Subregion Complementarity in AdS/CFT

Seiji Terashima **(YITP)**

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"Quantum Information, Quantum Matter and Quantum Gravity" at YITP

mainly based on the following papers:

with Sotaro Sugishita JHEP11(2022)041, 2207.06455 [hep-th], 2309.04231 [hep-th]

Introduction

One way to study quantum gravity is AdS/CFT duality

Maldacena

Quantum gravity on AdS

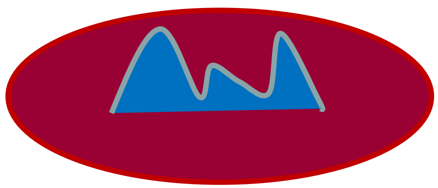
= conformal field theory (CFT)

In operator formalism, AdS/CFT is

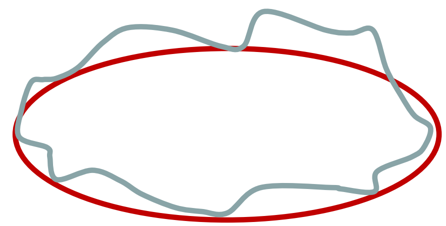
equivalence between
Hilbert spaces and Hamiltonians
of gravity on AdS and CFT

Low energy states of bulk theory and CFT

Bulk theory



CFT

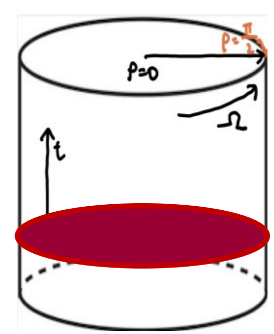


1 to 1

states on disk

states on S^1

They are on a fixed time slice of AdS or cylinder



We will focus on Large N limit, which is essential for AdS/CFT duality

We can EXPLICITLY show that

$$\text{Low energy spectrum of large } N \ CFT_d$$

$\Updownarrow$ **equivalent!**

$$\text{Spectrum of free gravity on } AdS_{d+1}$$

We have explicit map between these two

Using this map, I will explain below

- **violation of entanglement wedge reconstruction**

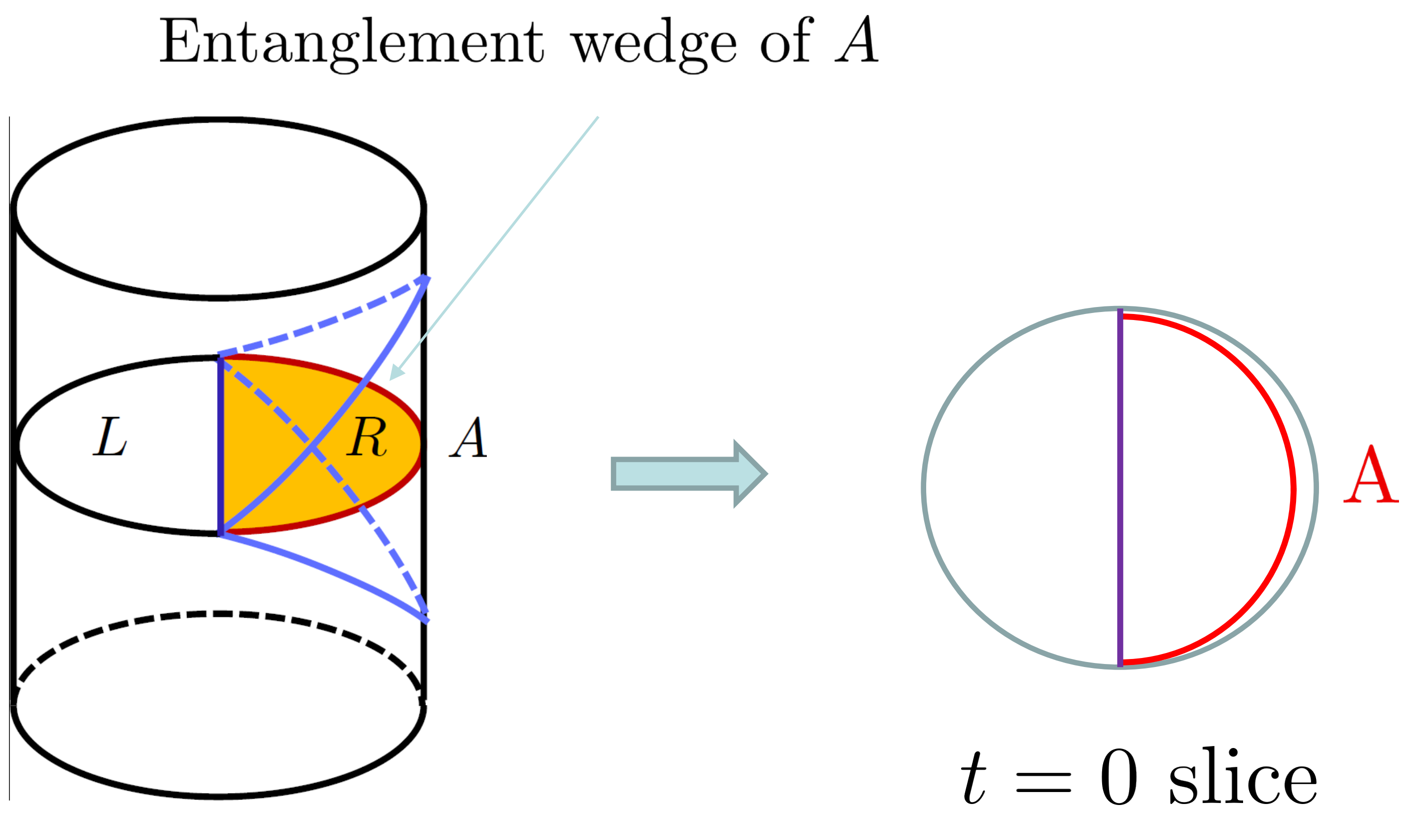
- **how AdS/CFT for subregion is realized**

- **Black hole complementarity like property (subregion complementarity) is important.**

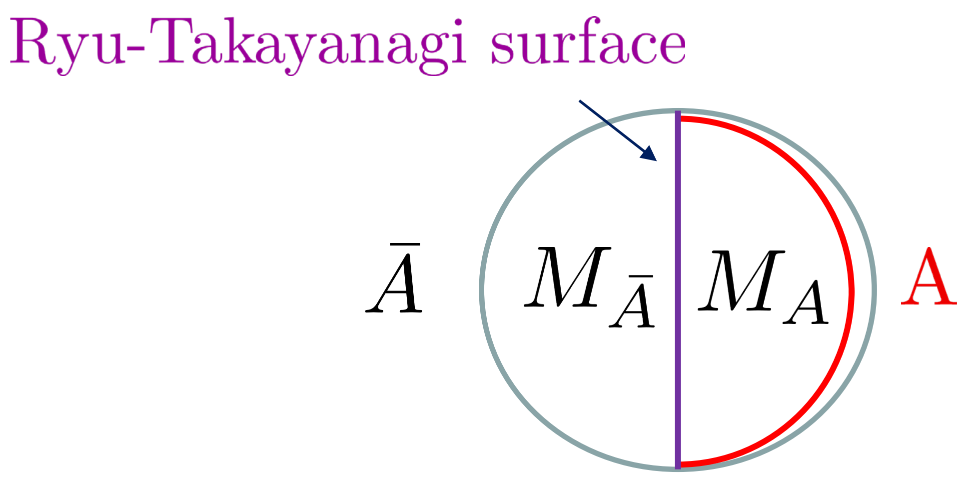
Plan

1. **Introduction**
2. **Violation of entanglement wedge reconstruction**
3. **AdS/CFT for subregion**
4. **Conclusion**

Counter example of entanglement wedge reconstruction (and subregion duality)


Entanglement wedge of A
L
R
A
A
$t = 0$ slice

Decompositions for Bulk and CFT



$$\text{CFT space } (= S^{d-1}) = A + \bar{A}$$

$$\text{Bulk space} = M_A + M_{\bar{A}}$$

$$M_A = \text{Entanglement wedge of } A$$

Subregion duality:

For density matrices ρ, σ,

$$\rho_A = \sigma_A \Leftrightarrow \rho_{M_A} = \sigma_{M_A}$$

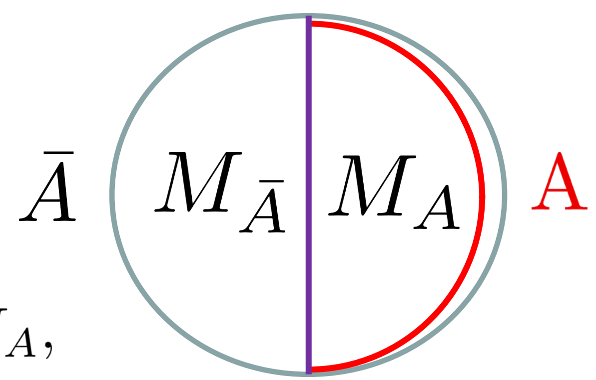
where $\rho_A = \mathrm{t}r_{\bar{A}}(\rho), \rho_{M_A} = \mathrm{t}r_{M_{\bar{A}}}(\rho)$

Entanglement wedge reconstruction:

Dong-Harlow-Wall

For low enegy state $|\phi\rangle$,

$$^{\forall}\mathcal{O}_{M_A}|\phi\rangle = {}^{\exists}\mathcal{O}_A|\phi\rangle,$$



$\mathcal{O}_{M_A}$ is bulk operator supported in M_A,

$\mathcal{O}_A$ is CFT operator supported in A

”Derivation”

Jafferis-Lewkowycz-Maldacena-Suh (JLMS) showed

CFT relative entropy = bulk relative entropy

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Subregion duality

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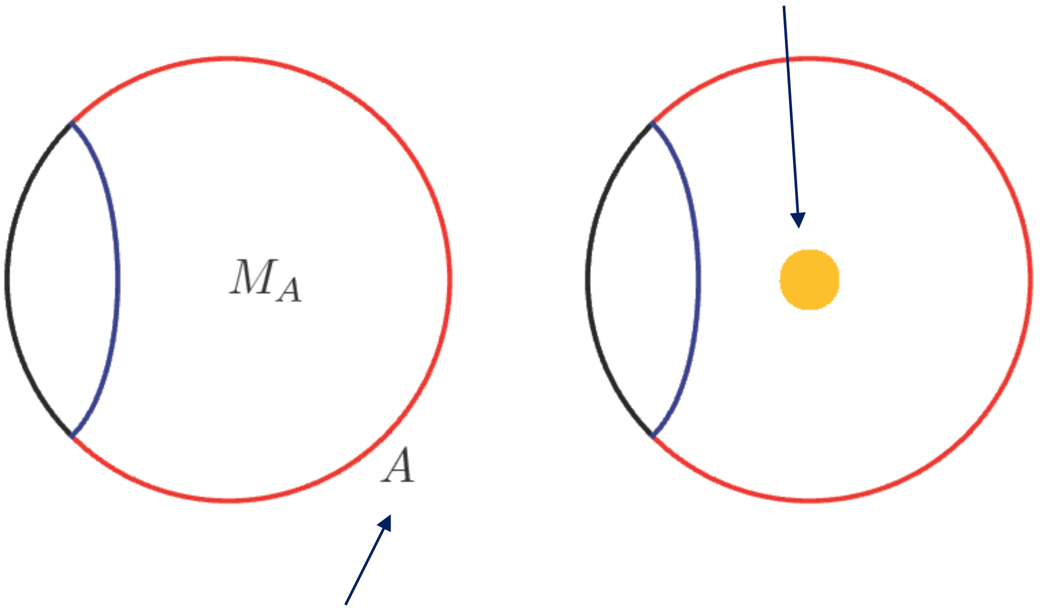
Entanglement wedge reconstruction

Couterexample of entanglement wedge reconstruction

bulk operator ϕ supported on small region around the center such that ϕ is spherically symmetric and Hermite



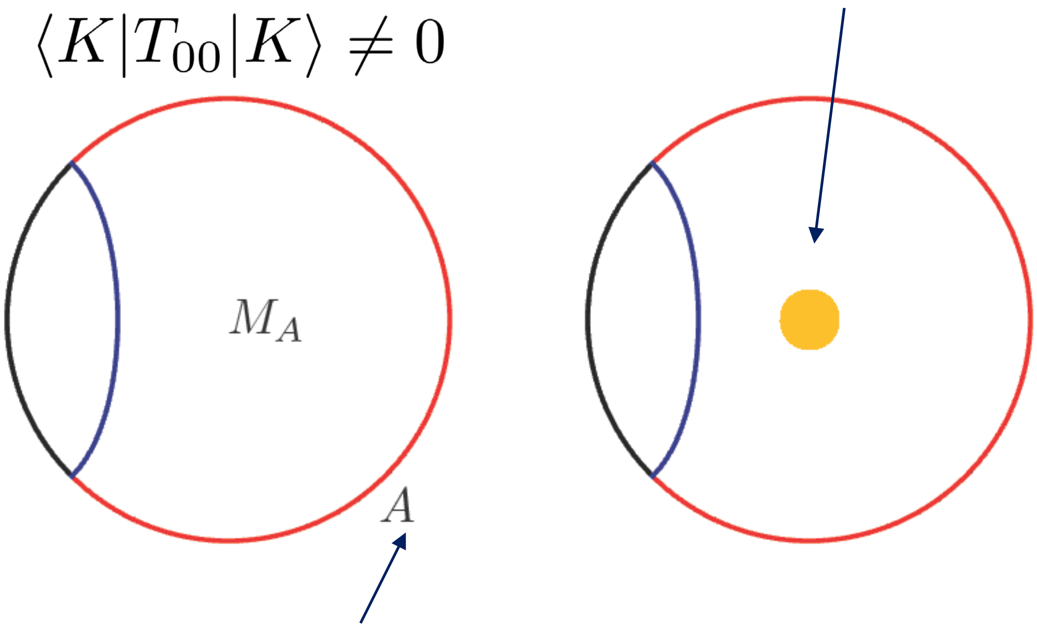
a CFT subregion A such that its causal wedge M_A includes the bulk region around the center

Consider the state $|K\rangle = e^{i\phi}|0\rangle$.

Violation of entanglement wedge reconstruction

Clearly, expectation value of the CFT stress tensor for $|K\rangle$ is spherical symmetric and nonzero everywhere.

i.e. $\langle K|T_{00}|K\rangle \neq 0$



Can we find CFT operator O^R, which is supported only on A, such that $e^{iO^R}|0\rangle = |K\rangle$?

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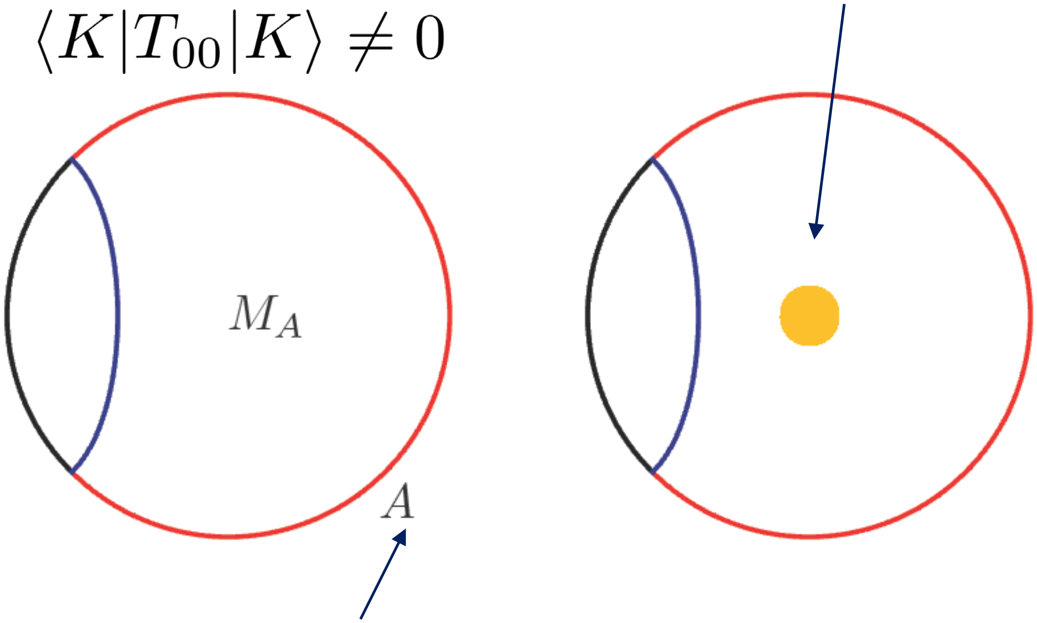
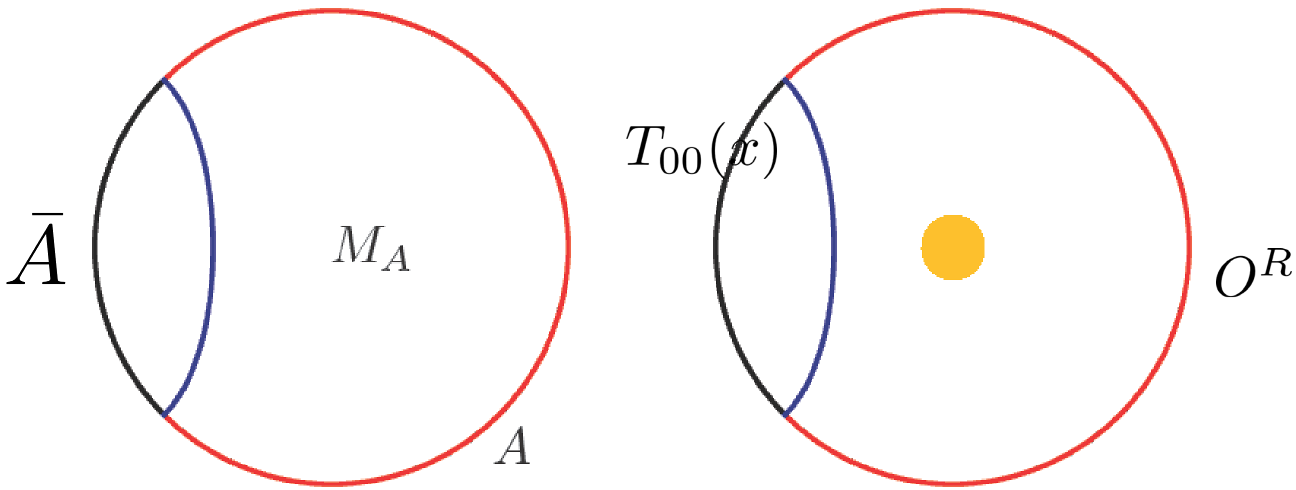


Can we find CFT operator O^R, which is supported only on A, such that $e^{iO^R}|0\rangle = |K\rangle$?

The answer is NO!

Violation of entanglement wedge reconstruction

The answer is no because
$\langle 0|e^{-iO^R} T_{00}(x) e^{iO^R}|0\rangle = 0$ where $x \in \bar{A}$
by the causality $[T_{00}(x), O^R] = 0$.



There is no O^R such that $e^{iO^R}|0\rangle = e^{i\phi}|0\rangle$.
Thus, the entanglement wedge reconstruction is not valid.

Remarks:

Everythings here is $\mathcal{O}(N^0)$.

By smearing of the operators in spacetime,
everything here is considered low energy.

The difference in energy fluxes means
difference in 3-point functions $\langle 0|[\phi, [T_{00}(x), \phi]]|0\rangle$.

Reeh–Schlieder th. gives any state, but not an operator.

ϕ and O^R can be represeted by
global and AdS-Rindler HKLL reconstructions.
Then, $e^{iO^R}|0\rangle \neq e^{i\phi}|0\rangle$. means
the two HKLL reconstructions give different operators
at same bulk point.

Why does this happens?

Bulk gravity theory is invalid if we consider a subregion of spacetime, which implies that there are "horizons".

This is because of the UV cut-off,
typically the Planck mass, of this effective theory.

We stress that this can be seen by considering finite N because 1/N expansion (i.e. semi-classical expansion) is based on the leading order spectrum.
In this sense, this is non-perturbative quantum gravity effect.

Why does this happens?

Bulk free theory is only the low energy
and large N limit of the (finite N) CFT.

Free theory on the bulk Rindler patch M_A is incorrect
as an approximation of the CFT, i.e. the quantum gravity,
in particular on "horizon".

Failure of low-energy effective theory(=bulk gravity)!
i.e. failure of asymptotic $1/N$ expansion

Jafferis-Lewkowycz-Maldacena-Suh used 1-loop correction of Ryu-Takayanagi formula (Faulkner-Lewkowycz-Maldacena), which assume that the bulk gravity theory is valid even for the subregion.

But, this is not justified.
(In particular, the entanglement entropy depends on the boundary of the subregion, i.e. “horizon”.)

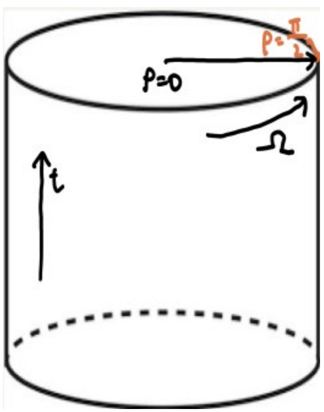
Such a violation is an essential property of (black hole) horizon, which is universal to general black hole horizons.
(related to “Brick wall”)

AdS/CFT for subregion

(Global) AdS_{d+1}

The metric of global AdS_{d+1} ($l_{AdS} = 1$) is

$$ds^2_{AdS} = \frac{1}{\cos^2(\rho)}\left(-dt^2 + d\rho^2 + \sin^2(\rho) d\Omega^2_{d-1}\right)$$



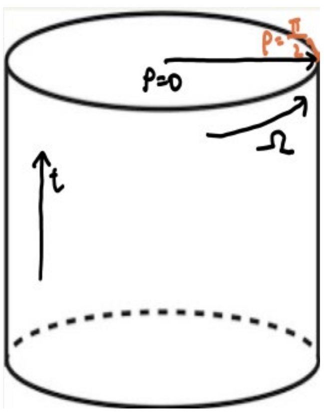
where $0 \leq \rho < \pi/2$

$$z \equiv \pi/2 - \rho$$

Boundary of AdS is at $z = 0$

Boundary of (Global) AdS_3

The boundary of AdS_3 is the cylinder



$$ds^2_{cylinder} = -dt^2 + d\theta^2$$

(Global) HKLL bulk reconstruction

Bulk local fied $\phi(t_0, z_0, \theta_0)$
is related to fields at boundary $\phi(t, z=0, \theta)$
using free e.o.m.

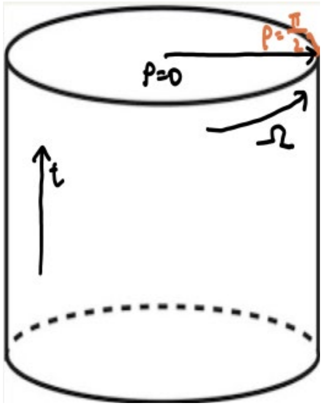
Then, using BDHM relation $\lim_{z\to 0} \frac{\phi(t,z,\theta)}{z^{\Delta}} \sim \mathcal{O}(t,\theta)$

$\Downarrow$

Bulk local fied is given by CFT field:

$$\phi(t, z, \theta) \leftrightarrow \int dt' d\theta' K(\theta', t') \mathcal{O}(\theta', t')$$

$z \equiv \pi/2 - \rho$



(Global) HKLL bulk reconstruction

We will denote the CFT operator as

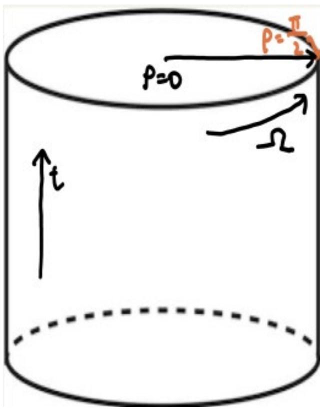
$$\phi^G(t,z,\theta) \equiv \int dt' d\theta' K(\theta',t')\mathcal{O}(\theta',t')$$

Then, we can show

$$\langle 0|\phi(t,z,\theta)\phi(t',z',\theta')|0\rangle = \langle 0|\phi^G(t,z,\theta)\phi^G(t',z',\theta')|0\rangle$$

i.e. bulk correlation function

is reproduced by CFT oeprator



Rindler AdS_3

The metric of Rindler patch of AdS_3 ($l_{AdS} = 1$) is

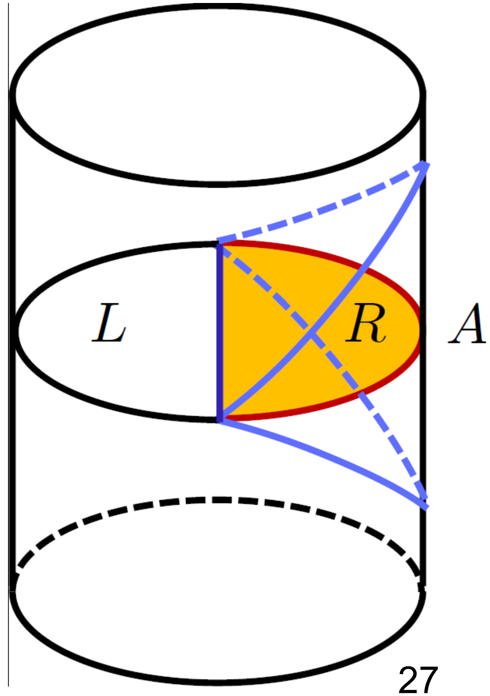
$$ds^2 = -\xi^2 dt_R^2 + \frac{d\xi^2}{1+\xi^2} + (1+\xi^2)d\chi^2$$

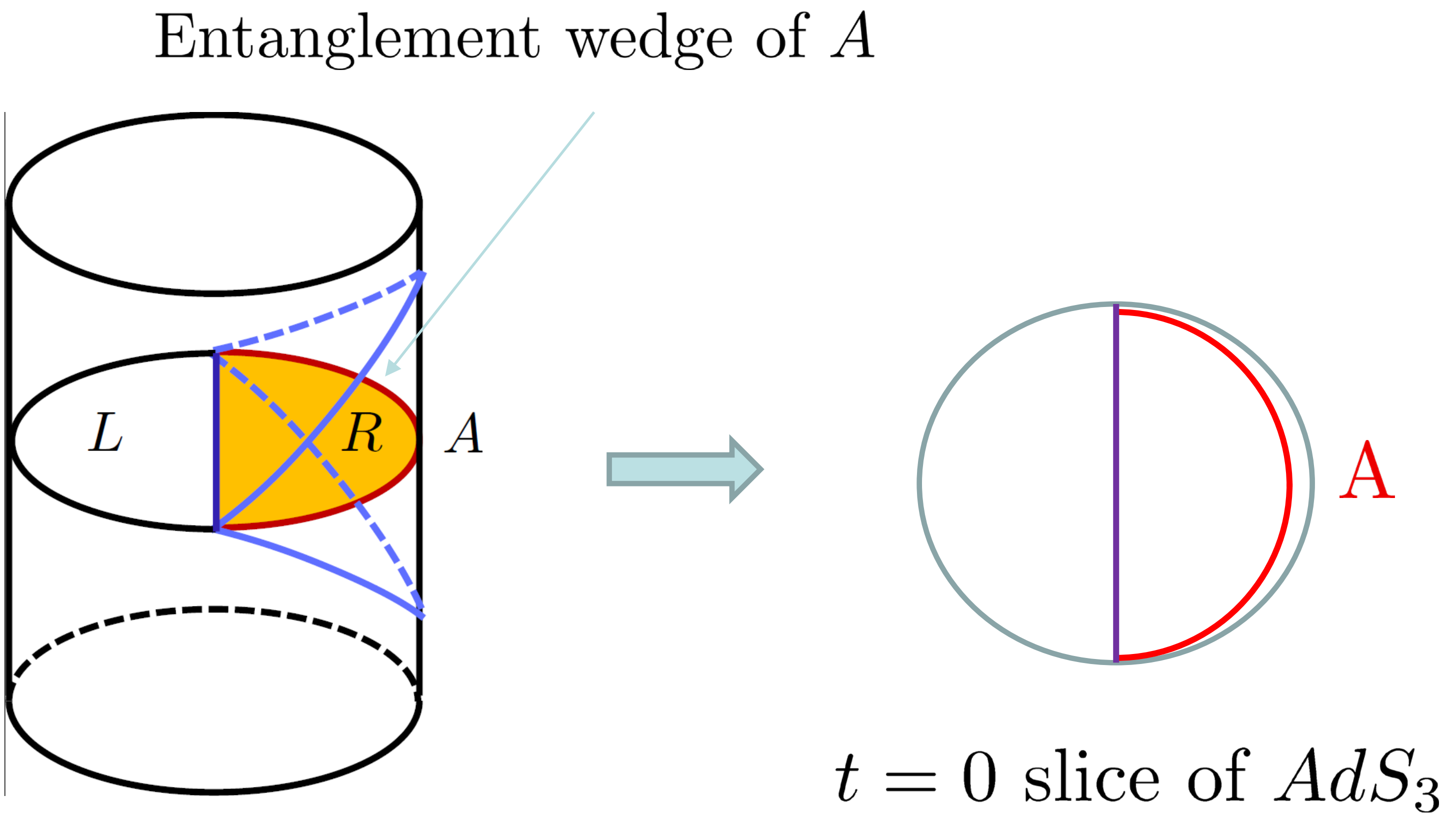
where $-\infty < t_R < \infty$, $-\infty \leq \chi < \infty$

$$0 \leq \xi < \infty$$

Boundary of AdS_3 is located at $\xi = \infty$

Rindler horizon is at $\xi = 0$





Entanglement wedge of A
L
R
A
A
$t = 0$ slice of AdS_3

AdS-Rindler HKLL bulk reconstruction

As for global case, we can reconstruct bulk operator.

We will denote the CFT operator as

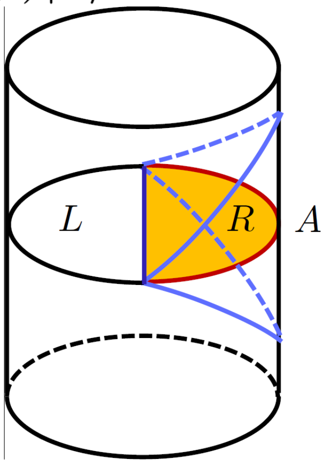
$$\phi^R(t_R, \xi, \chi) \equiv \int dt'_R d\chi' K(\chi', t'_R)\mathcal{O}(\chi', t'_R)$$

Then, if $\{t_R, \xi, \chi\}$ are in AdS-Rindler patch,

$$\langle 0|\phi(t_R, \xi, \chi)\phi(t'_R, \xi', \chi')|0\rangle = \langle 0|\phi^R(t_R, \xi, \chi)\phi^R(t'_R, \xi', \chi')|0\rangle$$

i.e. bulk correlation function in M_A

is reproduced by CFT operator in A



Subregion complementarity

Both of ϕ^G, ϕ^R, reconstructed by CFT operator, give bulk correlation function for $X, X' \in M_A$:

$$\langle 0|\phi(X)\phi(X')|0\rangle = \langle 0|\phi^G(X)\phi^G(X')|0\rangle = \langle 0|\phi^R(X)\phi^R(X')|0\rangle$$

Thus, both describe bulk physics in M_A.

Nevertheless, these are different: $\phi^R(X)|0\rangle \neq \phi^G(X')|0\rangle$

(Note that $\langle 0|\phi(X)\phi(X')|0\rangle \neq \langle 0|\phi^G(X)\phi^R(X')|0\rangle$)

Same bulk operator in different coordinate patches are realized in CFT differently!

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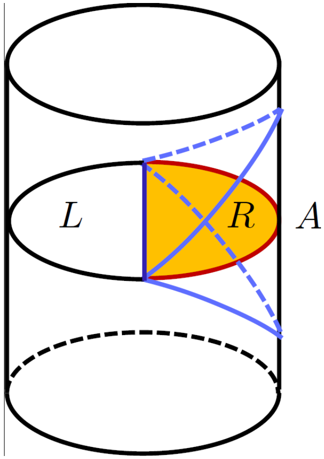
Subregion complementarity

(similar(?) to Black hole complementarity)



What is black hole complementarity

Consider a black hole, and two observers:

Infalling observer:

According to an infalling observer, nothing special happens at the event horizon itself

Outside observer:

infalling information heats up the stretched horizon, which then reradiates it as Hawking radiation

Black hole complementarity:

These two observers see different physics, but no problems because they can not communicate.

Subregion complementarity may be similar to this.

Conclusion

- **Entanglement wedge reconstruction (and subregion duality) is not valid.**
- **AdS/CFT for subregion works even though the subregion duality does not work.**
- **Black hole complementarity like property (subregion complementarity) is important.**

Future directions

- **There are lots of important things to investigate and understand, at least for me.**

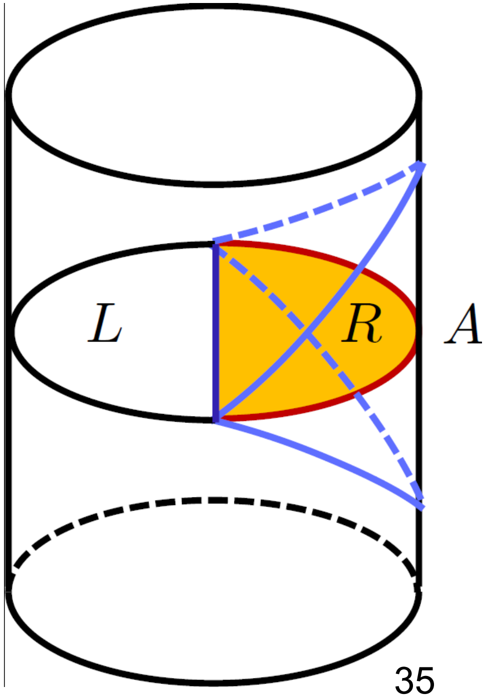
backups

Boundary of Rindler AdS_3

The boundary of Rindler patch of AdS_3 is (conformally) Minkowski space

$$ds^2 = e^{2\Phi}(-dt_R^2 + d\chi^2)$$

where $-\infty < t_R < \infty$, $-\infty \leq \chi < \infty$



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⇩

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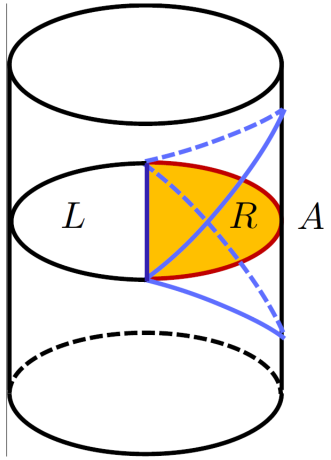

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i.e. bulk correlation function
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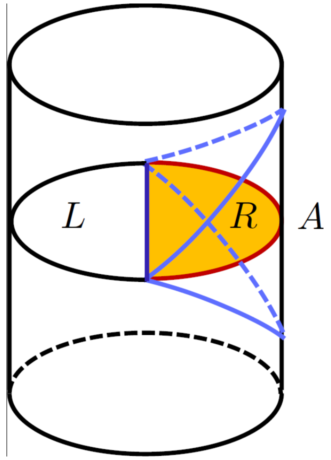


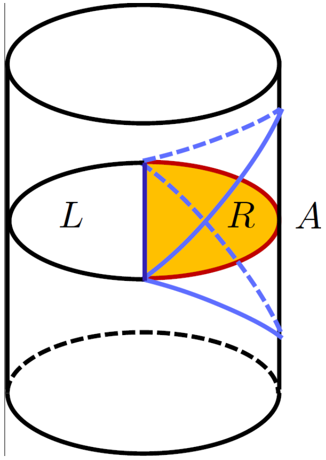
Difference between bulk semi-classical gravity theory and finite N CFT

CFT operator in Rindler patch

By the conformal transformation,

CFT primary operator in Rindler patch is same as

CFT primary operator in Minkowski space



Free scalar in Bulk Rindler patch

We expand ϕ by the modes $v_{\omega,\lambda,\mu}(t_R, \xi, \chi)$,

$$\phi(t_R,\xi,\chi) = \int_0^\infty d\omega \int_{-\infty}^{\infty} d\lambda \frac{1}{\sqrt{2\pi}} \tilde{\psi}_{\omega,\lambda}(\xi) \left[a_{\omega,\lambda} e^{-i\omega t_R + i\lambda\chi} + a^\dagger_{\omega,\lambda} e^{i\omega t_R - i\lambda\chi} \right].$$

Modes are given as

$$\tilde{\psi}_{\omega,\lambda}(\xi) = \frac{N_{\omega,\lambda}}{\Gamma(\nu+1)} \xi^{i\omega} (1+\xi^2)^{-\frac{i\omega}{2} - \frac{\Delta}{2}} \; {}_2F_1\left(\frac{i\omega - i\lambda + \nu + 1}{2}, \frac{i\omega + i\lambda + \nu + 1}{2}; \nu+1; \frac{1}{1+\xi^2} \right)$$

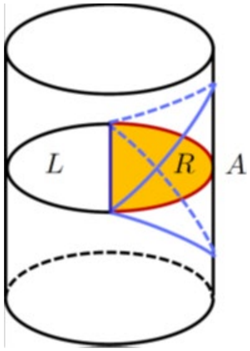
$$N_{\omega,\lambda} = \frac{\left|\Gamma\left(\frac{i\omega - i\lambda + \nu + 1}{2}\right)\right| \left|\Gamma\left(\frac{i\omega + i\lambda + \nu + 1}{2}\right)\right|}{\sqrt{4\pi\omega} |\Gamma(i\omega)|}$$

Here, energy ω is any real number

BDHM relates Bulk and CFT pictures

In global and Rindler, fields are identical.

$$\lim_{\xi\to\infty} \xi^{\Delta}\phi(t_R,\xi,\chi) = O_{\Delta}(t_R,\chi).$$



$$\Downarrow$$

$$O_{\Delta}(t_R,\chi) = \int_0^{\infty} d\omega \int_{-\infty}^{\infty} d\lambda \frac{N_{\omega,\lambda}}{\sqrt{2\pi}\Gamma(\nu+1)} \left[a_{\omega,\lambda}e^{-i\omega t_R + i\lambda\chi} + a^{\dagger}_{\omega,\lambda}e^{i\omega t_R - i\lambda\chi}\right]$$

Modes with $\omega < |\lambda|$ are tachyonic!

Thus, different from CFT primary field

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Failure of low-energy effective theory(=bulk gravity)!
Asymptotic $1/N$ expansion vs Unitarity

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