Scalable quantum simulation for topological quantum phases on noisy quantum devices

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RIKEN iTHEMS

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Outline

- Quantum simulation in NISQ era & its scalability
- Scalable quantum simulation of symmetry-protected topological (SPT) states [arXiv:**2303.17187**]
- Scalable quantum simulation of intrinsic topologically ordered (TO) states [arXiv:**2210.14662**]
- Summary

Quantum simulation for quantum many-body systems

"…Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy…"

—— *R. P. Feynman, Int. J. Theor. Phys.* ***21****, 467 (1982)*

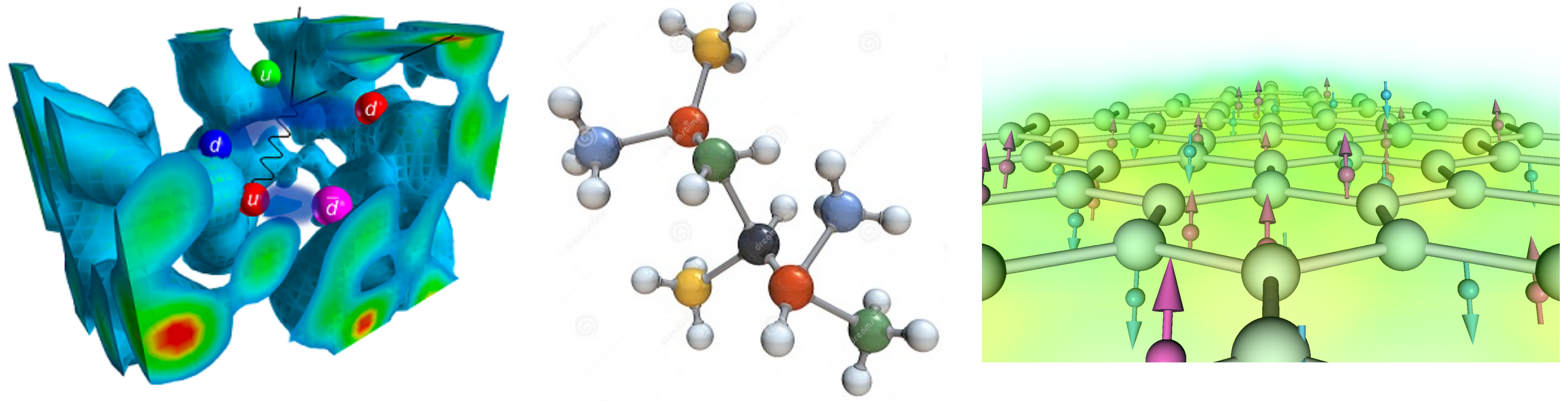
R. P. Feynman

classical computer

exponentially large
resource required

↔

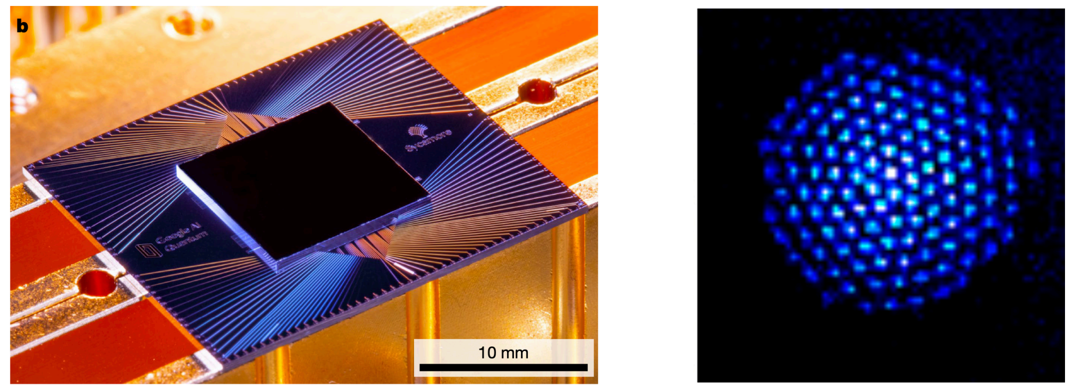
quantum many-body systems



exponentially large degrees of freedom + strong quantum entanglements

↔

other quantum systems
(quantum computer / simulator)

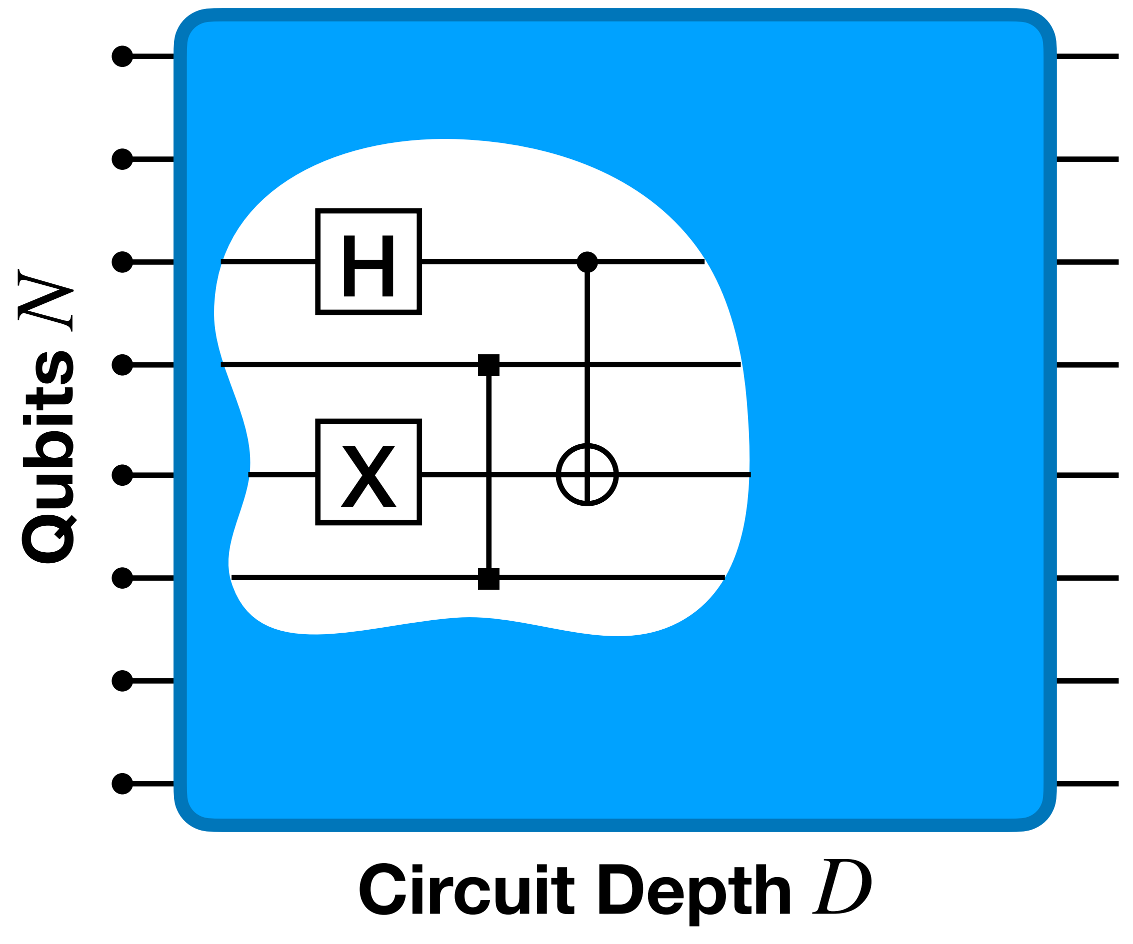


exponentially large
resource available

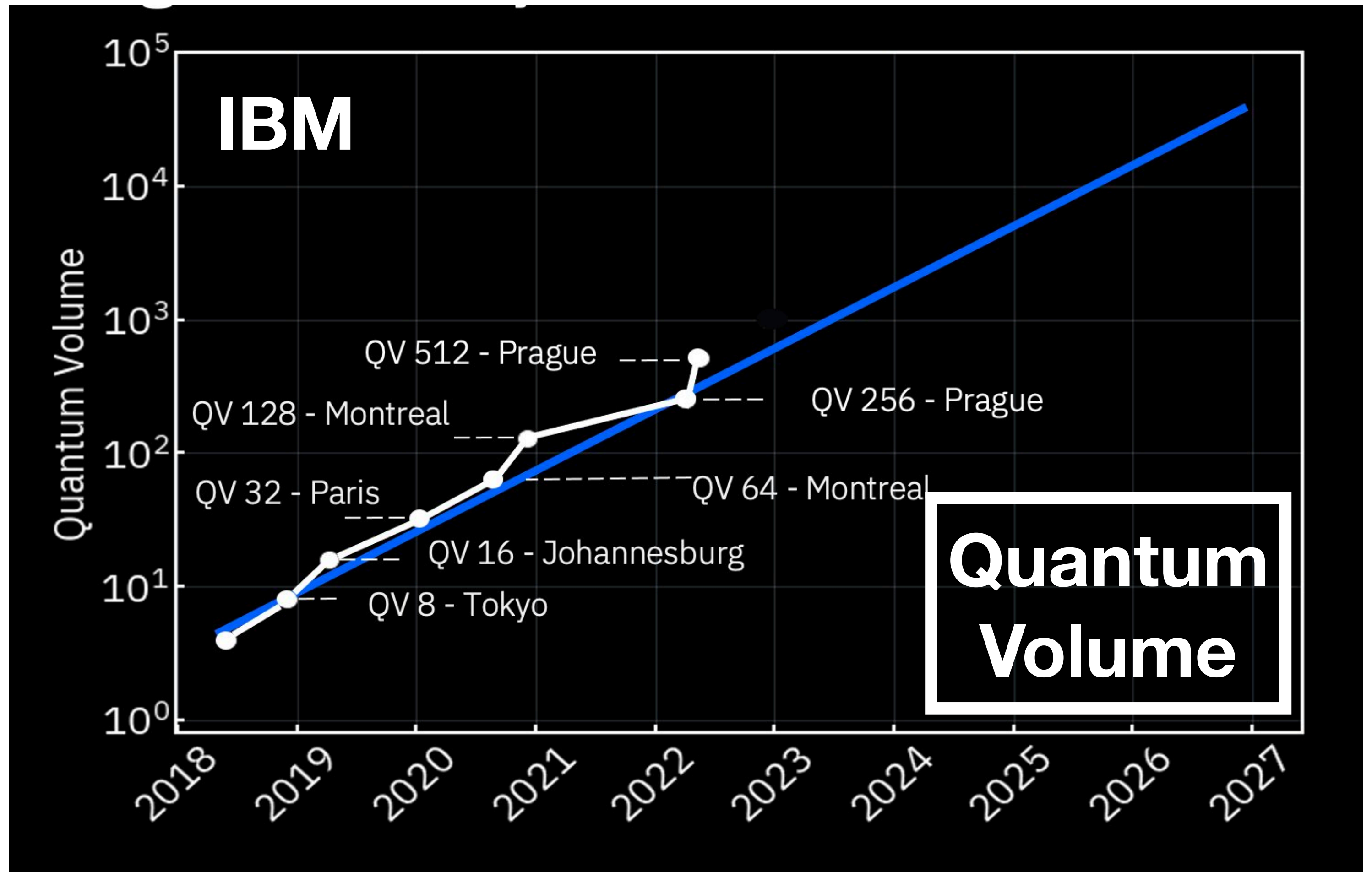
Simulating a quantum system by using other quantum systems (i.e. controllable quantum devices)

Noisy intermediate-scale quantum (NISQ) device

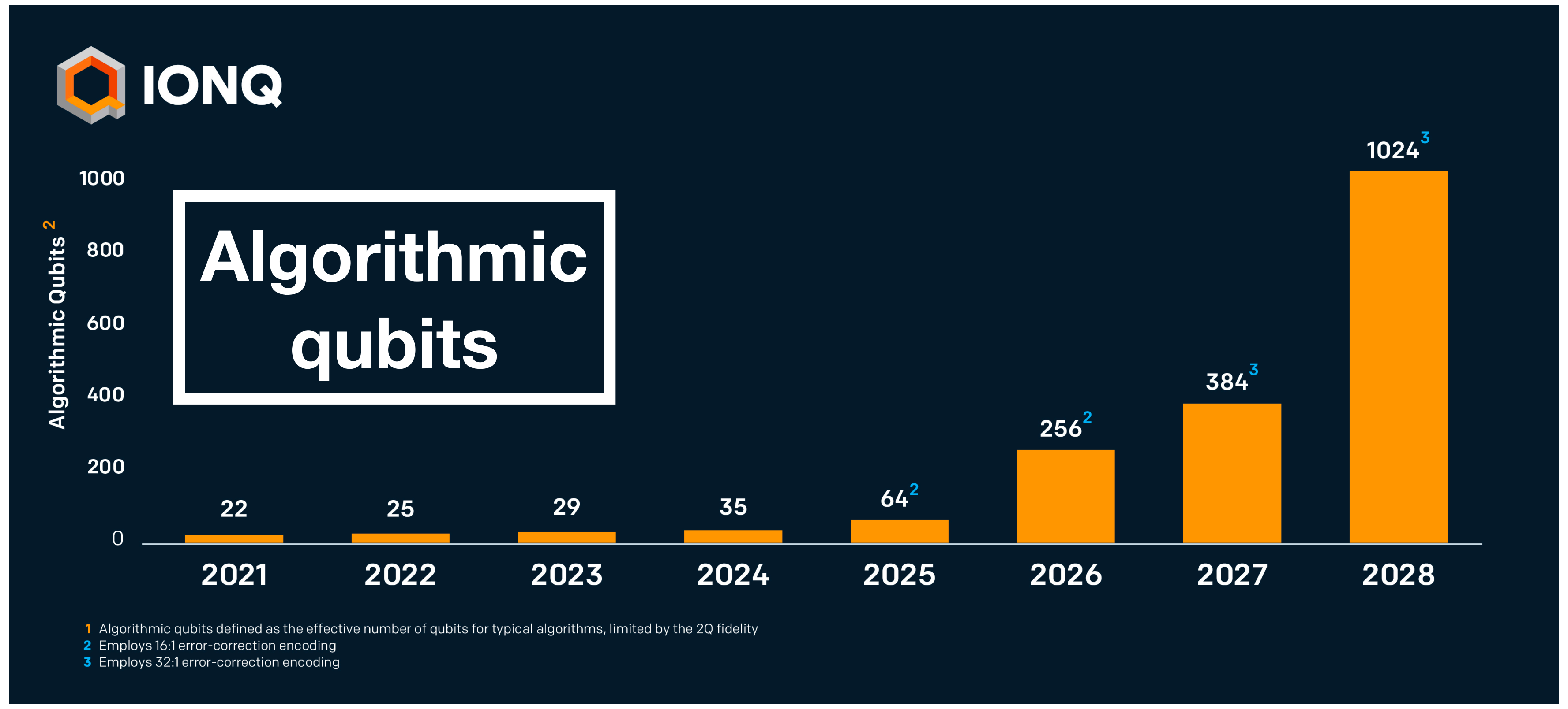
Quantum circuit



Capacity metrics of NISQ devices



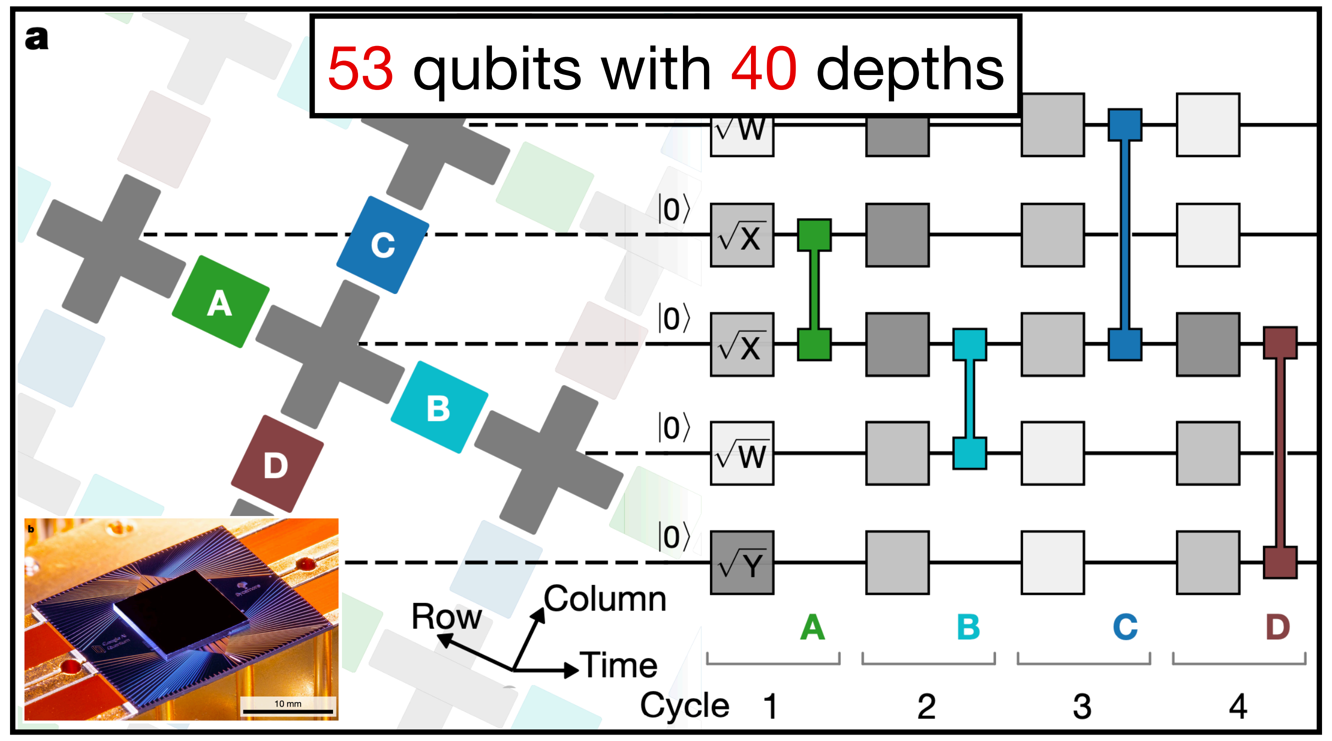
$$\log_2 V_Q = \arg\max\{\min[N, D(N)]\}$$



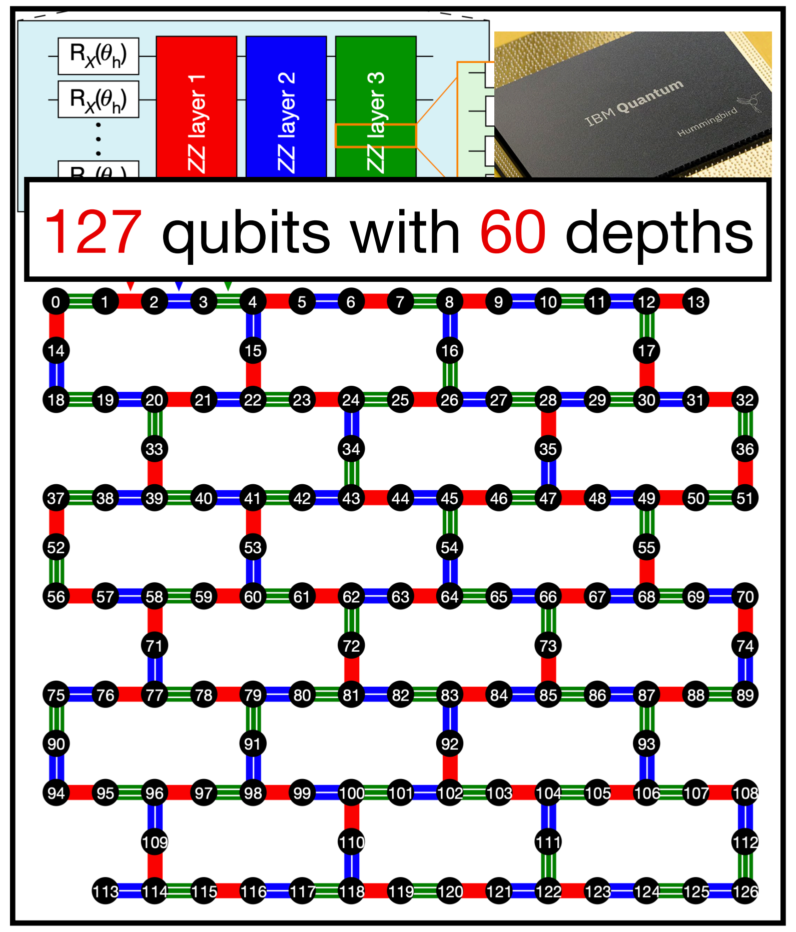
#AQ $= N$: successful performing N^2 two-site gates

Noisy intermediate-scale quantum (NISQ) device

- A few $\mathcal{O}(10^1 \sim 10^2)$ qubits without error correction
- A few $\mathcal{O}(10^0 \sim 10^1)$ circuit depths

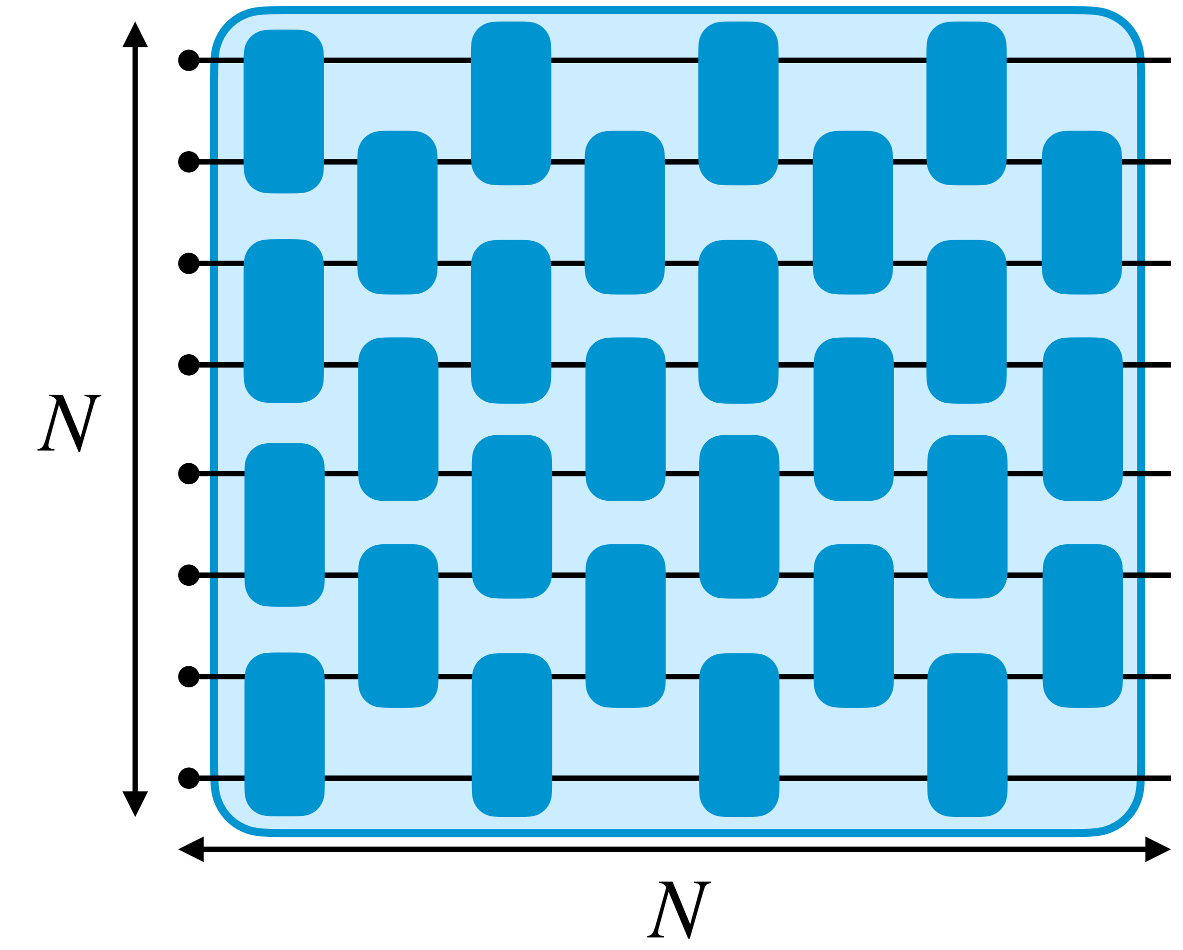


Nature **574**, 7779 (2019)



Nature **618**, 7965 (2023)

NISQ device scalability



Variational quantum eigensolver (VQE) algorithm

VQE & its scalabilities

$$\hat{H}|\Psi_G\rangle = E_G|\Psi_G\rangle$$

quantum computer

Parametrized quantum circuit (PQC)

variational quantum state:
PQC *Ansatz*

$$\Psi(\{\theta_i\})\rangle = \hat{U}(\{\theta_i\})|0\rangle =$$

unitary operator

$|0\rangle$, $|0\rangle$, $|0\rangle$, $|0\rangle$, $|0\rangle$ → $\hat{U}(\{\theta_i\})$

expectation value of energy

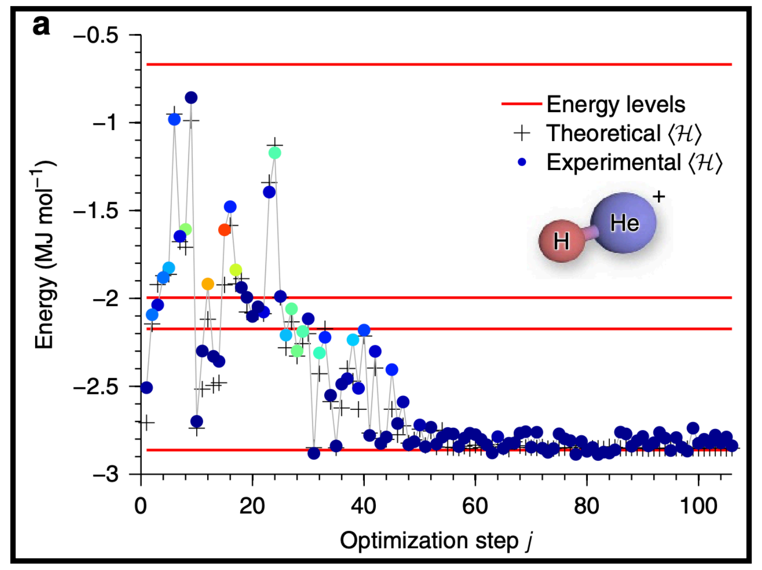
$$E(\{\theta_i\}) = \sum_k \langle \hat{H}_k \rangle_{\{\theta_i\}}$$

classical computer

optimization of variational parameters $\{\theta_i\}$

$$\theta_i \leftarrow \theta_i - \lambda \partial E(\{\theta_i\}) / \partial \theta_i$$

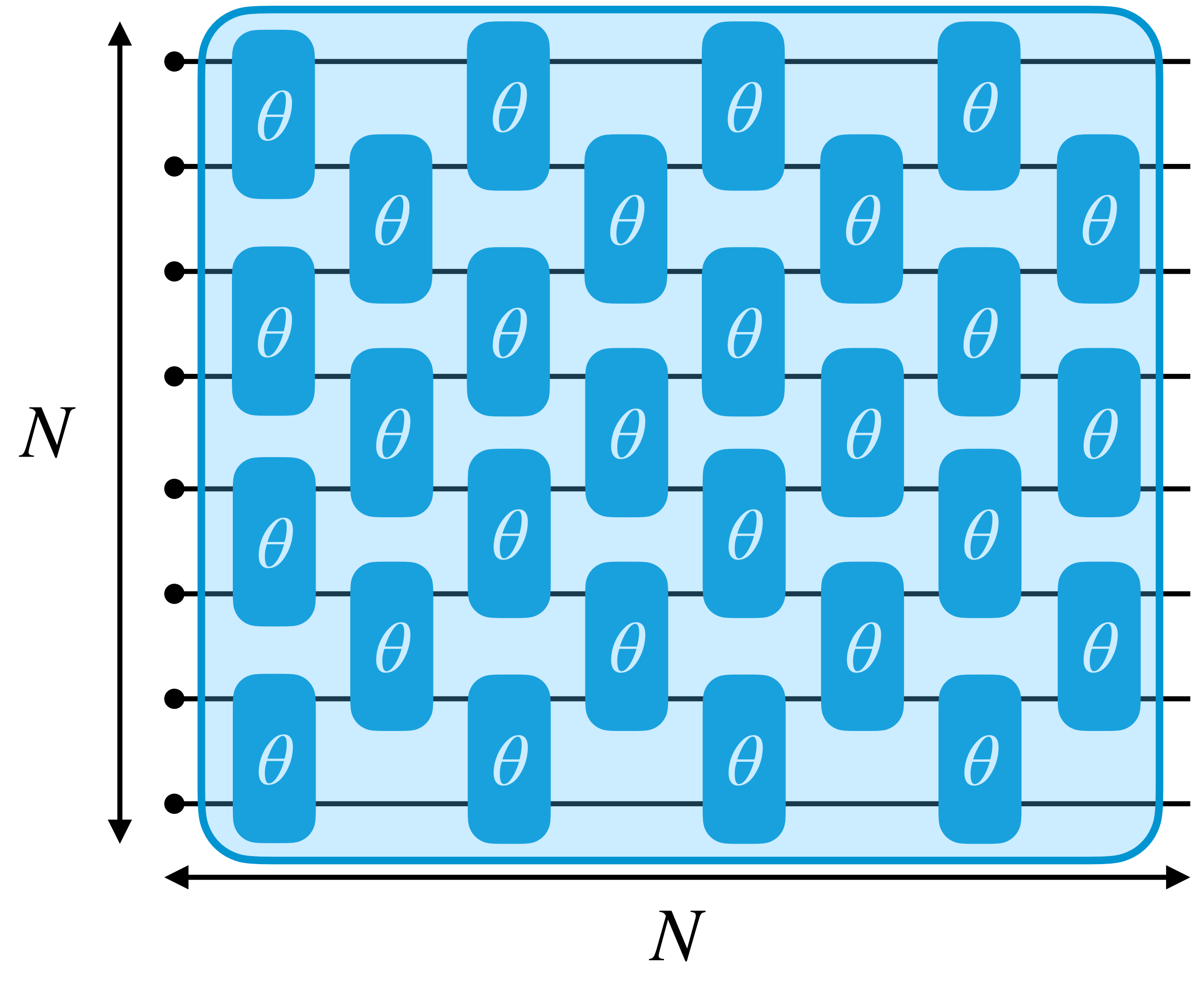
Gradient Descent, SPSA, COBYLA, …



$$|\Psi(\{\theta_i\}_{\text{opt}})\rangle \approx |\Psi_G\rangle$$

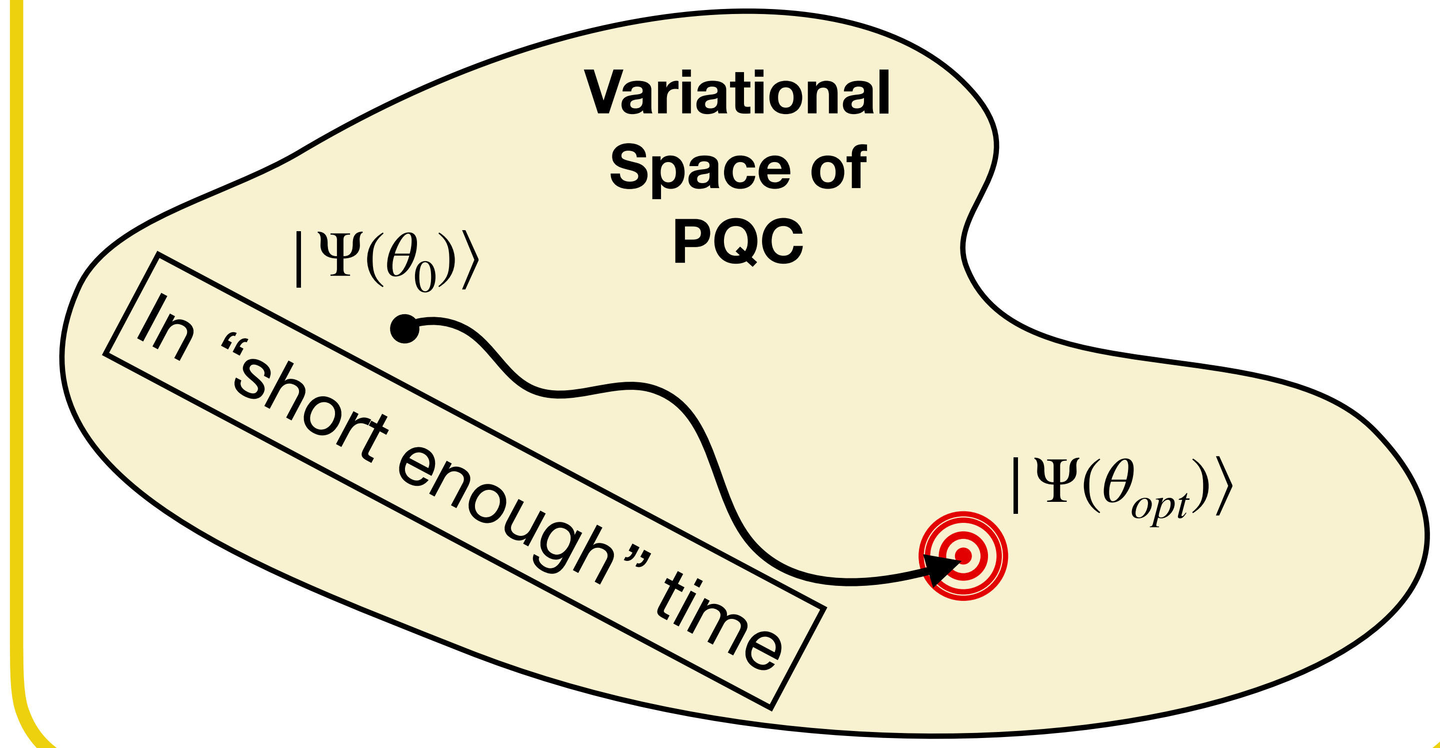
Peruzzo et. al., Nat. Commun. **5**, *4213 (2014)*

Scalable PQC *Ansatz*



Volume of PQC $\leq O(N^2)$

Scalable classical optimizer



Scalable quantum simulation of symmetry-protected topological states

R.-Y. Sun, *T. Shirakawa, and S. Yunoki, Phys. Rev. B* ***108****, 075127 (2023)* [arXiv:**2303.17187**]

Scalable PQC *Ansatz* for topological phases

Quantum circuit definition of topological phases

Topological phases: Symmetry-protected topological (SPT) state & intrinsic topological order (TO)

PHYSICAL REVIEW B **82**, 155138 (2010)

Local unitary transformation, long-range quantum entanglement, wave function renormalization, and topological order

Xie Chen,[1] Zheng-Cheng Gu,[2] and Xiao-Gang Wen[1]
[1]*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
[2]*Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, USA*
(Received 28 July 2010; revised manuscript received 21 September 2010; published 26 October 2010)

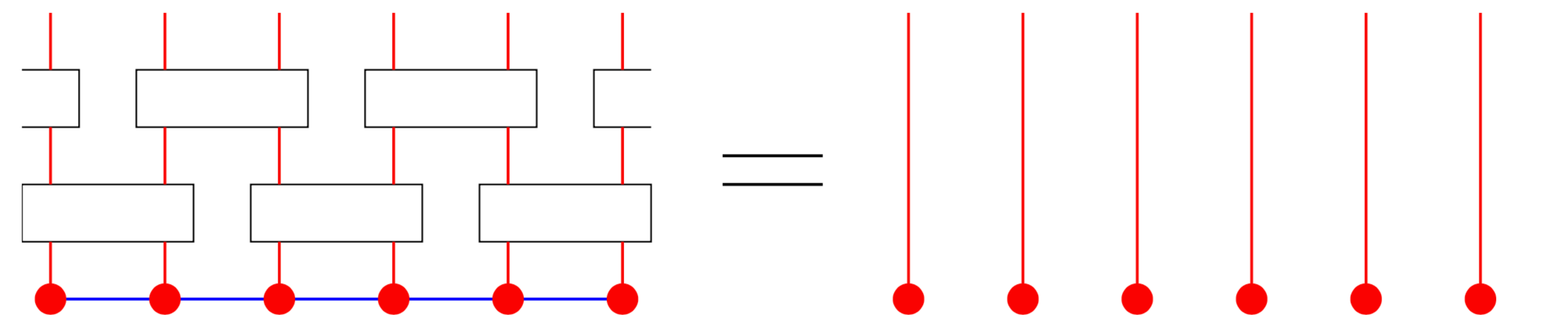
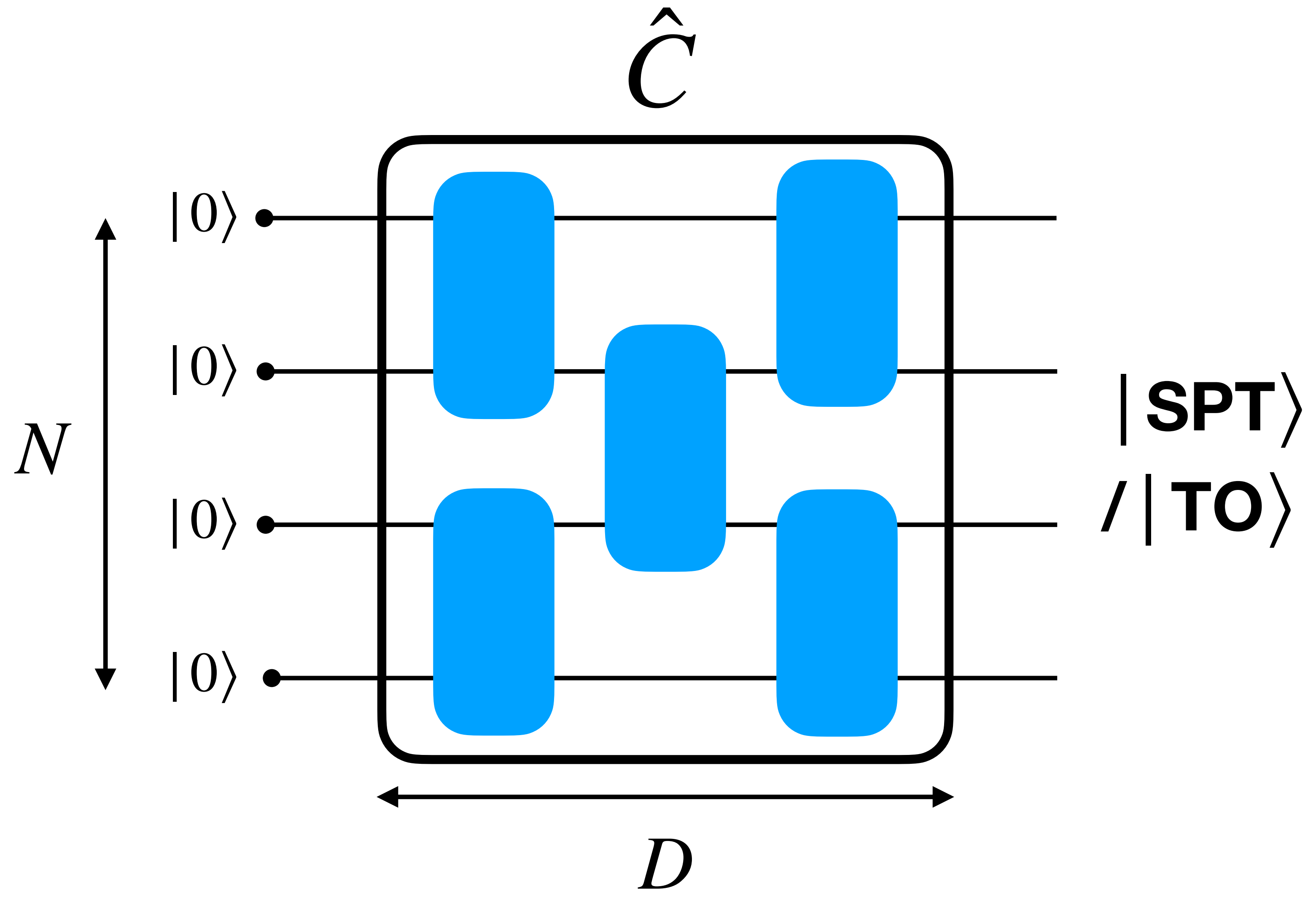


FIG. 4. (Color online) A finite depth quantum circuit can transform a state $|\Phi\rangle$ into a direct-product state, if and only if the state $|\Phi\rangle$ has no long-range quantum entanglement. Here, a dot repre-

would like to show that *two gapped states* $|\Phi(0)\rangle$ and $|\Phi(1)\rangle$ are in the same phase, if and only if they are related by a local unitary (LU) evolution. We define a LU evolution as a



SPT (short-range entangled)

- $D \propto N$ for symmetric $\hat{C}$
- Exist constant D for unsymmetric $\hat{C}$

TO (long-range entangled)

- $D \propto N$ for any $\hat{C}$

- $\hat{C}$ is NISQ **scalable**
- In the same phase <=> connected by **finite** depth quantum circuit

Scalable PQC *Ansatz* for topological phases

Two-layer structured PQC *Ansatz*

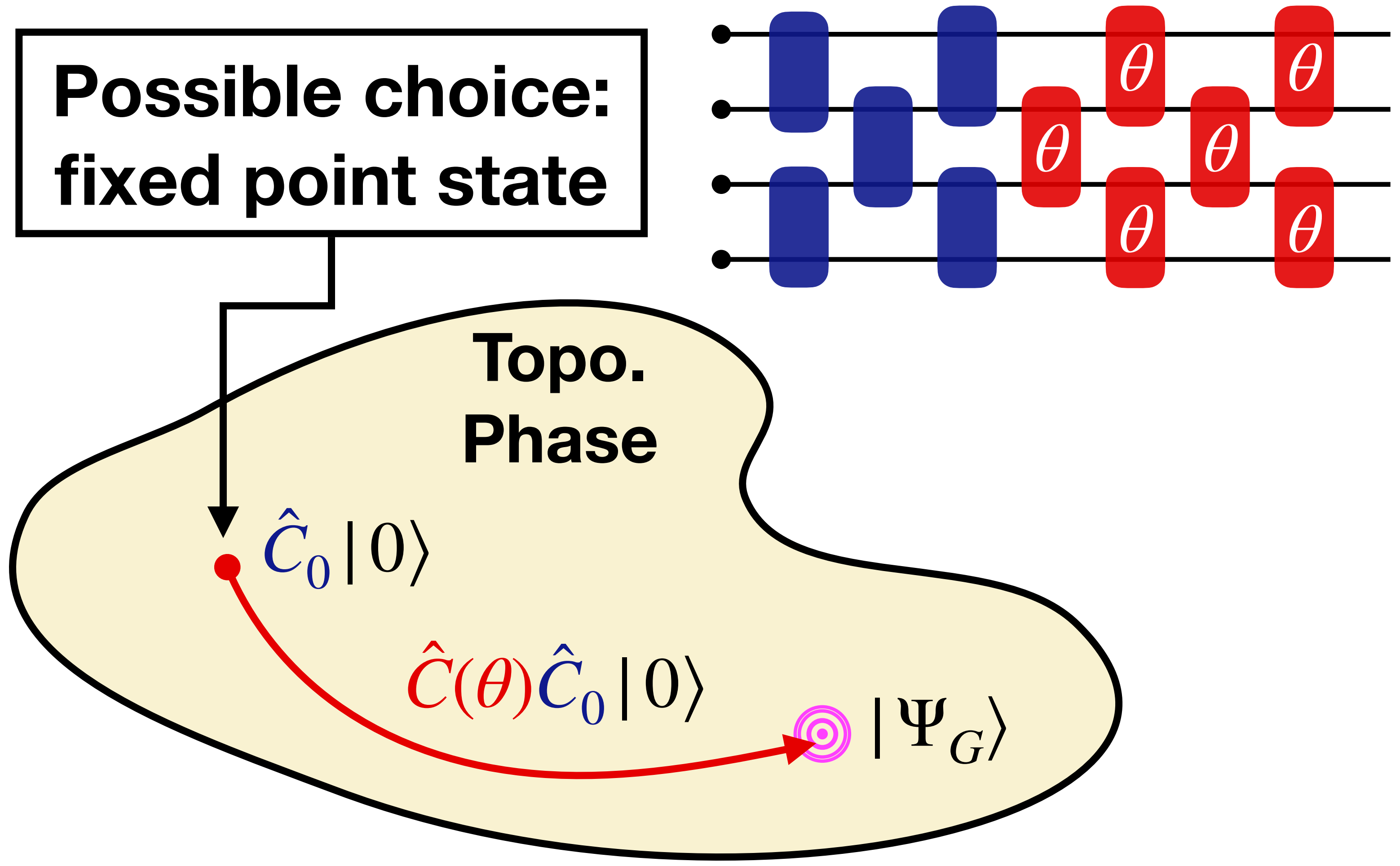
$$|\Psi(\theta)\rangle = \hat{C}(\theta)\hat{C}_0|0\rangle$$

Scalable initialization layer $\hat{C}_0$

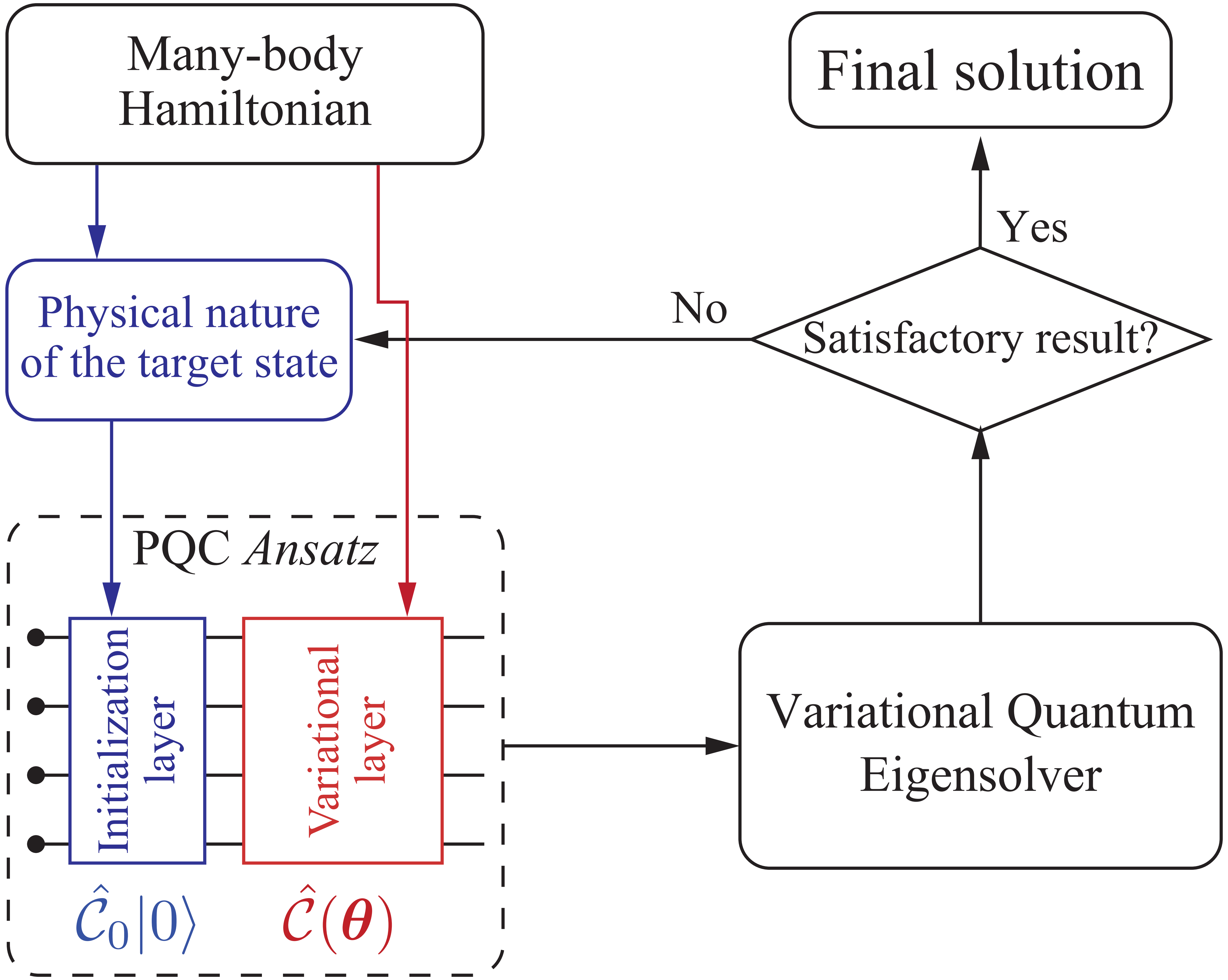
- **Constant** depth for SPT
- **Linear** depth for TO

Scalable variational layer $\hat{C}(\theta)$

- **Constant** depth



Computation scheme



***R.-Y. Sun**, T. Shirakawa & S. Yunoki, PRB **108**, 075127 (2023)*

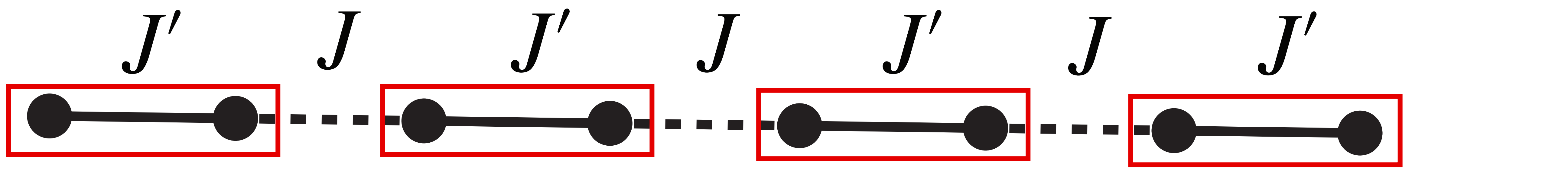
VQE study of SPT spin chain

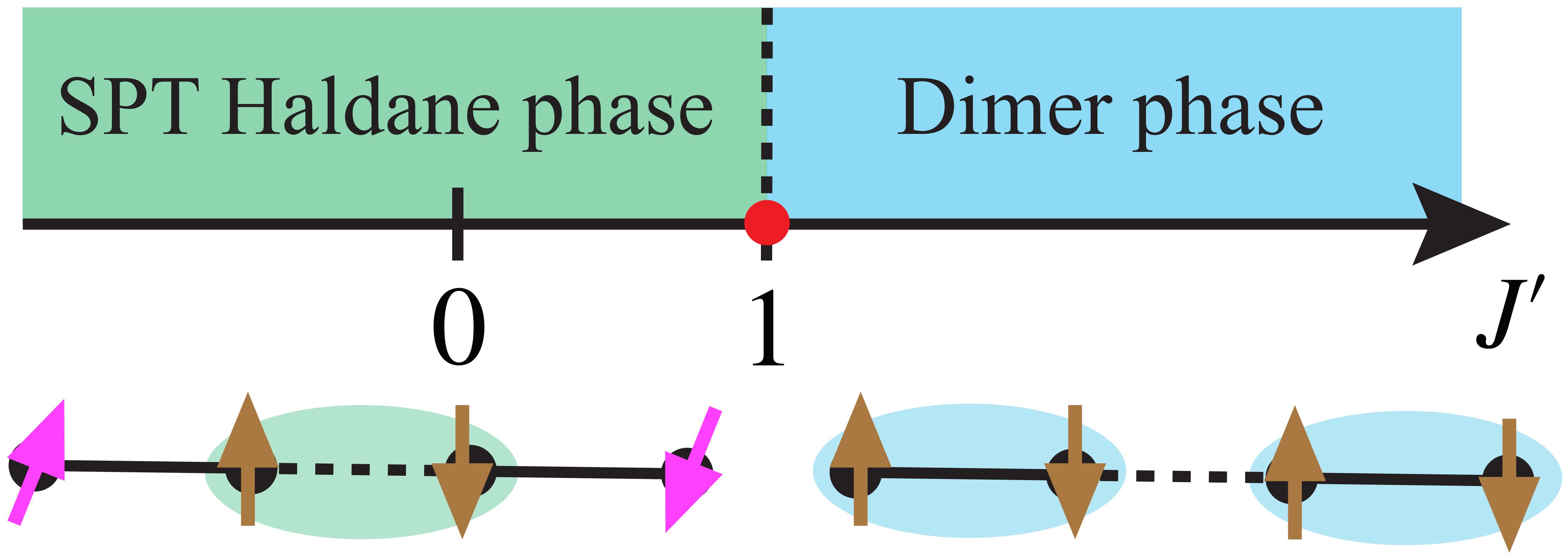
$S = 1/2$ alternating Heisenberg chain (AHC)

Hida, PRB **45**, 5 (1992)

$$H = \sum_{i=0}^{L/2-1} \left(J'\mathbf{S}_{2i} \cdot \mathbf{S}_{2i+1} + J\mathbf{S}_{2i+1} \cdot \mathbf{S}_{2(i+1)} \right)$$

$J = 1$





$J' < 1$ SPT Haldane phase

- Fixed point: $J' = 0$
- $S = 1$ Haldane chain: $J' = -\infty$

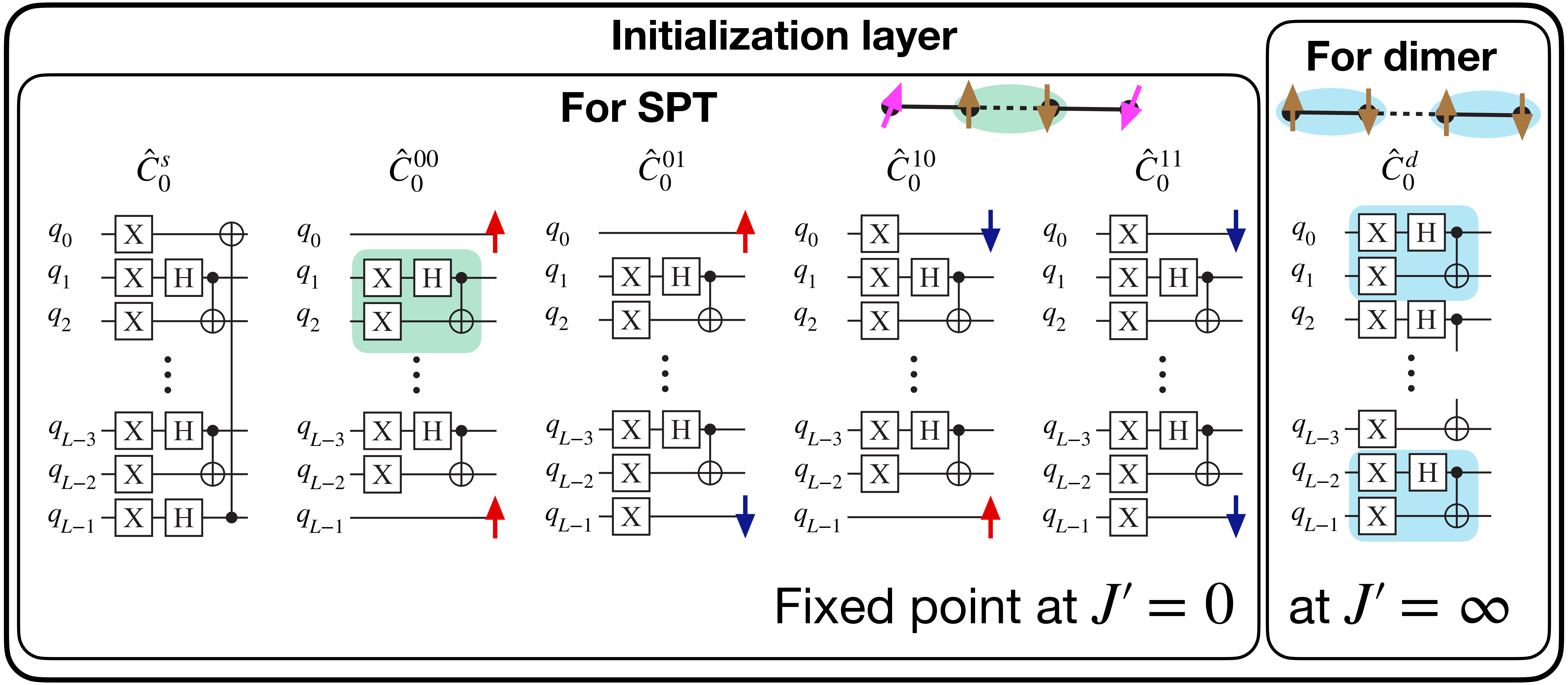
$J' > 1$ trivial dimer phase

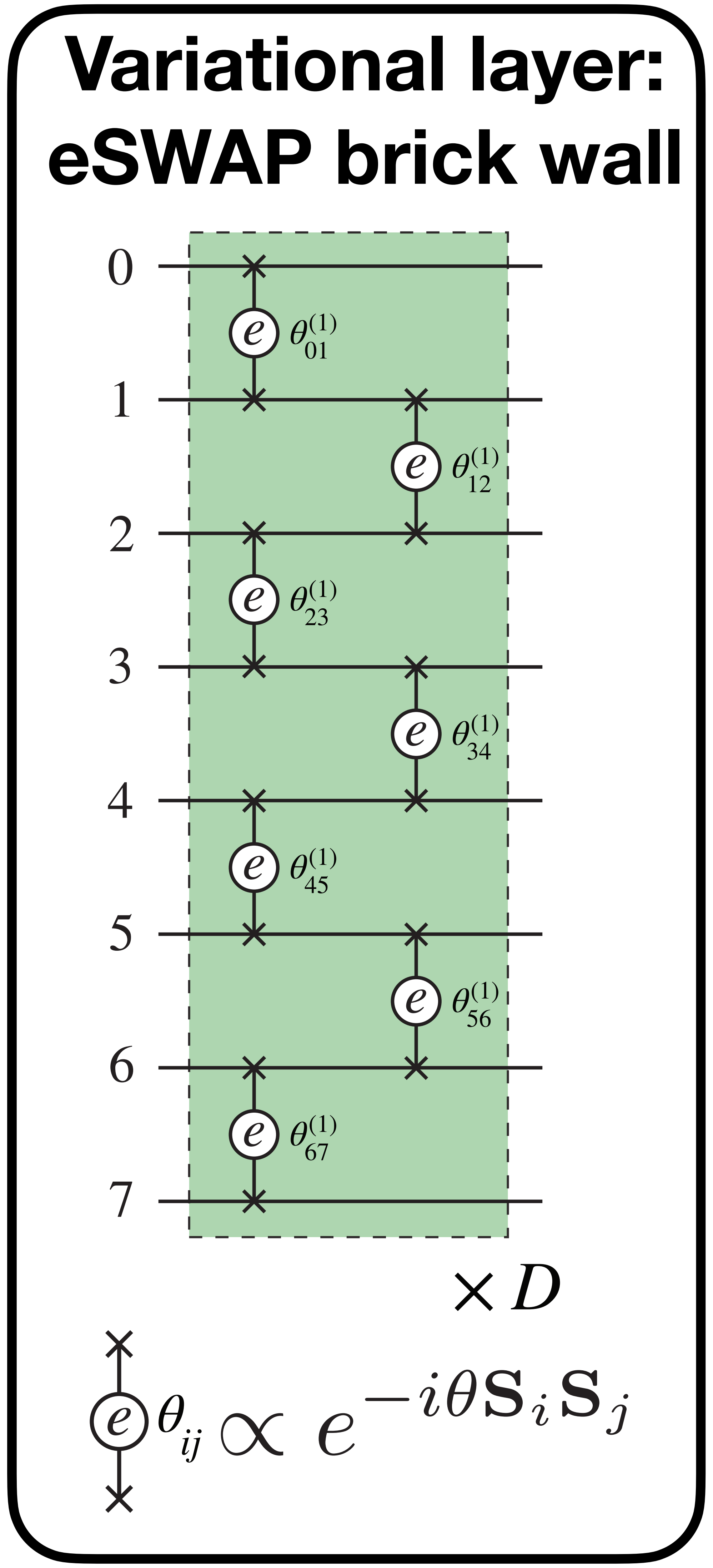
- Fixed point: $J' = \infty$

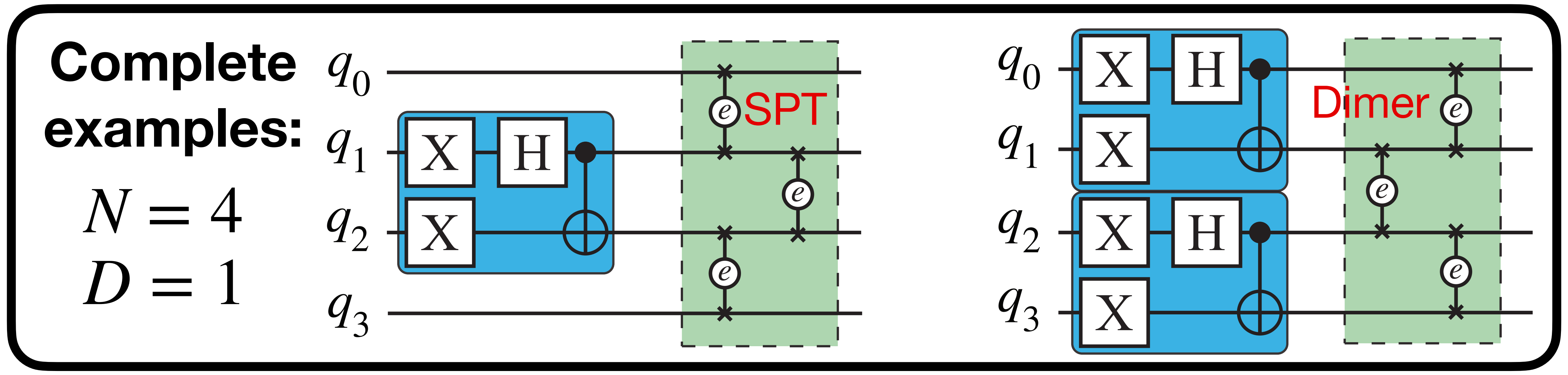
In general, non-exactly solvable

VQE study of SPT spin chain

PQC *Ansatz* for AHC





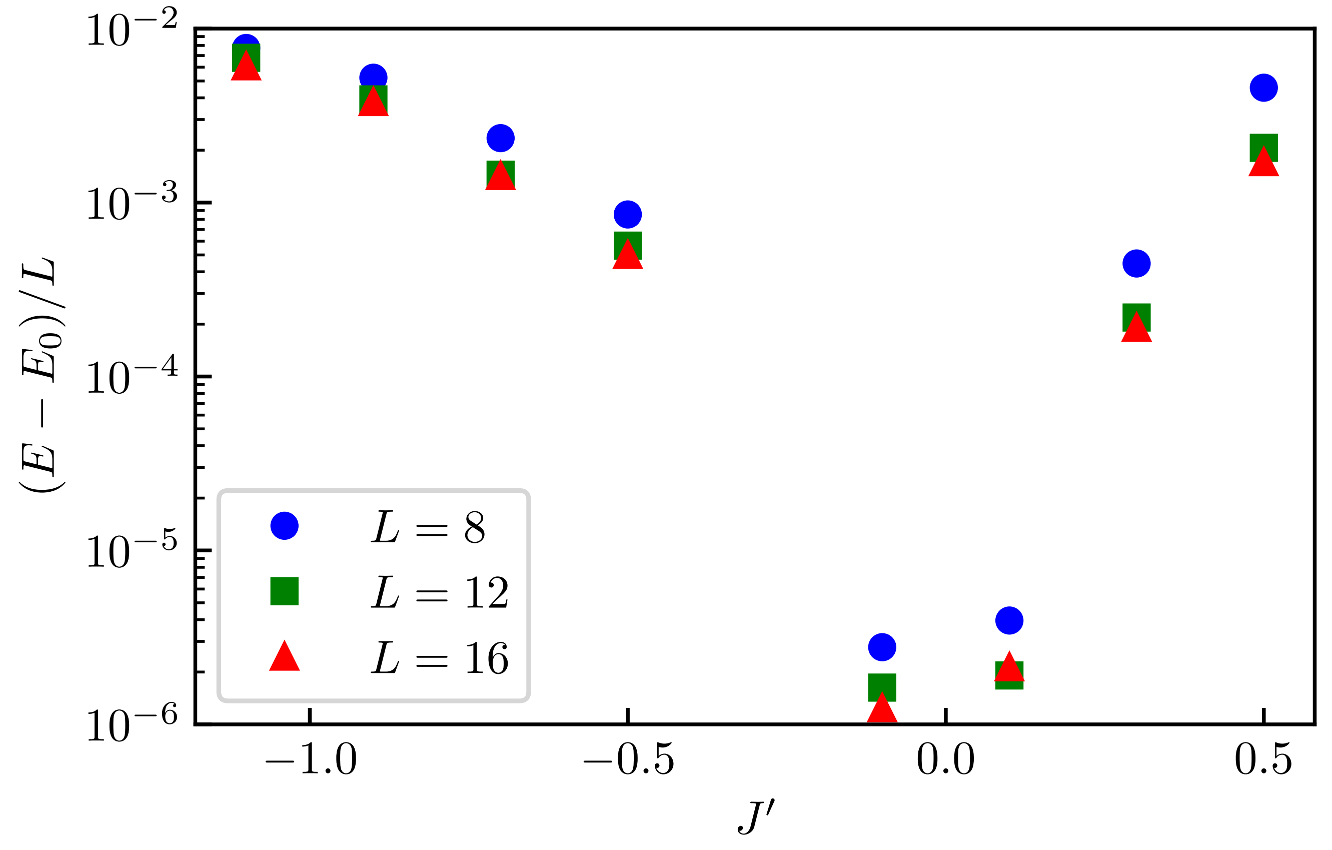


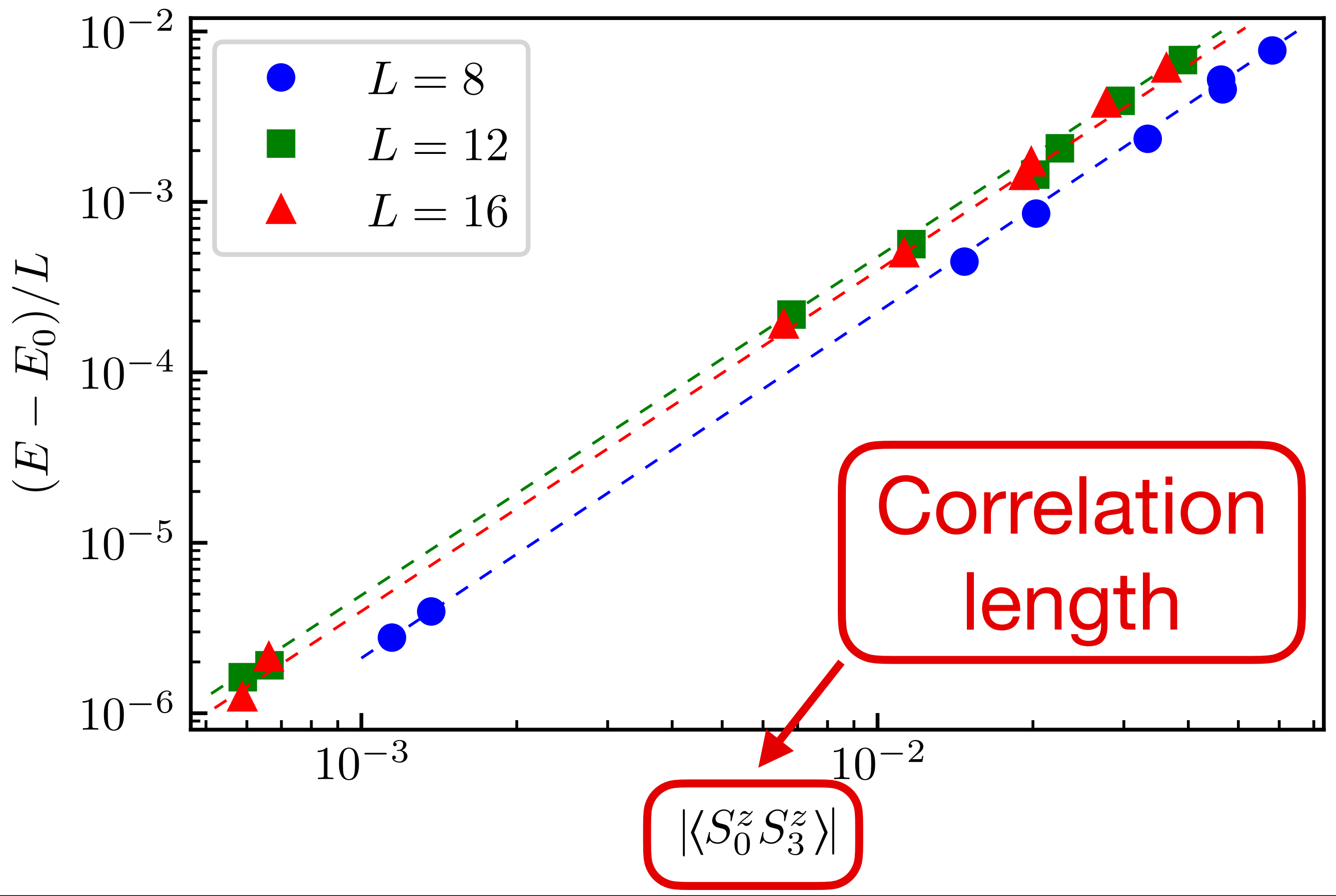
R.-Y. Sun, T. Shirakawa & S. Yunoki, PRB **108**, 075127 (2023)

VQE study of SPT spin chain

VQE simulation of AHC using $D = 1$ shallow circuit *Ansatz*

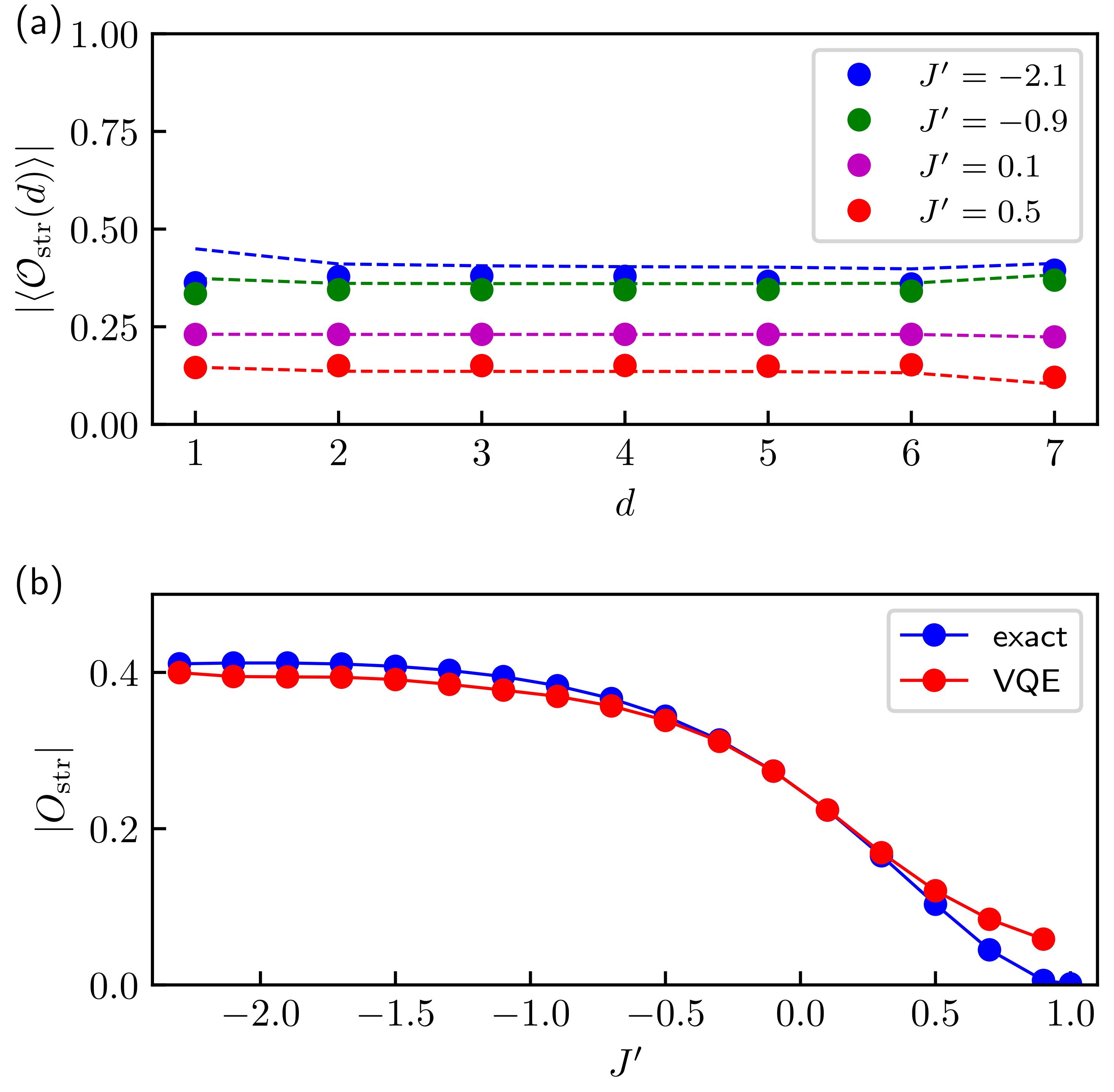
Ground state energy in SPT



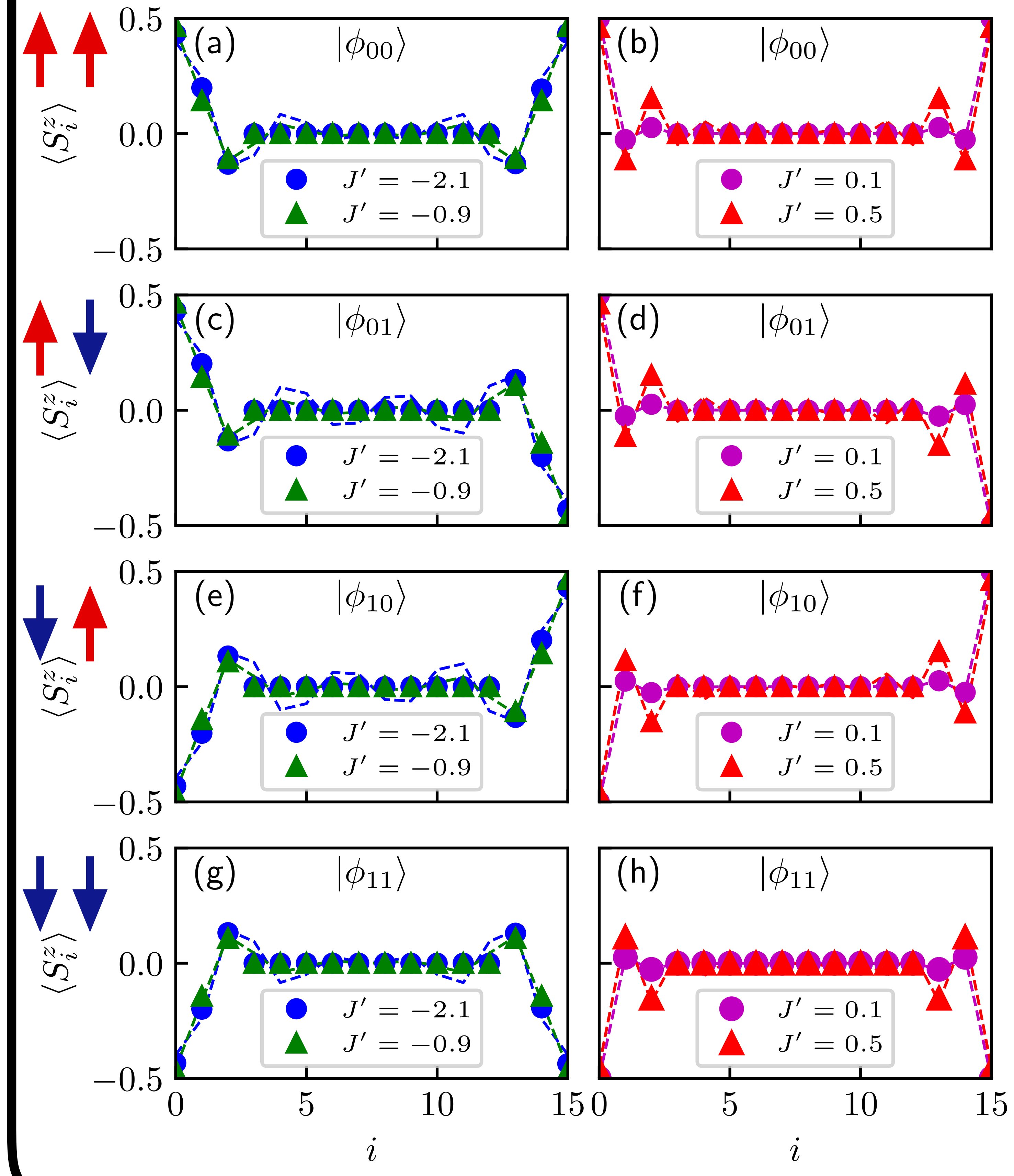


String order in SPT

$$\mathcal{O}_{\mathrm{str}}(d) = \mathcal{S}_k^z \left(\prod_{l=k+1}^{k+d-1} \exp\left(i\pi \mathcal{S}_l^z \right) \right) \mathcal{S}_{k+d}^z$$



Edge spin patterns in SPT



Scalable: Keeping accuracy with increasing system size

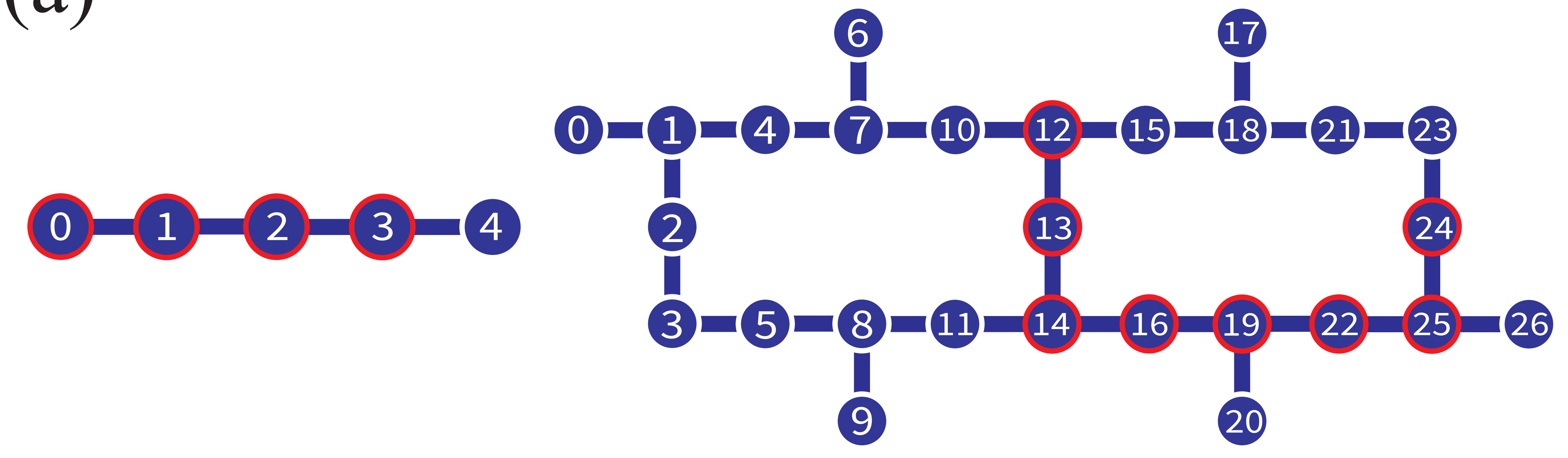
***R.-Y. Sun**, T. Shirakawa & S. Yunoki, PRB **108**, 075127 (2023)*

VQE study of SPT spin chain

Realize SPT state on real quantum computer

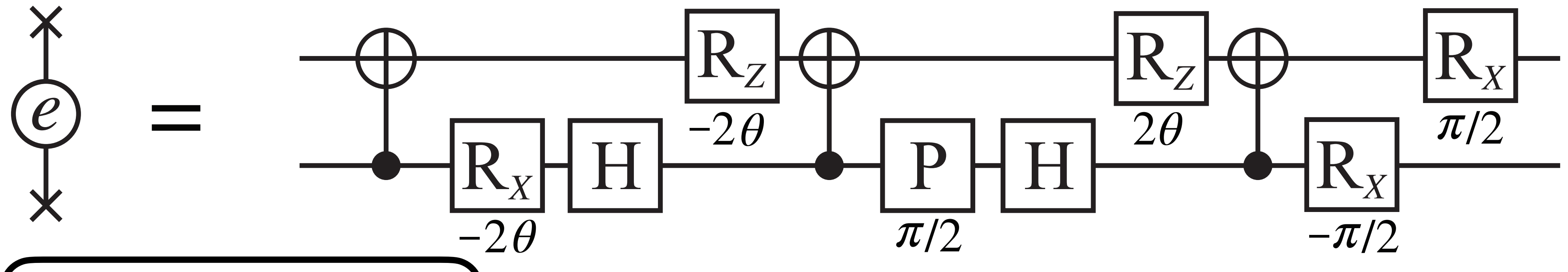
Quantum devices & gate realization

(a)



ibmq_manila   ibm_kawasaki

(b)



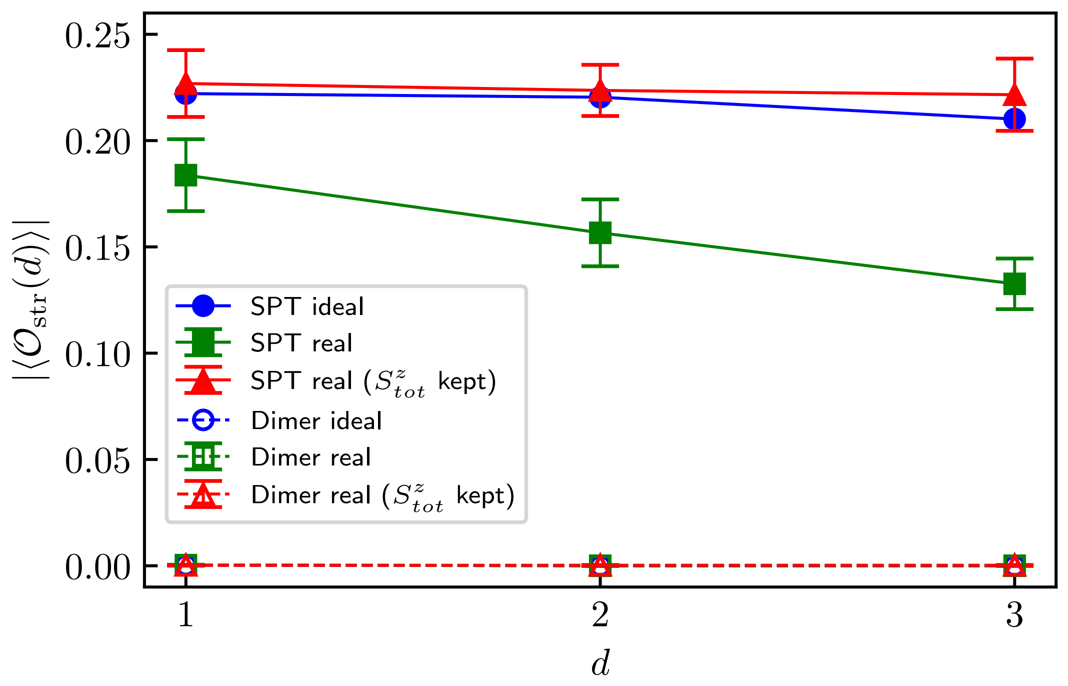
3 CNOT layers

$6 \times D + 1$ CNOT layers

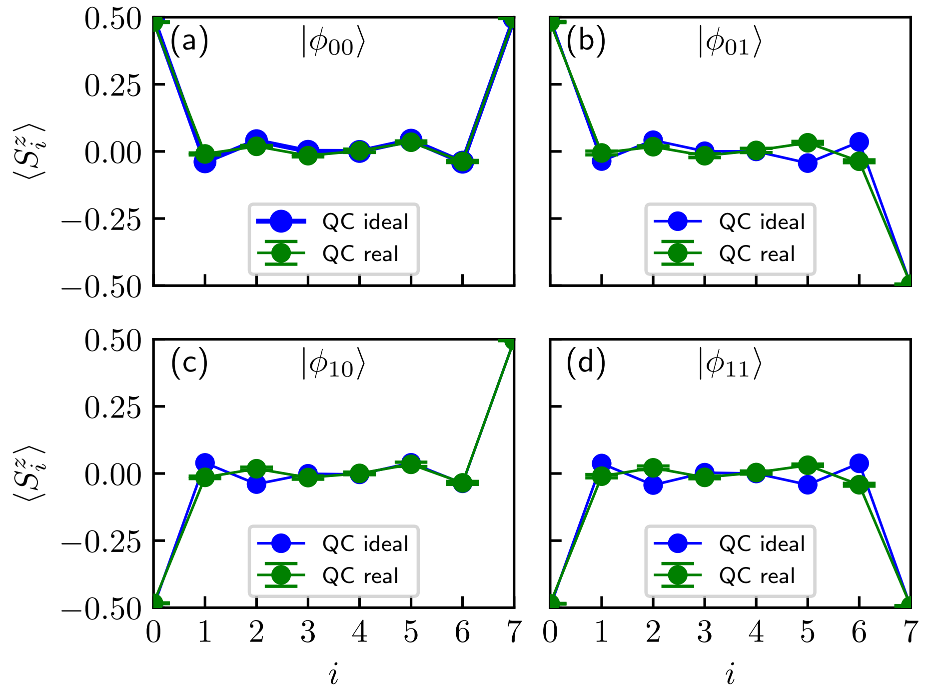
Constant scaling with L

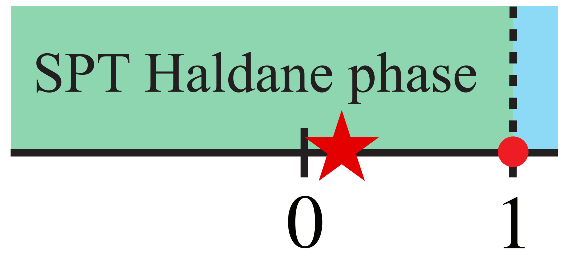
Real quantum experiments with $D = 1$ Ansatz

String order



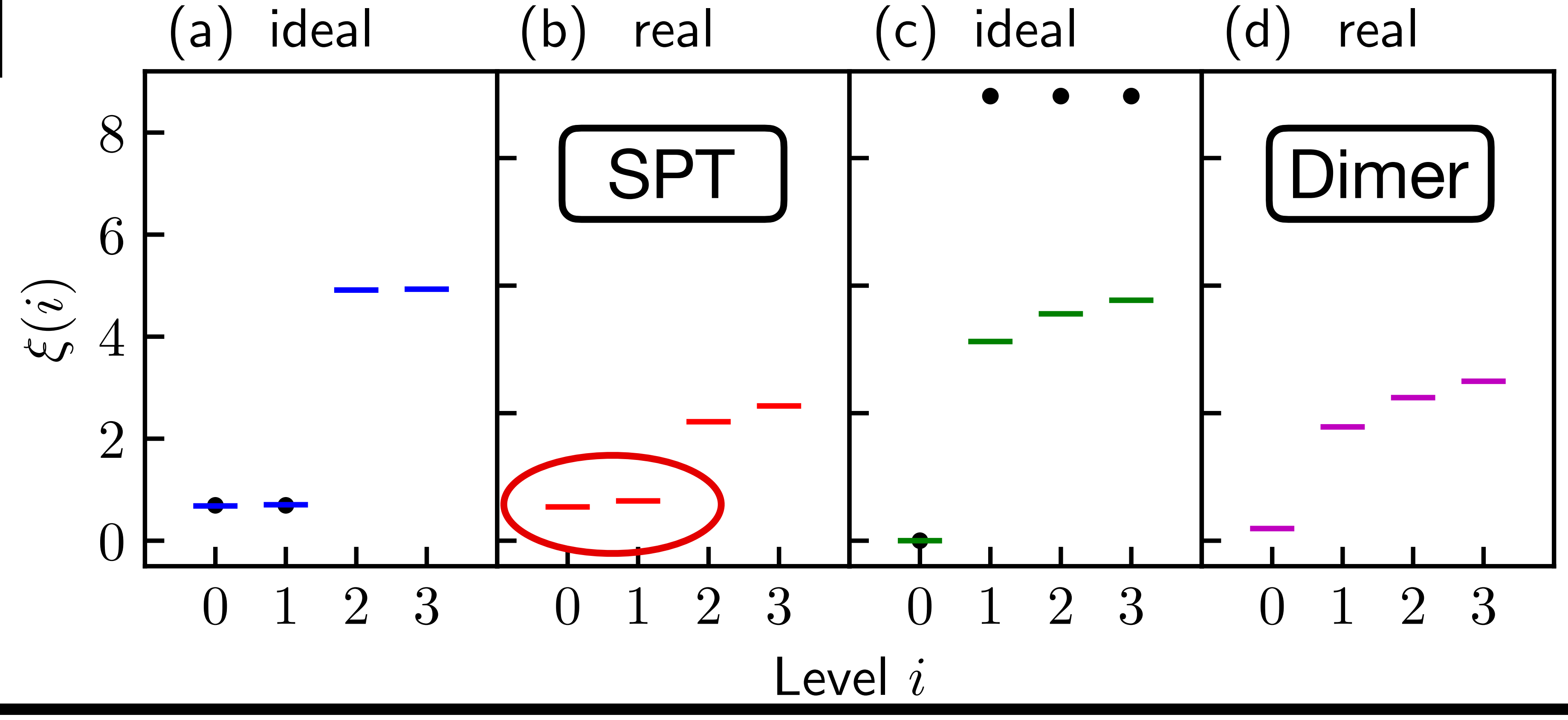
Edge spins





$J' = 0.15$

Entanglement spectrum



R.-Y. Sun, *T. Shirakawa & S. Yunoki, PRB* ***108****, 075127 (2023)*

Scalable quantum simulation of intrinsic topologically ordered states

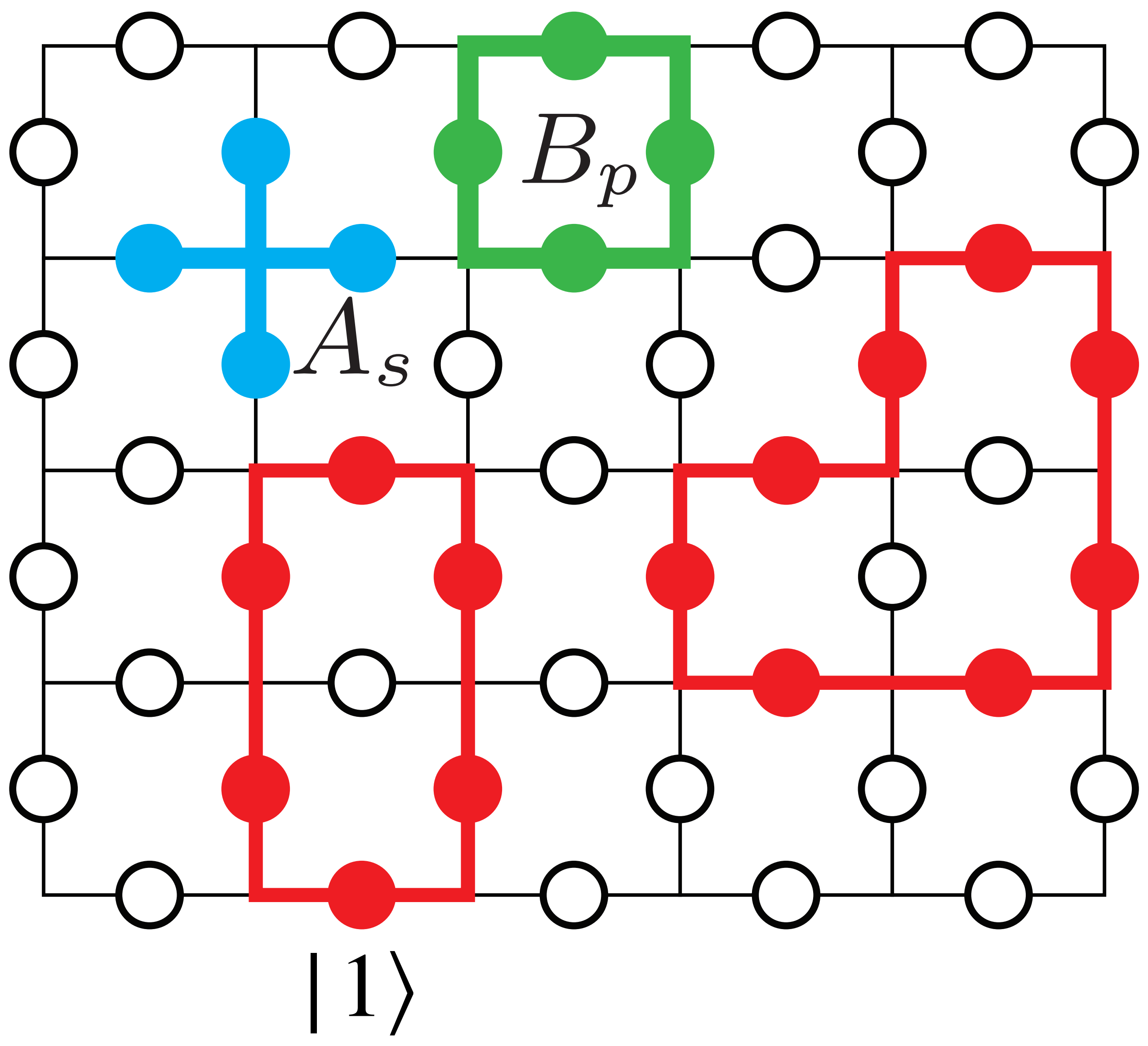
***R.-Y. Sun**, T. Shirakawa, and S. Yunoki, Phys. Rev. B **107**, L041109 (2023)* [arXiv:**2210.14662**]

Scalable PQC Ansatz for quantum loop gas

Toric code model and its realization on NISQ device

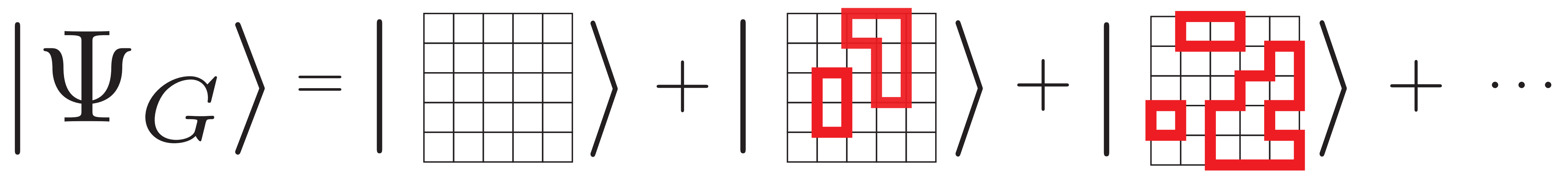
Toric code model

$$H_{\mathrm{TC}} = -\sum_s A_s - \sum_p B_p$$

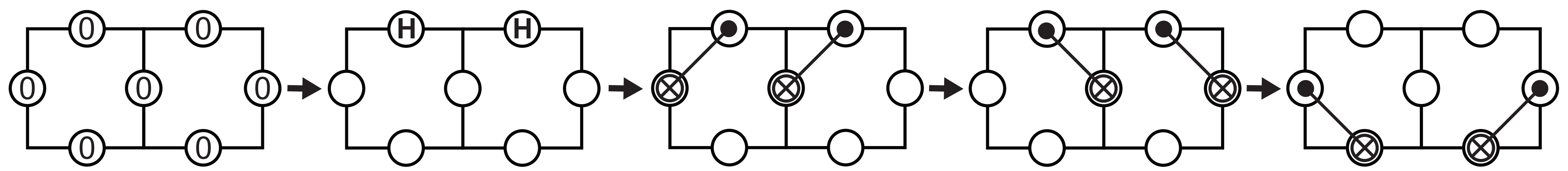


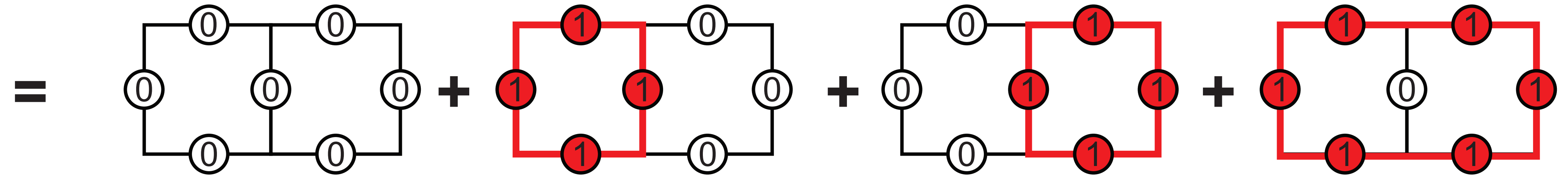
$$A_s = \prod_{i \in s} \sigma_i^z \qquad B_p = \prod_{i \in p} \sigma_i^x$$

- Intrinsic topological order
- Quantum loop gas with equal weights

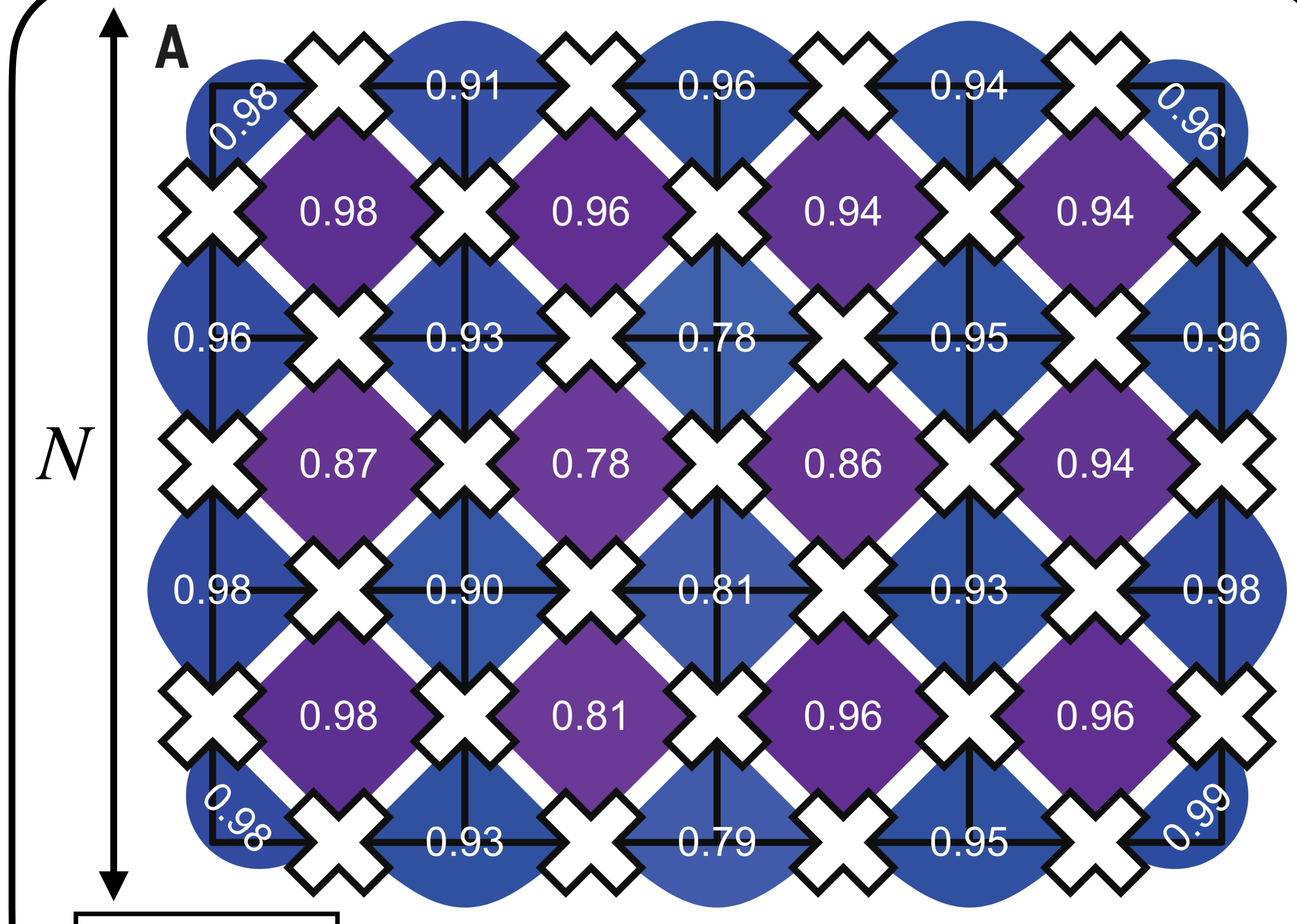


NISQ realization of toric code state





PRX Quantum **3**, 040315 (2022)



Science **374**, 1237 (2021)

CNOT

Hadamard gate

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

NISQ **scalable**:

$3 + 2[(N-1)/2]$

CNOT layers

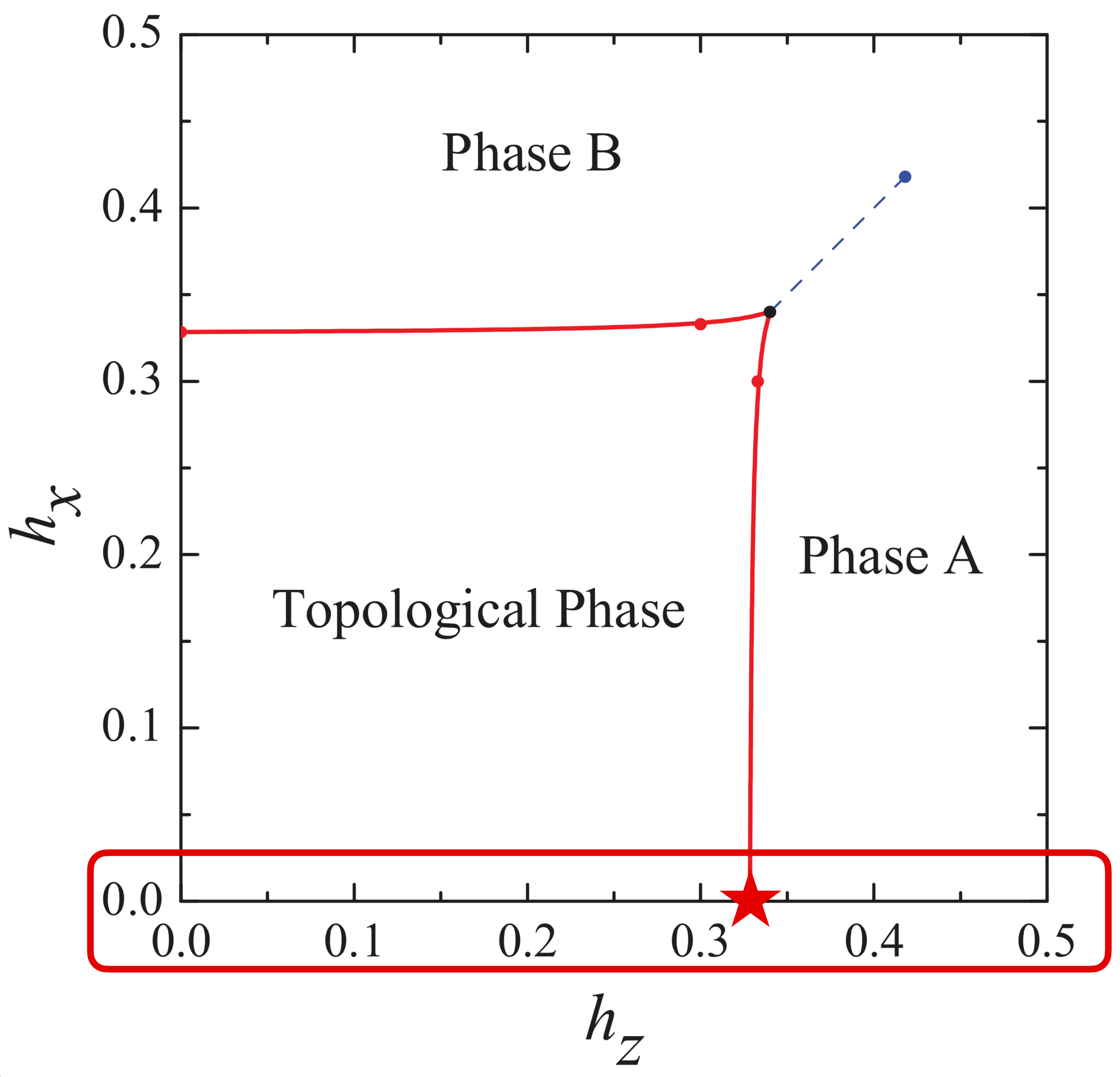
Scalable PQC Ansatz for quantum loop gas

Toric code model in a magnetic field

$$H_{\mathrm{TCM}}(x) = (1-x)H_{\mathrm{TC}} - x\sum_{i=1}^{N}\sigma_i^z \qquad x\in[0,1]$$

Non-exactly solvable

QMC phase diagram



$$h_z = \frac{x}{1-x}$$

Topological order to ferromagnetic order: $x_c \sim 0.25$

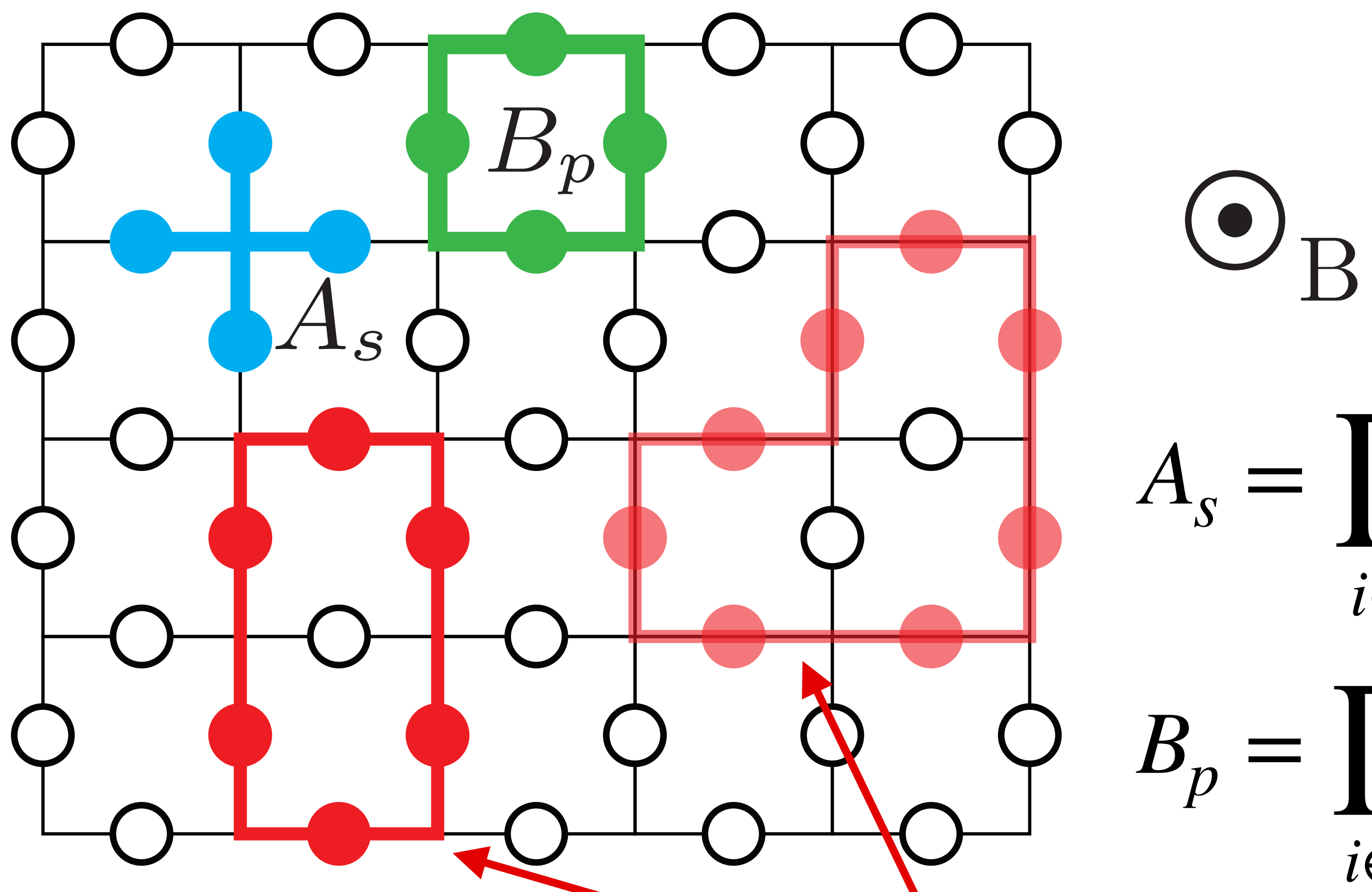
PRB **85**, *195104 (2012)*

Goal: simulate $H_{\mathrm{TCM}}(x)$ on quantum computer

PRL **98,** 070602 (2007) PHYSICAL REVIEW LETTERS week ending 16 FEBRUARY 2007

Breakdown of a Topological Phase: Quantum Phase Transition in a Loop Gas Model with Tension

Simon Trebst,[1] Philipp Werner,[2] Matthias Troyer,[3] Kirill Shtengel,[4] and Chetan Nayak[1,5]

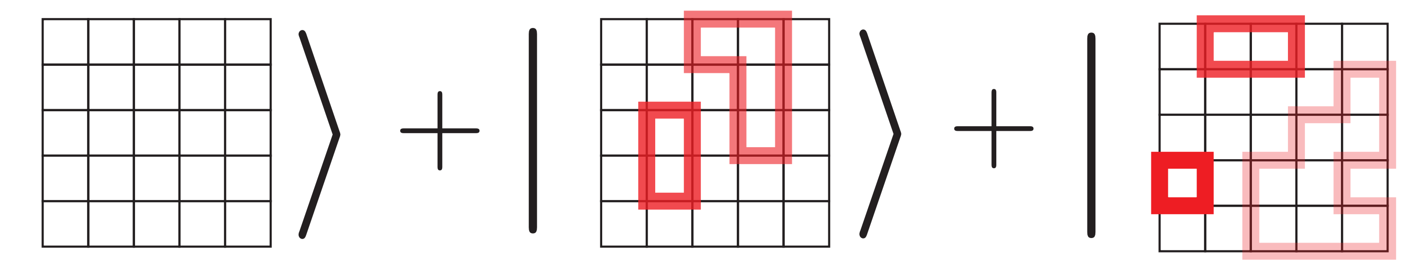


$$A_s = \prod_{i\in s}\sigma_i^z$$

$$B_p = \prod_{i\in p}\sigma_i^x$$

Loop weights

$|\Psi_G\rangle = |\ \rangle + |\ \rangle + |\ \rangle + \cdots$

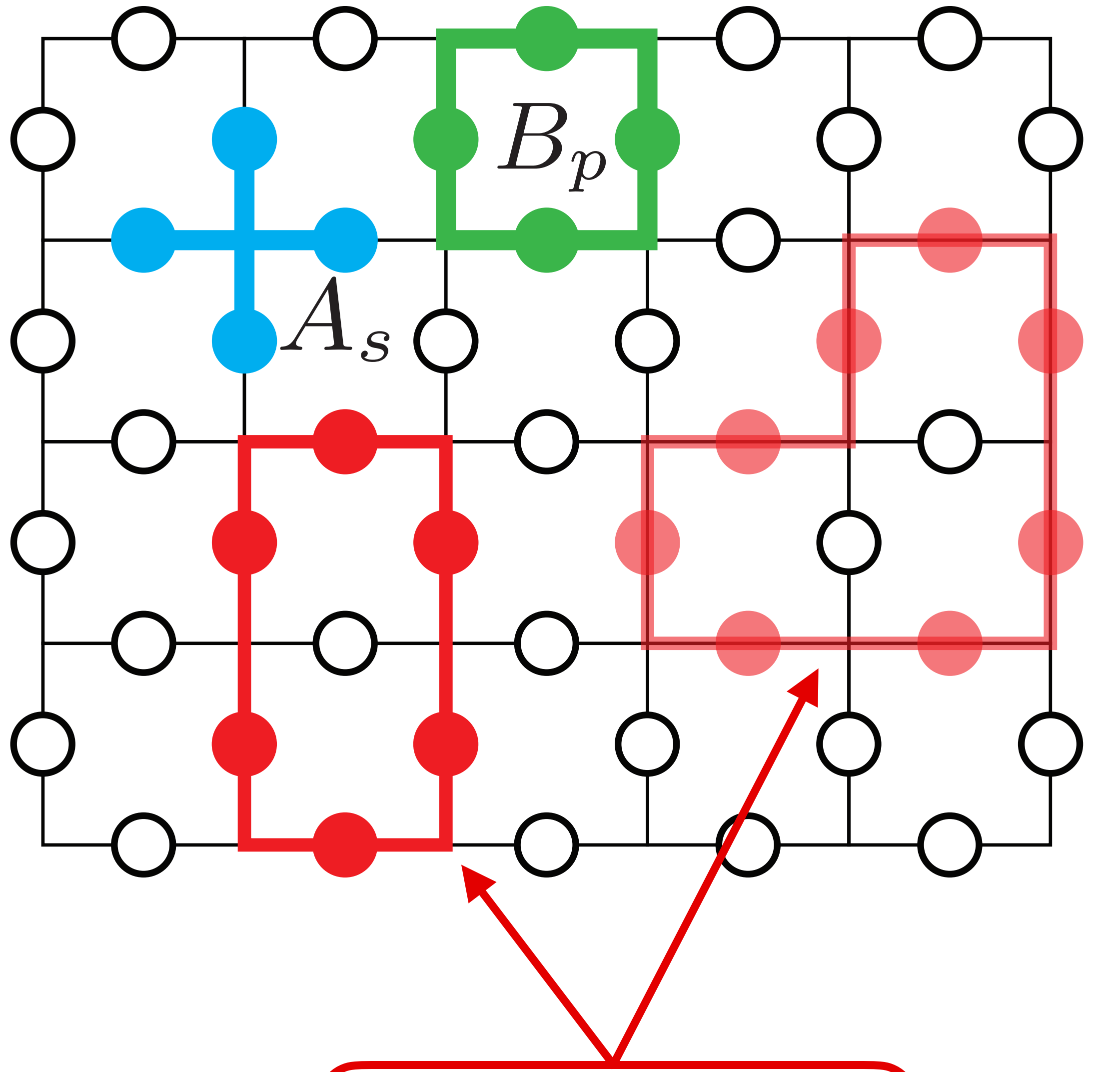


Scalable PQC Ansatz for quantum loop gas

Parametrized loop gas circuit (PLGC)

$$H_{\mathrm{TCM}}(x) = (1-x)H_{\mathrm{TC}} - x\sum_{i=1}^{N}\sigma_i^z$$

$x \in [0,1]$



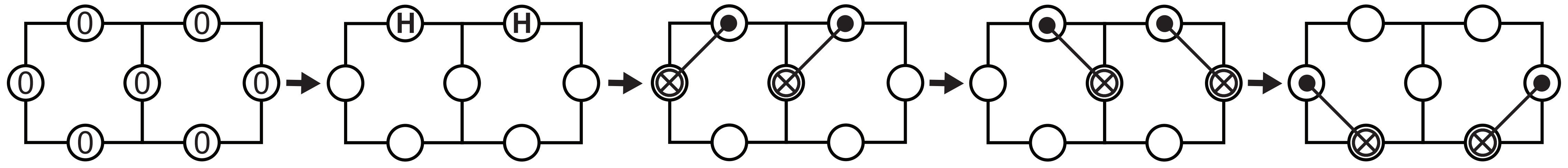
$\odot$ B

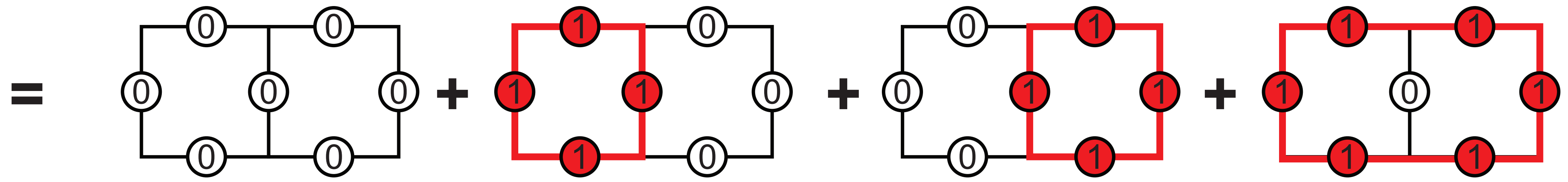
$$A_s = \prod_{i\in s}\sigma_i^z$$

$$B_p = \prod_{i\in p}\sigma_i^x$$

GS: Quantum loop gas with non-equal weights

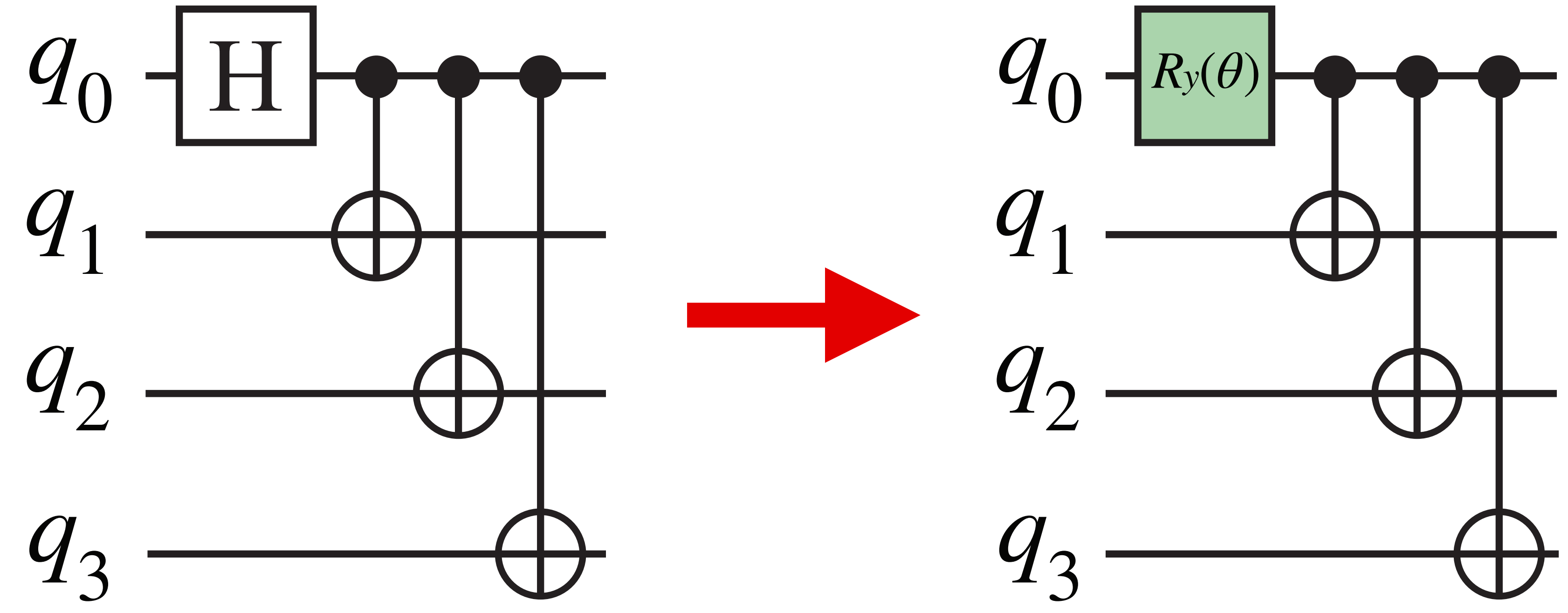
PLGC *Ansatz*: weight-adjustable loop gas





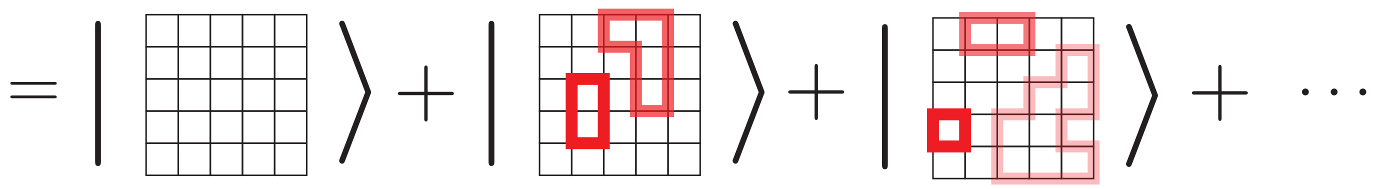
$$H|0\rangle = R_y(\pi/2)|0\rangle$$

$$R_y(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$



# of parameters = # of plaquettes

$$|\Psi(\boldsymbol{\theta})\rangle = \prod_{p=1}^{N_p}\left(\cos(\theta_p/2)I + \sin(\theta_p/2)B_p\right)|00\cdots0\rangle$$



Scalable on NISQ devices

No additional complexity

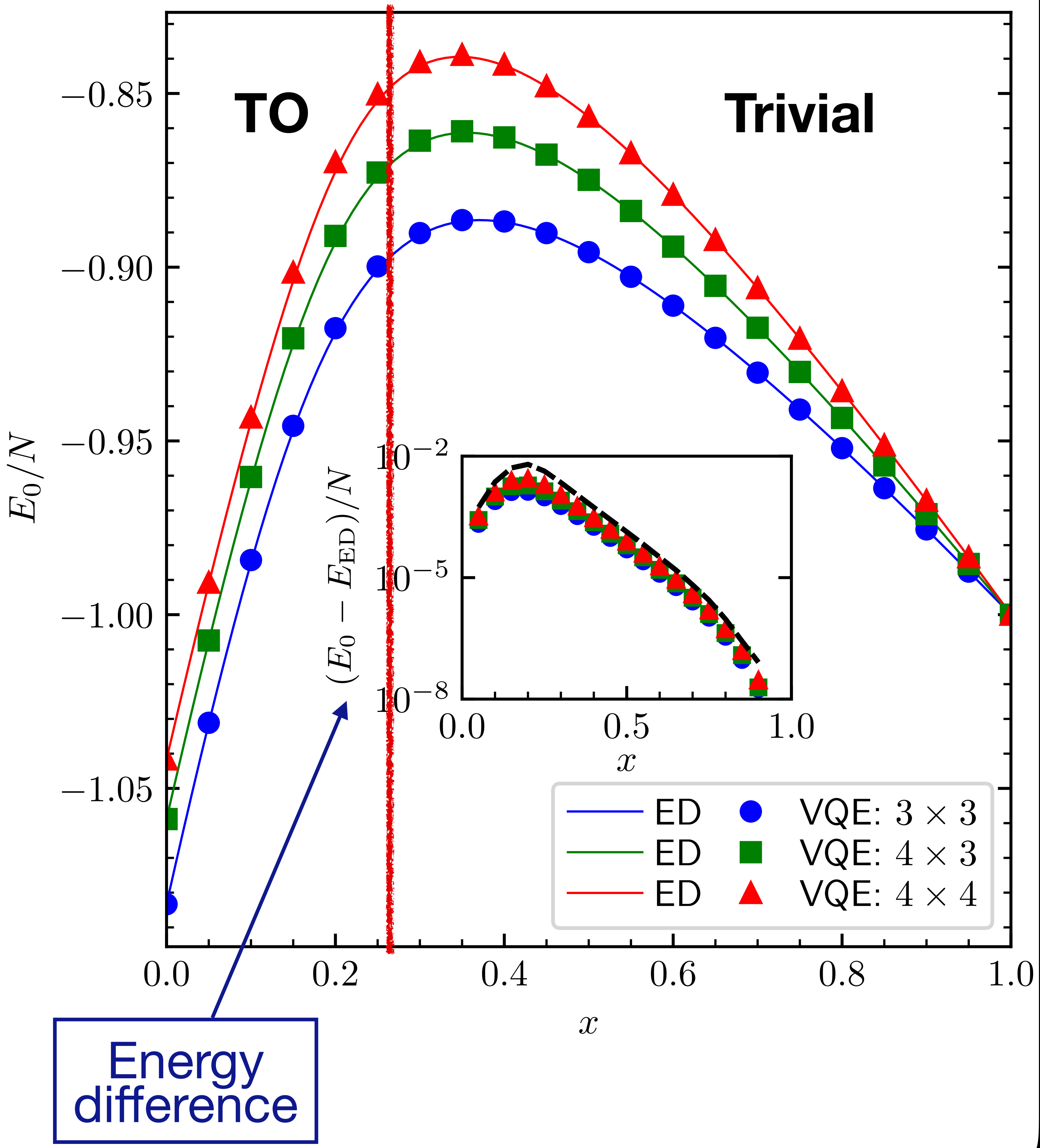
***R.-Y. Sun**, T. Shirakawa & S. Yunoki, PRB **107**, L041109 (2023)*

VQE study of toric code model in a magnetic field

VQE simulation of $H_{\mathrm{TCM}}(x)$ using PLGC *Ansatz*

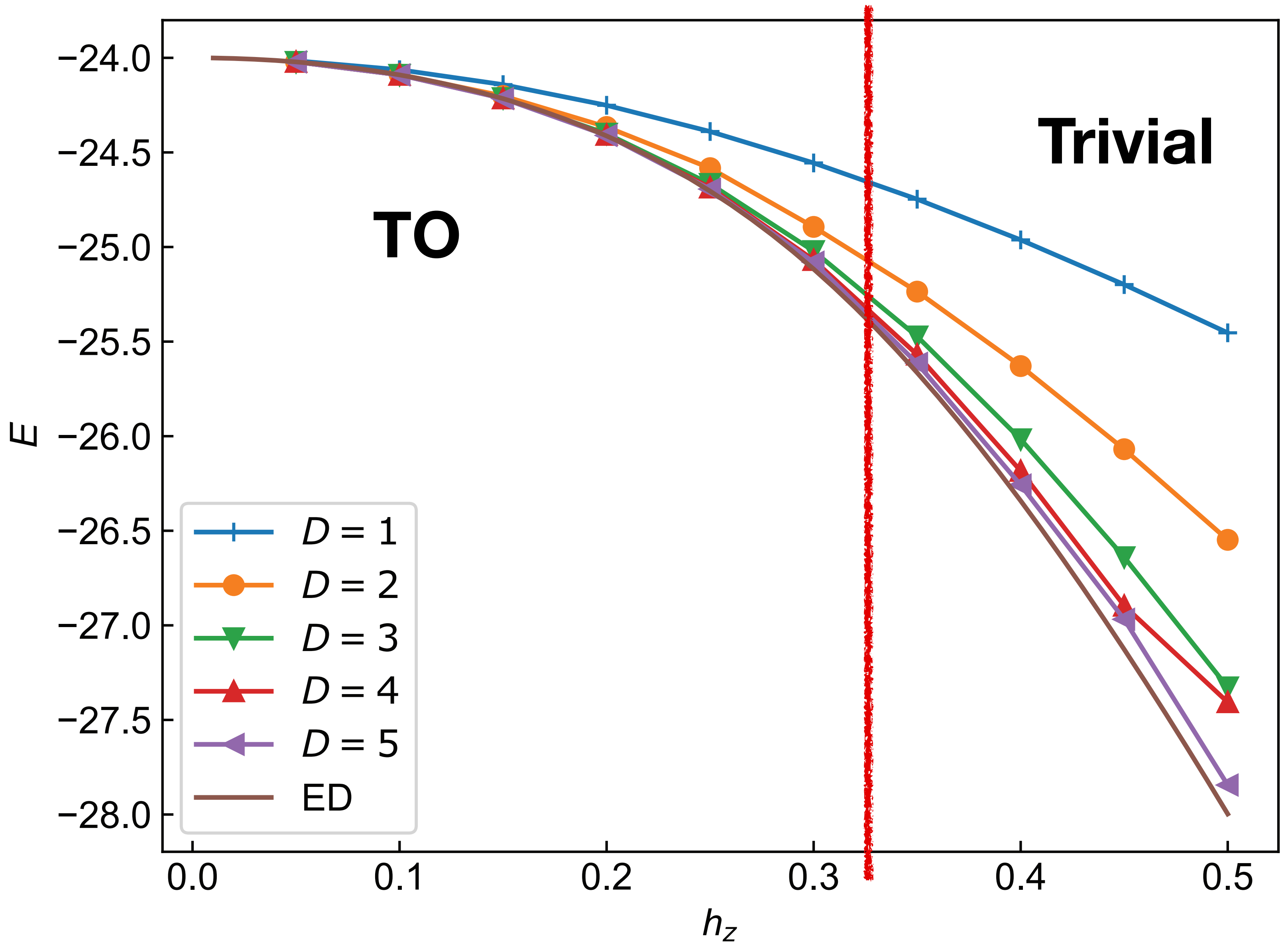
Ground state energy

PLGC (our work)



QAOA-type (previous result)

$$|\Psi(\theta_l^0, \theta_l^1)\rangle = \prod_{l=1}^{D} \left[e^{-i\theta_l^0 H_{TC}} e^{-i\theta_l^1 H_h} \right] |\Psi_0\rangle$$

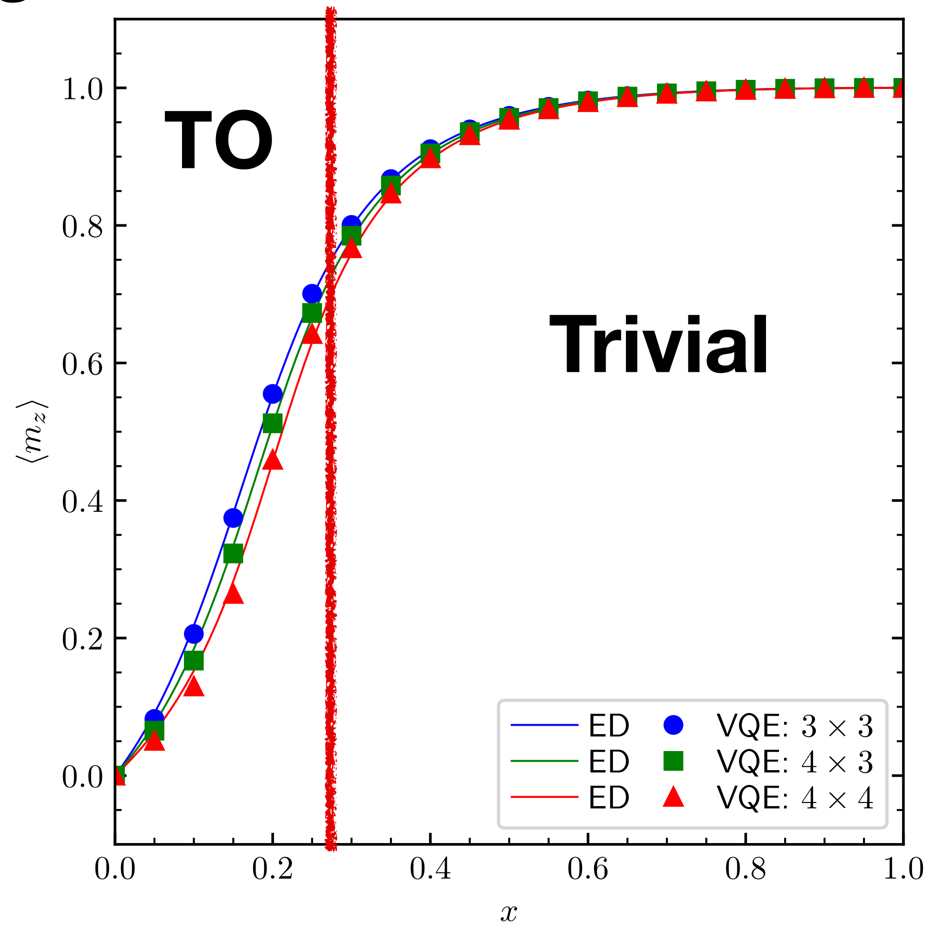


arXiv:2202.02909

Realize scalable VQE simulations

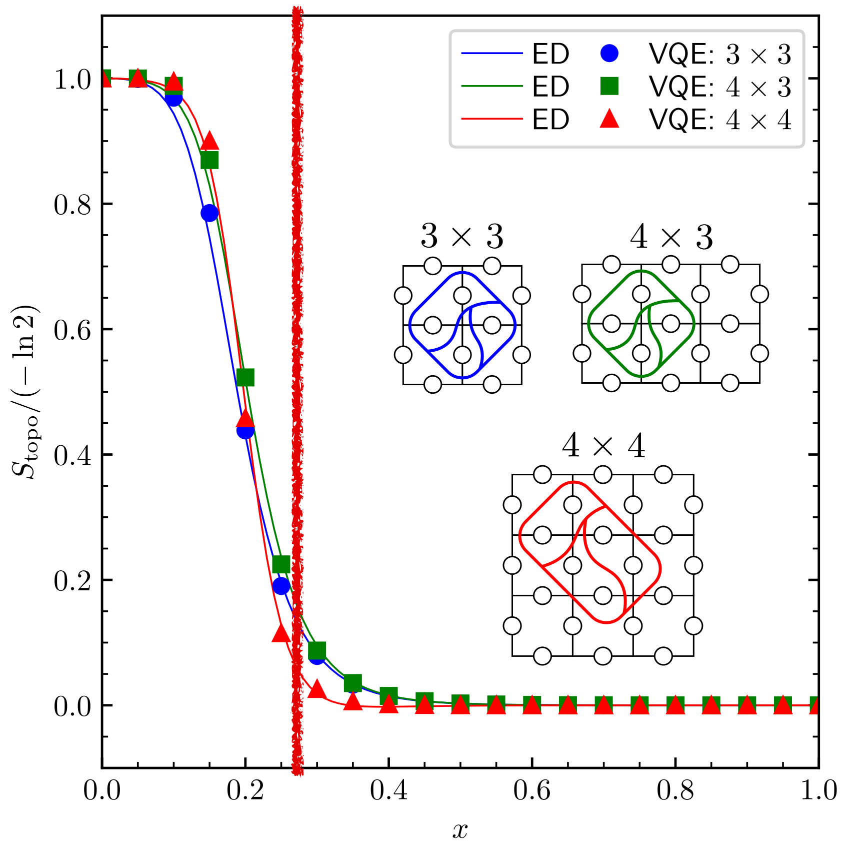
Magnetization

$$\langle m_z \rangle = \frac{1}{N} \sum_{i=1}^{N} \langle \sigma_i^z \rangle$$



Topological entanglement entropy

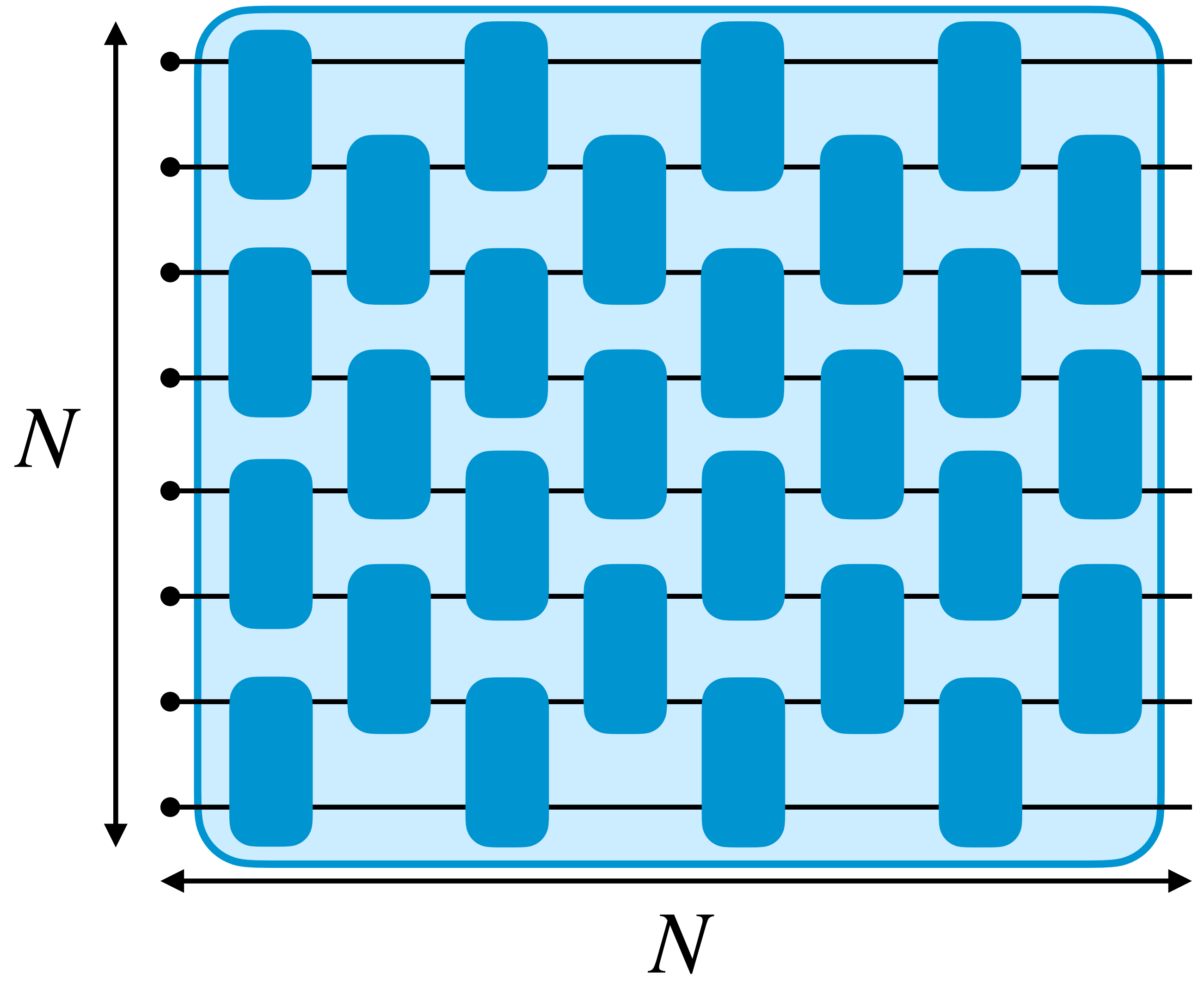
$$S_{\mathrm{topo}} = S_{\mathrm{A}} + S_{\mathrm{B}} + S_{\mathrm{C}} - S_{\mathrm{AB}} - S_{\mathrm{BC}} - S_{\mathrm{AC}} + S_{\mathrm{ABC}}$$



*R.-Y. Sun, T. Shirakawa & S. Yunoki, PRB **107**, L041109 (2023)*

Take home message

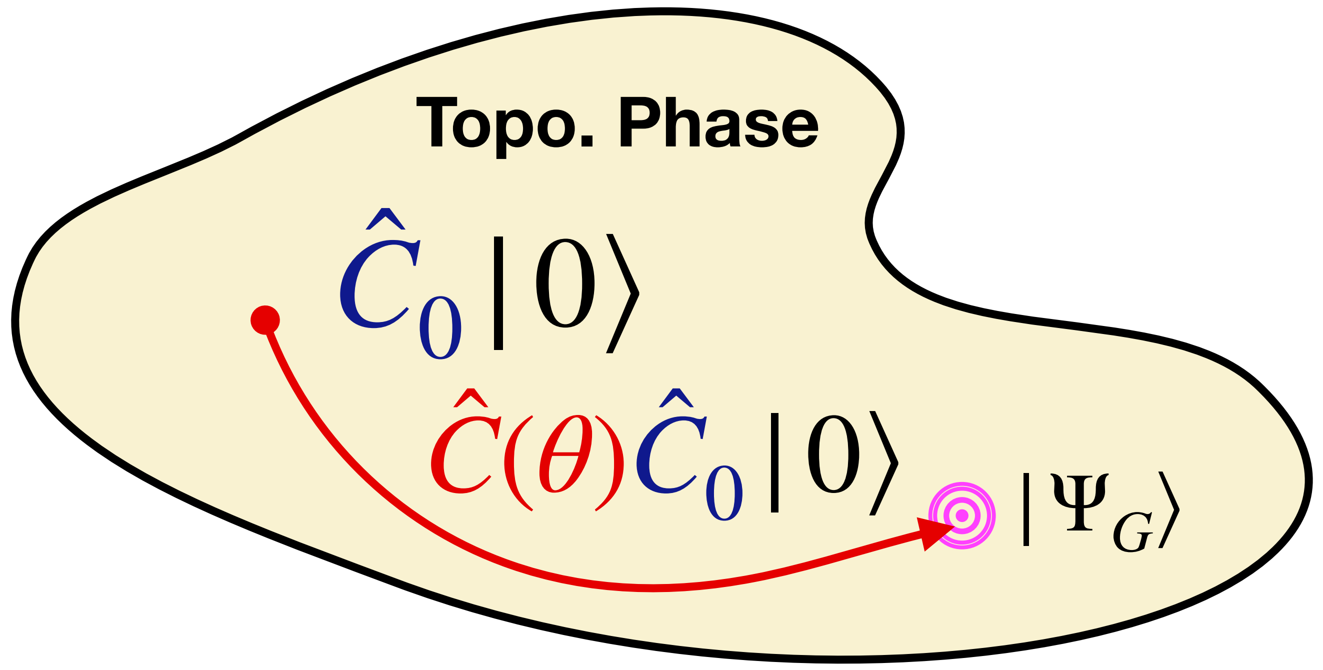
$N \times N$ NISQ playground



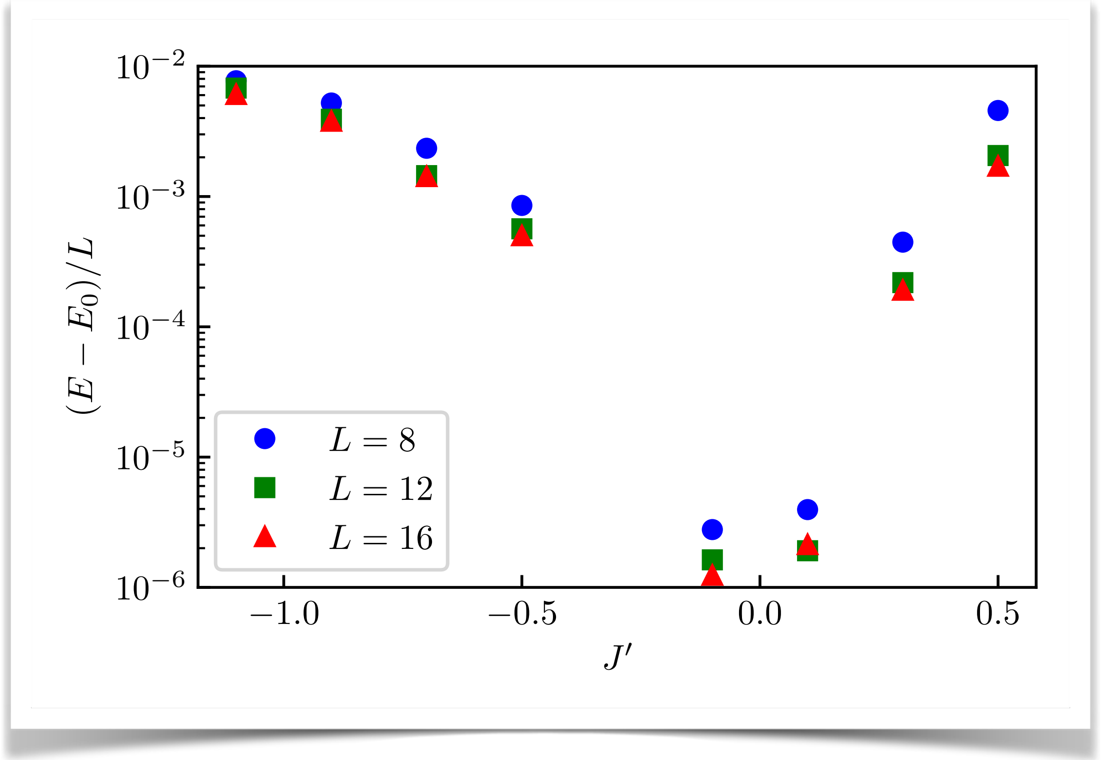
- NISQ scalability: $N \times N$ circuit
- Scalable PQC: Volume $\leq O(N^2)$
- Enough for topological states
- Promising in the near future

Two-layer structured PQC *Ansatz*

$$|\Psi(\theta)\rangle = \hat{C}(\theta)\hat{C}_0|0\rangle$$

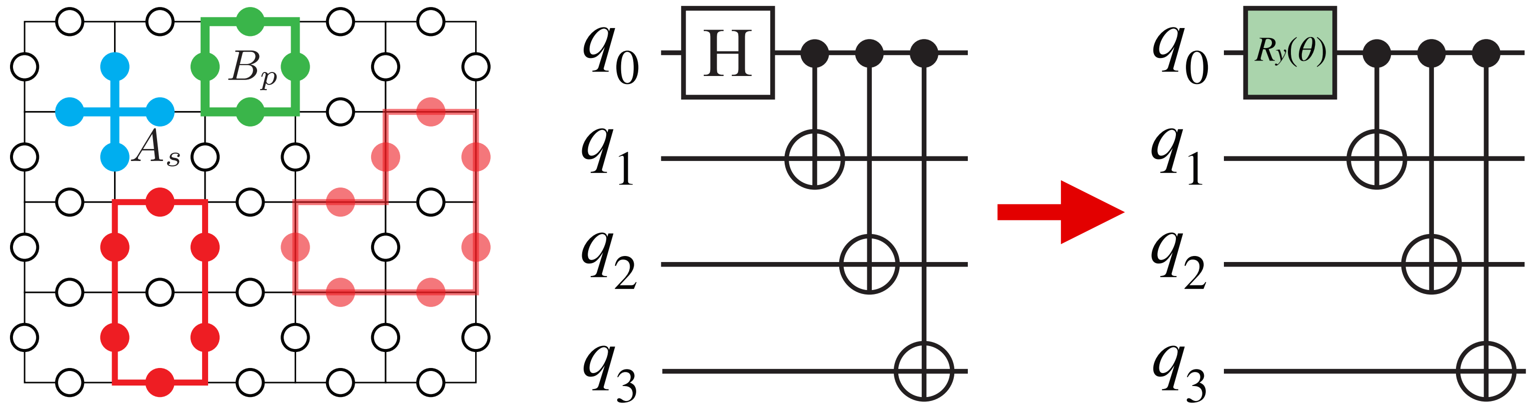


Scalable VQE for 1D SPT

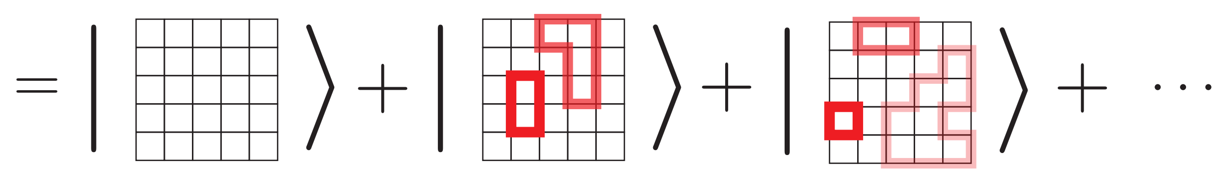


*R.-Y. Sun et. al., arXiv:**2303.17187***

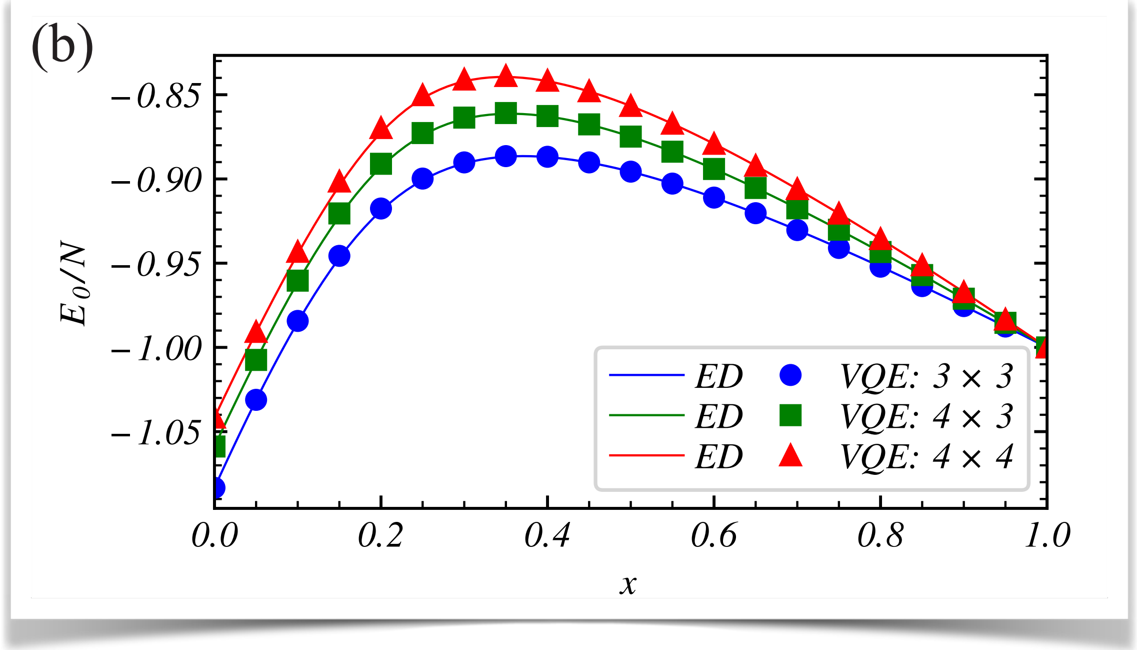
Parametrized loop gas circuit (PLGC)



$$|\Psi(\boldsymbol{\theta})\rangle = \prod_{p=1}^{N_p}\Big(\cos(\theta_p/2)I + \sin(\theta_p/2)B_p\Big)|00\cdots0\rangle$$



Scalable VQE for TC + magnetic field



*R.-Y. Sun et. al., arXiv:**2210.14662***

Thank you for your attention!