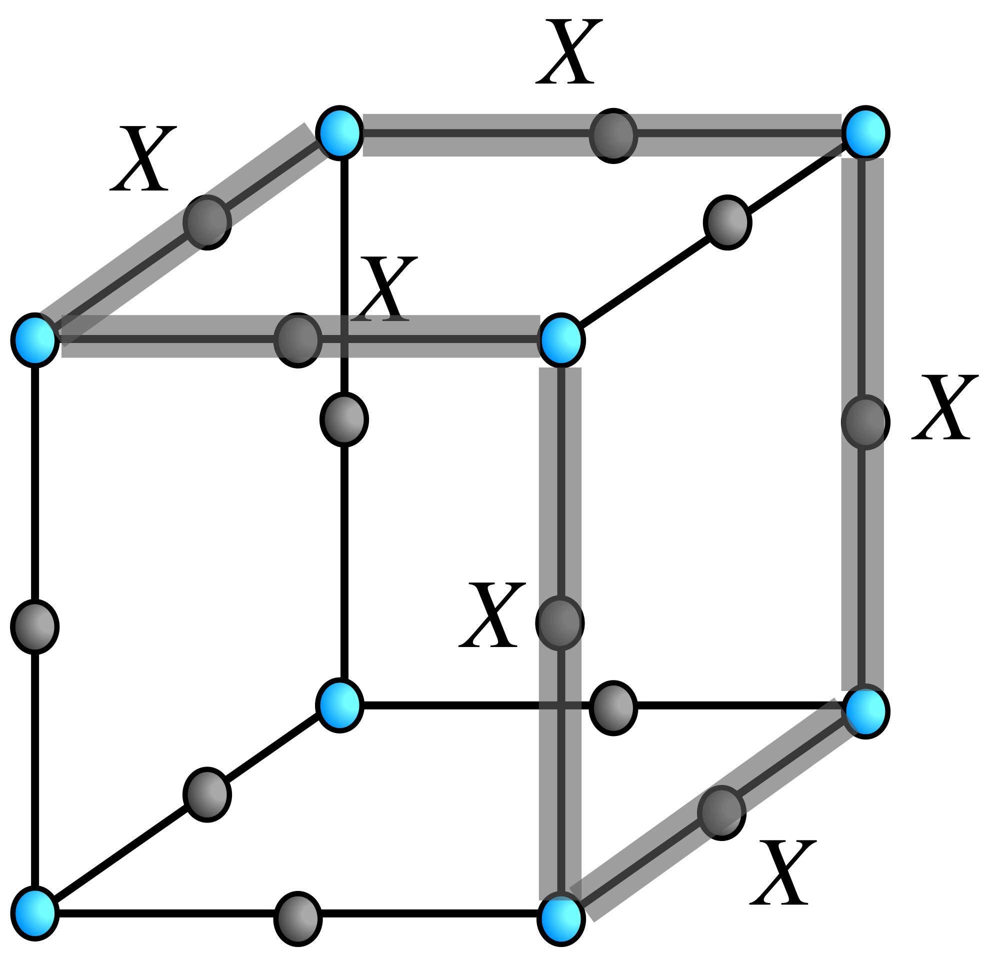


Aspects of symmetries in MBQS
SPT and holographic interplay

Higher-form symmetries in gCS

$(d, n) = (3,1)$

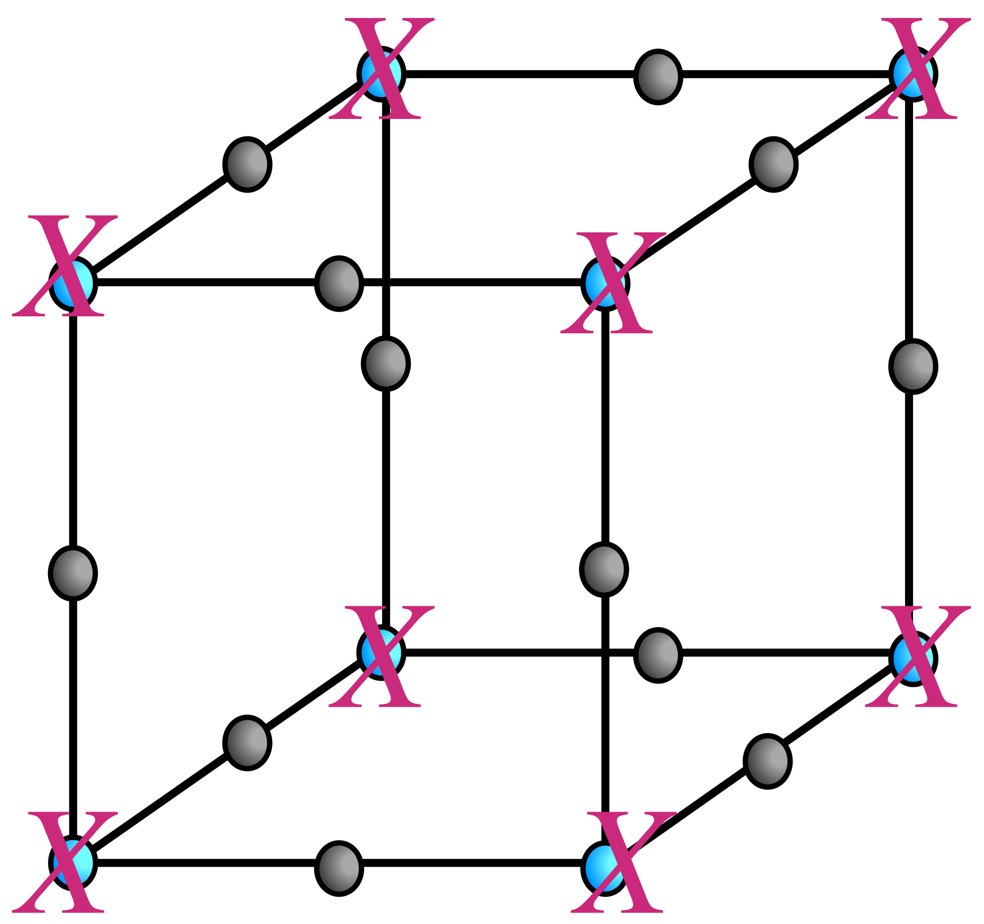
$(d-n) = 2$-form symmetry



$X(\breve{z}_1)$

$\partial \breve{z}_1 = 0$

$(n-1) = 0$-form symmetry



$X(\breve{z}_3^*)$

$\partial^* \breve{z}_3^* = 0$

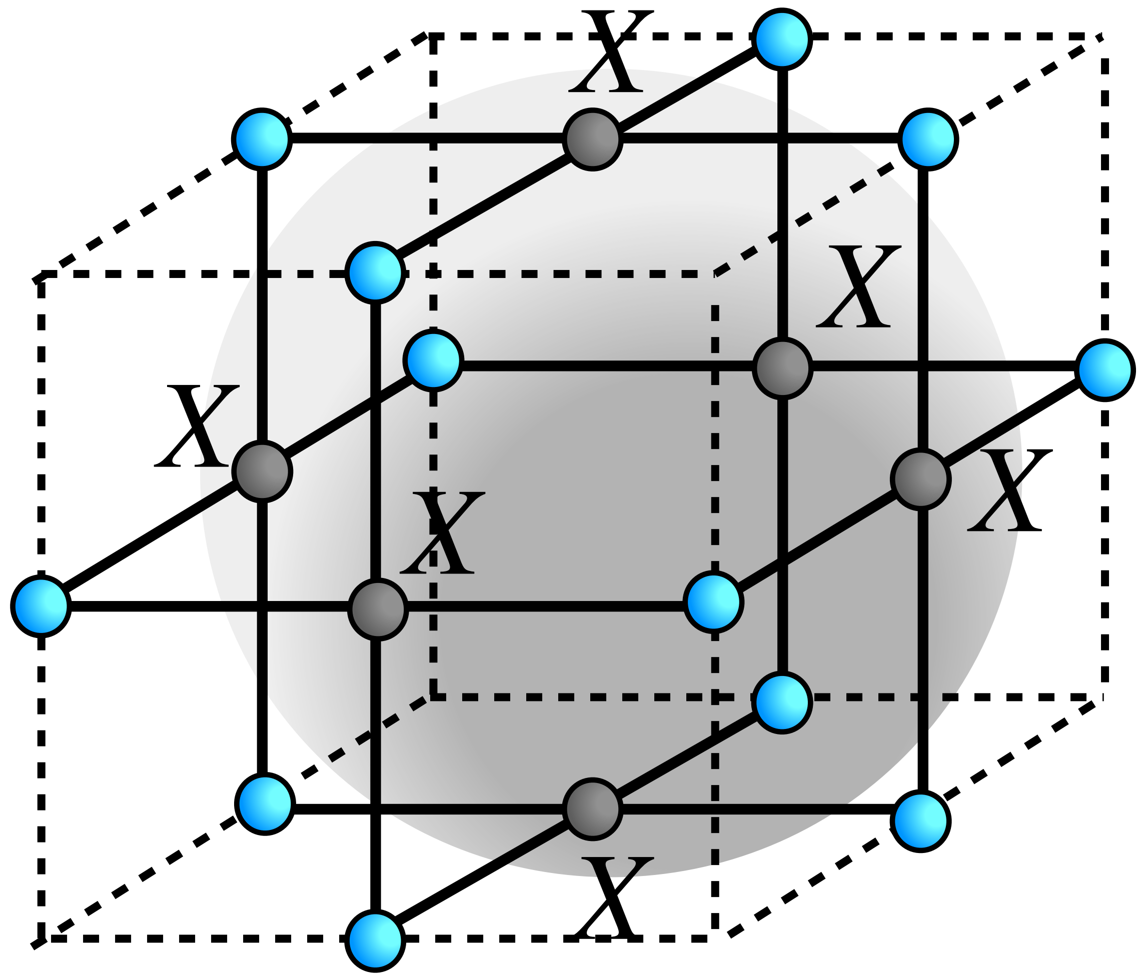
Higher-form symmetries in gCS

$(d, n) = (3,2)$

$(d - n) = 1$-form symmetry

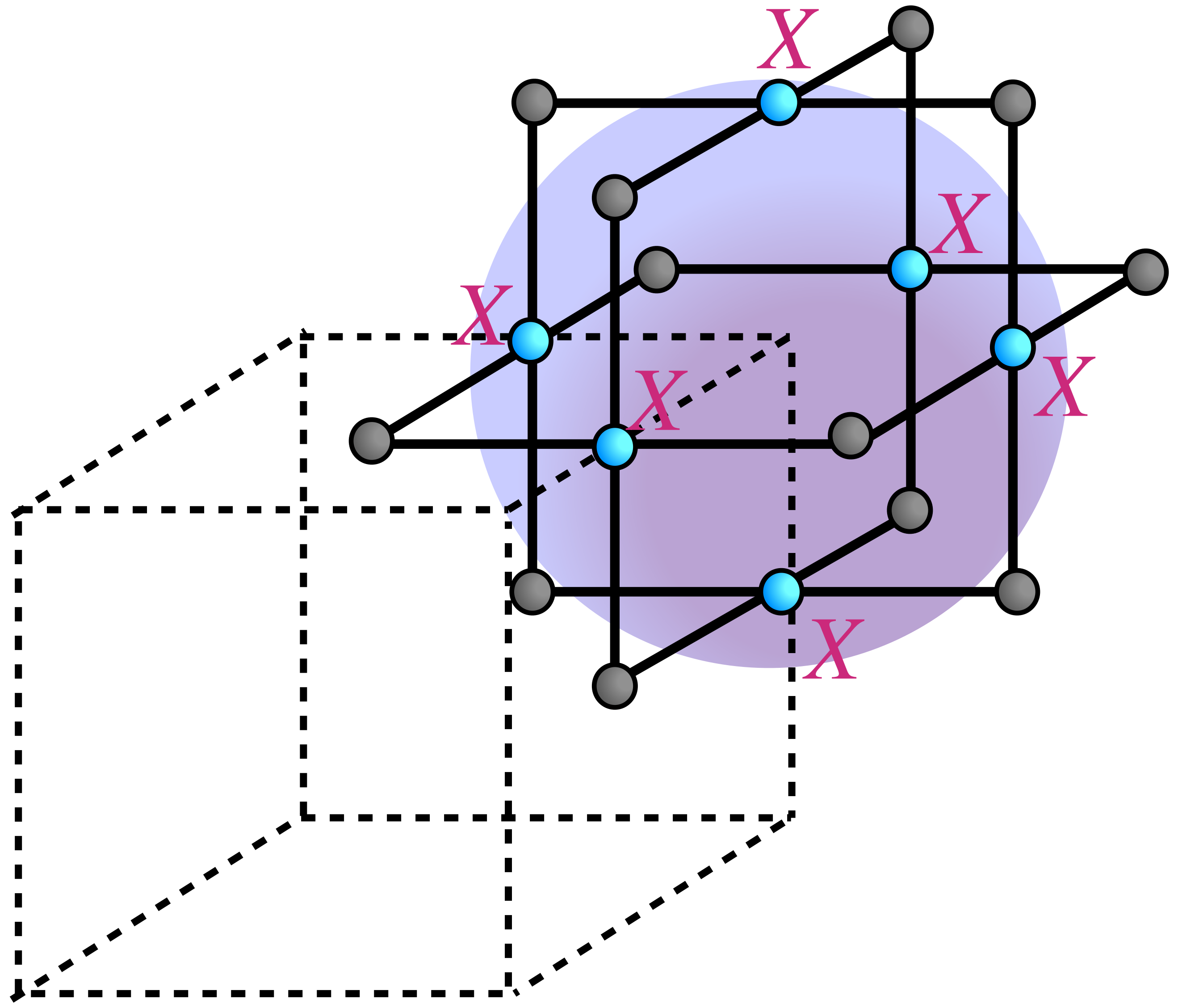
$(n - 1) = 1$-form symmetry

$X(\breve{z}_1)$



$\partial \breve{z}_1 = 0$

$X(\breve{z}_2^*)$



$\partial^* \breve{z}_2^* = 0$

Higher-form symmetries in gCS

$(d-n)$-form and $(n-1)$-form symmetry:

$$|\mathrm{gCS}\rangle = X(\breve{z}_n)|\mathrm{gCS}\rangle = X(\breve{z}^*_{d-n+1})|\mathrm{gCS}\rangle$$

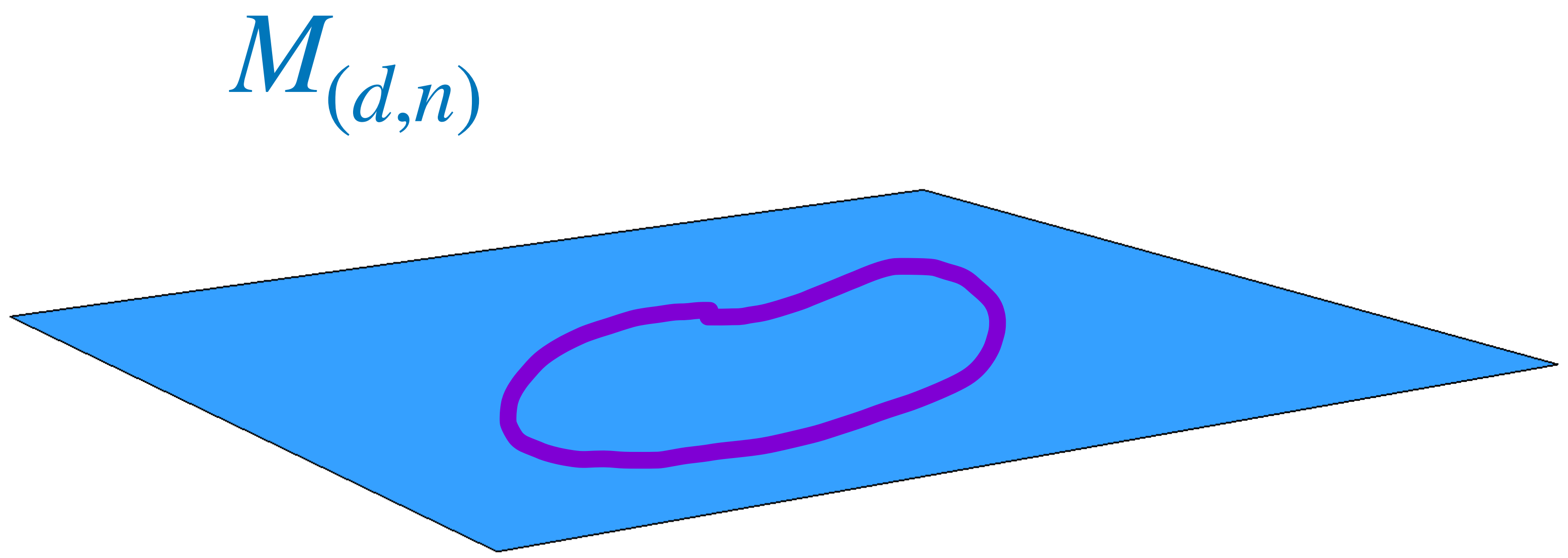
with $M_{d-n} = \{\breve{z}_n | \partial \breve{z}_n = 0\}$, $M'_{n-1} = \{\breve{z}^*_{d-n+1} | \partial^* \breve{z}^*_{d-n+1} = 0\}$.

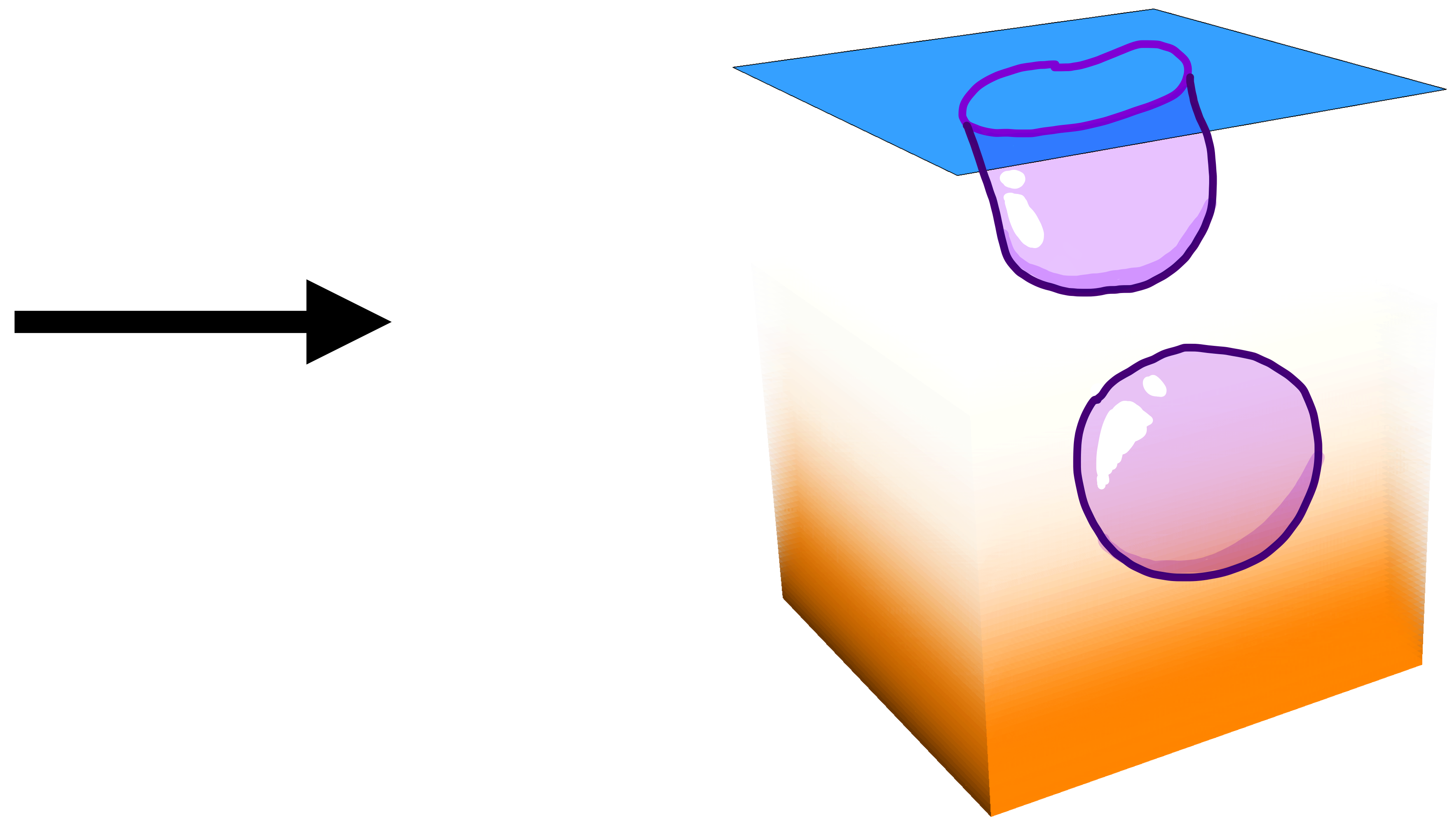
SPT order in gCS

gCS$_{(d,n)}$ has an SPT order protected by $(d-n)$-form $\mathbb{Z}_2$ and $(n-1)$-form $\mathbb{Z}_2$

- Two symmetry generators act projectively at the boundaries of the lattice → SPT. [Yoshida (2016), Roberts-Kubica-Yoshida-Bartlett (2017)].
- The simulated state as an edge state of an SPT. Cf. [Miyake (2010)]
- *Open Question*: Is the quantum simulation possible with any state in the SPT phase?

Bulk/boundary symmetries in MBQS





Boundary symmetry generator $X(z^*_{d-n})$

Bulk symmetry generator $X(\breve{z}^*_{d-n+1})$ with $\partial^* \breve{z}^*_{d-n+1} = 0$ or $= z^*_{d-n}$.

(3,1) Ising — 0-form symmetry $X(z^*_2) = \prod_{v \in V} X_v$ → 0-form symmetry $X(\breve{z}^*_3) = \prod_{\breve{v} \in \breve{V}} X_{\breve{v}}$

(3,2) gauge — Electric 1-form symmetry $X(z^*_1)$ → 1-form symmetry $X(\breve{z}^*_2)$

Bulk/boundary symmetries in MBQS

For comparison, consider a d-dimensional bulk (ungauged) Hamiltonian

$$H = -\sum Z(\partial\breve{\sigma}_n)\ ,$$

which is symmetric under the transformation with the **global** $(n-1)$-form, $X(\breve{z}^*_{d-n+1})$.

Cluster state gCS:

It is described by the local stabilizer conditions:

$$X(\breve{\sigma}_n)Z(\partial\breve{\sigma}_n)\,|\,\mathrm{gCS}_{(d,n)}\rangle = X(\breve{\sigma}_{n-1})Z(\partial^*\breve{\sigma}_{n-1})\,|\,\mathrm{gCS}_{(d,n)}\rangle = |\,\mathrm{gCS}_{(d,n)}\rangle\ ,$$

i.e., the ground state of the **gauged version** of the above Hamiltonian,

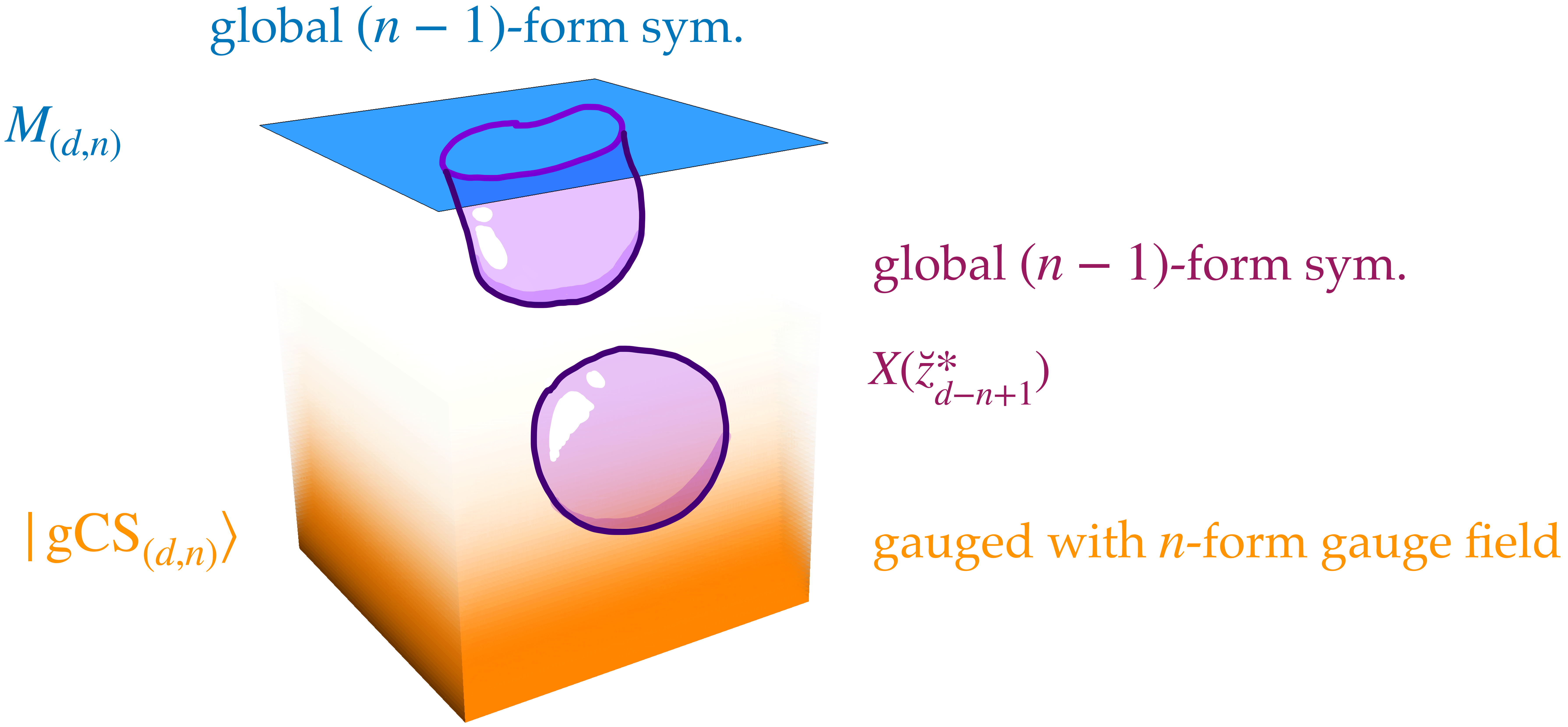
$$H_{\mathrm{gauged}} = -\sum X(\breve{\sigma}_n)Z(\partial\breve{\sigma}_n)\ ,$$

with the local gauge constraint $X(\breve{\sigma}_{n-1})Z(\partial^*\breve{\sigma}_{n-1}) = 1$ $(\forall\breve{\sigma}_{n-1})$.

(The global symmetry generator $X(\breve{z}^*_{d-n+1})$ is a product of local stabilizers $X(\breve{\sigma}_{n-1})Z(\partial^*\breve{\sigma}_{n-1})$.)

Bulk/boundary symmetries in MBQS

In other words, the boundary global symmetry is promoted to the bulk(+boundary) global symmetry $X(\breve{z}^*_{d-n+1})|\psi_C\rangle = |\psi_C\rangle$*, and it is gauged in the cluster state.*



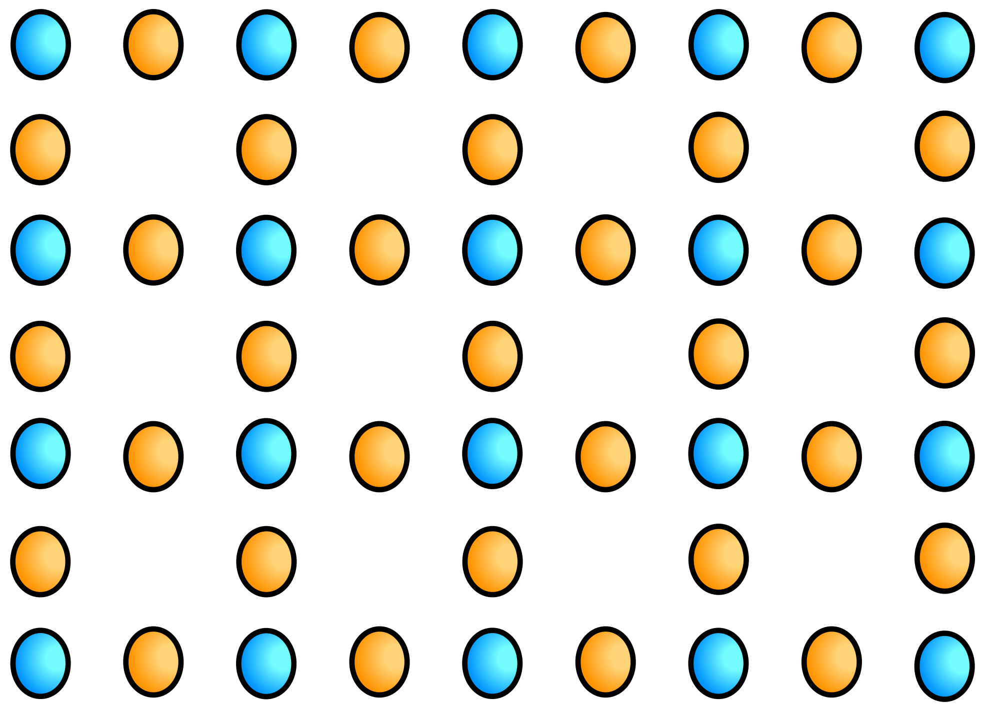
"Holographic interplay"

From cluster states to Euclidean path integral of simulated models

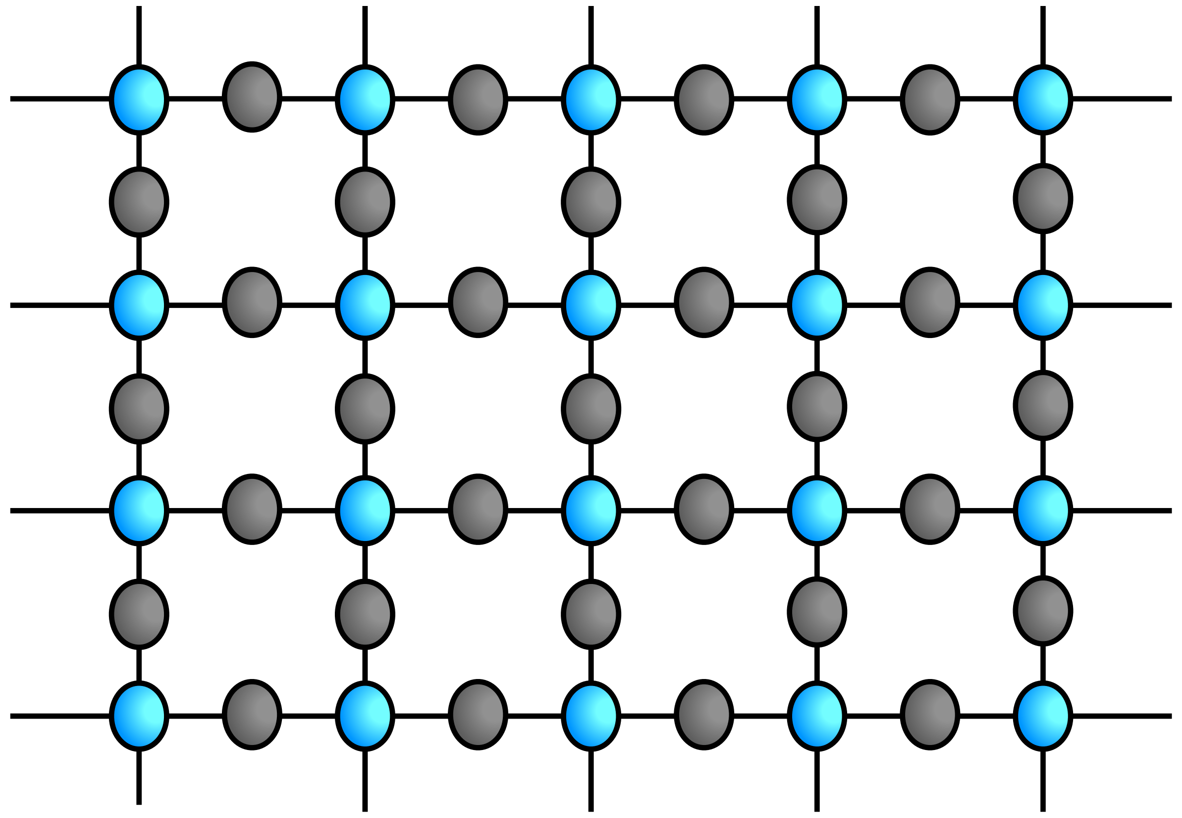
Strange correlator

Our MBQS measurement pattern is related to the *overlap formula* below:

$$Z_{(2,1)} = \mathcal{N} \times \Big\langle$$



$$\Big|$$



$$\Big\rangle$$

2d ***classical*** Ising partition function

$\langle 0 | e^{-KX}$

$\langle + |$

(K : real)

$\mathrm{gCS}_{(2,1)}$

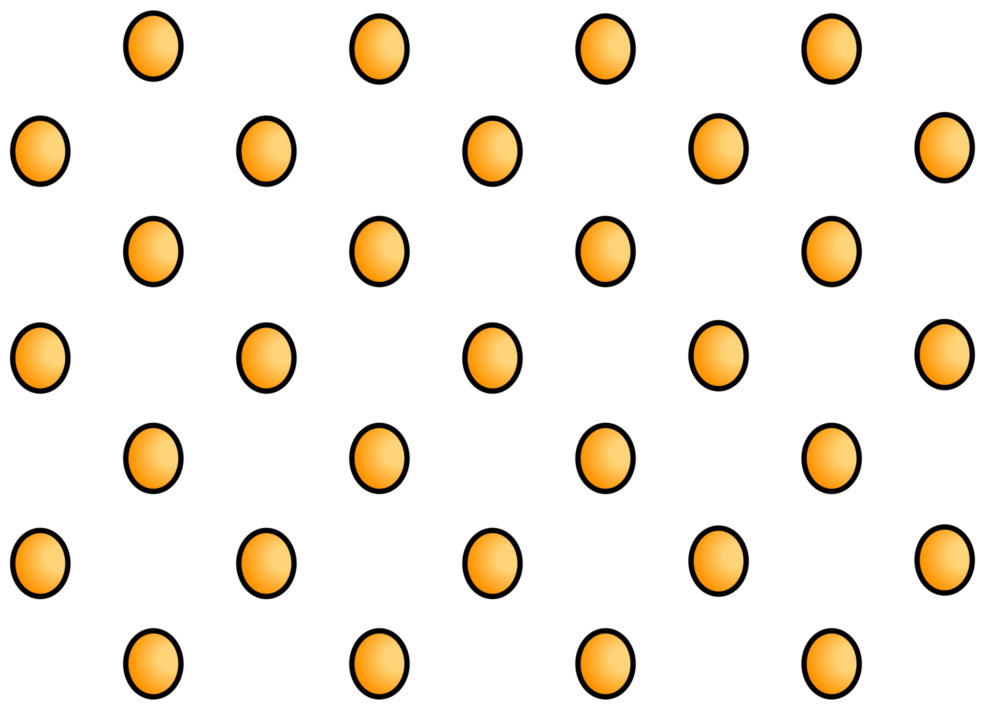
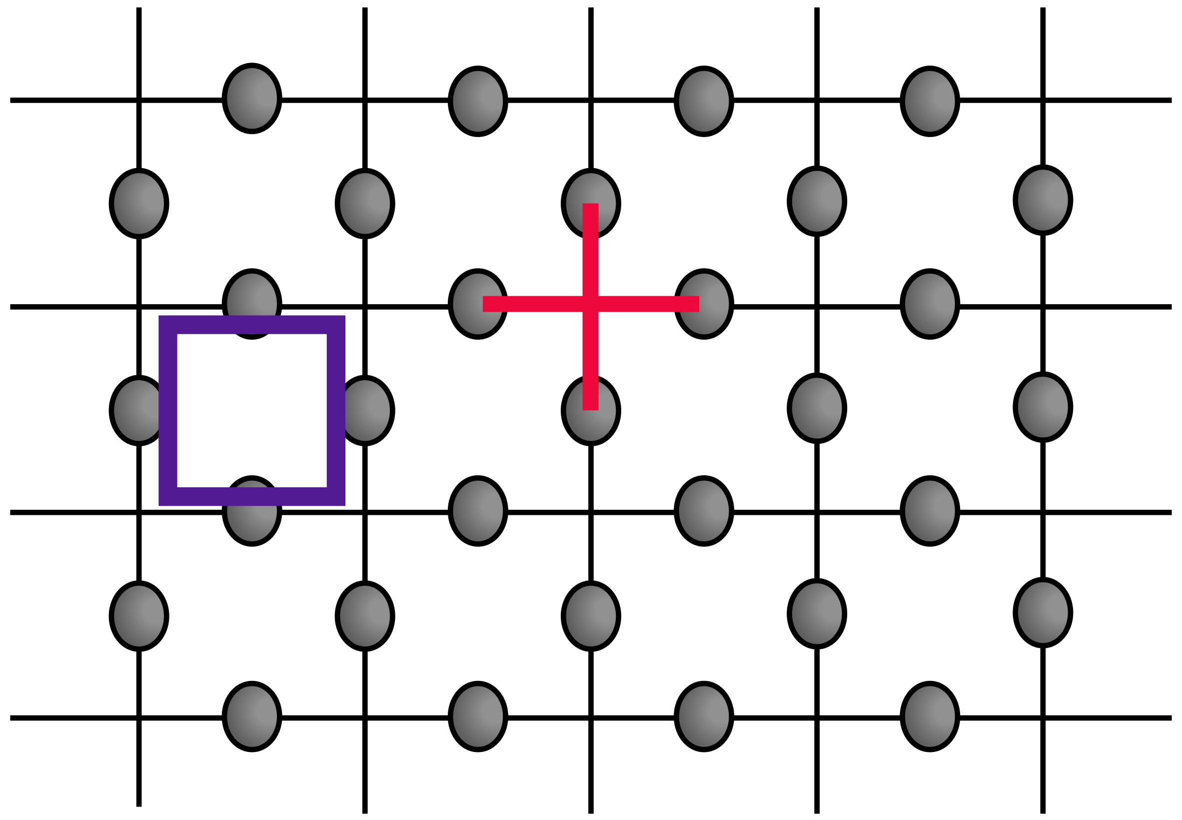
Resource state for (1+1)d transverse-field Ising model

It is known as a classical-quantum correspondence [Van den Nest-Dur-Briegel (2008)] relating a 2d quantum state and a 2d classical statistical model.

See also [Lee-Ji-Bi-Fisher (2022)]

Strange correlator

Rewriting it further,

$Z_{(2,1)} = \mathcal{N} \times \Big\langle$  $\Big|$  $\Big\rangle$

2d ***classical*** Ising partition function

● $\langle 0 | e^{-KX}$

(K : real)

Toric code

= partially "measuring" out $\mathrm{gCS}_{(2,1)}$

This generalizes to between $Z_{(d,n)}$ (Wegner's model) and the generalized toric code ground state in d-dimensions.

A map from TQFT_{d+1} state to a d-dim classical spin system; *strange correlator*.

[M. Bal et al. (2018), Chen et al. (2022) etc.] [More generalizations in Aswin's talk on Friday]

Generalization

Generalizations [Okuda-Parayil Mana-HS, *to appear*]

- Local lattice CSS code $S_X = \{A_\alpha\}_{\alpha=1,\ldots,|S_X|}$ (made of X), $S_Z = \{B_\beta\}_{\beta=1,\ldots,|S_Z|}$ (made of Z).

  $\longrightarrow$ *foliated* graph (cluster) state $|\psi_{\mathscr{C}}\rangle$ [Bolt et al. (2016)]

- Consider a Hamiltonian

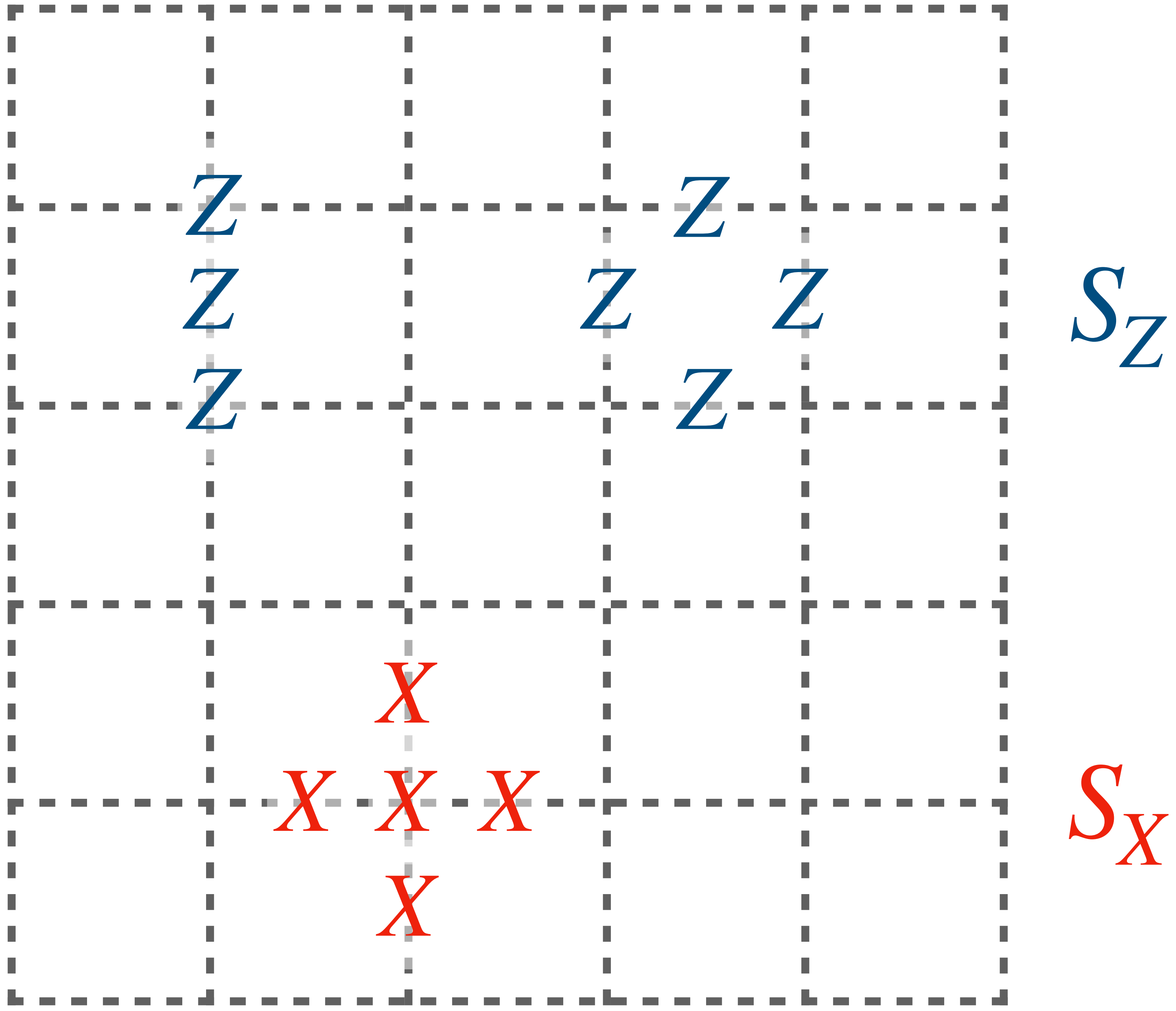
$$H = -\sum_{i\in\text{qubits}} X_i - \lambda \sum_{\beta} B_\beta \quad \text{with a gauge constraint } A_\alpha = 1\ .$$

- Time evolution $U(t) = e^{-itH}$ can be simulated on the state $|\psi_{\mathscr{C}}\rangle$ by measurements.
- When the logical code space of the (local) CSS code on a lattice is non-trivial, it can be seen as having a mixed 't Hooft anomaly in the ground states.
- Accordingly, the foliated cluster state has an SPT order. The global symmetries that protect the bulk SPT order can be higher-form or subsystem-like.
- A CSS generalization of the Kramers-Wannier-Wegner duality is obtained for statistical models corresponding to CSS codes via the overlap formula etc.

Fradkin-Shenker model [Okuda-Parayil Mana-HS, *to appear*]

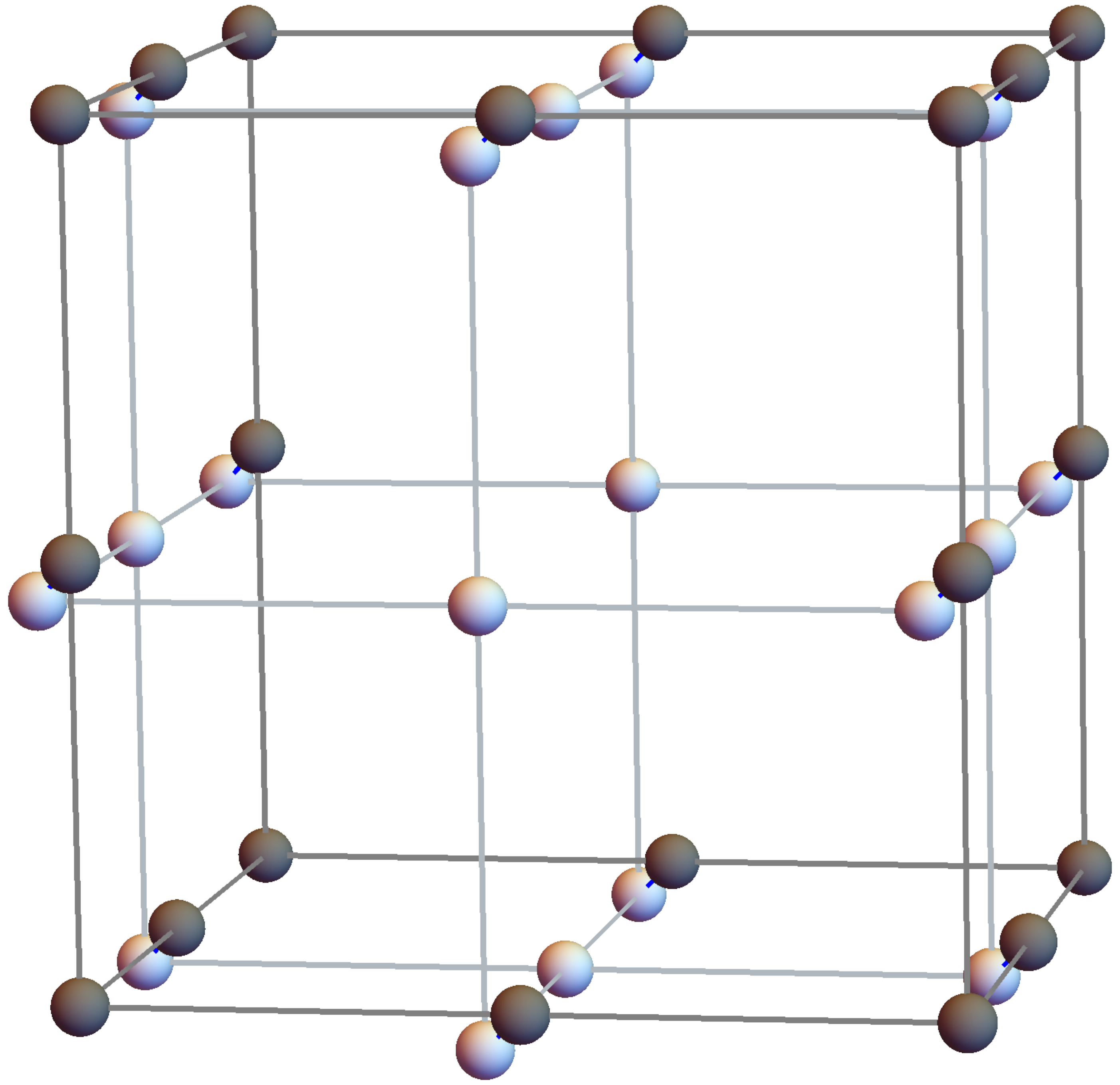
$\mathbb{Z}_2$ gauge theory coupled to Ising matter field in (2+1)d

$$H = -\sum_{i\in\text{site}} X_i - \sum_{i\in\text{link}} X_i - \lambda \sum \begin{matrix} Z \\ Z \\ Z \end{matrix} - g \sum \begin{matrix} & Z & \\ Z & & Z \\ & Z & \end{matrix} \quad \text{with a gauge constraint} \quad \begin{matrix} & X & \\ X & X & X \\ & X & \end{matrix} = 1\,.$$



Foliation of CSS code

$\longrightarrow$



$|\psi_{\mathscr{C}}^{\text{FS}}\rangle$

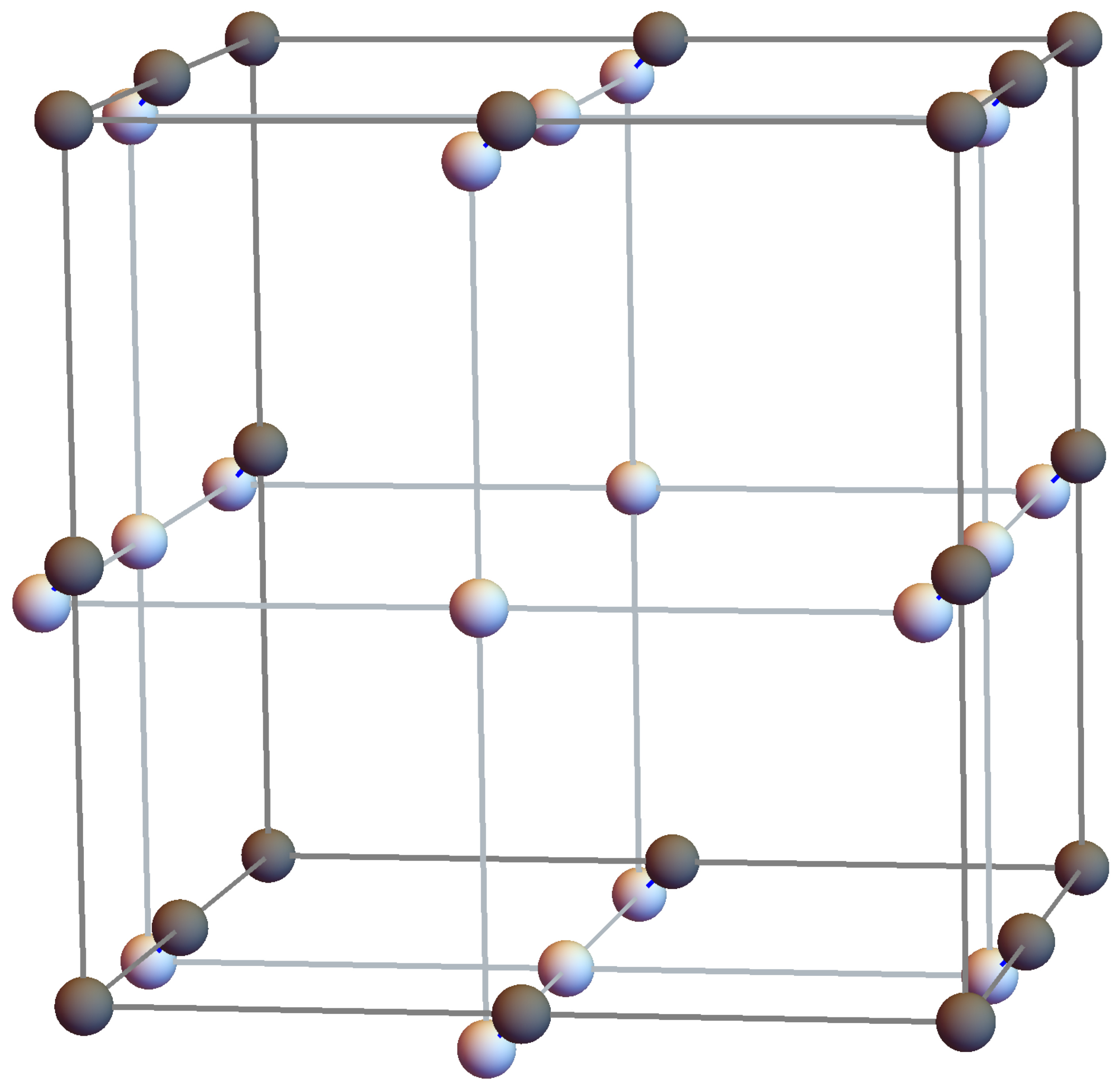
Fradkin-Shenker model [Okuda-Parayil Mana-HS, *to appear*]

- By measuring the foliated cluster state $|\psi_{\mathscr{C}}^{\mathrm{FS}}\rangle$, one can perform the quantum simulation of the Fradkin-Shenker model.
- Overlap formula:

$$Z_{\mathrm{FS}}(J,K) = \langle \Omega(J,K) | \psi_{\mathscr{C}}^{\mathrm{FS}} \rangle$$

$Z_{\mathrm{FS}}(J,K)$: the Euclidean lattice path integral of the FS model.
$\langle \Omega(J,K) |$: a product state.
(J,K) : coupling parameters

- One can reproduce the self-duality of the FS model by manipulating stabilizers of $|\psi_{\mathscr{C}}^{\mathrm{FS}}\rangle$. Cf. [van den Nest-Dur-Briegel (2007)]



Summary

- Entanglement structure of $\mathrm{gCS}_{(d,n)}$ $\rightleftarrows$ Spacetime structure of $M_{(d,n)}$.

- Single-qubit measurement on $\mathrm{gCS}_{(d,n)}$ $\rightleftarrows$ Hamiltonian quantum simulation of $M_{(d,n)}$.

- Overlap between a product state and $\mathrm{gCS}_{(d,n)}$ $\rightleftarrows$ Partition function of $M_{(d,n)}$.

- The gCS possesses $(n-1)$- and $(d-n)$-form global symmetries.
    1. A state of $M_{(d,n)}$ as an edge state of an SPT
    2. Boundary $(n-1)$-from symmetry is promoted to bulk $(n-1)$-from symmetry, which is gauged in gCS.

Recipe

- $Z_{\text{simulated}} = \langle \Omega(K) | \psi_{\text{resource}} \rangle$: *Euclidean* path integral.
- Projection to $\Omega(K)$ requires post-selections.
- Rotate $\Omega(K)$ by $K \to iK$.
- $\Omega(K)$ becomes one of the vectors in a "good" measurement basis.
- Measurement on $|\psi_{\text{resource}}\rangle$ and feedforward
- $\to$ Deterministic *real-time* evolution with $H_{\text{simulated}}$.

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SPT in gCS

[Chen-Gu-Wen]

- A state has a **long-range entanglement** iff it is not short-range entangled.
- A state $|\Phi\rangle$ has a **short-range entanglement** iff there is (finite-depth) local unitary evolution such that $|\Phi\rangle = U|\Phi_{\text{prod}}\rangle$

Long-range entangled state

Product state

SPT in gCS

[Chen-Gu-Wen]

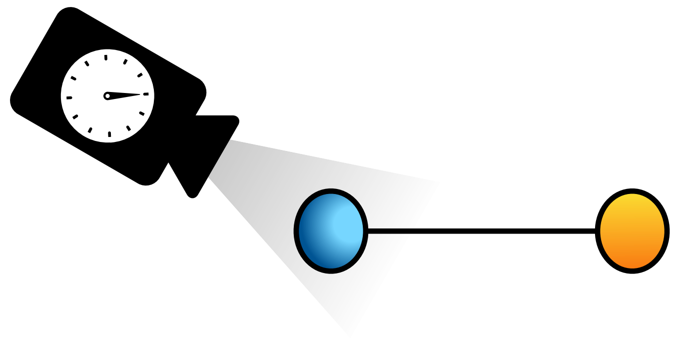
- A state has a **nontrivial SPT order** if it is SRE and it is not a trivial SPT.
- A symmetric state $|\Phi\rangle$ has a **trivial SPT order** with respect to a symmetry G iff there is (finite-depth) symmetric local unitary evolution such that $|\Phi\rangle = U_{\text{sym}}|\Phi_{\text{prod}}\rangle$

SPT ordered state

Symmetric-SRE

Product state

MBQC: (0+1)dimensions



- Consider a one-qubit "initial state" $|\psi\rangle$
- Prepare a "resource state" $CZ_{a,b}|\psi\rangle_a|+\rangle_b$
- Measure the first qubit with the basis $\{e^{i\xi Z}|+\rangle, e^{i\xi Z}|-\rangle\}$, i.e., $Z^s e^{i\xi Z}|+\rangle \quad (s=0,1)$

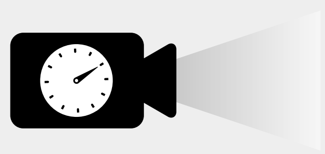
$$\langle +|_a e^{-i\xi Z_a} Z_a^s \cdot CZ_{b,a}|\psi\rangle_a|+\rangle_b = \frac{1}{\sqrt{2}} e^{-i\xi X} X^s H|\psi\rangle_b$$

$\rightarrow$ **Teleportation & rotation.**

Notation:

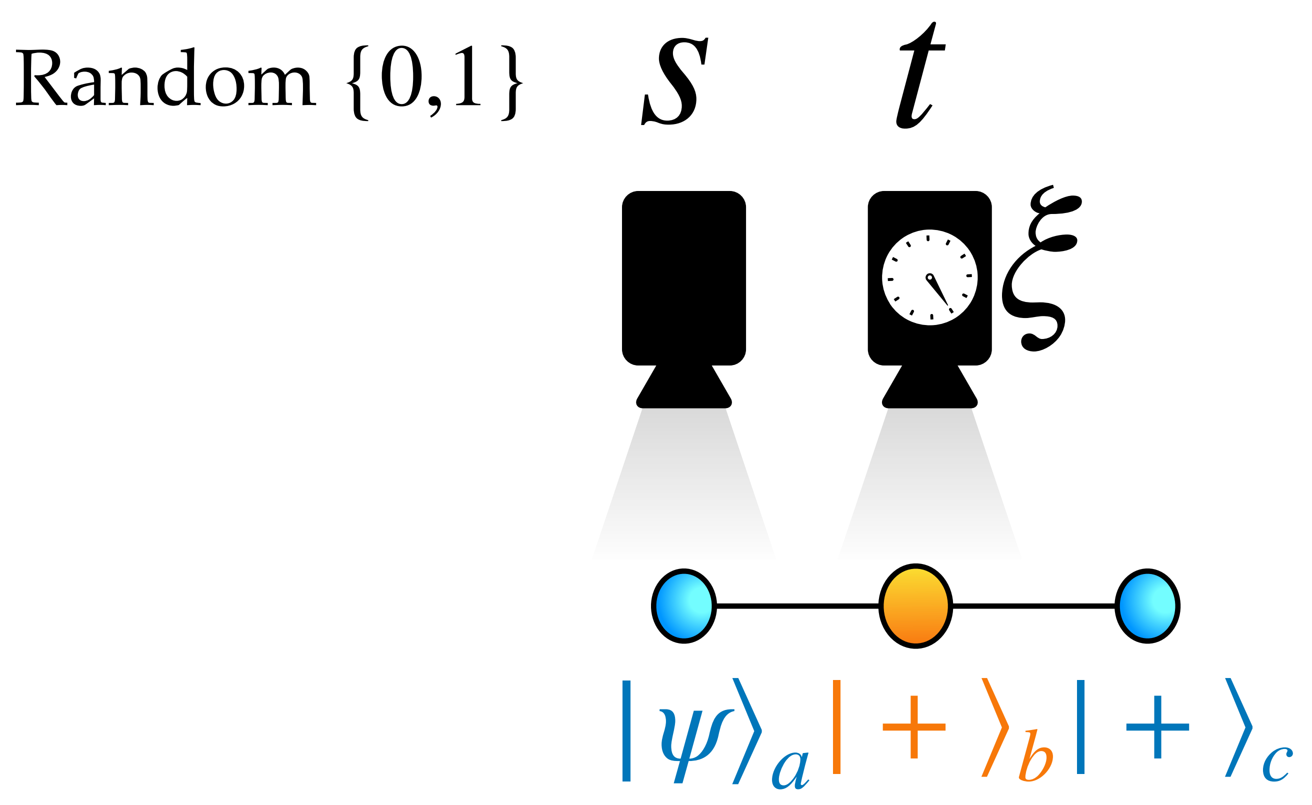
 $|+\rangle$

—— $CZ_{a,b} := |0\rangle\langle 0|_a \otimes I_b + |1\rangle\langle 1|_a \otimes Z_b = CZ_{b,a}$

 Single-qubit measurement

H : Hadamard transform, $H|0\rangle = |+\rangle, \quad H|1\rangle = |-\rangle, \quad HZH = X, \quad H^2 = I.$

MBQC: (0+1)dimensions



- Consider a one-qubit "initial state" $|\psi\rangle$
- Prepare a "resource state" $CZ_{a,b}CZ_{b,c}|\psi\rangle_a|+\rangle_b|+\rangle_c$
- Measure: the a qubit with the basis $Z^s|+\rangle \quad (s=0,1)$
- Measure: the b qubit with the basis $Z^t e^{i\xi Z}|+\rangle \quad (t=0,1)$

Notation:

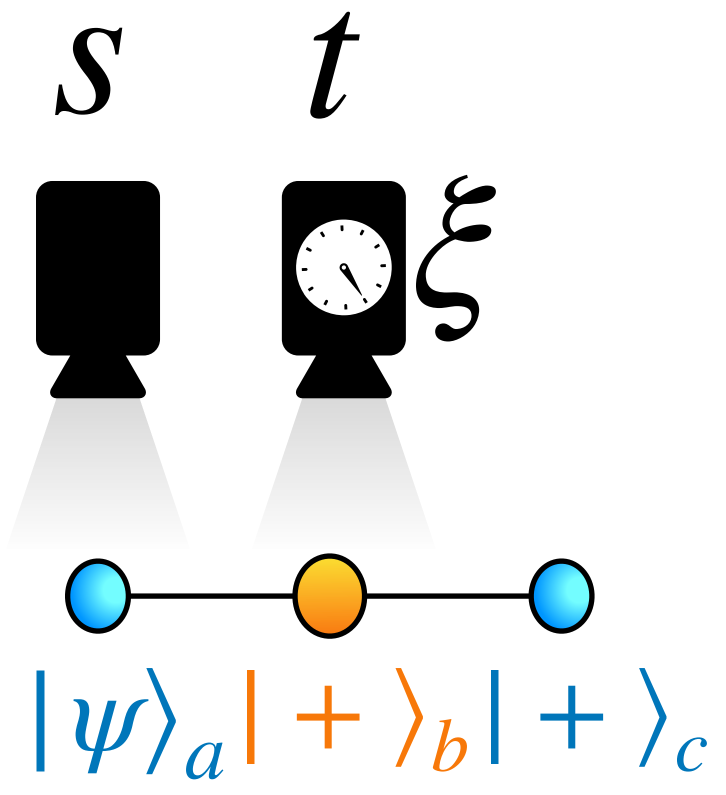
● $|+\rangle$

— $CZ_{a,b} := |0\rangle\langle 0|_a \otimes I_b + |1\rangle\langle 1|_a \otimes Z_b = CZ_{b,a}$

Single-qubit measurement

MBQC: (0+1)dimensions

Random {0,1}



- Consider a one-qubit "initial state" $|\psi\rangle$
- Prepare a "resource state" $CZ_{a,b}CZ_{b,c}|\psi\rangle_a|+\rangle_b|+\rangle_c$
- Measure: the a qubit with the basis $Z^s|+\rangle \quad (s=0,1)$
- Measure: the b qubit with the basis $Z^t e^{i\xi Z}|+\rangle \quad (t=0,1)$

$$\left[\langle +|_a Z_a^s \otimes \langle +|_b e^{-i\xi Z_b} Z_b^t\right] \times \left[CZ_{a,b}CZ_{b,c}|\psi\rangle_a|+\rangle_b\,|+\rangle_c\right] \propto X^t Z^s \cdot e^{-i(-1)^s \xi X}|\psi\rangle_c$$

Teleportation to the c qubit & rotation.

The rotation angle depends on the outcome of the first measurement s.

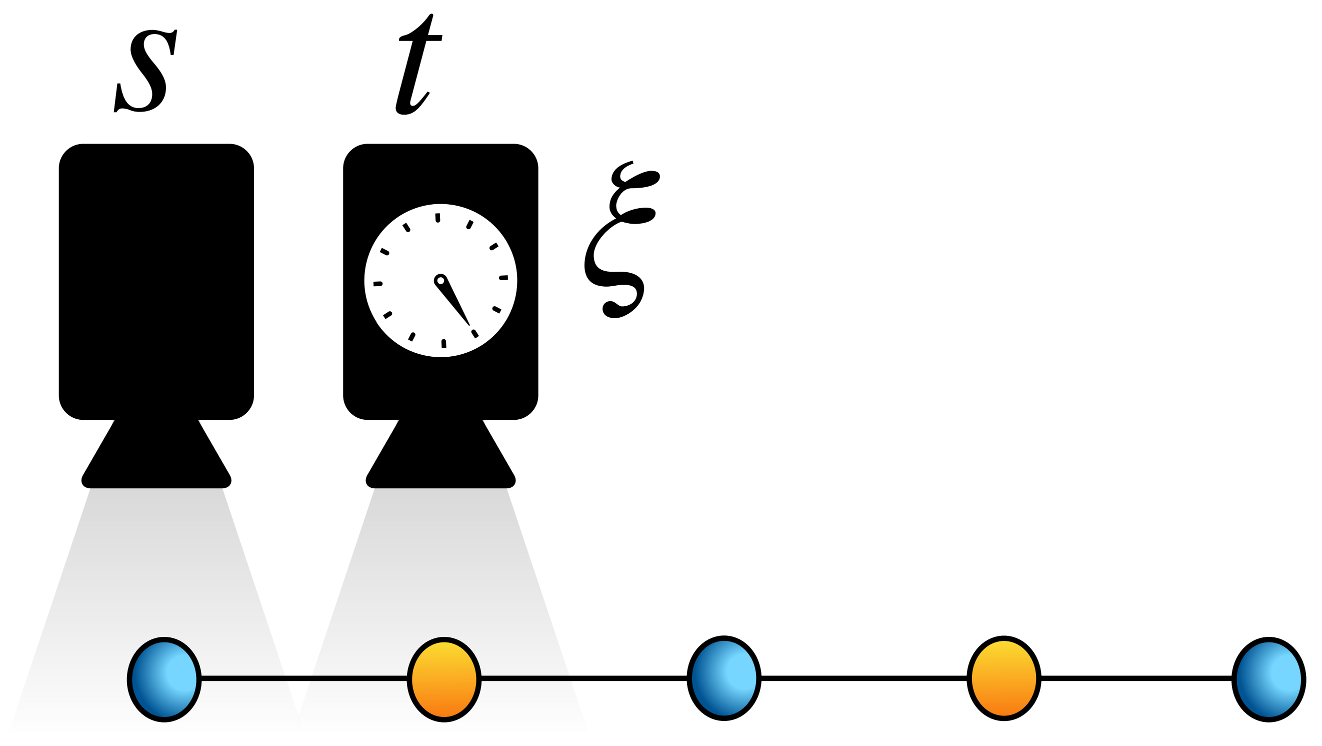
Notation:

○ $|+\rangle$

— $CZ_{a,b} := |0\rangle\langle 0|_a \otimes I_b + |1\rangle\langle 1|_a \otimes Z_b = CZ_{b,a}$

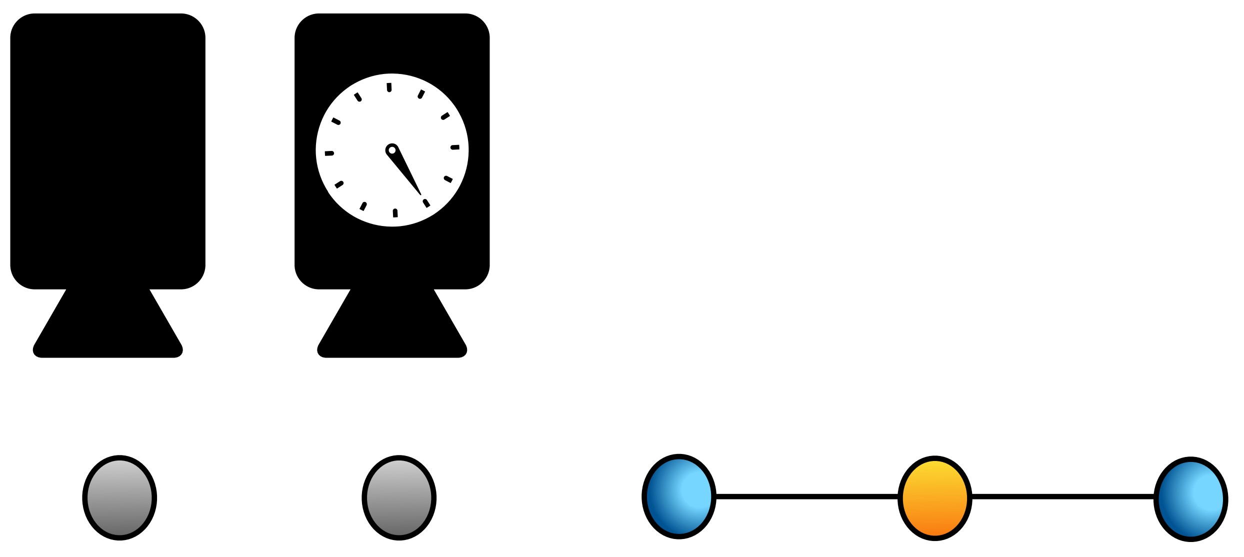
Single-qubit measurement

MBQC: (0+1)dimensions

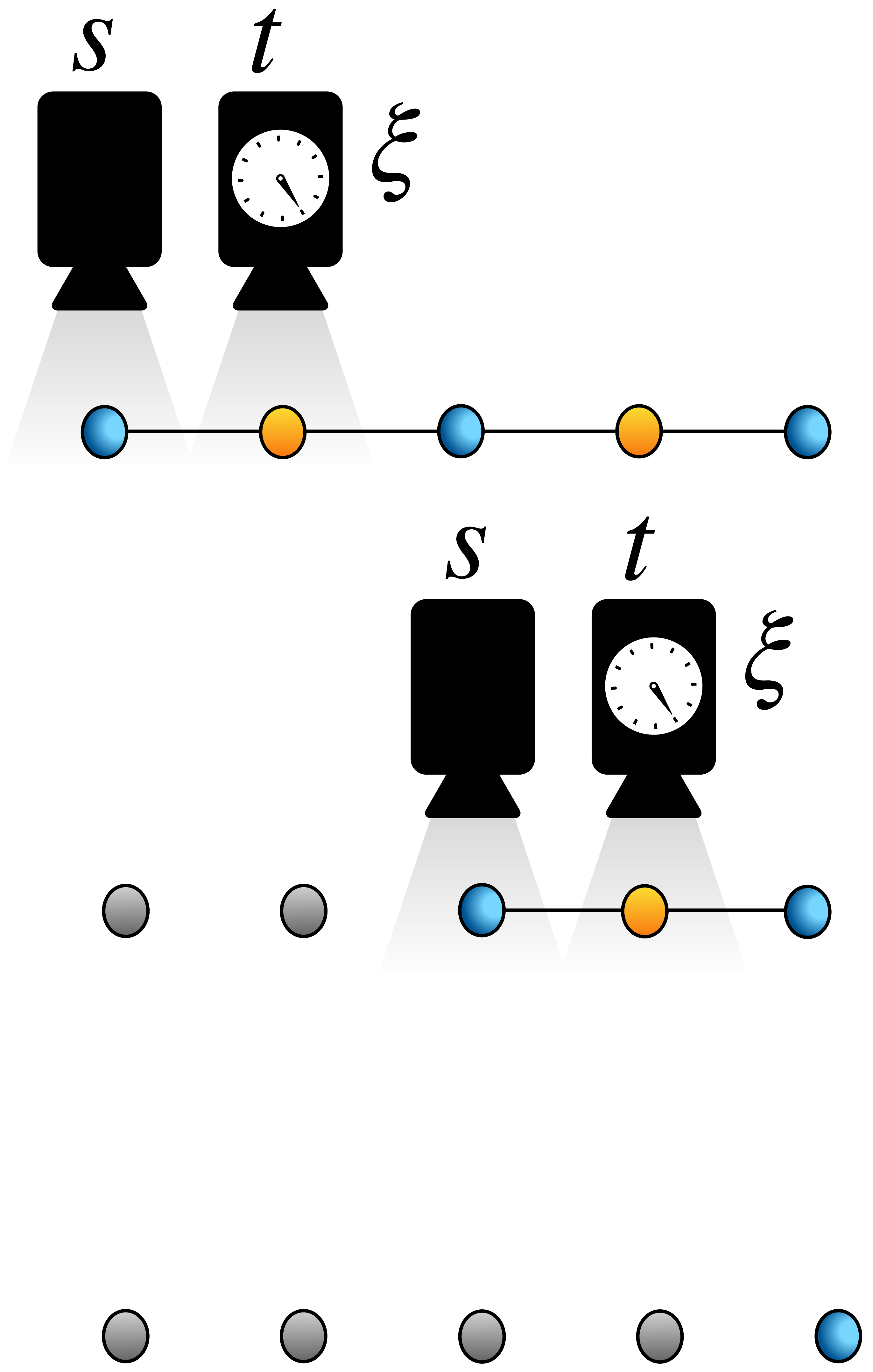


$$X^t Z^s \cdot e^{-i(-1)^s \xi X} |\psi\rangle \to X^t Z^s \cdot e^{-i\alpha X} |\psi\rangle$$

by choosing $\xi = (-1)^s \alpha$. (α : desired angle)



MBQC: (0+1)dimensions



$X^t Z^s \cdot e^{-i(-1)^s \xi X} | \psi \rangle \to X^t Z^s \cdot e^{-i\alpha X} | \psi \rangle$

by choosing $\xi = (-1)^s \alpha$. (α : desired angle)

One can choose angles $\{\xi_k\}$ adaptively to absorb effects from previous measurements.

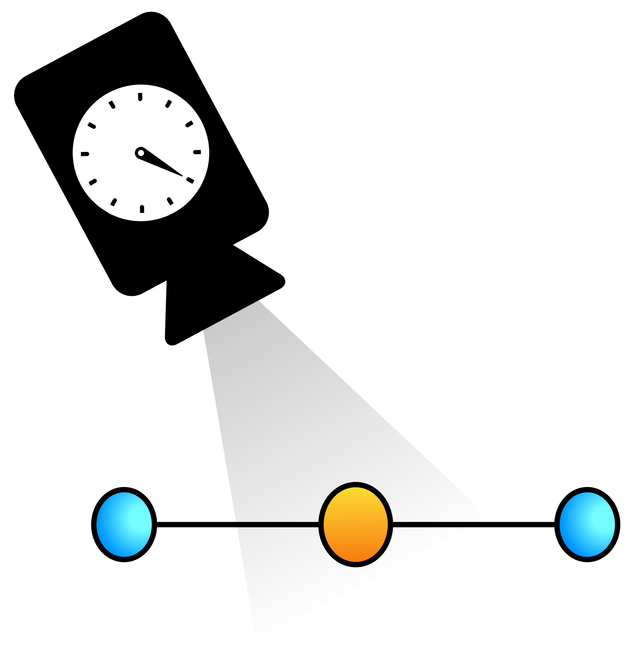
After measuring all qubits except the last one, we will be left with

$$X^{\#} Z^{\#} e^{-i\alpha_k X} \cdots e^{-i\alpha_2 X} e^{-i\alpha_1 X} | \psi \rangle_{\text{right bdry}}$$

can be removed

MBQC: multi-body interaction term

We make use of the following ingredient.



- Consider a general "initial state" $|\psi\rangle_{bc}$
- Prepare a "resource state" $CZ_{a,b}CZ_{a,c}|\psi\rangle_{bc}|+\rangle_a$
- Measure the middle qubit with $\{e^{i\xi X}|0\rangle, e^{i\xi X}|1\rangle\}$, i.e., $X^s e^{i\xi X}|0\rangle \quad (s=0,1)$

$$\langle 0|_a e^{-i\xi X_a} X_a^s \cdot CZ_{a,b}CZ_{a,c}|\psi\rangle_{bc}|+\rangle_a = e^{-i\xi Z_b Z_c}(Z_b Z_c)^s|\psi\rangle_{bc}$$

→ **Multi-qubit rotation.**