Quantum error correction in spin chains and related models

Third Annual Meeting of the ExU Collaboration

10:45 – 11:15, 11 September 2023

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Thermalization of quantum systems
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Kazuya YAMASHITA
Cold atom experiment

and many collaborators

[ExU PD]
Giacomo MARMORINI
Theory of cold atoms
Thermalization of quantum systems

Contents

- Summary of our research plan
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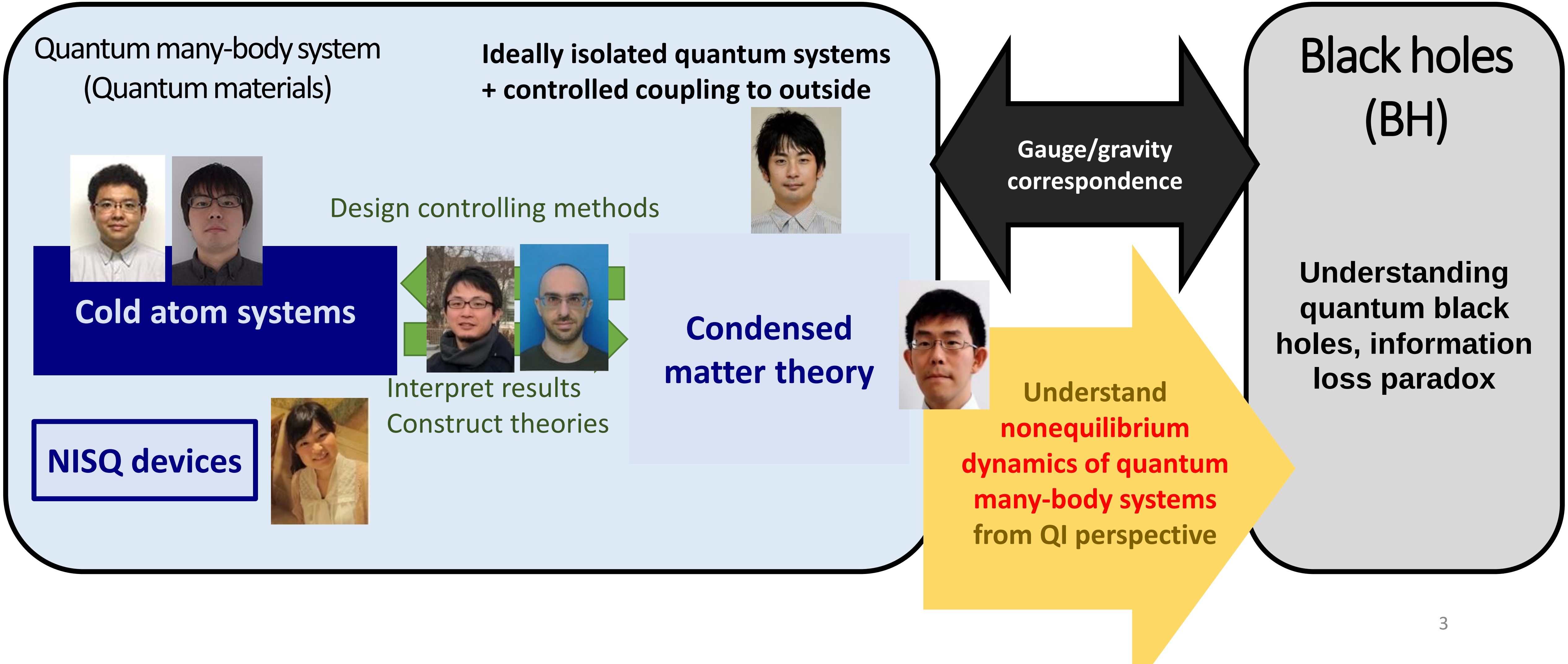
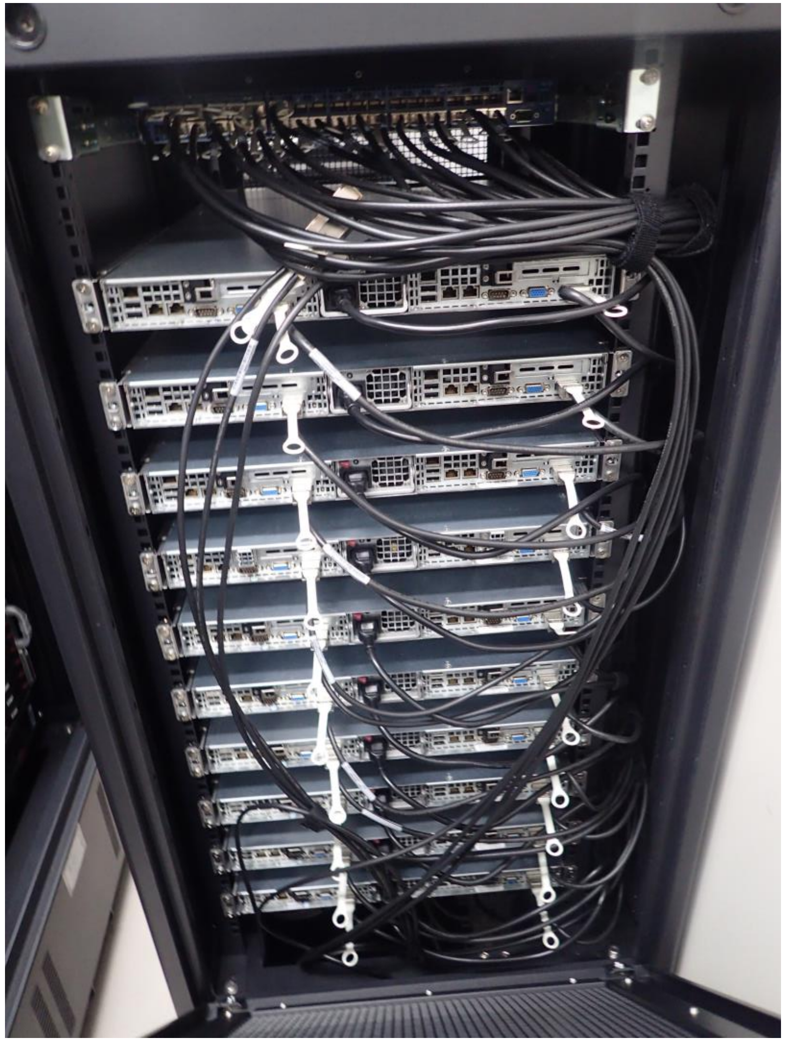
- Some of recent results & ongoing work
  - "Possible measurement of entanglement in a many-body quantum simulator via spiral quantum state tomography" → Giacomo Marmorini's talk on 29th
  - "Evaluation of Quantum Entanglement via Permutationally Invariant Quantum State Tomography" → Yuki Miyazaki's poster on 26th
  - **"Hayden-Preskill Recovery in Hamiltonian Systems"**
    - Y. Nakata (A01) and M. Tezuka, 2303.02010
  - "A model of randomly coupled Pauli spins"
    - M. Hanada, A. Jevicki, X. Liu, E. Rinaldi, and M. Tezuka, 2309.15349

Group B02: One-page summary



“Hayden-Preskill Recovery in Hamiltonian Systems” Y. Nakata (A01) and M. Tezuka, arXiv:2303.02010

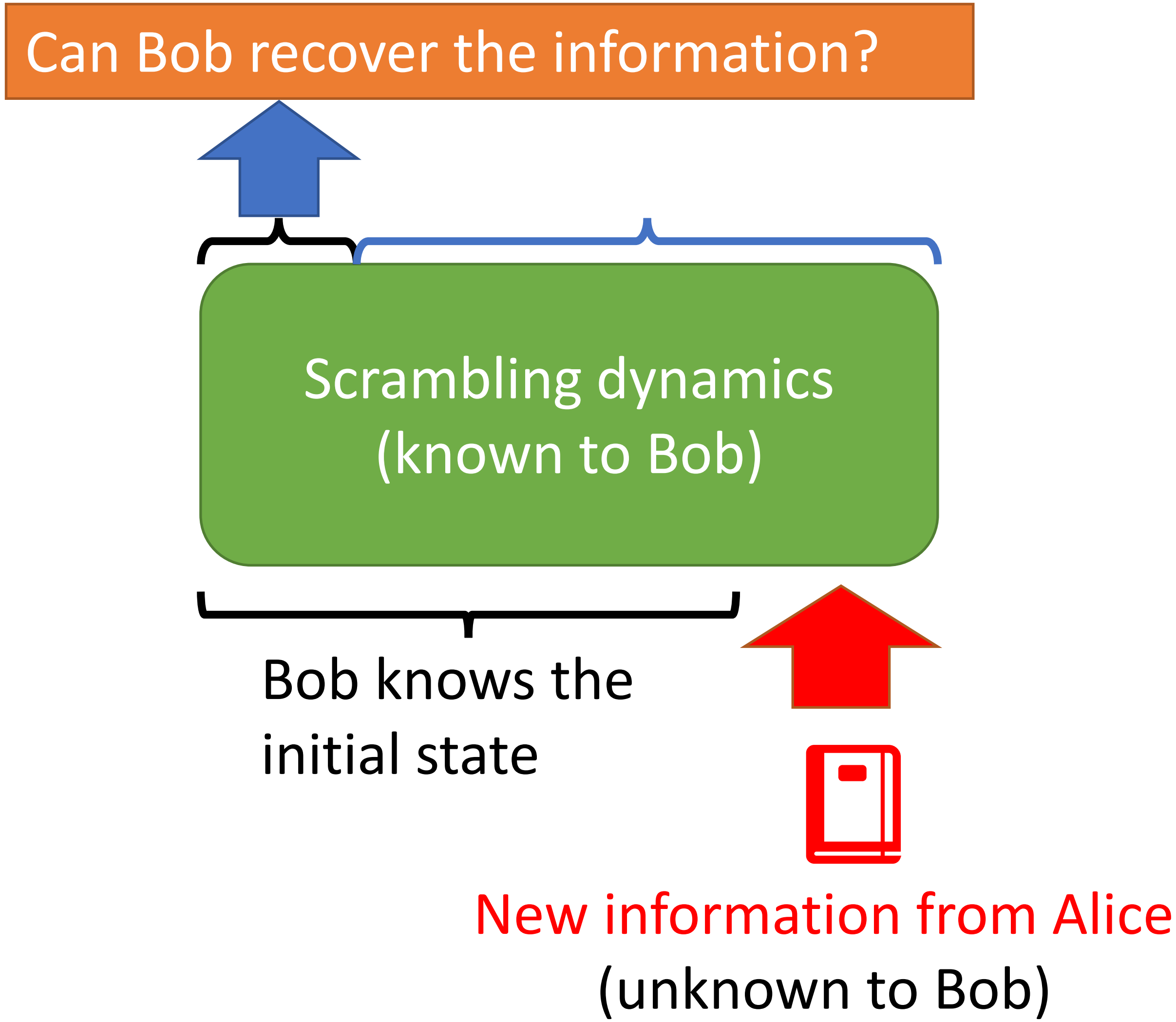
Error correction



- Embed the message in a **longer bit sequence** by **adding some redundancy**
- Consistency of the transmitted message can be checked by some algorithm
- **Error can be detected or corrected**

Quantum error correction

(also known as information scrambling)



The quantum information becomes delocalized

↓

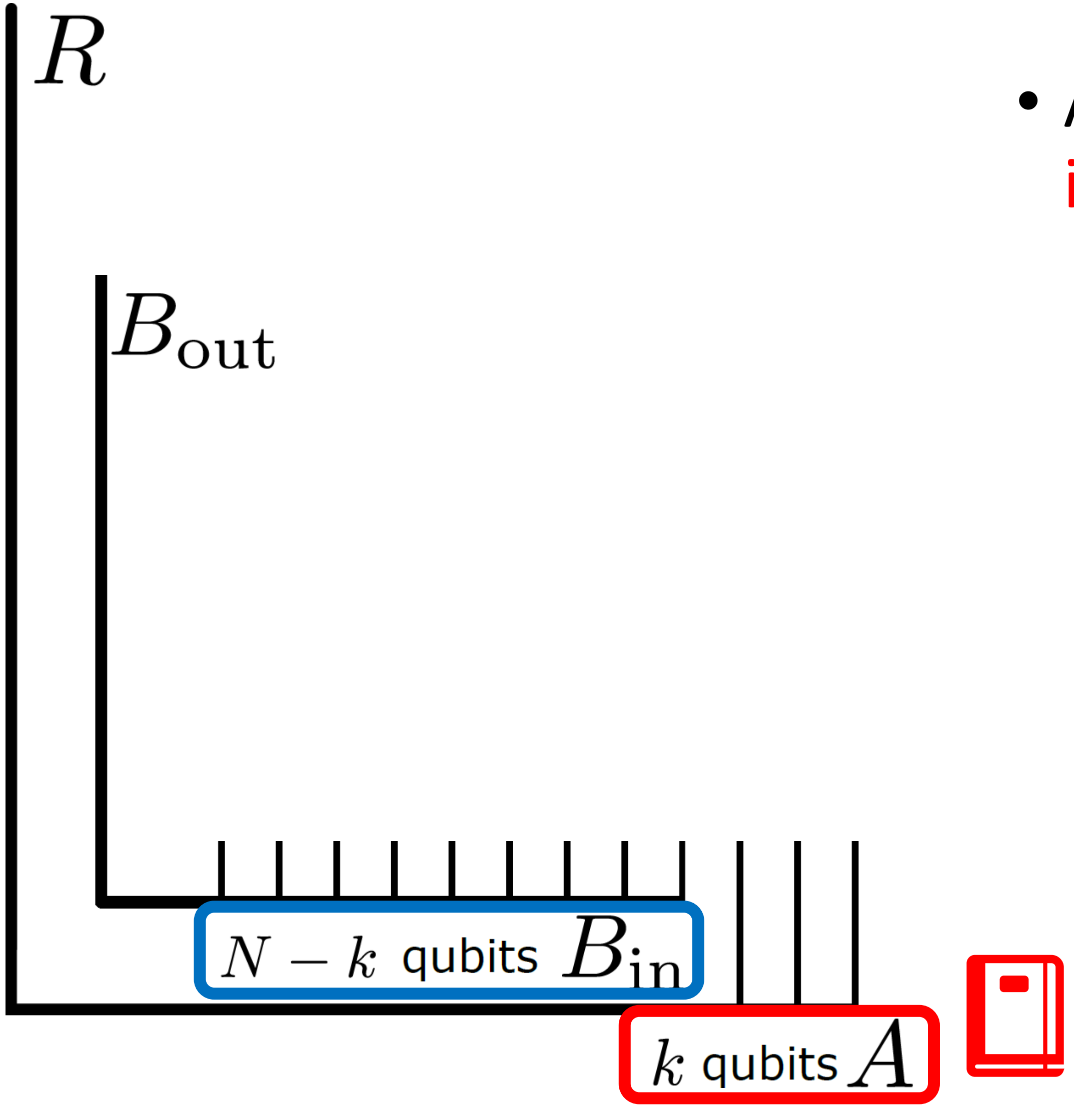
It can be recovered from a part of the system

No-cloning theorem:
It is not possible to create two accurate copies of arbitrarily given quantum information!

After the recovery process, **the reminder of the system** should **lose correlation with the input**!

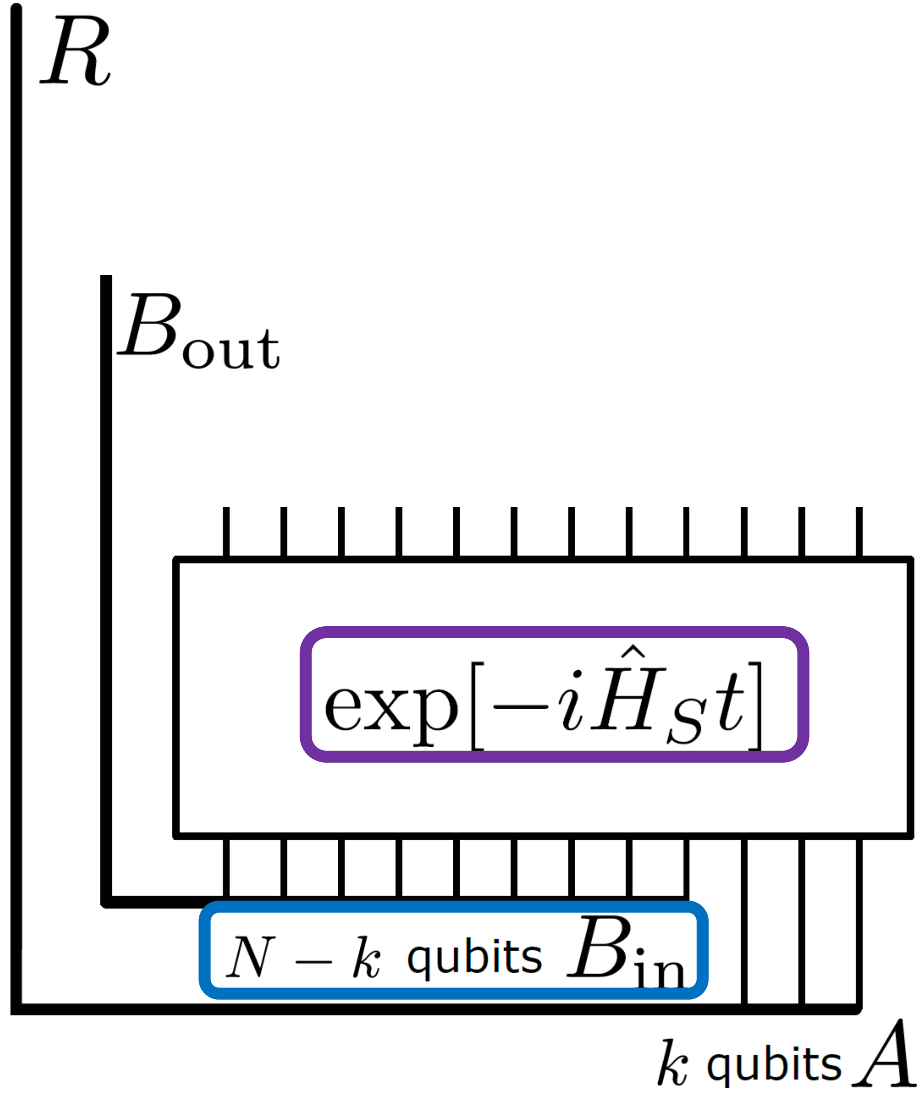
P. Hayden and J. Preskill, JHEP 2007

Quantum error correction: The Hayden-Preskill protocol



- Alice: throws k-qubit quantum information A into a box B_{in}

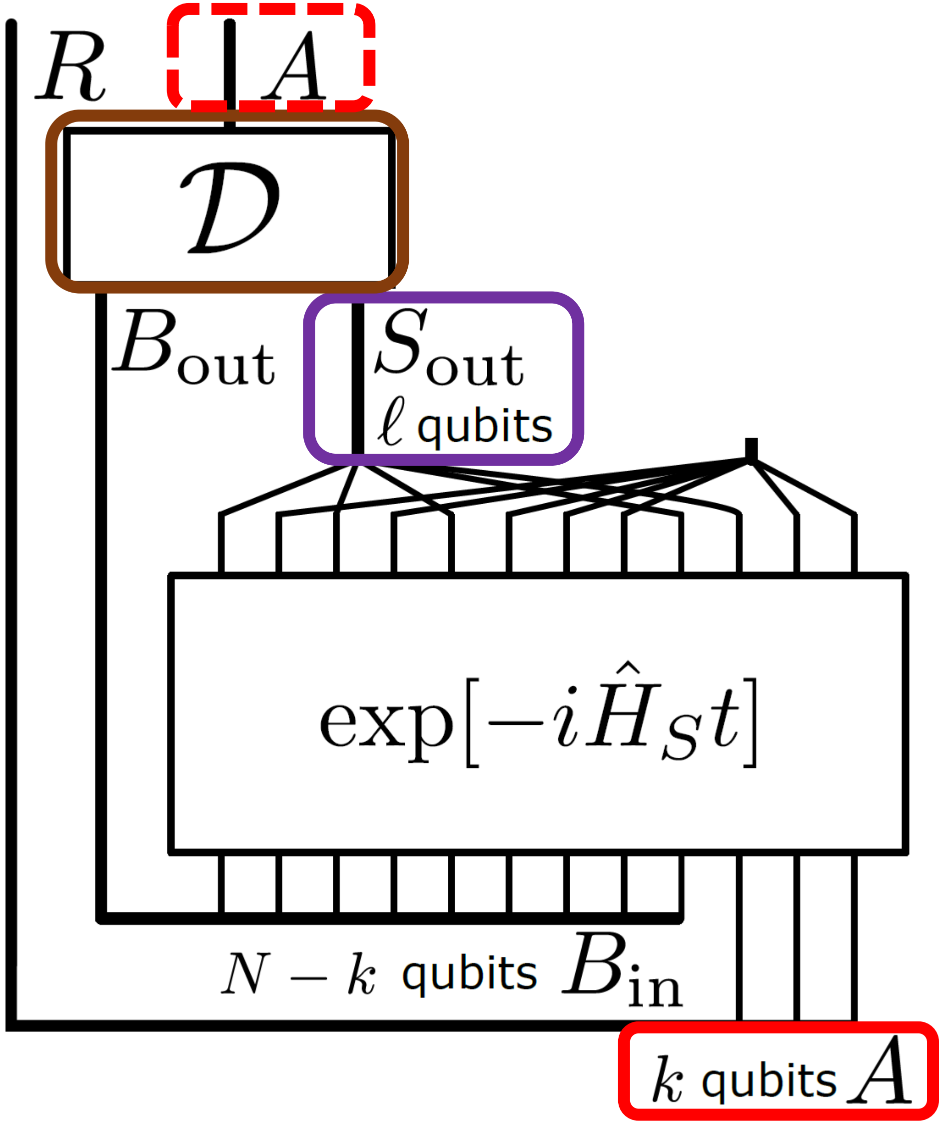
Quantum error correction: The Hayden-Preskill protocol



- Alice: throws k-qubit quantum information A into a box B_{in}
- Bob: knows the original state of B_{in} and the Hamiltonian $\hat{H}_S$ of $S = A + B_{\text{in}}$

P. Hayden and J. Preskill, JHEP 2007

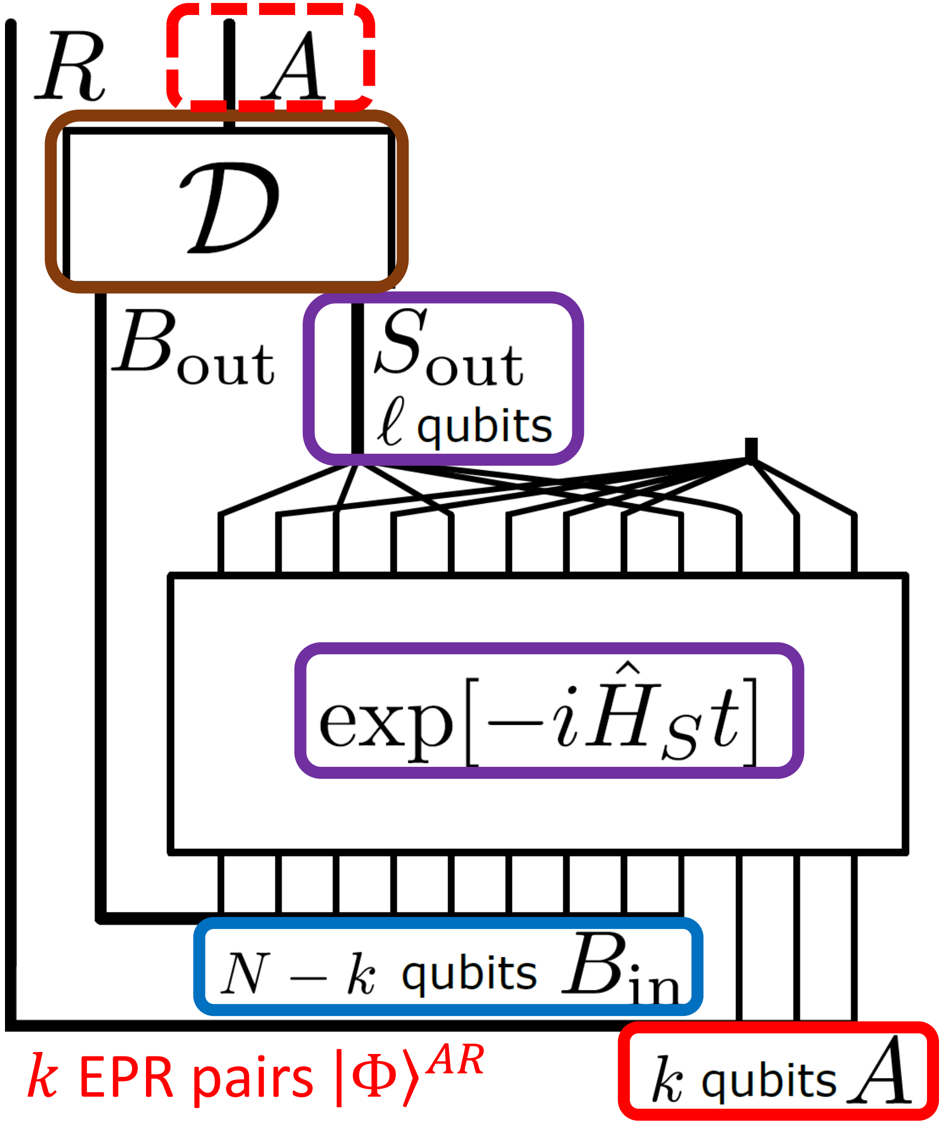
Quantum error correction: The Hayden-Preskill protocol



- Alice: throws k-qubit quantum information A into a box $B_{\rm in}$
- Bob: knows the original state of $B_{\rm in}$ and the Hamiltonian $\hat{H}_S$ of $S = A + B_{\rm in}$
- Bob obtains ℓ qubits $S_{\rm out}$ after time t. Can Bob decode ($\mathcal{D}$) Alice's secret?

P. Hayden and J. Preskill, JHEP 2007

Quantum error correction: The Hayden-Preskill protocol



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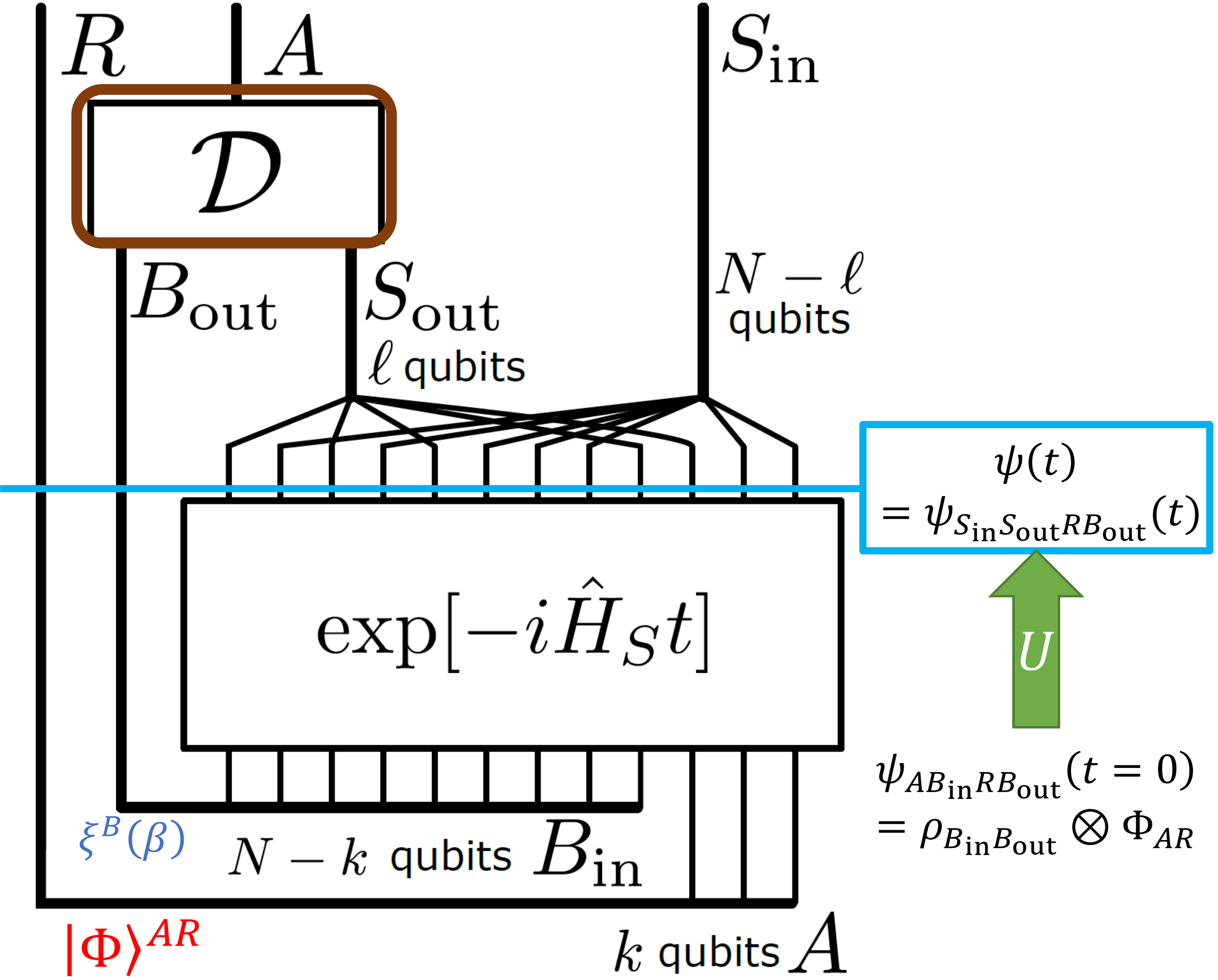
Black holes: information recovery for $\ell \sim k$

[Hayden and Preskill, JHEP 2007]

Circular unitary (Haar) ensemble was assumed

Quantum error correction: The Hayden-Preskill protocol



Recovery error $\Delta_{\hat{H}}(t,\beta)$ among any $\mathcal{D}$ is hard to compute…

Decoupling approach

For $\mathcal{D}$ to succeed, no correlation is allowed between $S_{\rm in}$ and R

$$\rho_{S_{\rm in}R} = {\rm Tr}_{B_{\rm out},S_{\rm out}} |\psi(t)\rangle\langle\psi(t)|$$

Decoding error estimate

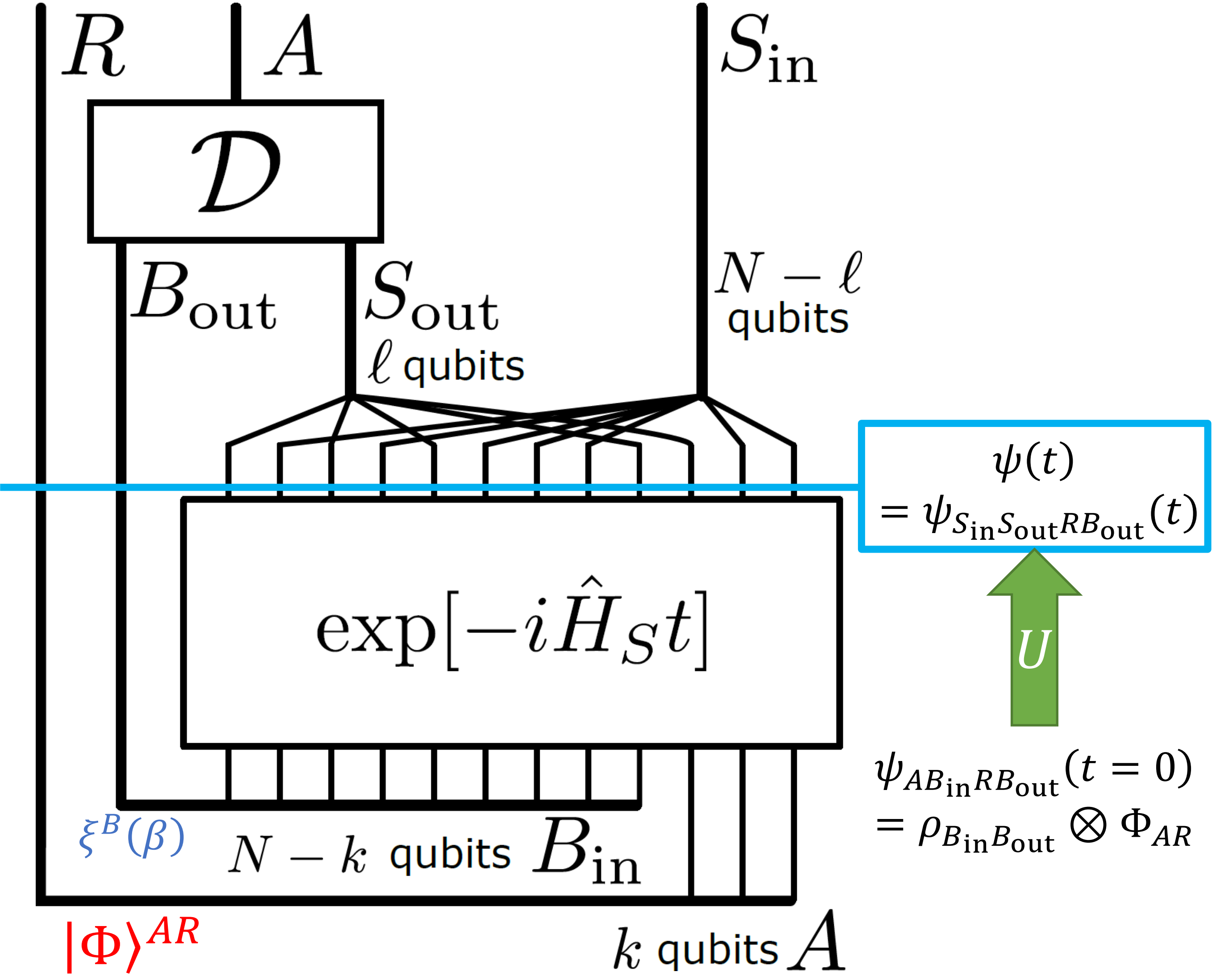
$$\bar{\Delta}_{\hat{H}}(t,\beta) \equiv \min\left\{1, \sqrt{\left\|\rho_{S_{\rm in}R} - \rho_{S_{\rm in}} \otimes \frac{I_R}{d_R}\right\|_1}\right\}$$

$(\geq \Delta_{\hat{H}}(t,\beta))$

$$\rho_{S_{\rm in}} = {\rm Tr}_R \rho_{S_{\rm in}R}$$

$$|M|_1 \equiv {\rm Tr}\sqrt{M^\dagger M}$$

Quantum error correction: The Hayden-Preskill protocol



Haar random unitary case:

$$\bar{\Delta}_{\text{Haar}}(\beta) = \min\left\{1, 2^{\frac{1}{2}(\ell_{\text{Haar,th}}(\beta)-\ell)}\right\}$$

$$\ell_{\text{Haar,th}}(\beta) = \frac{N+k-H(\beta)}{2} \xrightarrow{\beta\to 0} k$$

$H(\beta)$: Renyi-2 entropy of $\xi^B(\beta)$

$\bar{\Delta}_{\text{Haar}}$ exponentially decreases as function of ℓ after $\ell \approx k$ [HP recovery]

P. Hayden and J. Preskill, JHEP 2007

[Yoshifumi Nakata and MT, arXiv:2303.02010]

Our numerical study:

- SYK-type Hamiltonians
- One-dimensional spin chains

➔ Characterization of chaotic Hamiltonian dynamics

Error estimate for the SYK model

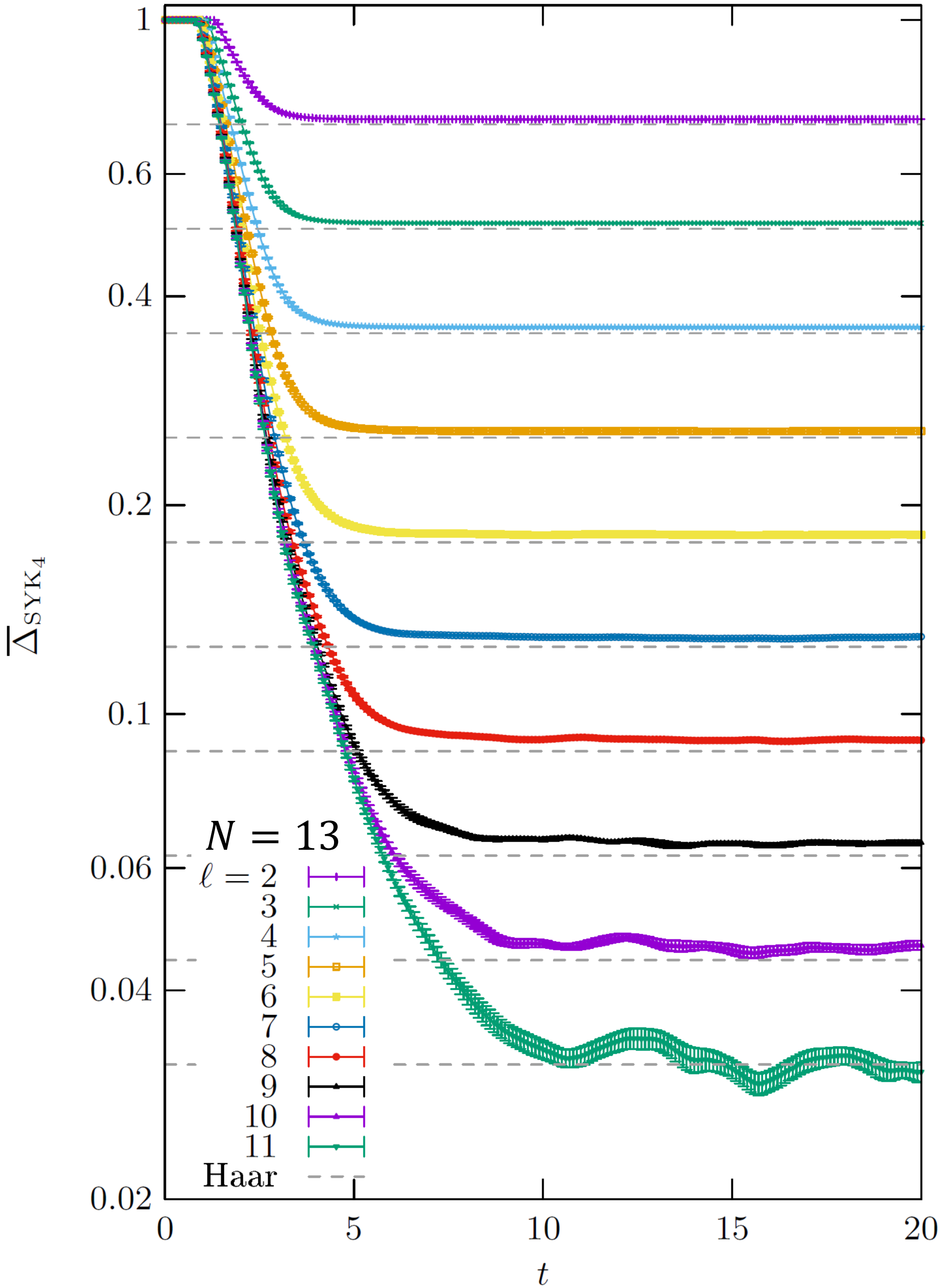
$$\hat{H} = \sum_{1 \le a<b<c<d \le 2N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$

[Kitaev 2015][Sachdev & Ye 1993]

$\hat{\chi}_{a=1,2,\ldots,2N}$: $2N$ Majorana fermions ($\{\hat{\chi}_a, \hat{\chi}_b\} = 2\delta_{ab}$)

J_{abcd} : independent Gaussian random couplings
($\overline{J_{abcd}{}^2} = J^2$, $\overline{J_{abcd}} = 0$);

Normalization hereafter: SYK half-bandwidth $\sqrt{\frac{\langle \mathrm{Tr}\,\hat{H}^2\rangle}{2^N}} = 1$



➔ $\overline{\Delta}$ reaches the Haar value quickly ($t \sim \sqrt{N}$)

Summary of our results for SYK-like models

SYK-like models with long-range couplings

Gaussian dense SYK_4

$$\hat{H} = \sum_{a<b<c<d} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$

sparsify →

Binary coupling sparse SYK

$$\hat{H} \propto \sum_{\substack{(a,b,c,d)\in P \\ K_{\mathrm{cpl}}=|P|\sim\mathcal{O}(N)}} (\pm 1)\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$

[Phys. Rev. B **107**, L081103 (2023)]

- $K_{\mathrm{cpl}} \lesssim N$: additional degeneracy, large error remains
- $K_{\mathrm{cpl}} \gtrsim 2N$: realize chaotic spectrum more efficiently than Gaussian

Error decays to ~ Haar value in $t \sim \sqrt{N}$

Quantum error correction by scrambling Hamiltonian dynamics
[Yoshifumi Nakata and MT, arXiv:2303.02010]

Add SYK_2 term ↓

SYK_{4+2}

$$\hat{H} = \cos\theta \sum_{a<b<c<d} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + \sin\theta \sum_{a<b} K_{ab}\hat{\chi}_a\hat{\chi}_b$$

[PRL **120**, 241603; PRR **3**, 013023; PRL **127**, 030601]

- $\delta \propto \tan\theta \ll 1$: SYK_4
- $\delta = \mathcal{O}(1)$: chaotic spectrum but eigenstates restricted in Fock space; entanglement entropy has plateau
- $\delta \gg 1$: localization of many-body eigenfunctions

Error increases before Fock space localization

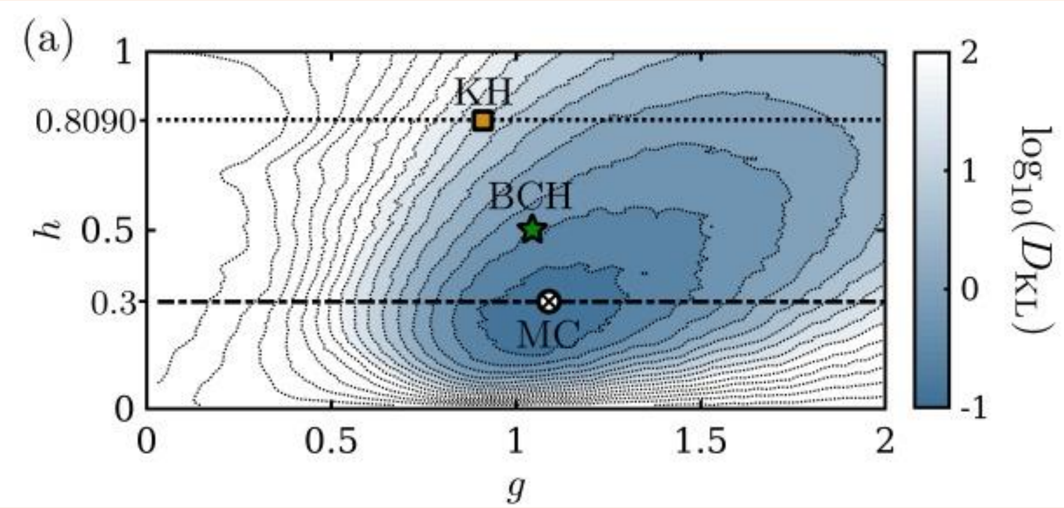
One-dimensional spin chains ($S = 1/2$)

- Ising model + uniform magnetic field

$$\hat{H}_{\text{Ising}} = -J \sum_{\langle j,k \rangle} S_j^z S_k^z - g \sum_j S_j^x - h \sum_j S_j^z$$

- $(g, h) = (g, 0), (0, h)$: integrable

BCH: often studied as being **far from integrability**
MC: “Most chaotic” in terms of entanglement entropy distribution [Rodriguez-Nieva, Jonay, Khemani 2305.11940]



- Heisenberg model + random field

$$\hat{H}_{\text{XXZ}} = J \sum_{\langle j,k \rangle} S_j \cdot S_k + \sum_j h_j S_j^z ,$$

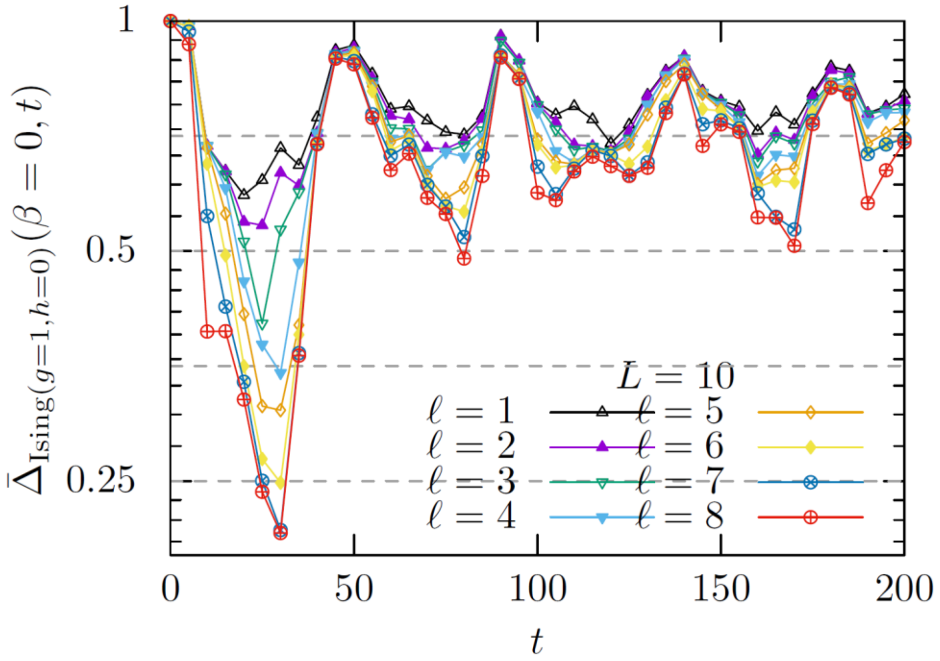
$$h_j \in [-W, W]$$

- $W = 0$: integrable
- $W \sim J$: **“ergodic”**
- $W \gtrsim 4J$: “MBL”

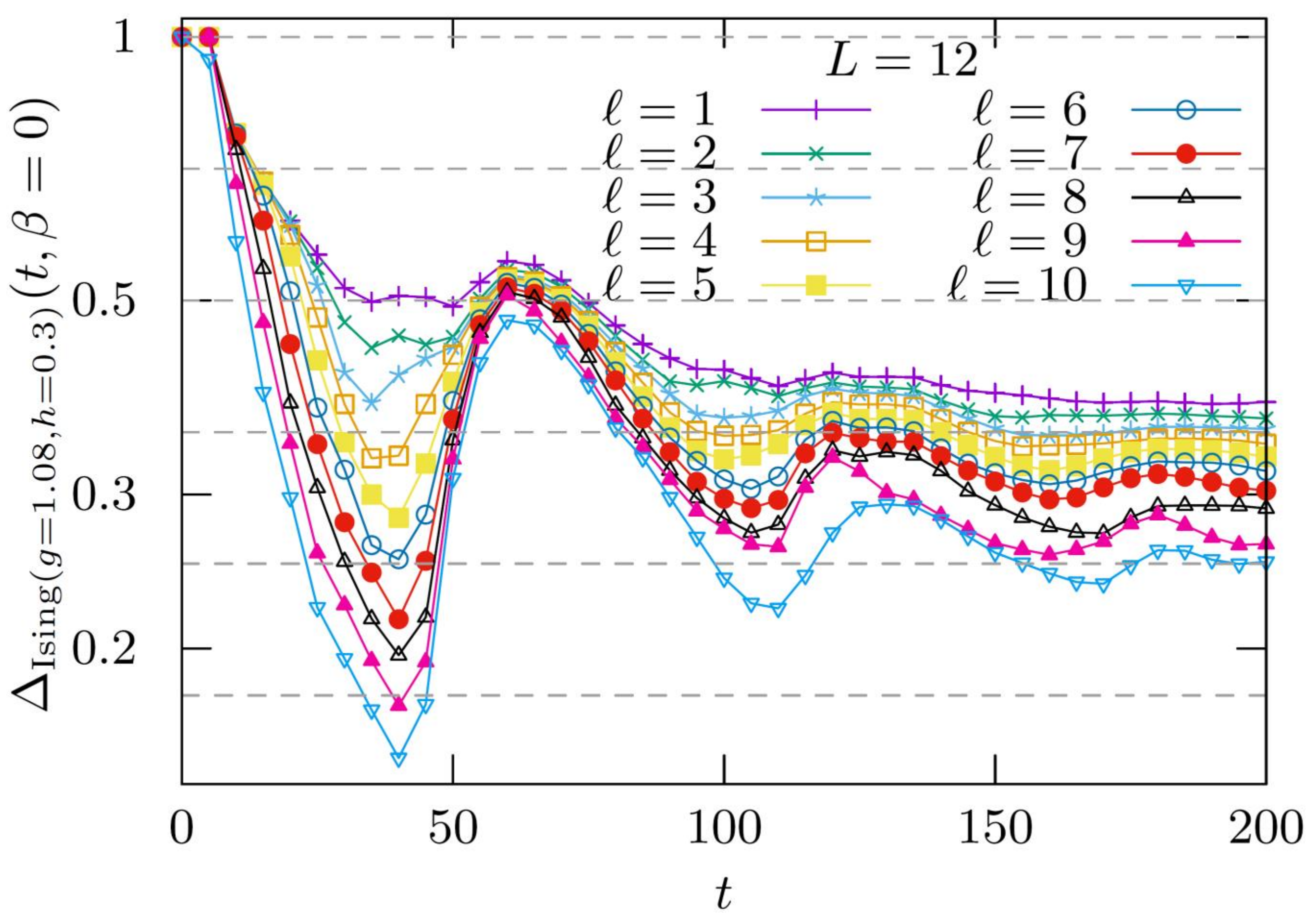
(though recently debated; see *e.g.* Morningstar *et al.*, PRB **105**, 174205 (2022))

Ising model + uniform magnetic field

- $H \parallel x$: integrable



- Non-integrable case

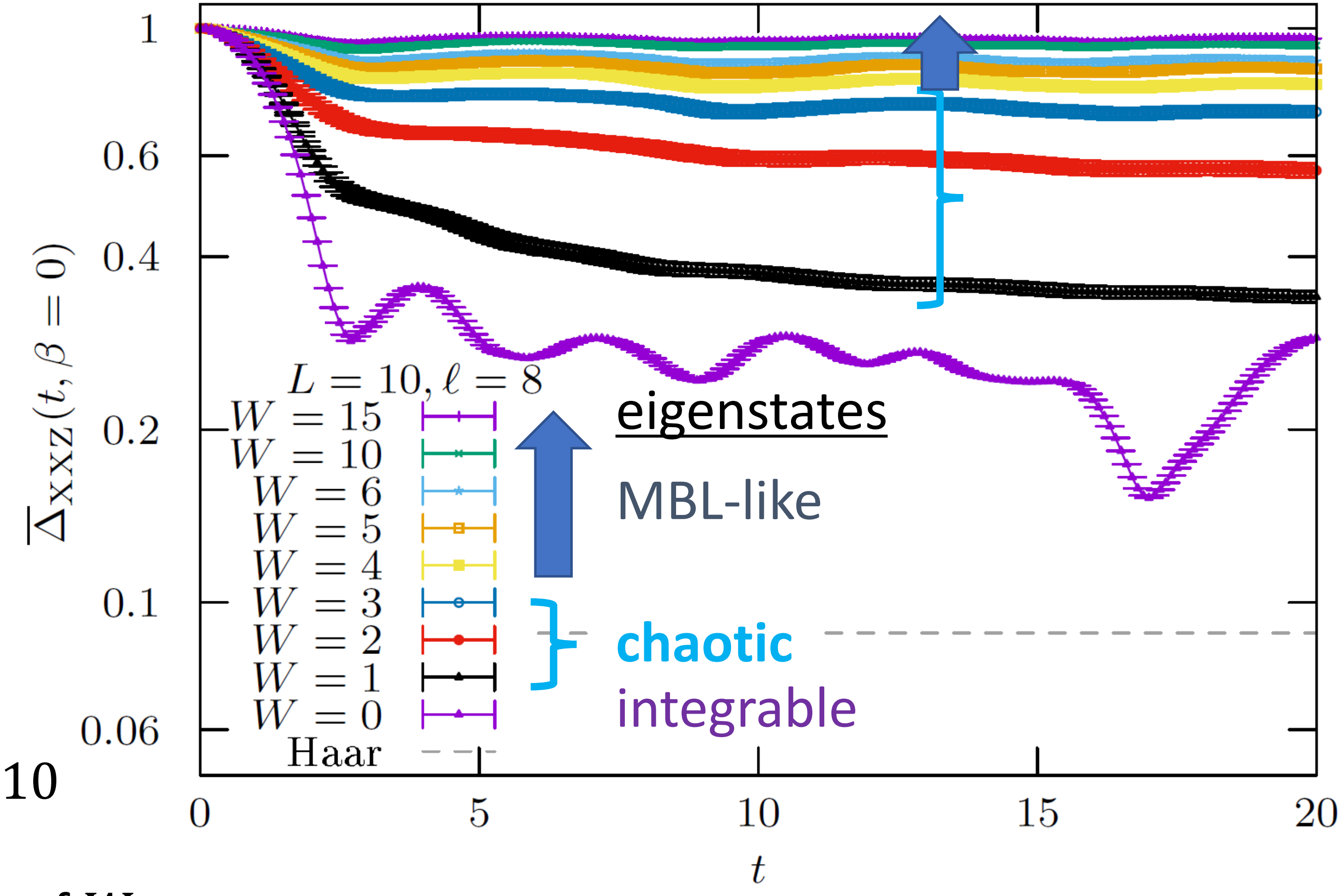


- **Very large error remains at long times in both cases**

Heisenberg model + random field

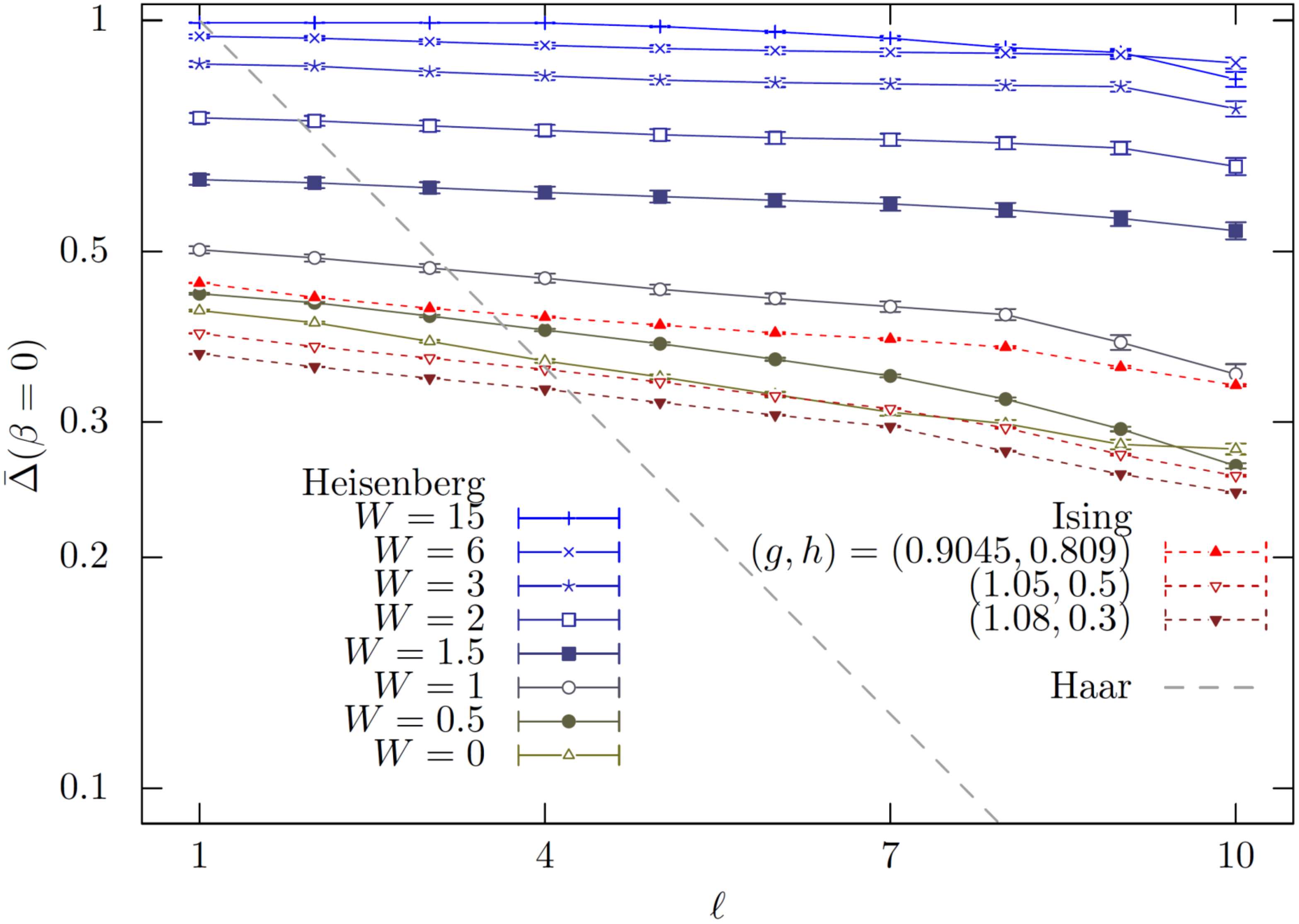
$$\hat{H}_{\mathrm{XXZ}} = J \sum_{\langle j,k \rangle} S_j \cdot S_k + \sum_j h_j S_j^z \,,$$

$$h_j \in [-W, W]$$



- Sample-averaged error stabilizes after $t \sim 10$
- **The Haar value is not reached**
- Error increases monotonically as a function of W

Late-time values



[M. Hanada, A. Jevicki, X. Liu, E. Rinaldi, and M. Tezuka, 2309.15349]

A model of randomly coupled Pauli spins

Consider N quantum spins ($S = 1/2$) with all-to-all interactions

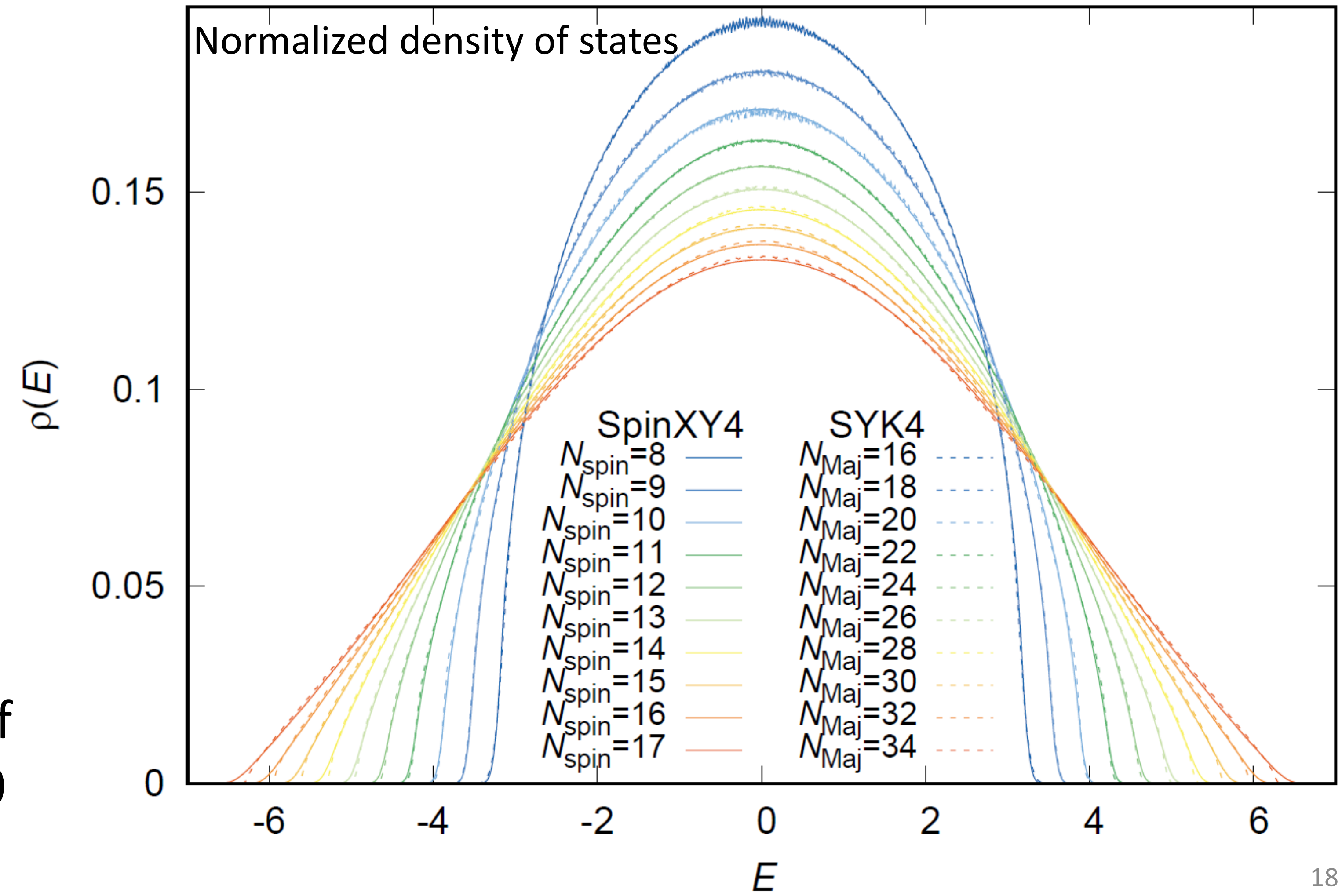
$$\hat{H} = \sum_{1 \le a < b < c < d \le 2N} i^{\eta_{abcd}} J_{abcd} \hat{O}_a \hat{O}_b \hat{O}_c \hat{O}_d$$

$$\hat{O}_{2j-1} = \hat{S}_{j,x},\ \hat{O}_{2j} = \hat{S}_{j,y}$$

η_{abcd}: number of pairs of indices on the same spin

➔ Random-matrix behavior with density of states similar to the SYK_4 model

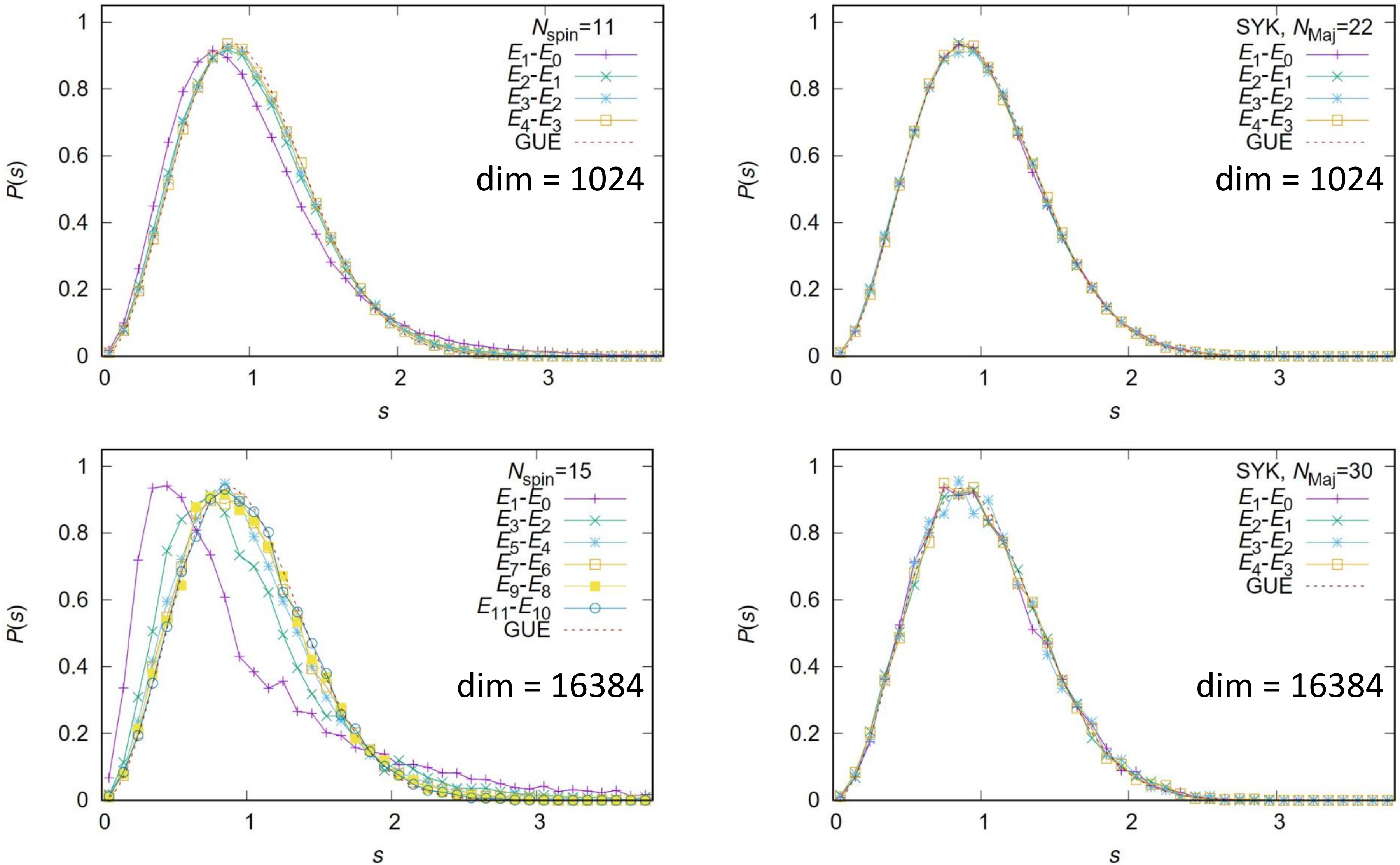
➔ Also, we may change the number of interacting spins, sparcify, forbid $\eta > 0$ terms, etc.



Normalized gap distribution

Parity conserved
➔ Look at each parity sector

Matches the random matrix theory (GUE) distribution except for a few gaps at the edge



Spectral form factor

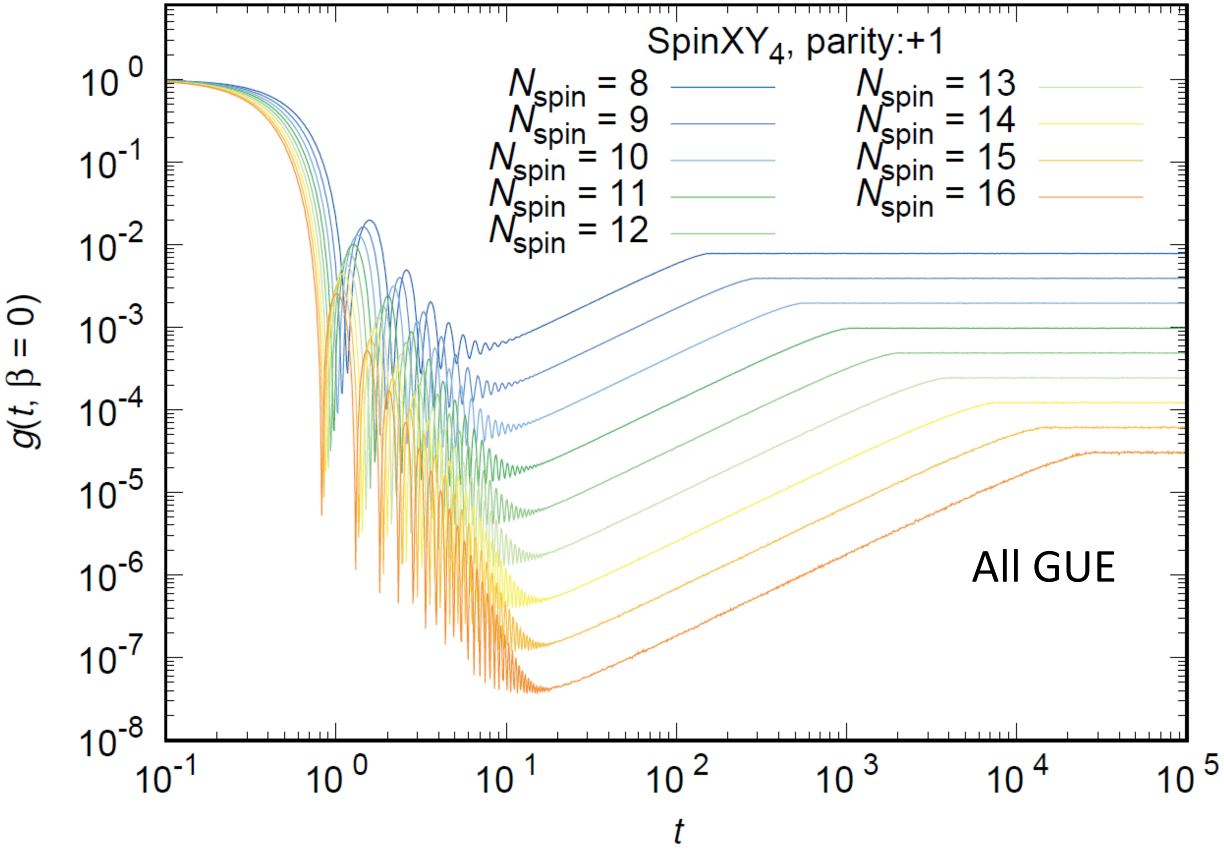
$$g(\beta,t) = \frac{\langle |Z(\beta,t)|^2 \rangle_{\{J\}}}{\langle Z(\beta) \rangle_{\{J\}}{}^2}$$

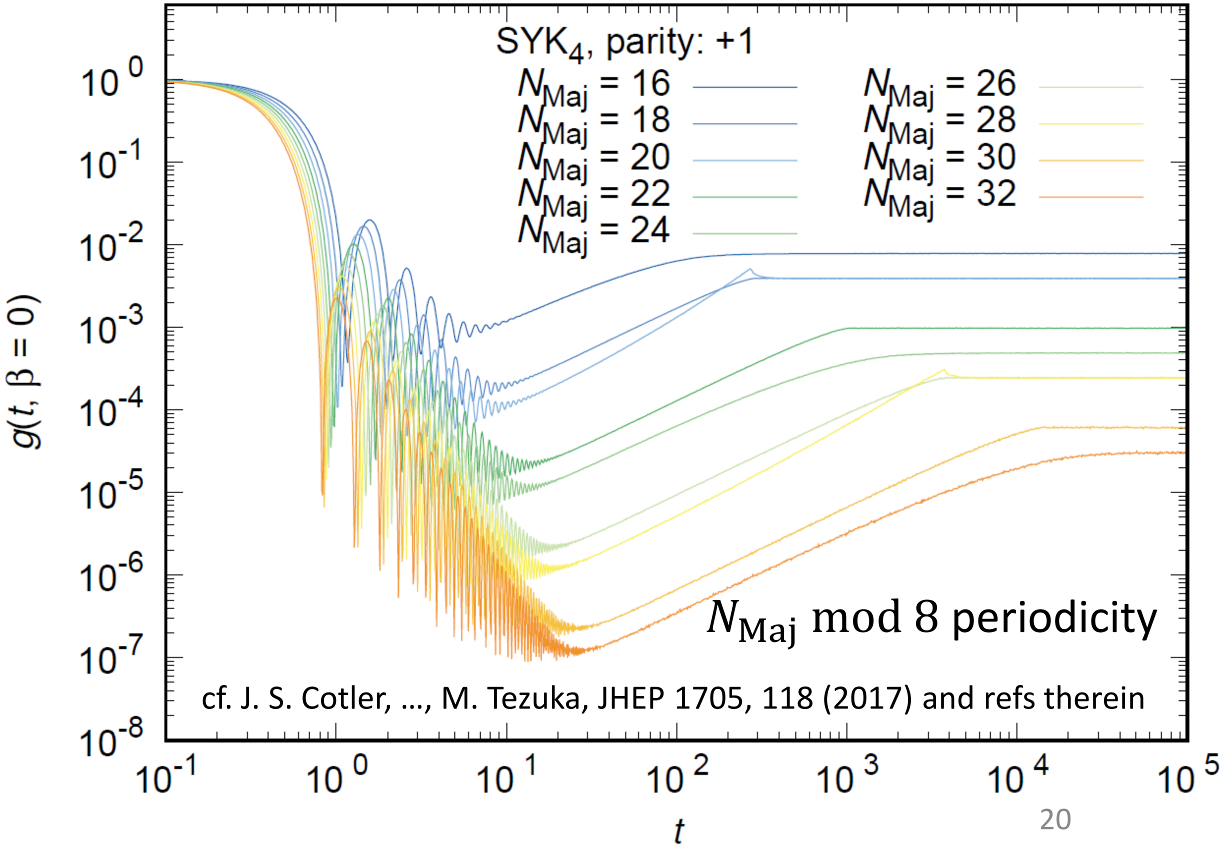
Partition function

$$Z(\beta,t) = Z(\beta + \mathrm{i}t) = \mathrm{Tr}\left(\mathrm{e}^{-\beta \hat{H} - \mathrm{i}\hat{H}t}\right)$$

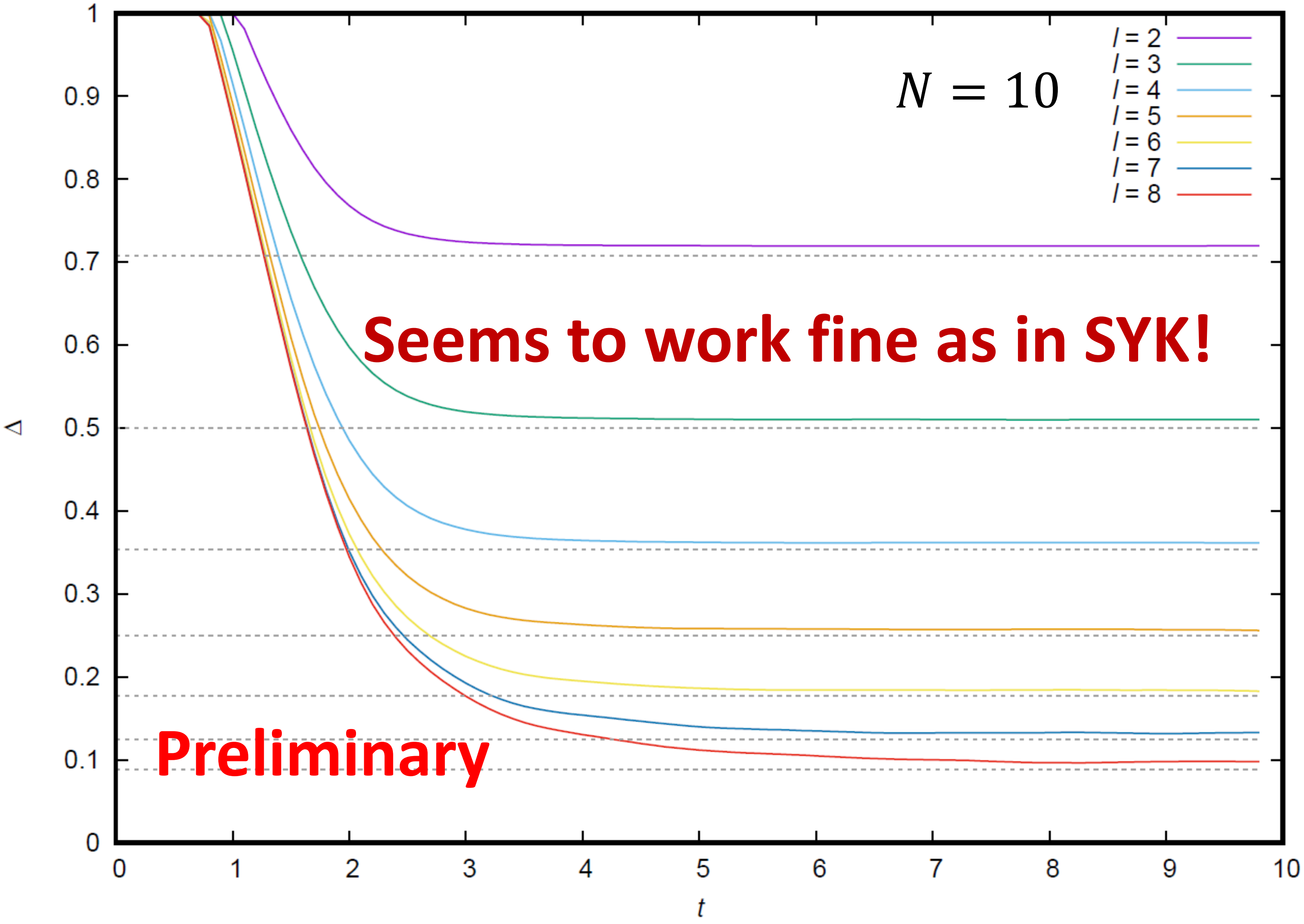
Fourier transform of the spectrum

Ramp $\propto t^1$: random-matrix level correlation over the energy spectrum





Quantum error correction with the spin model?



Summary

- Status of our cold atom experiment → **Kazuya Yamashita**'s poster on 26th
  - ✓Molecule BEC
  - ✓Introduction of 2d optical lattice
- Theoretical results
  - **Quantum state tomography** (QST)
    - Spiral QST → **Giacomo Marmorini**'s talk on 29th
    - Permutationally Invariant QST → **Yuki Miyazaki**'s poster on 26th
  - Hayden-Preskill Recovery in Hamiltonian Systems [**Y. Nakata** (A01) and M. Tezuka, 2303.02010]
    - **SYK and binary-coupling sparse SYK**: efficient recovery (quantum error correction) realized in short time
    - **Chaotic spin chains: efficient recovery not realized**
  - All-to-all coupling model of quantum spins [M. Hanada, A. Jevicki, X. Liu, E. Rinaldi, and M. Tezuka, 2309.15349]
    - Random-matrix like behavior, surprisingly similar to SYK (but with some differences)