Cost-optimal quantum error mitigation based on universal cost bound

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Outline

- Introduction: quantum error mitigation (QEM)
  - What is the theoretical limit of the sampling cost of QEM?
  - What is the cost-optimal QEM method?
- Method: quantum estimation theory
  - Optimal measurement on noisy quantum systems
- Main results: two lower bounds on the sampling cost of QEM
  - First bound: exponential growth with circuit depth
  - Second bound: exponential growth with circuit depth and qubit count
- Application: cost-optimal QEM method
  - Rescaling the measurement results is cost-optimal

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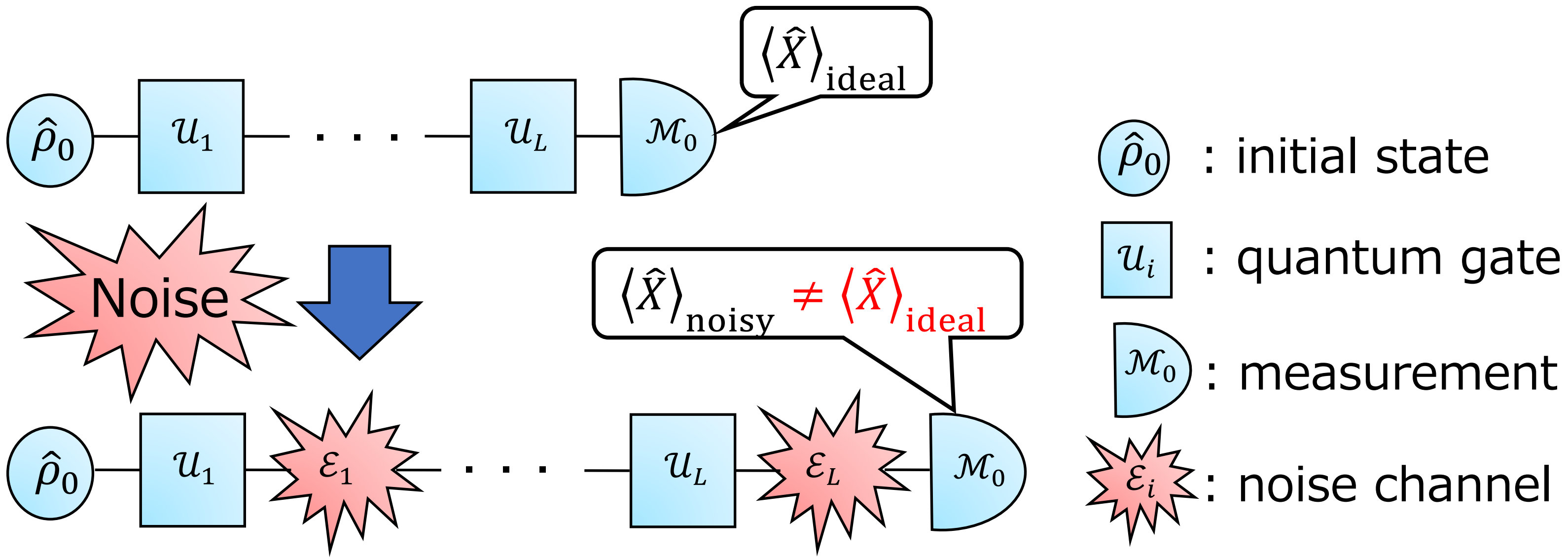
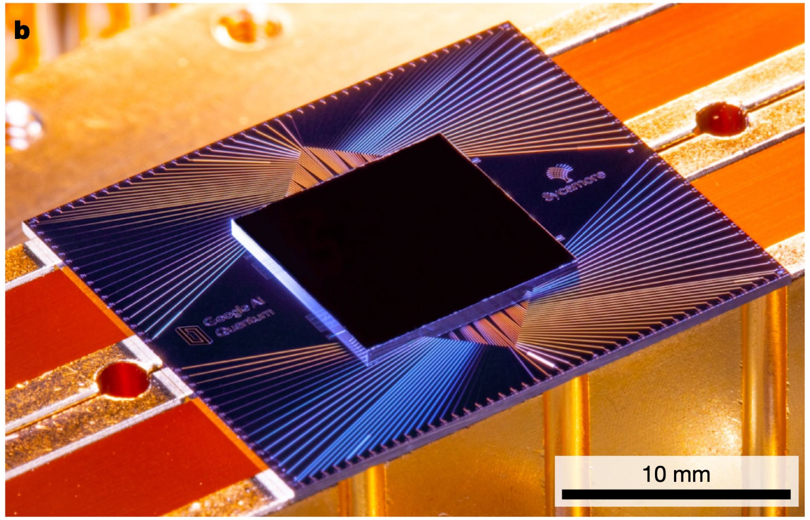
Noise of quantum computers

- Quantum computers are affected by noise.
  - Errors occur in computation results.



We need ways to combat noise.

Google Sycamore processor



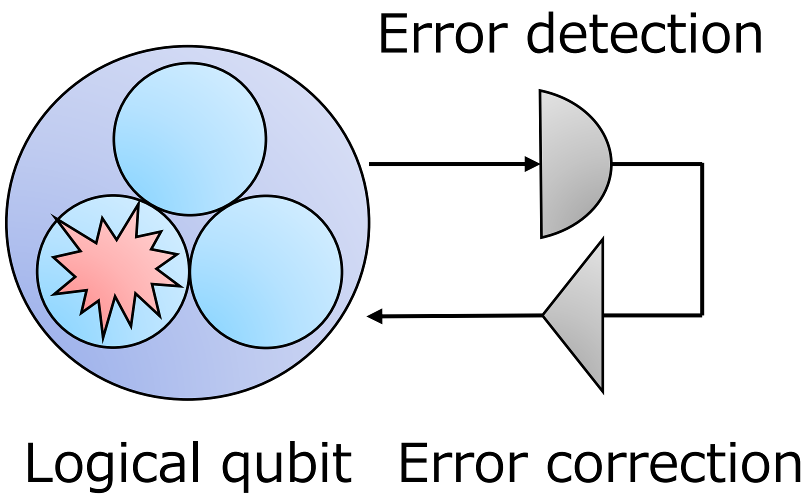
F. Arute et al., 2019.

Pauli and measurement errors

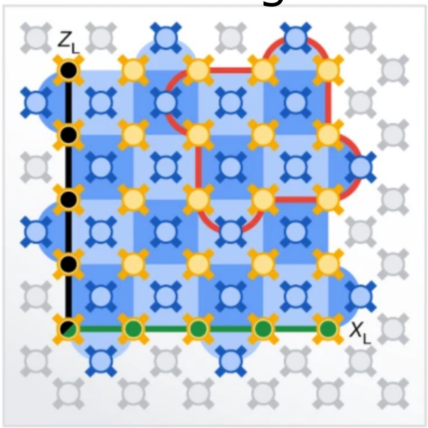
Average error	Isolated	Simultaneous
Single-qubit (e_1)	0.15%	0.16%
Two-qubit (e_2)	0.36%	0.62%
Two-qubit, cycle (e_{2c})	0.65%	0.93%
Readout (e_r)	3.1%	3.8%

Countermeasure 1: quantum error correction (QEC)

- By encoding multiple physical qubits into a single logical qubit, errors are detected and **corrected** during the computation.



Single logical qubit on superconducting hardware



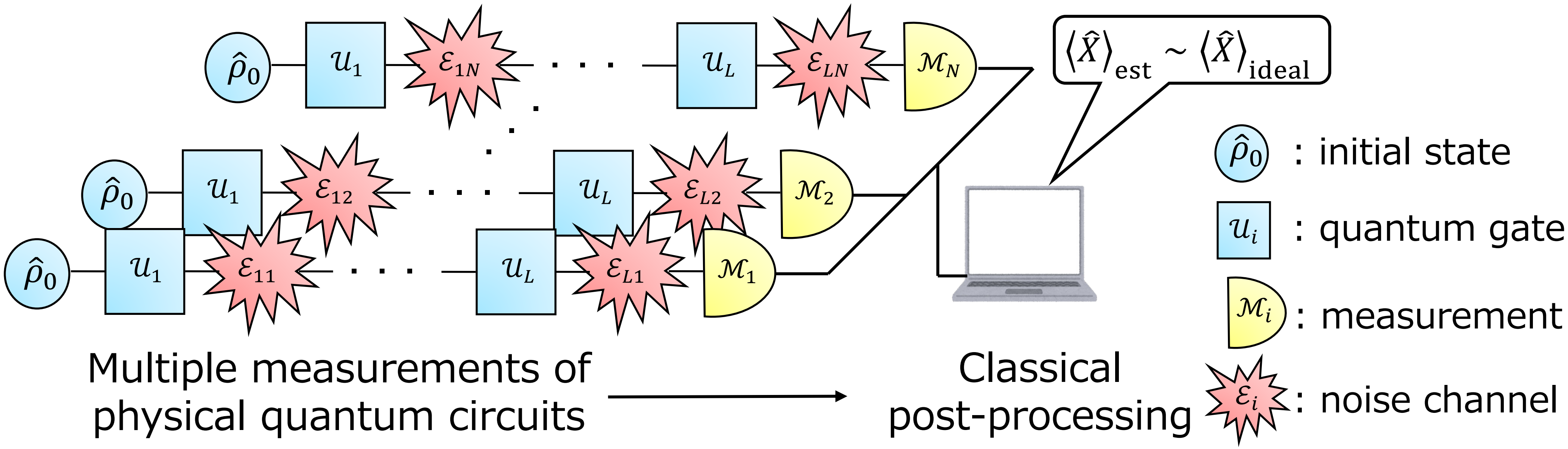
Google Quantum AI, 2023.

- Requires a lot of qubits

⇒ Can we combat noise more hardware-friendly?

Countermeasure 2: quantum error mitigation (QEM)

- The effect of errors is **mitigated** by increasing the number of samples and performing appropriate post-processing.

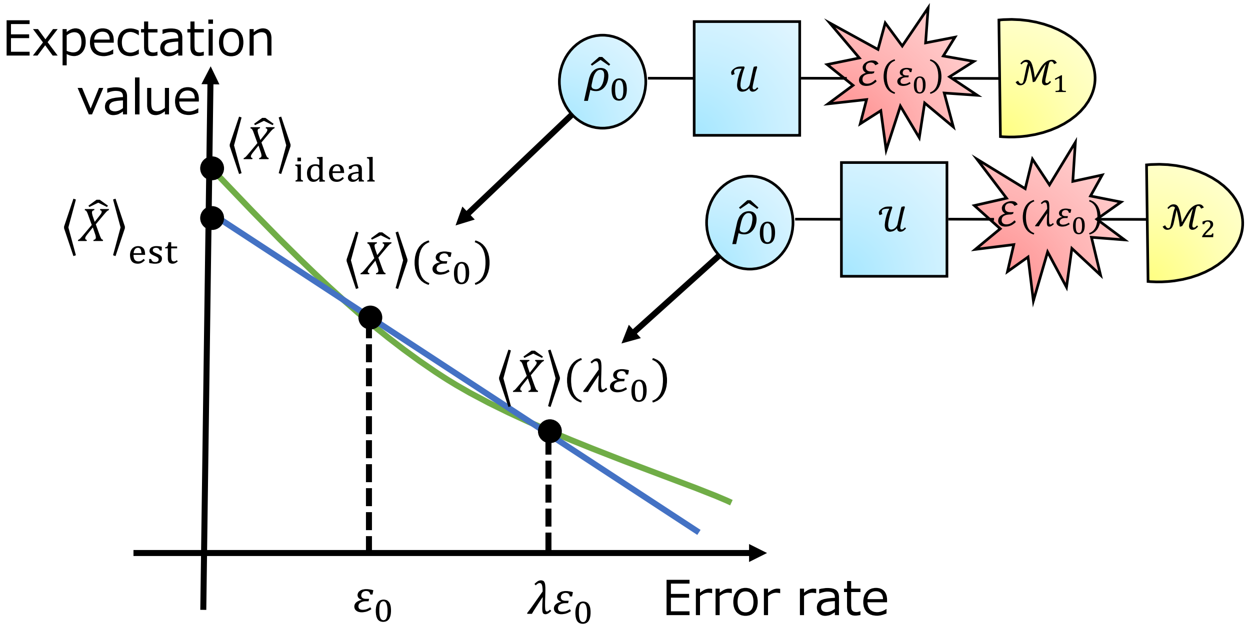


- No qubit overhead

⇒ Can be performed on existing hardware.

Methods of QEM : Zero-noise extrapolation

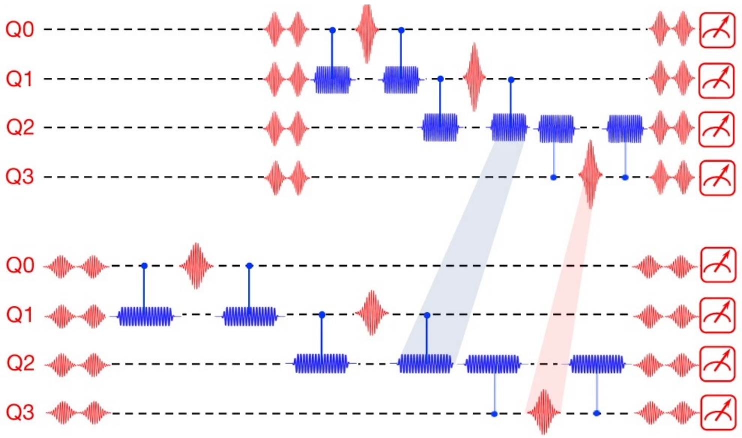
- Mitigates errors by increasing the noise rate and extrapolating noisy expectation values Y. Li et al., 2017. K. Temme et al., 2017.



Increasing noise rate in quantum circuits

Pulse time: τ
Error rate : ε_0

Pulse time: $\lambda\tau$
Error rate : $\lambda\varepsilon_0$



A. Kandala et al., 2019.

$$\langle\hat{X}\rangle_{\text{est}} = \frac{\lambda}{\lambda-1}\langle\hat{X}\rangle(\varepsilon_0) - \frac{1}{\lambda-1}\langle\hat{X}\rangle(\lambda\varepsilon_0)$$

Increase in the variance

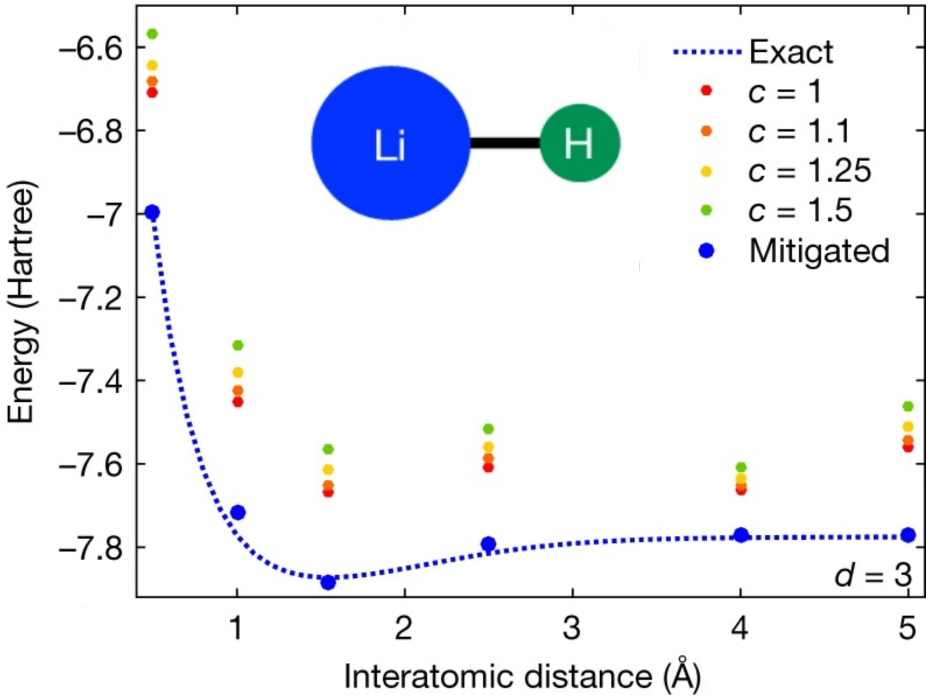
Increase in the sampling cost

$$\left(\sum_{m=1}^{M}\left|\prod_{k\neq m}\frac{\lambda_k}{\lambda_k-\lambda_m}\right|\right)^2$$

Methods of QEM : Zero-noise extrapolation

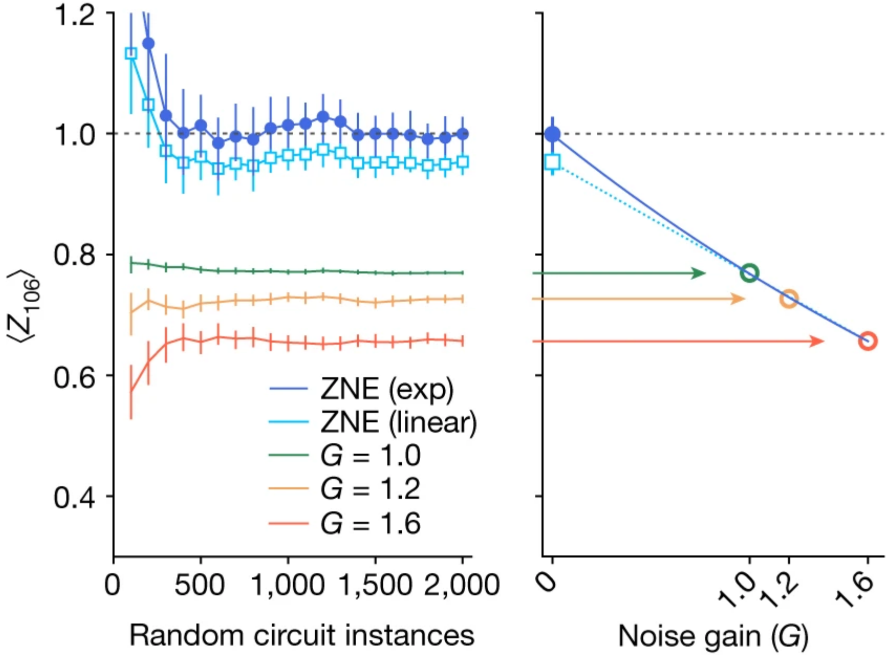
■ Performing Zero-noise extrapolation in experiments

VQE on 4-qubit hardware



A. Kandala et al., 2019.

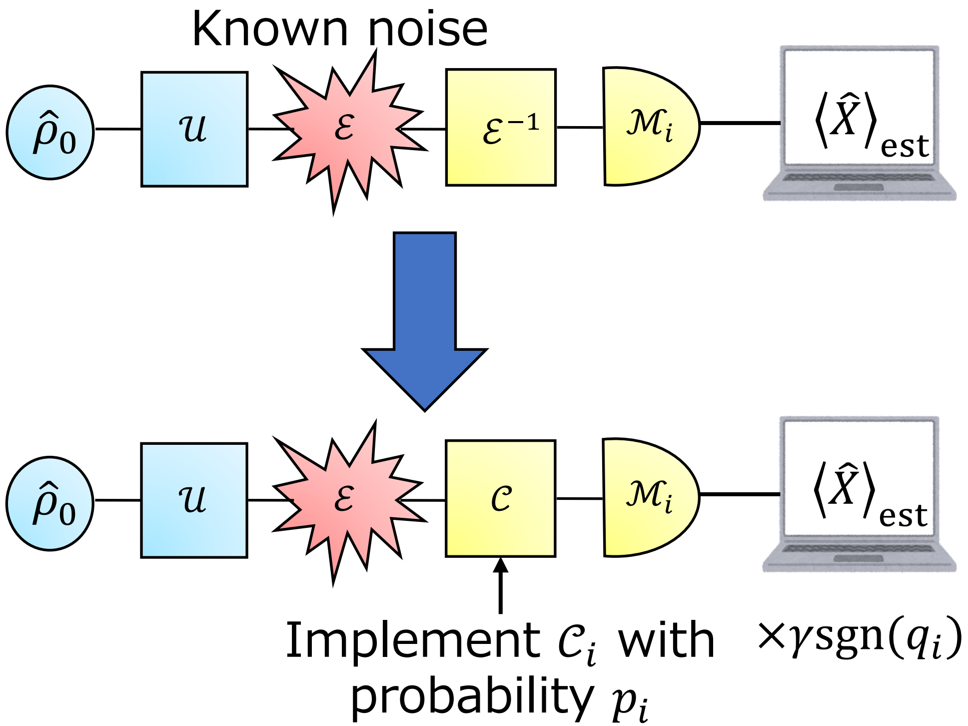
Trotter circuits on 127-qubit hardware



Y. Kim et al., 2023.

Methods of QEM : Probabilistic error cancellation

- Mitigates errors by virtually implementing the inverse of the noise channel K. Temme et al., 2017.



$$\mathcal{E}^{-1} = \sum_i q_i \mathcal{C}_i \quad \begin{pmatrix} \mathcal{C}_i\text{: implementable operation} \\ q_i\text{: quasi-probability distribution} \end{pmatrix}$$

$$= \gamma \sum_i p_i \mathrm{sgn}(q_i) \mathcal{C}_i \quad \begin{pmatrix} \gamma = \Sigma_i |q_i| \\ p_i = |q_i|/\gamma \end{pmatrix}$$

Increase in the variance

$$\langle \hat{X} \rangle_{\mathrm{est}} = \gamma \sum_i p_i \mathrm{sgn}(q_i) \mathrm{tr}[\mathcal{C}_i \circ \mathcal{E}(\hat{\rho}) \hat{X}]$$

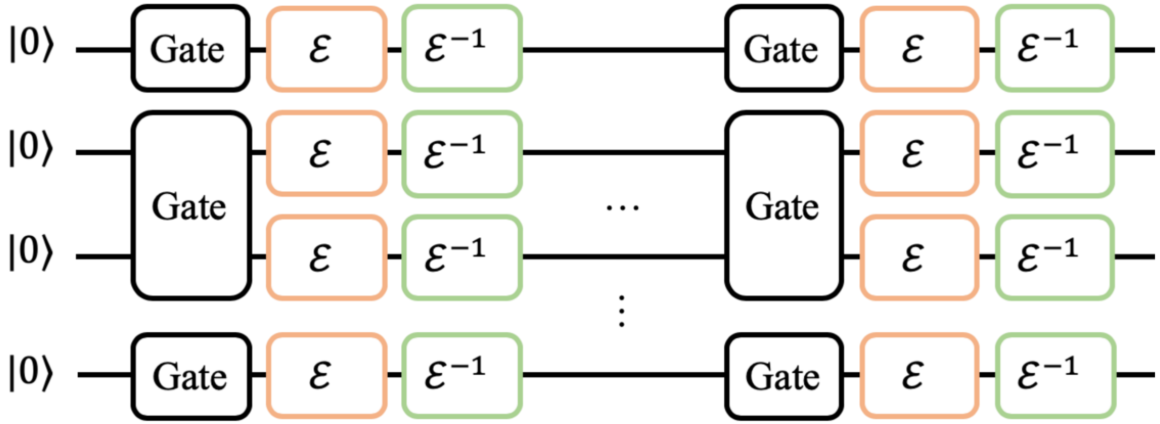
$$= \mathrm{tr}[\mathcal{E}^{-1} \circ \mathcal{E}(\hat{\rho}) \hat{X}]$$

$$= \mathrm{tr}[\hat{\rho} \hat{X}]$$

Obtain unbiased estimator with sampling overhead γ^2

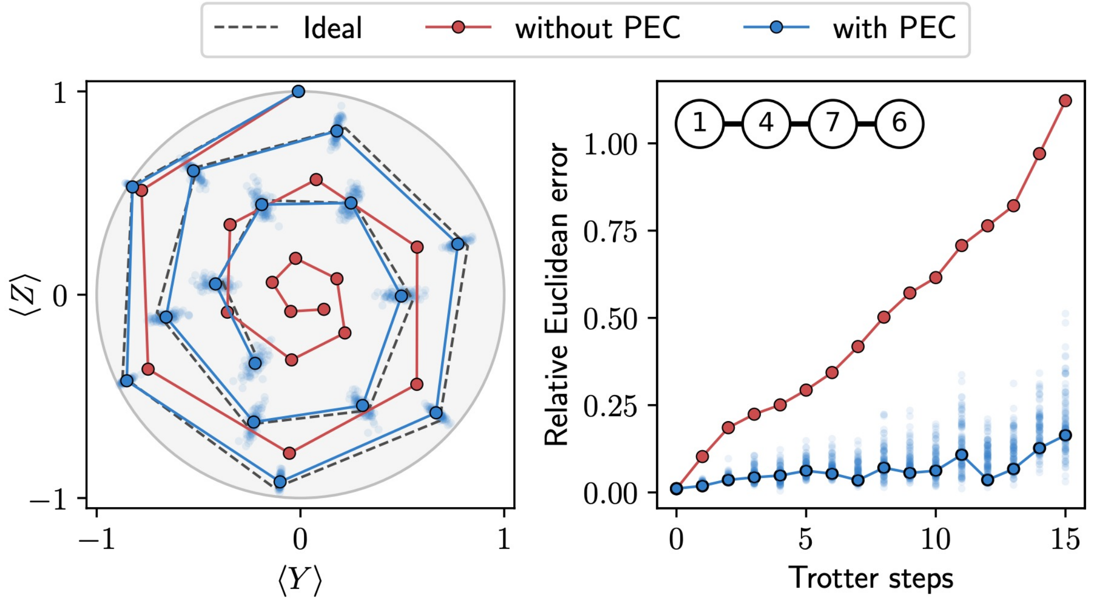
Methods of QEM : Probabilistic error cancellation

■ Performing Zero-noise extrapolation in experiments



S. Endo et al., 2020.

Time evolution of Ising model on 4-qubit hardware

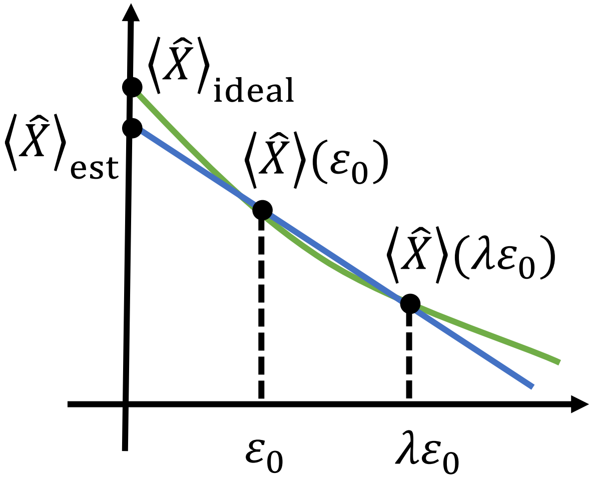
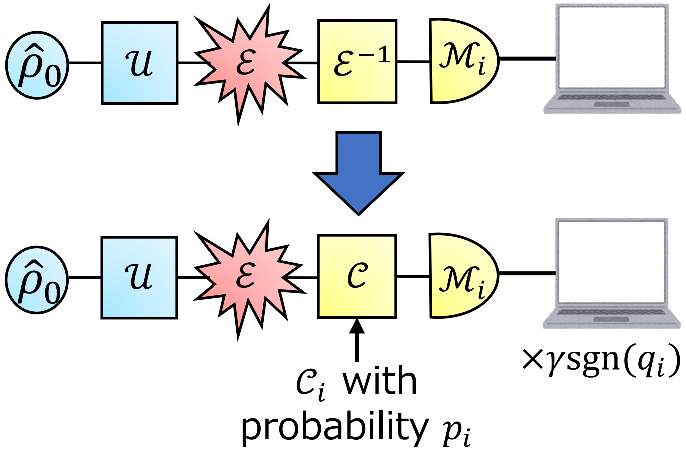


Ewout van den Berg et al., 2022.

Sampling overhead

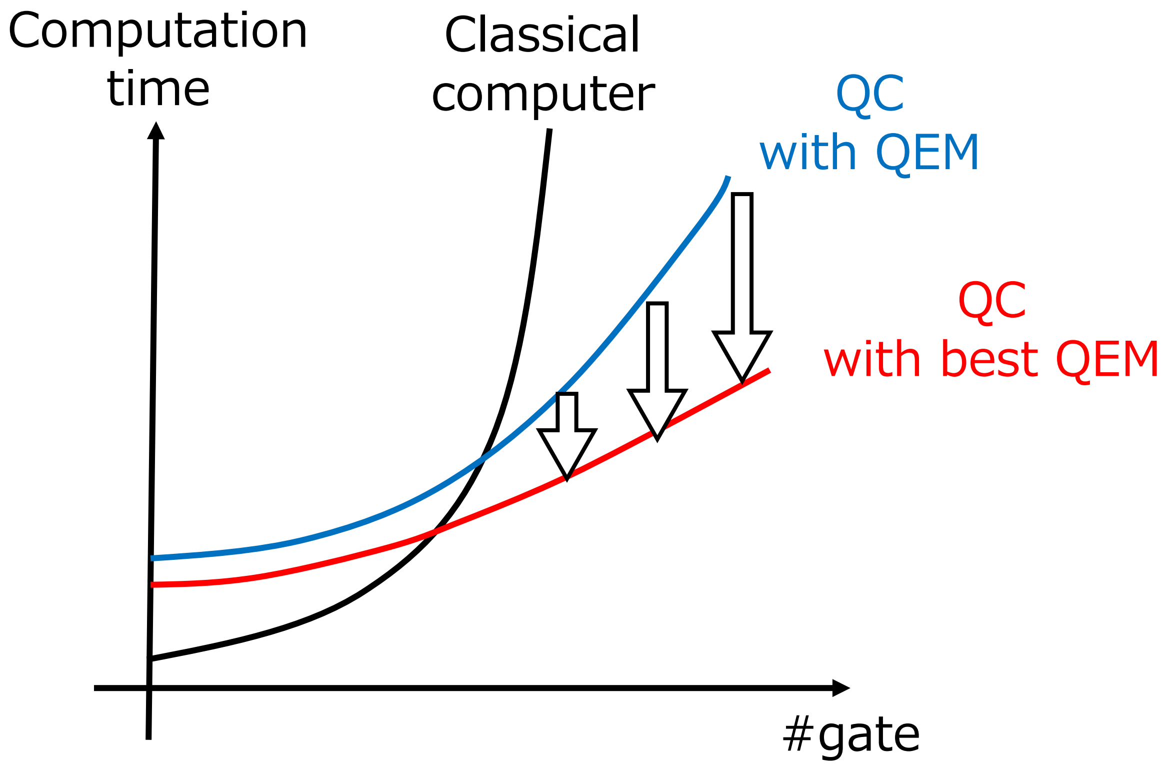
$$O\left(\gamma^{2\#\text{noise}}\right)$$

Comparing QEM methods

<table>
<tr><th></th><th>Zero-noise extrapolation</th><th>Probabilistic error cancellation</th><th>Best QEM</th></tr>
<tr><td>Idea</td><td>
</td><td>
</td><td></td></tr>
<tr><td>Hardware constraint</td><td>Boosting noise rate</td><td>Learning noise</td><td>Easily implementable</td></tr>
<tr><td>Bias（Accuracy)</td><td>Large bias for large ε_0</td><td>Unbiased</td><td>Unbiased</td></tr>
<tr><td>Sampling cost</td><td>$\left(\sum_{m=1}^{M}\left|\prod_{k\neq m}\frac{\lambda_k}{\lambda_k-\lambda_m}\right|\right)^2$</td><td>$\gamma^{2\#\text{noise}}$</td><td>Low</td></tr>
</table>

Sampling cost of QEM

- The expected sampling overhead : $2^{O(\#\text{noise})} \sim 2^{O(\#\text{gate})}$



What is the exact scaling achievable with best = cost-optimal QEM?

Theoretical limits of QEM

- What is the theoretical limit of the sampling cost of unbiased QEM?

- Exponential growth with circuit depth
  - For general circuit

$$N \geq O(\gamma^{2L})$$

L: circuit depth
$\gamma > 1$:Noise-dependent constants

- Exponential growth with qubit count
  - For scrambling circuit

$$\mathbb{E}_U[N] \geq O(\nu^{nL})$$

n: qubit count
$\nu > 1$:Noise-dependent constants

- What is the cost-optimal QEM method achieving the limit?

Rescaling the measurement result is cost-optimal.

$$\langle \hat{X} \rangle_{\mathrm{est}} = (1-p)^{-k_{\mathrm{mean}}L} \langle \hat{X} \rangle_{\mathrm{noisy}}$$

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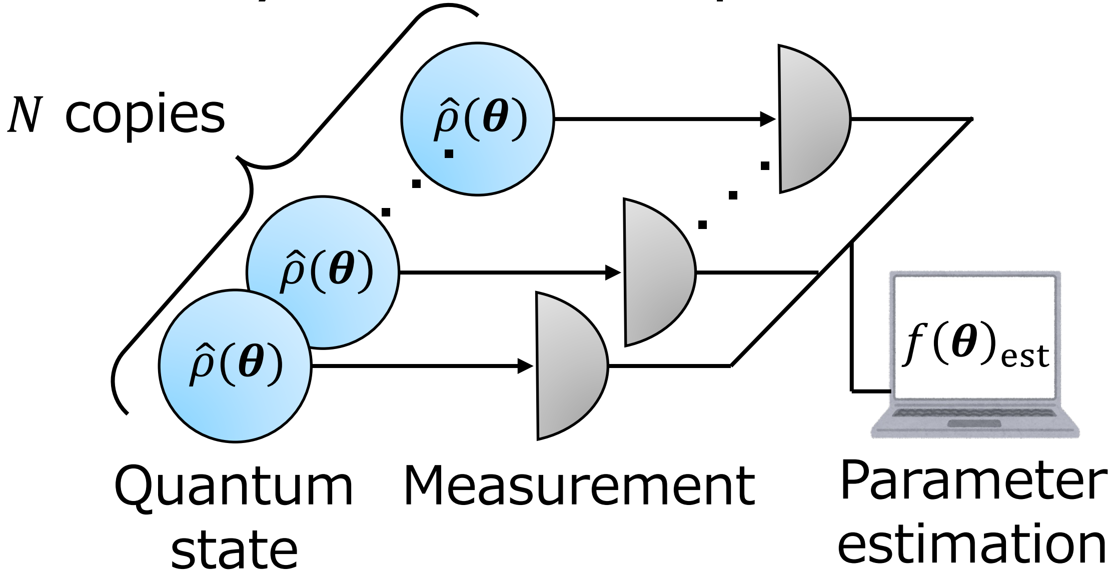
Quantum estimation theory

- Characterize the accuracy and cost of parameter estimation.



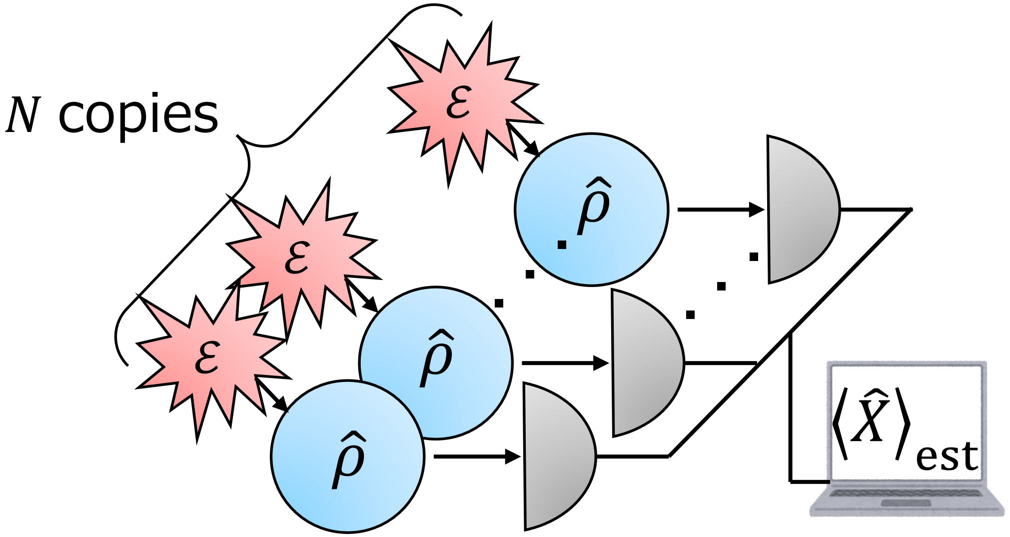
- Quantum Cramér-Rao bound: C. W. Helstrom, 1969.
  - The accuracy limit of the estimator is bounded by the amount of information in the quantum state.

$$\mathrm{Var}(f(\boldsymbol{\theta})_{\mathrm{est}}) \geq \frac{1}{N}(\nabla_{\boldsymbol{\theta}} f(\boldsymbol{\theta}))^T J^{-1}(\nabla_{\boldsymbol{\theta}} f(\boldsymbol{\theta}))$$

Accuracy | Cost | Quantum Fisher information matrix

Application: Estimating expectation values in noisy state

- How much cost do we need to estimate the expectation value of an observable X from the measurement of the noisy state? Y. Watanabe et al., 2010.



Noisy state
$\mathcal{E}(\hat{\rho})$

- Parametrize quantum state using **generalized Bloch vector $\boldsymbol{\theta}$** $\in \mathbb{R}^{4^n-1}$

$$\hat{\rho}(\boldsymbol{\theta}) = \frac{1}{2^n}\hat{I} + \frac{1}{2^{(n+1)/2}}\boldsymbol{\theta} \cdot \hat{\boldsymbol{P}}$$

$$\mathcal{E}(\hat{\rho}(\boldsymbol{\theta})) = \frac{1}{2^n}\hat{I} + \frac{1}{2^{(n+1)/2}}(A\boldsymbol{\theta} + \boldsymbol{c}) \cdot \hat{\boldsymbol{P}}$$

n : qubit count
$\hat{\boldsymbol{P}}$: Array of nontrivial Pauli operators
$A_{ij} = 2^{-n}\mathrm{tr}\left[\hat{P}_i \mathcal{E}(\hat{P}_j)\right]$: Unital part of the PTM of $\mathcal{E}$
$c_i = 2^{(1-3n)/2}\mathrm{tr}\left[\hat{P}_i \mathcal{E}(\hat{I})\right]$: Non-unital effect of $\mathcal{E}$

Application: Estimating expectation values in noisy state

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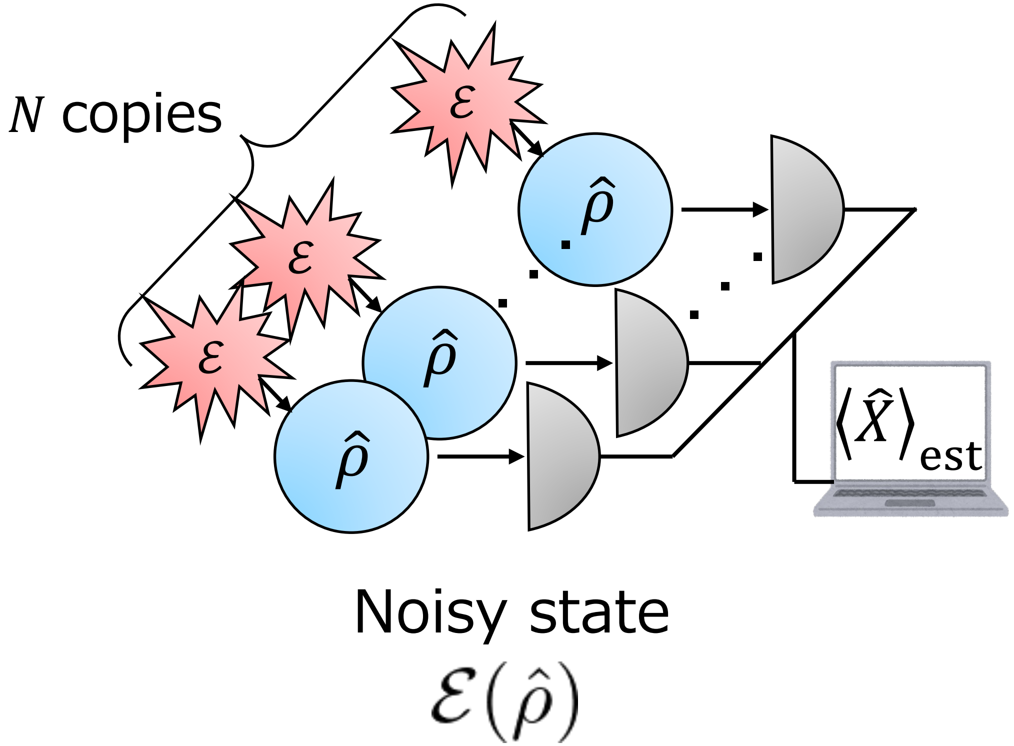
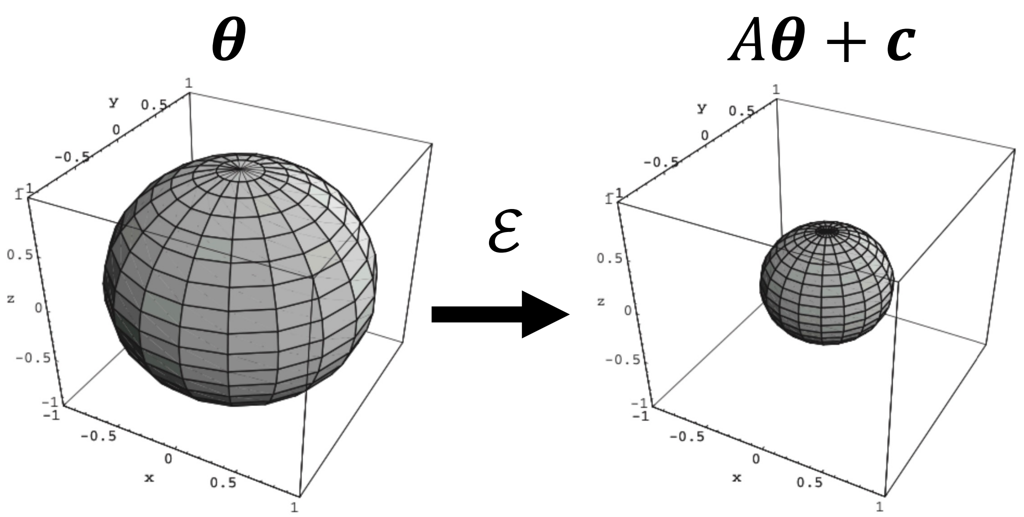


- Visualization of the effect of noise $\mathcal{E}$ on generalized Bloch vector $\boldsymbol{\theta}$



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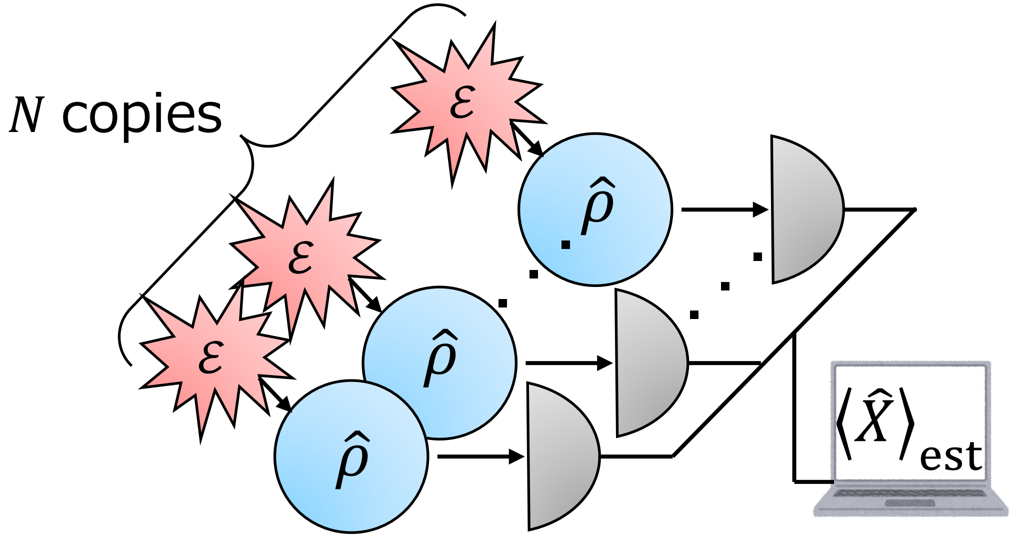

Noisy state
$\mathcal{E}(\hat{\rho})$

- SLD Quantum Fisher information matrix of $\mathcal{E}(\hat{\rho}(\boldsymbol{\theta}))$:

$$J(\mathcal{E}(\hat{\rho}(\boldsymbol{\theta}))) = A^T C_{\mathcal{E}(\hat{\rho})}^{-1} A$$
$$(\lesssim ||A||^2 I)$$

$A_{ij} = 2^{-n}\mathrm{tr}\left[\hat{P}_i \mathcal{E}(\hat{P}_j)\right]$: Unital part of the PTM of $\mathcal{E}$

$C_{\mathcal{E}(\hat{\rho})}$: Covariance matrix of $\hat{\boldsymbol{P}}$ for $\mathcal{E}(\hat{\rho})$

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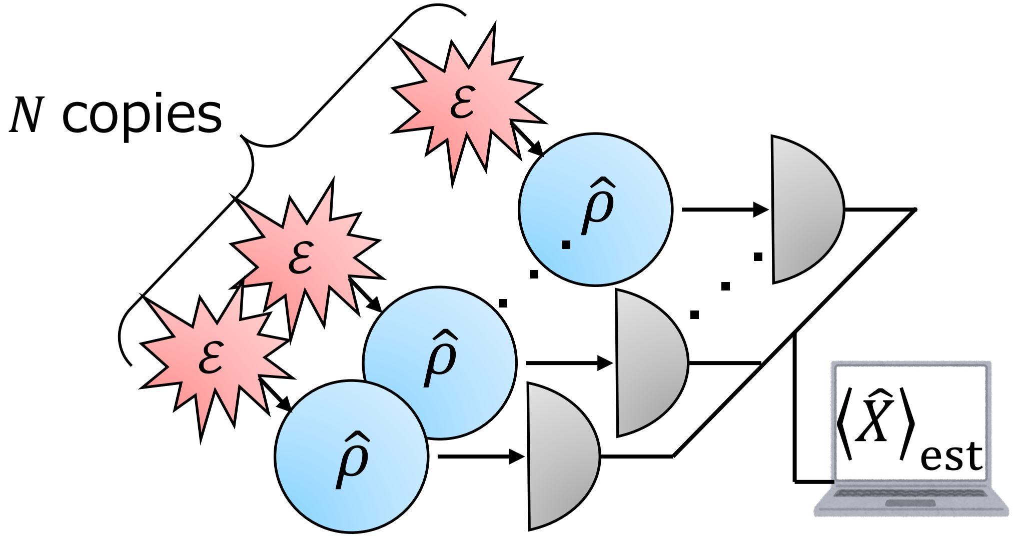

Noisy state
$\mathcal{E}(\hat{\rho})$

- Quantum Cramér-Rao bound

$$\mathrm{Var}(f(\boldsymbol{\theta})_{\mathrm{est}}) \geq \frac{1}{N}(\nabla_{\boldsymbol{\theta}} f(\boldsymbol{\theta}))^T J^{-1} (\nabla_{\boldsymbol{\theta}} f(\boldsymbol{\theta}))$$

$$J(\mathcal{E}(\hat{\rho}(\boldsymbol{\theta}))) = A^T C_{\mathcal{E}(\hat{\rho})}^{-1} A$$

$$\mathrm{Var}(\langle \hat{X} \rangle_{\mathrm{est}}) \geq \frac{1}{N}\left(\mathrm{tr}[\mathcal{E}(\hat{\rho})\hat{Y}^2] - \mathrm{tr}[\mathcal{E}(\hat{\rho})\hat{Y}]^2\right)$$

$$\hat{Y} = (\mathcal{E}^{-1})^\dagger(\hat{X})$$
$$\mathrm{tr}[\mathcal{E}(\hat{\rho})\hat{Y}] = \mathrm{tr}[\hat{\rho}\hat{X}]$$

Measuring $\mathcal{E}(\rho)$ with $\hat{Y}$ is cost-optimal

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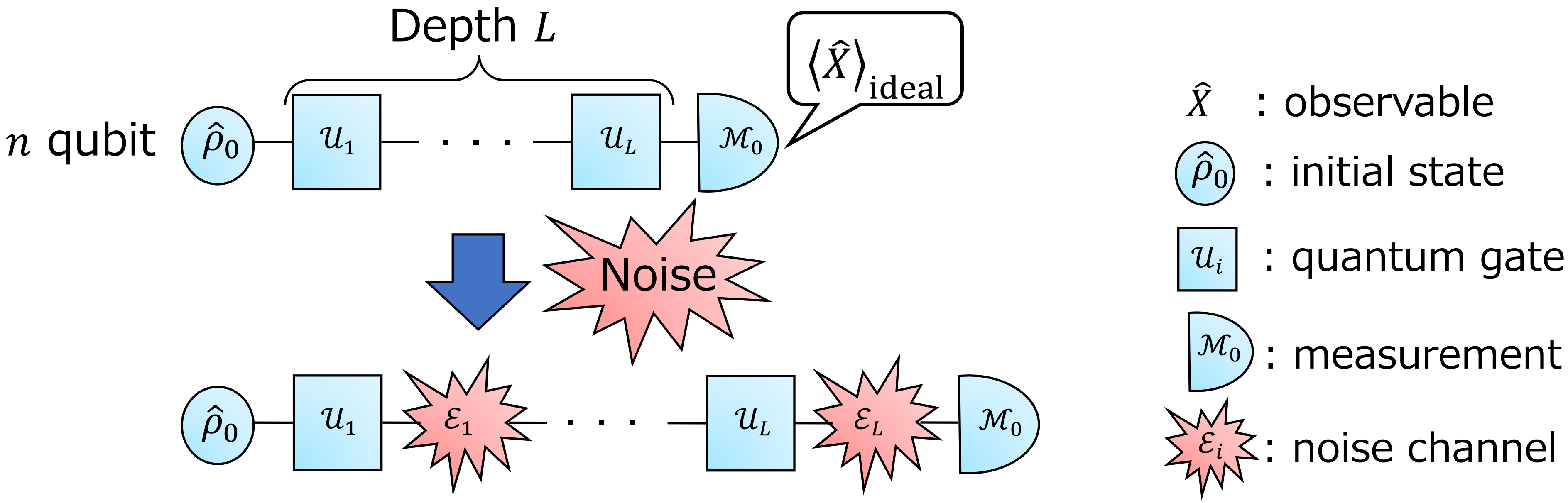
Problem setup

- Quantum circuit affected by noise:

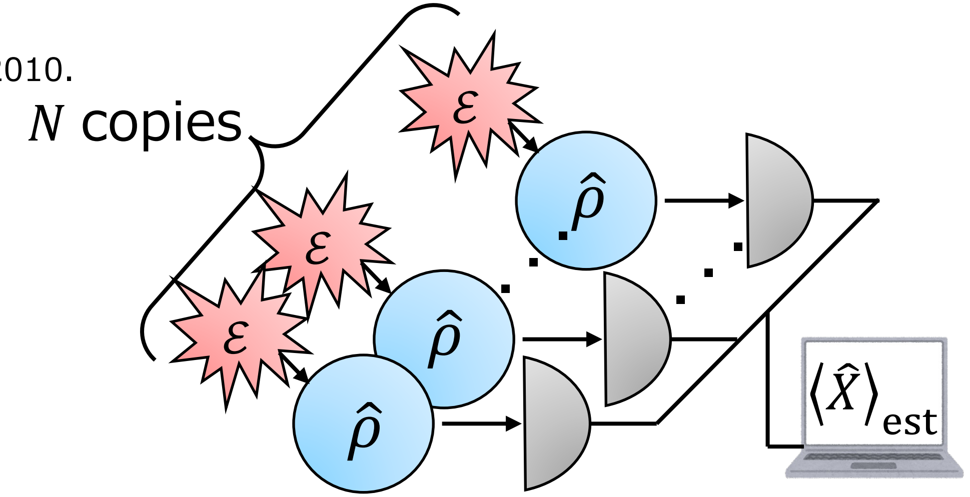


- What are the lower bounds on the sampling cost of estimating $\langle \hat{X} \rangle_{\mathrm{ideal}}$ without bias through QEM?

Main results: two lower bounds on the sampling cost of QEM

- **Quantum estimation theory** Y. Watanabe et al., 2010.

$$J(\mathcal{E}(\hat{\rho}(\boldsymbol{\theta}))) = A^T C^{-1}_{\mathcal{E}(\hat{\rho})} A$$
$$(\lesssim ||A||^2 I)$$



- **Exponential growth with circuit depth**
  - For general circuit

$$N \geq O(\gamma^{2L})$$

L: circuit depth
$\gamma > 1$:Noise-dependent constants

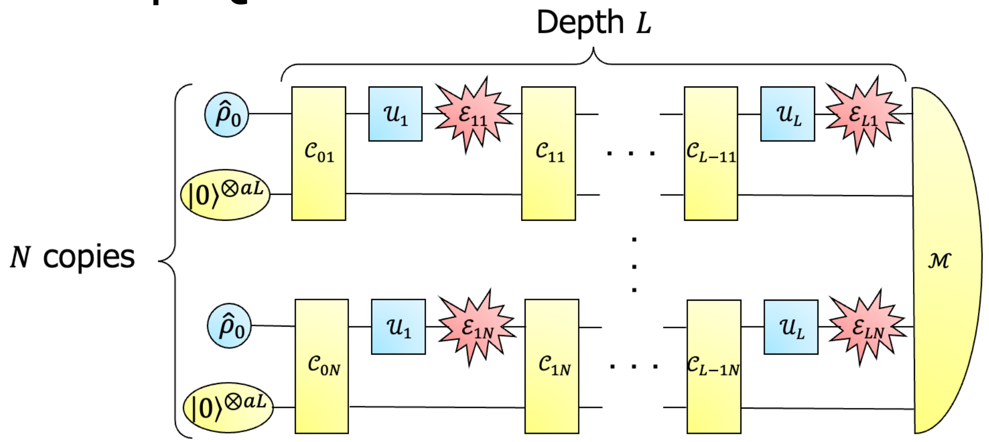
- **Exponential growth with qubit count**
  - For scrambling circuit

$$N \geq O(\nu^{nL})$$

n: qubit count
$\nu > 1$:Noise-dependent constants

Roadmap for obtaining the lower bound

① Map QEM into virtual circuit



② Define QFIM of the virtual circuit

$$\hat{\rho}(\boldsymbol{\theta}) = \frac{1}{2^n}\hat{I} + \frac{1}{2^{(n+1)/2}}\boldsymbol{\theta}\cdot\hat{\boldsymbol{P}}$$

$$J\left(\Pi_{m=1}^{N}\mathcal{E}'_m\left(\hat{\rho}(\boldsymbol{\theta})\otimes|0^{aL}\rangle\langle 0^{aL}|\right)\right)$$

③ Bound QFIM of the virtual circuit

$$\begin{aligned} J(\mathcal{E}'(\hat{\rho}(\boldsymbol{\theta}))) &= A'^T C^{-1}_{(\mathcal{E}'(\hat{\rho}))} A' \\ &\lesssim ||A'||^2 I \\ &\lesssim ||A||^{2L} I \end{aligned}$$

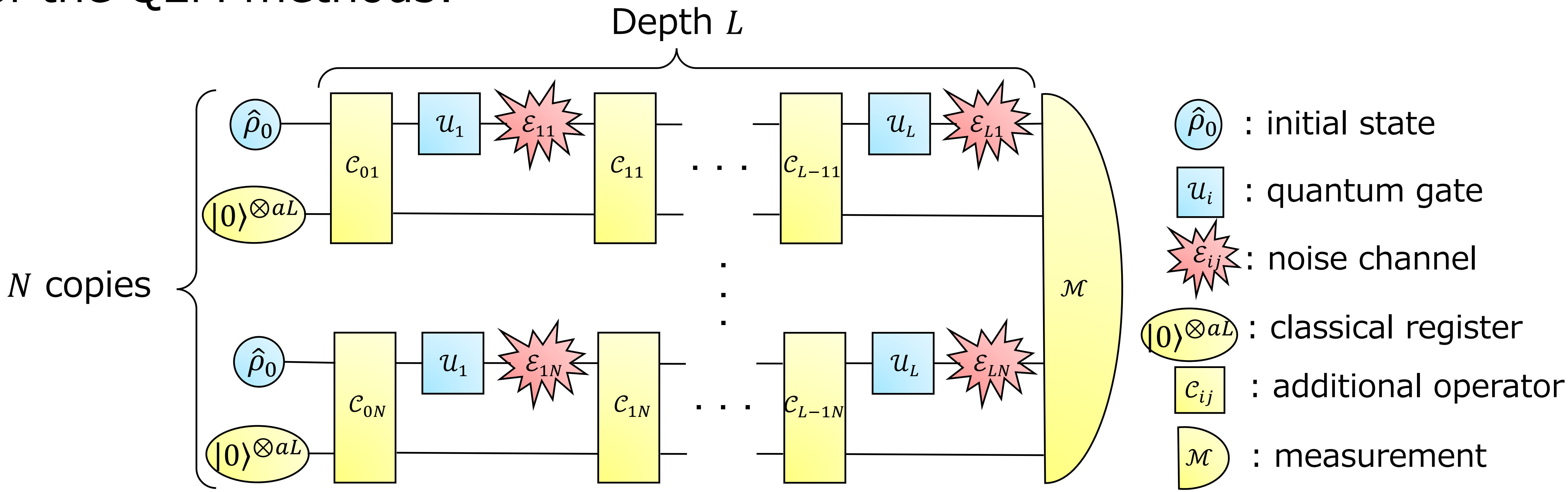
④ Obtain the lower bound

$N \geq O(\gamma^{2L})$ for general circuit

$N \geq O(\nu^{nL})$ for scrambling circuit

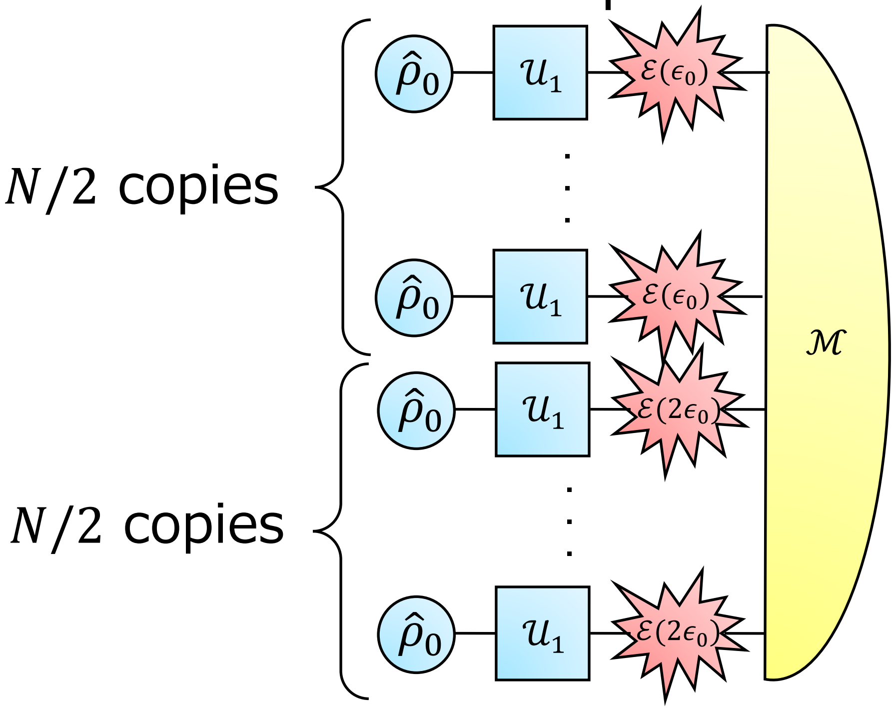
① Mapping QEM to virtual circuit

- The virtual quantum circuit that gives an equivalent representation of the QEM methods:

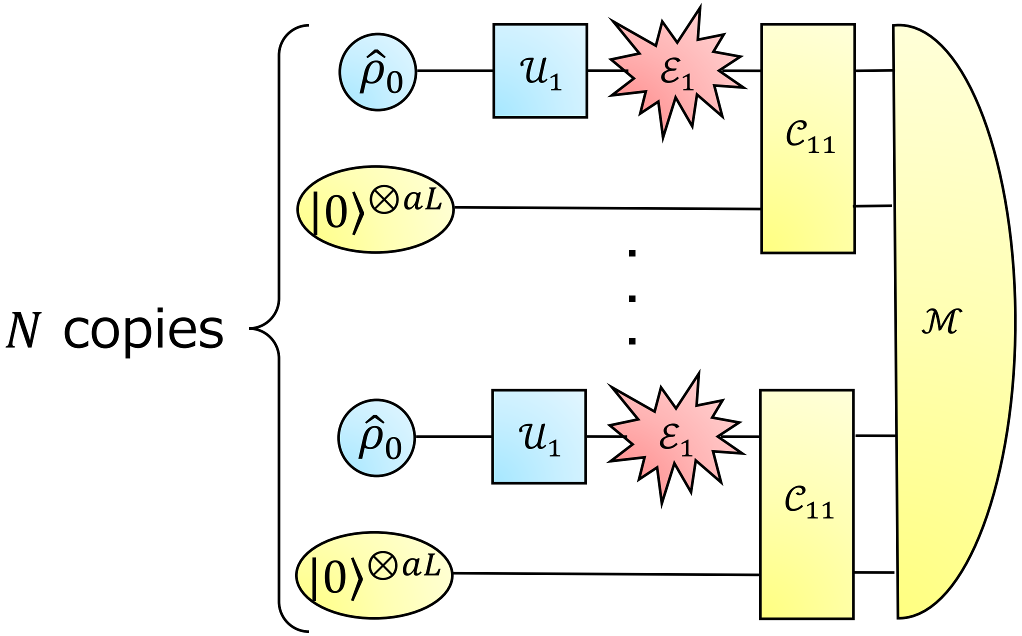


① Mapping QEM to virtual circuit

- The virtual quantum circuit that gives an equivalent representation of the QEM methods:
  - Zero-noise extrapolation



  - Probabilistic error cancellation



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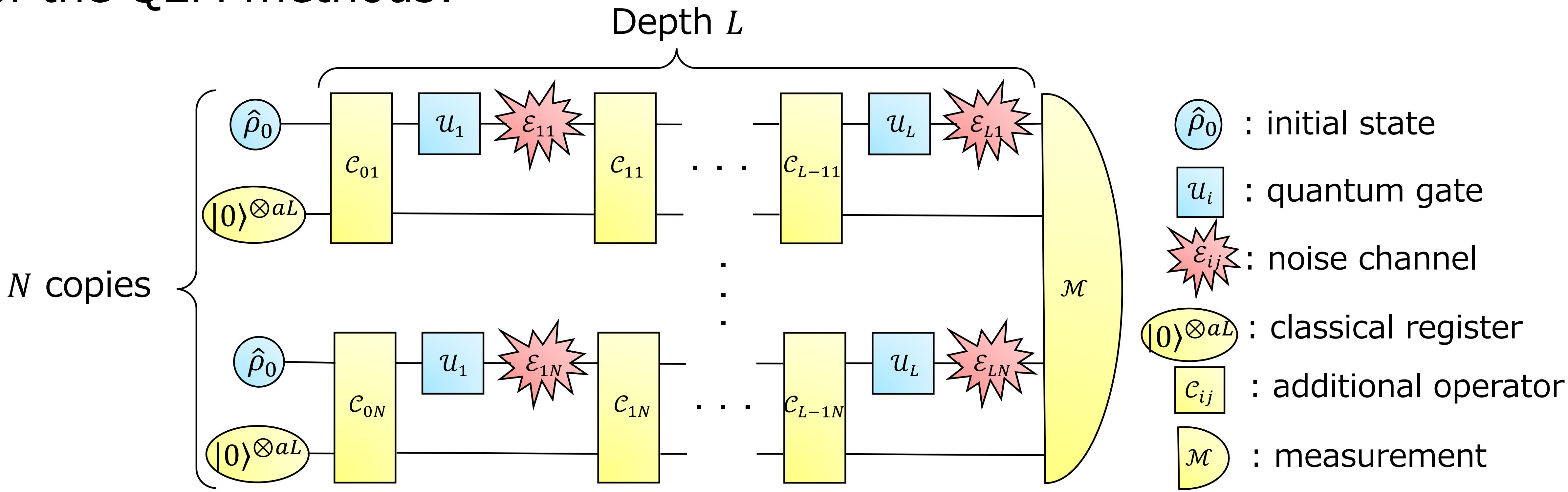
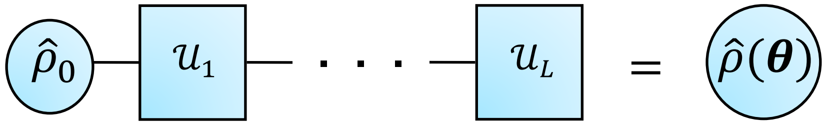


- Number of copies N: sampling cost of QEM.

② Defining QFIM of the virtual circuit

- Parameterize the noiseless quantum state with the generalized Bloch vector. G. Kimura, 2003.



$$= \frac{1}{2^n}\hat{I} + \frac{1}{2^{(n+1)/2}}\boldsymbol{\theta}\cdot\hat{\boldsymbol{P}}$$

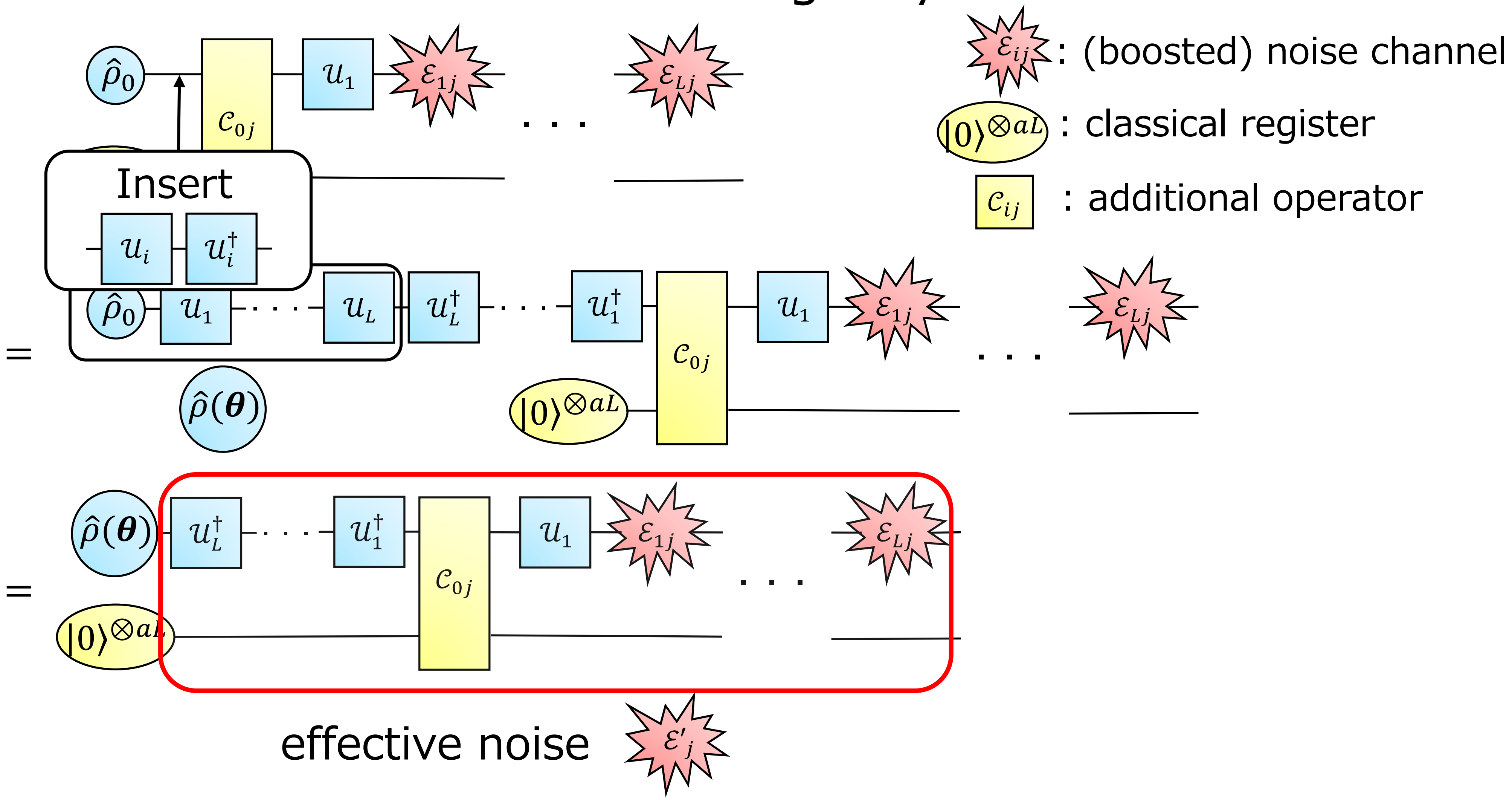
n : Number of qubits

$\hat{\boldsymbol{P}}$: Array of nontrivial Pauli operators

$\boldsymbol{\theta} \in \mathbb{R}^{4^n-1}$: Generalized Bloch vector

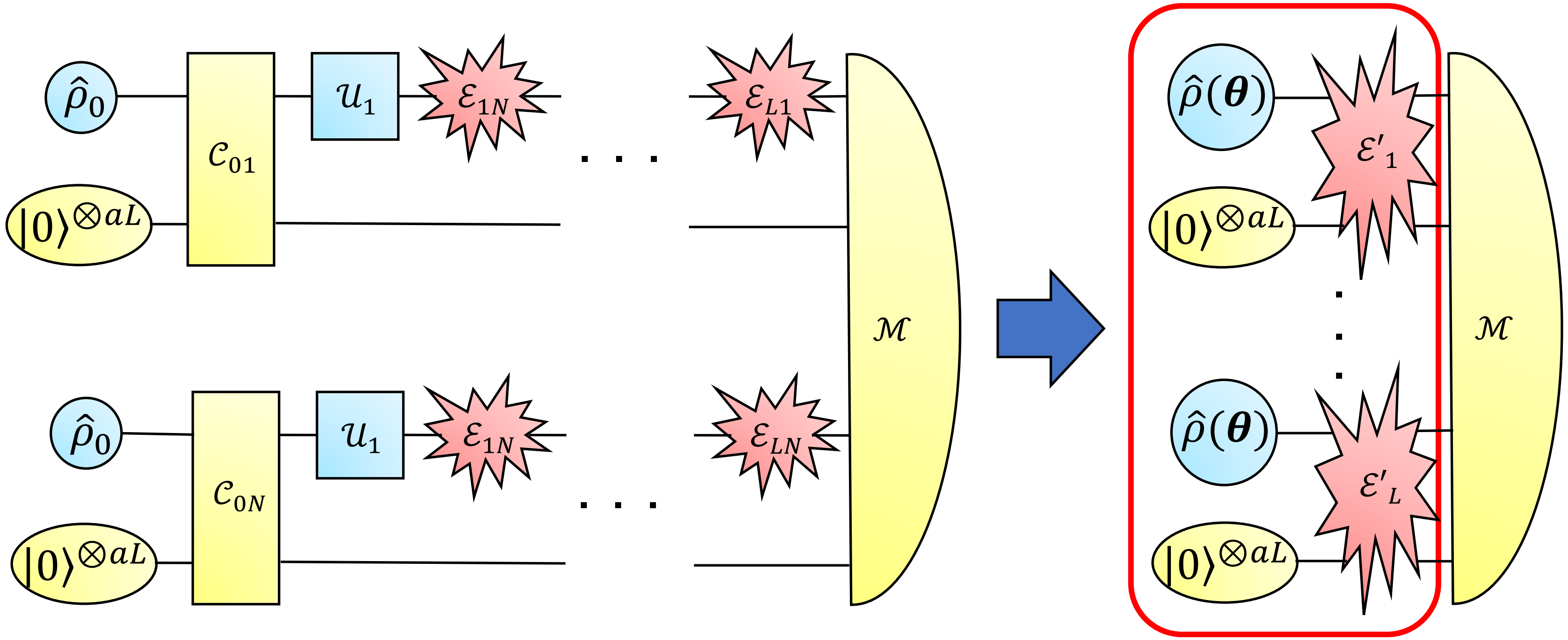
② Defining QFIM of the virtual circuit

- Rewrite the virtual circuit as in the following way.



② Defining QFIM of the virtual circuit

- Define the QFIM of the state before the measurement of the virtual circuit.

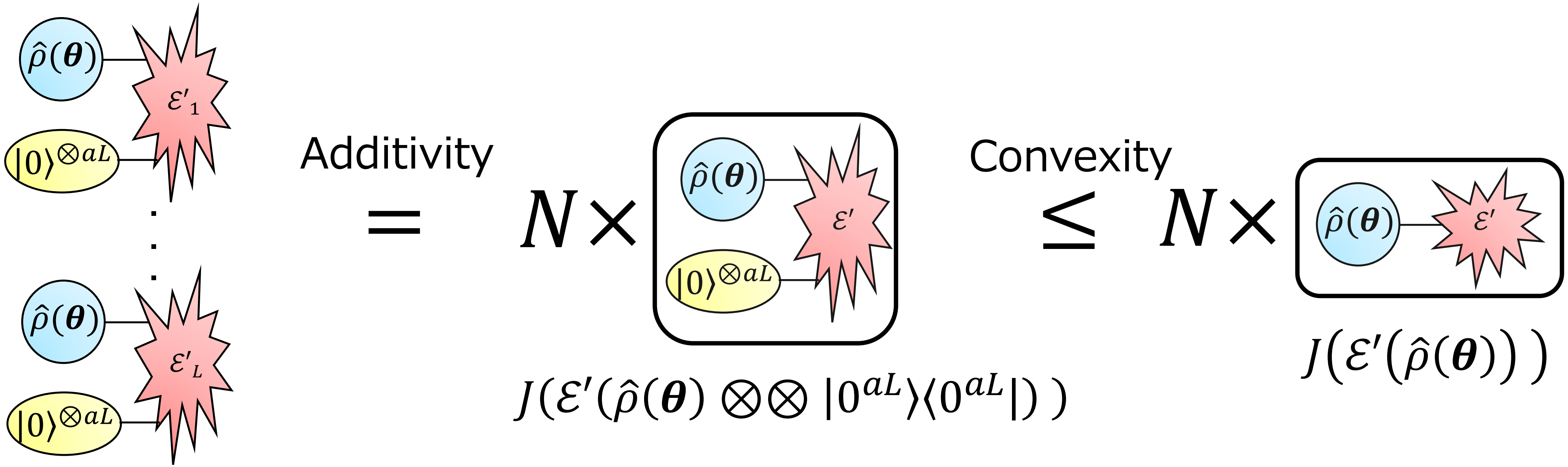


Quantum Fisher information matrix

$$J(\otimes_{m=1}^{N} \mathcal{E}'_m(\hat{\rho}(\boldsymbol{\theta}) \otimes |0^{aL}\rangle\langle 0^{aL}|))$$

③ Bounding QFIM of the virtual circuit

- Simplify QFIM using additivity and convexity of QFIM.



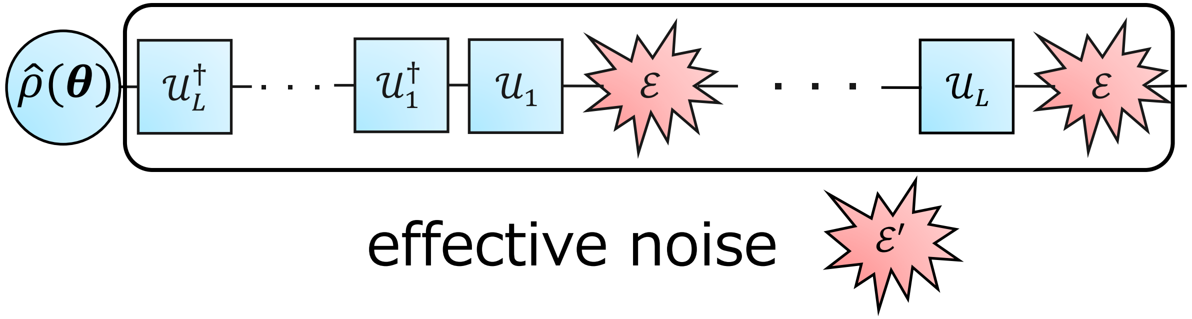
$$J(\otimes_{m=1}^{N} \mathcal{E}'_m(\hat{\rho}(\boldsymbol{\theta}) \otimes |0^{aL}\rangle\langle 0^{aL}|))$$

$$N \times J(\mathcal{E}'(\hat{\rho}(\boldsymbol{\theta}) \otimes\otimes |0^{aL}\rangle\langle 0^{aL}|))$$

$$N \times J\left(\mathcal{E}'\left(\hat{\rho}(\boldsymbol{\theta})\right)\right)$$

③ Bounding QFIM of the virtual circuit

- Analyze $J\left(\mathcal{E}'\left(\hat{\rho}(\boldsymbol{\theta})\right)\right)$.



- From the result of Y. Watanabe et al., 2010,

$$
\begin{aligned}
J(\mathcal{E}'(\hat{\rho}(\boldsymbol{\theta}))) &= A'^T C^{-1}_{(\mathcal{E}'(\hat{\rho}))} A' \\
&\lesssim ||A'||^2 I \\
&\lesssim ||A||^{2L} I
\end{aligned}
$$

$C_{\mathcal{E}'(\hat{\rho})}$: Covariance matrix of $\hat{\boldsymbol{P}}$ for $\mathcal{E}'(\hat{\rho})$

A' : Unital part of the PTM of $\mathcal{E}'$

A : Unital part of the PTM of $\mathcal{E}$

④ Obtaining the lower bound for general circuit

- Bound on QFIM J of the virtual circuit

$$J \leq O(N\gamma^{-2L})$$

Quantum Cramér-Rao bound

$$\mathrm{Var}(f(\boldsymbol{\theta})_{\mathrm{est}}) \geq \frac{1}{N}(\nabla_{\boldsymbol{\theta}} f(\boldsymbol{\theta}))^T J^{-1} (\nabla_{\boldsymbol{\theta}} f(\boldsymbol{\theta}))$$

- Exponential growth with circuit depth
  - For general circuit

$$N \geq O(\gamma^{2L})$$

L: circuit depth
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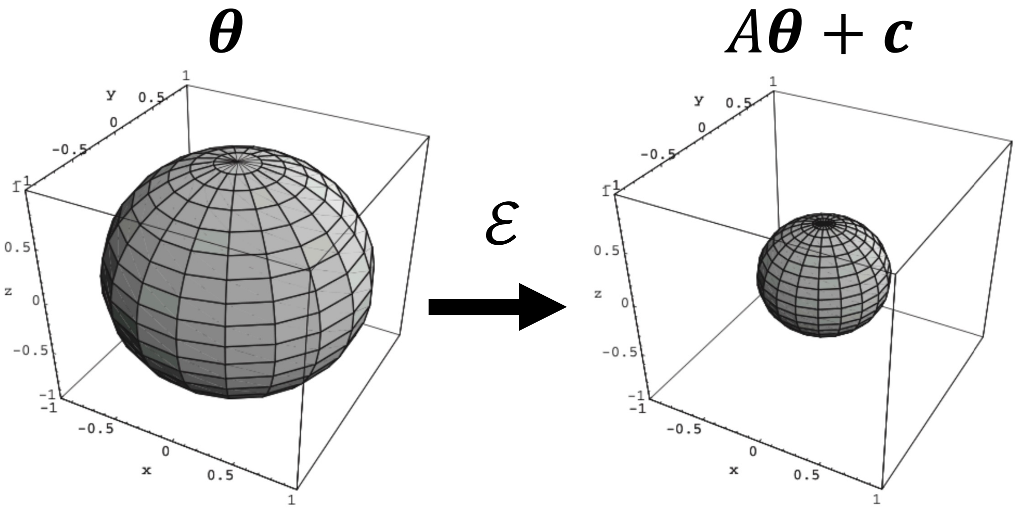


$$\gamma = ||A||^{-1}$$

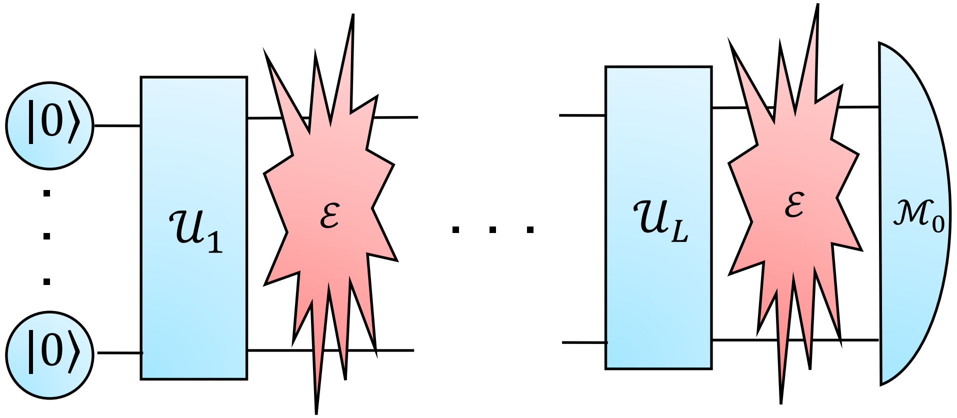
Minimal degree of shrinkage of the generalized Bloch sphere

Applying general bound to some noise models

Global depolarizing noise

- Correlated uniform noise: $\boldsymbol{\theta} \mapsto (1-p)\boldsymbol{\theta}$
- $\gamma = (1-p)^{-1}$

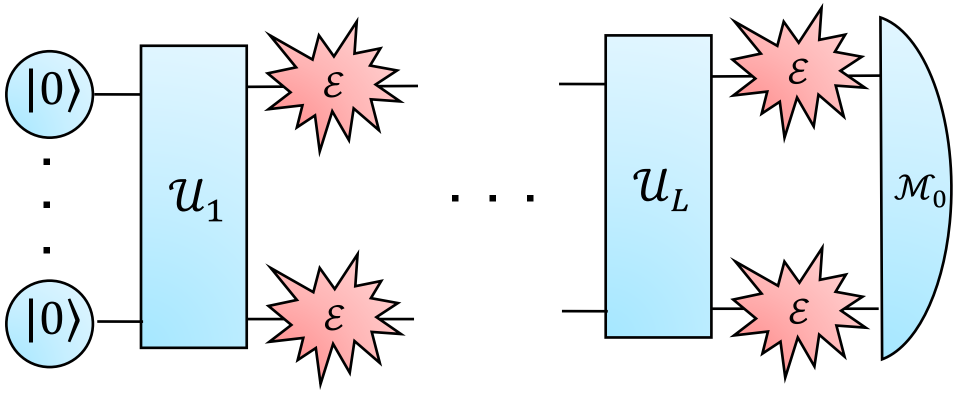
$$N \gtrsim \left(\frac{1}{1-p}\right)^{2L}$$



Local depolarizing noise

- Noise acting independently on each qubit
- $\gamma = (1-p)^{-1}$

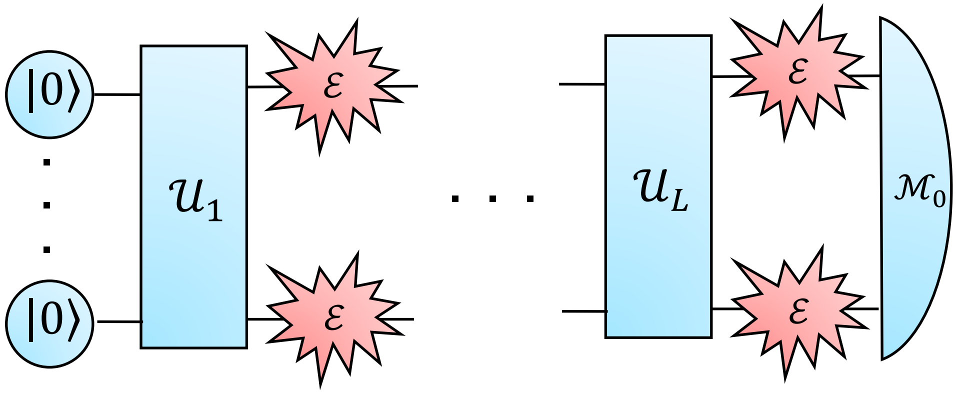
$$N \gtrsim \left(\frac{1}{1-p}\right)^{2L}$$



Problem with the general bound

- Local depolarizing noise
  - Noise acting independently on each qubit
  - $\gamma = (1-p)^{-1}$

$$N \gtrsim \left(\frac{1}{1-p}\right)^{2L}$$



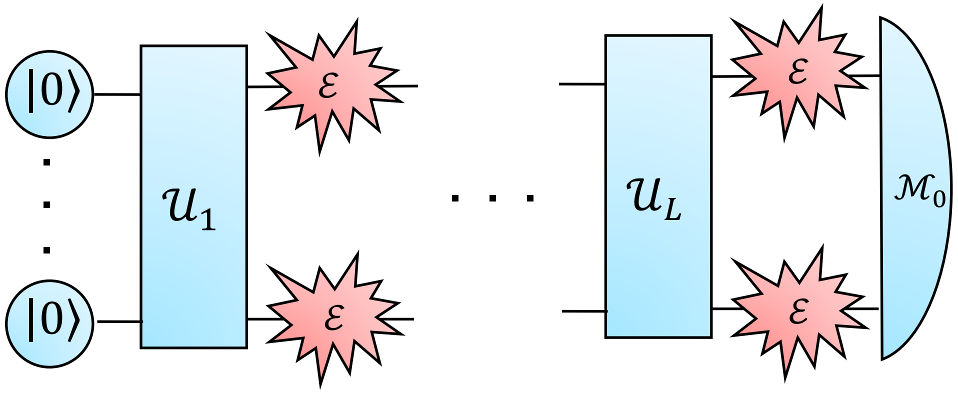
- Bound for general circuit
  - Includes the case where all $\mathcal{U}_i$ are single-qubit gate

- Expected lower bound under entanglement: $2^{O(\#\mathrm{noise})} = 2^{O(nL)}$

Second bound: exponential growth with qubit count

Scrambling circuit

- Noise : local noise $\mathcal{E}^{\otimes n}$
- Gates : drawn from unitary 2-design



QFIM J of the virtual circuit

$$J \leq N A'^T C^{-1}_{\mathcal{E}'(\hat{\rho})} A'$$

$\mathbb{E}_U$ ⇒

Exponential growth with qubit count

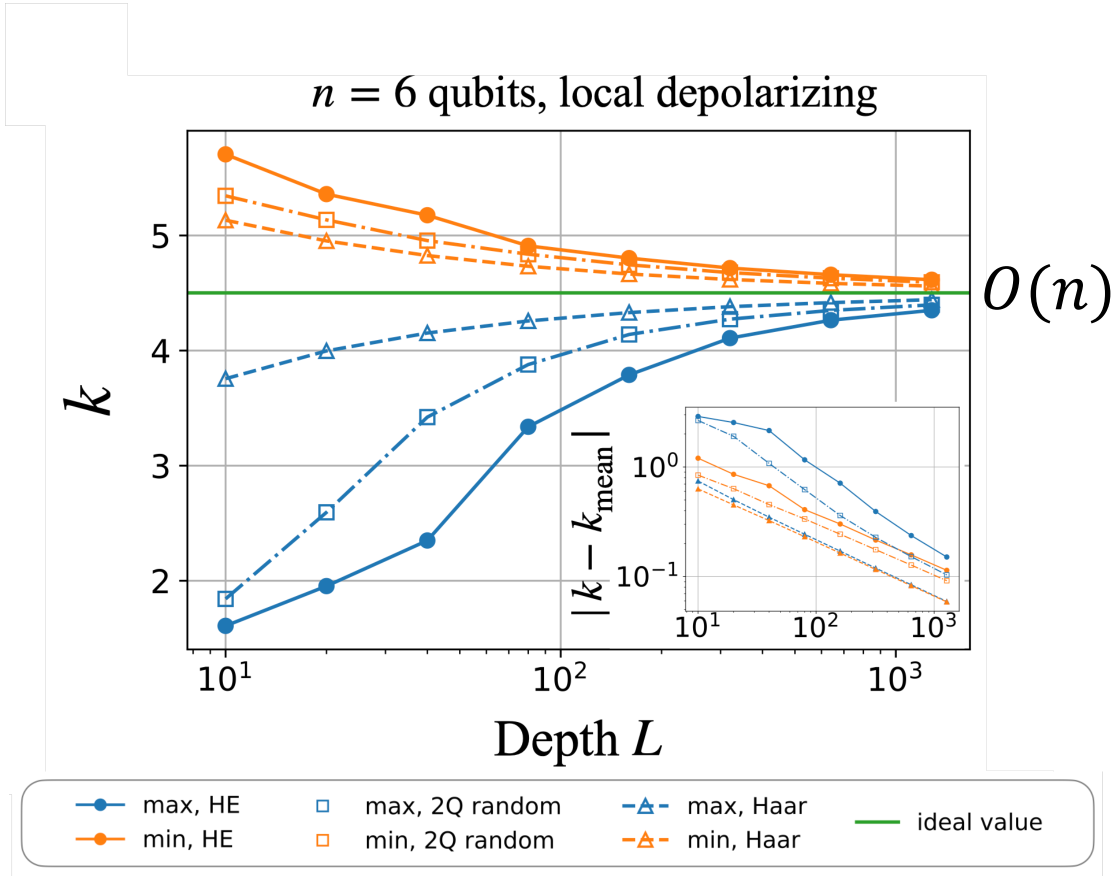
- For scrambling circuit

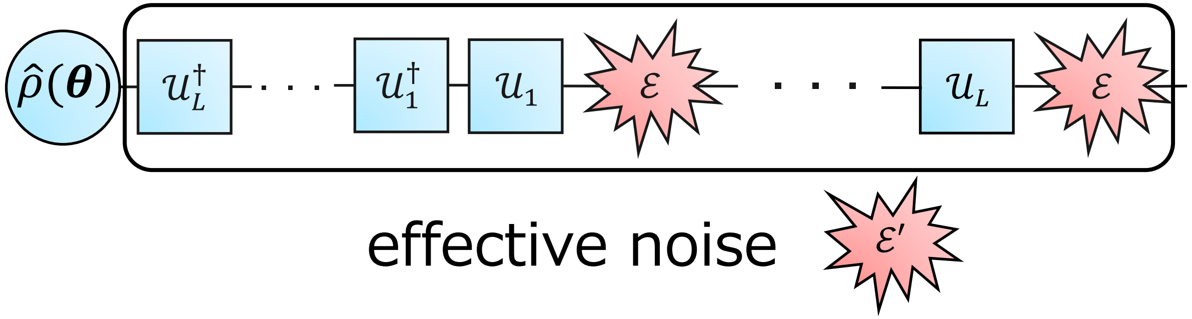
$$\mathbb{E}_U[N] \geq O(\nu^{nL})$$

n: qubit count
$\nu > 1$:Noise-depending constants

Lower bound under local randomness

- Even when gates are not drawn from unitary 2-design, we can still observe the exponential growth with n from numerics.





- Pauli transfer matrix A' of effective noise $\mathcal{E}'$
  - singular values: $(1-p)^{kL}$
- Convergence to $k = k_{\mathrm{mean}} = O(n)$

$\Rightarrow ||A'|| = 2^{-O(nL)}$

$N \geq 2^{O(nL)}$

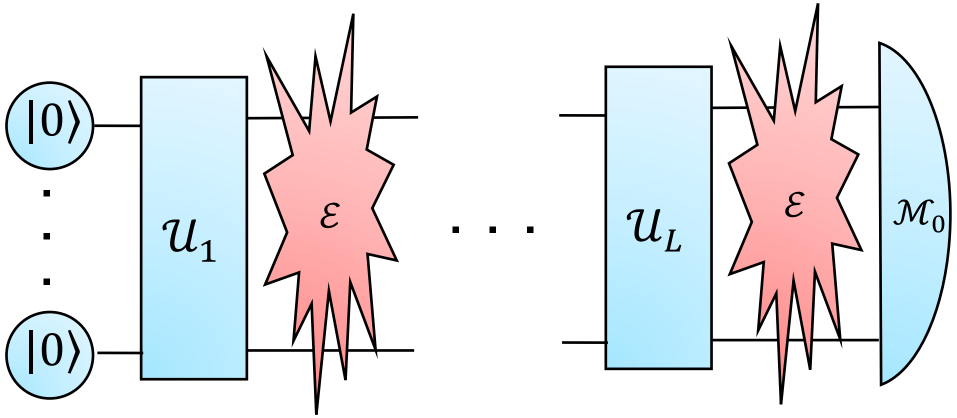
Outline

- Introduction: quantum error mitigation (QEM)
  - What is the theoretical limit of the sampling cost of QEM?
  - What is the cost-optimal QEM method?
- Method: quantum estimation theory
  - Optimal measurement on noisy quantum systems
- Main results: two lower bounds on the sampling cost of QEM
  - First bound: exponential growth with circuit depth
  - Second bound: exponential growth with circuit depth and qubit count
- Application: cost-optimal QEM method
  - Rescaling the measurement results is cost-optimal

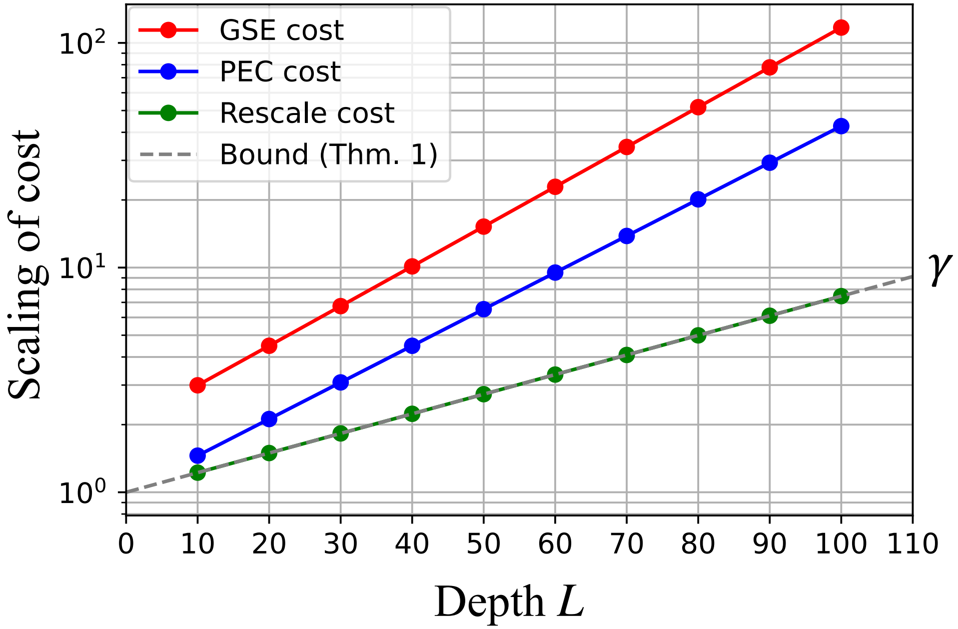
Cost-optimal QEM for Global depolarizing noise

Global depolarizing noise

- $\langle X\rangle_{\text{noisy}} = (1-p)^L \langle X\rangle_{\text{ideal}}$
- $\therefore \langle X\rangle_{\text{est}} = (1-p)^{-L} \langle X\rangle_{\text{noisy}} = \langle X\rangle_{\text{ideal}}$



QEM cost under global depol, n=2 qubits



$\gamma^{2L} = (1-p)^{-2L}$

GSE: Optimization in $\mathrm{SPAN}\{\rho^m\}$
N. Yoshioka, et al, (2022).

PEC: Quasiprobabilistic $\mathcal{E}^{-1}$
K. Temme, et al, (2017).

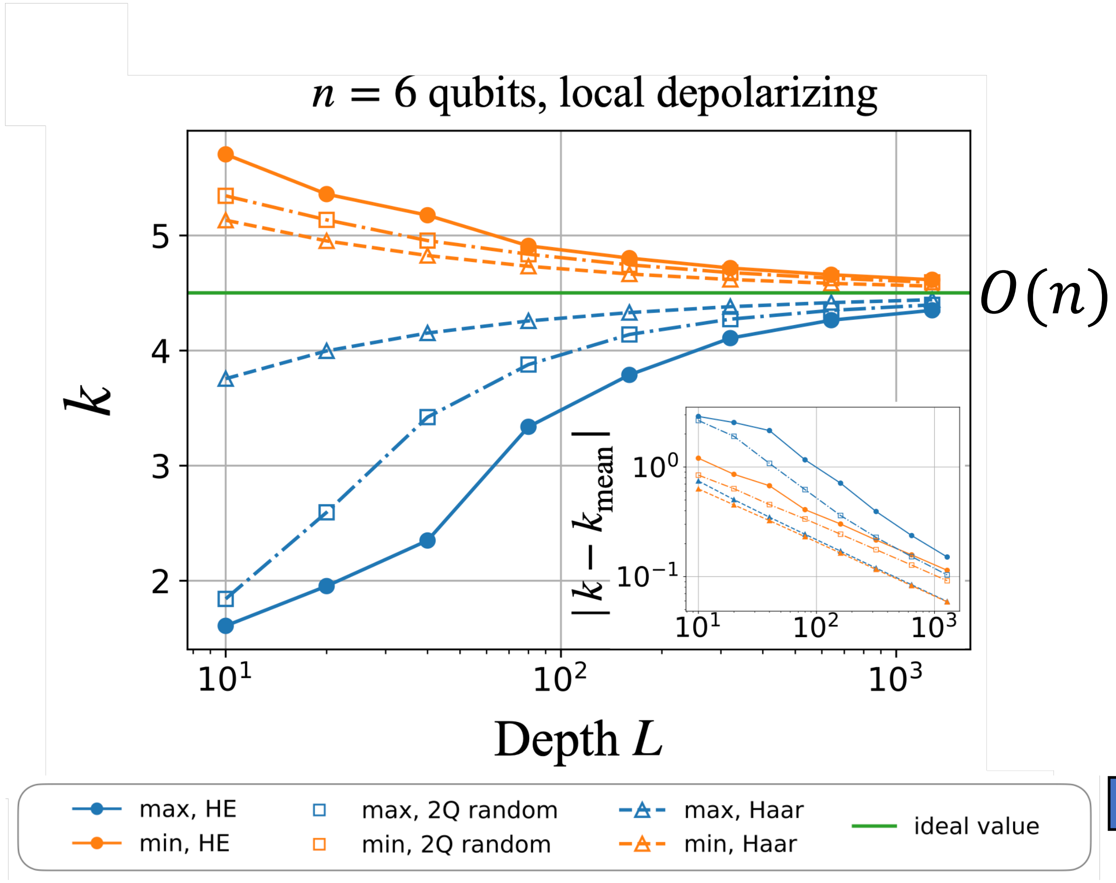
Rescaling: $\langle X\rangle_{\text{est}} = (1-p)^{-L} \langle X\rangle_{\text{noisy}}$

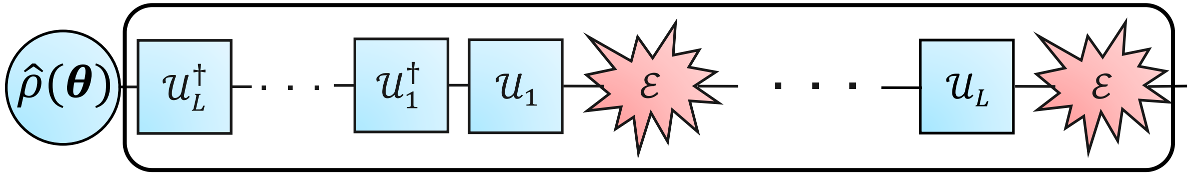
Cost-optimal

Convergence to global depolarizing noise

- Randomness causes noise to converge to global depolarizing noise.
  - Our numerics supports the convergence.

A. M. Dalzell et al., 2022.





effective noise 

- Pauli transfer matric A' of effective noise $\mathcal{E}'$
  - Singular values: $(1-p)^{kL}$
- Convergence to $k = k_{\mathrm{mean}} = \mathcal{O}(n)$ implies convergence to global depolarizing noise

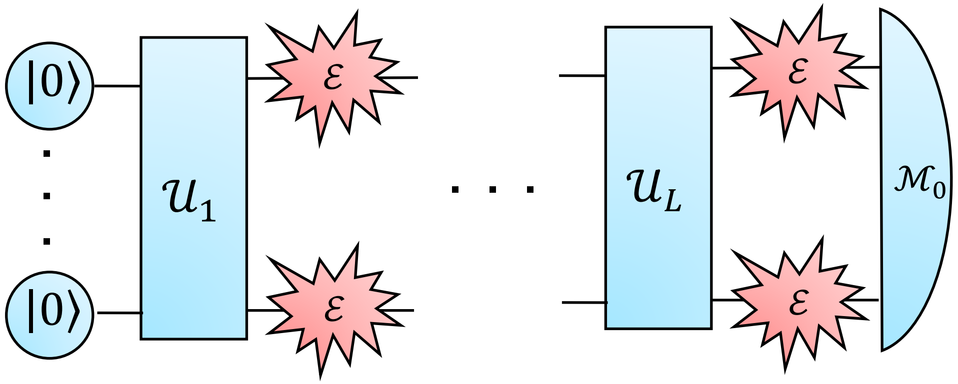
$$\langle X \rangle_{\mathrm{noisy}} \sim (1-p)^{k_{\mathrm{mean}}L} \langle X \rangle_{\mathrm{ideal}}$$

Implies the usefulness of the rescaling!

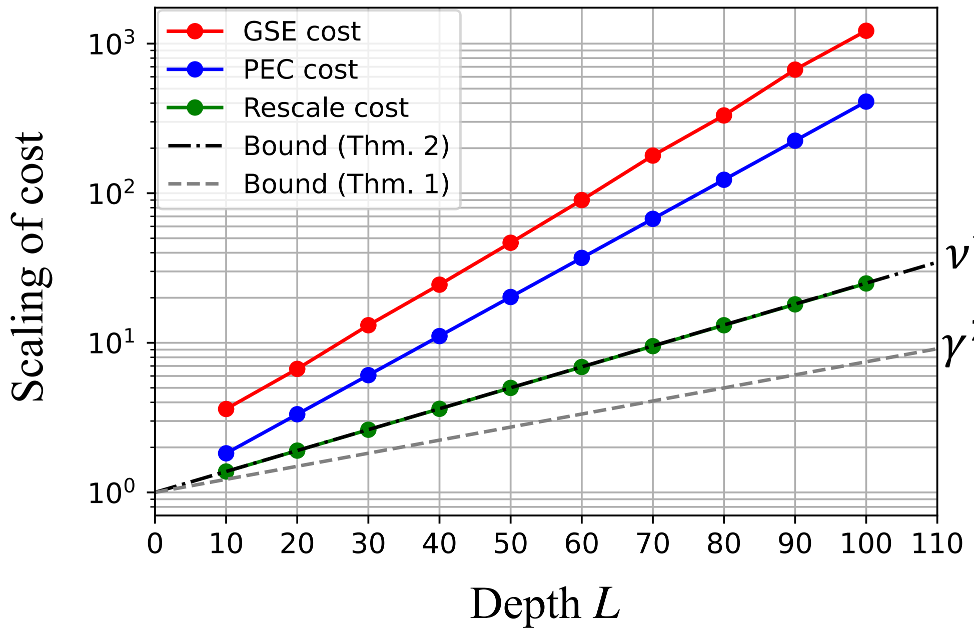
Cost-optimal QEM for scrambling circuit

Local depolarizing noise

- $\langle X\rangle_{\mathrm{noisy}} \sim (1-p)^{k_{\mathrm{mean}}L}\langle X\rangle_{\mathrm{ideal}}$
- $\therefore\ \langle X\rangle_{\mathrm{est}} = (1-p)^{-k_{\mathrm{mean}}L}\langle X\rangle_{\mathrm{noisy}} \sim \langle X\rangle_{\mathrm{ideal}}$



QEM cost under local depol., n=2 qubits



$\nu^{nL} = (1-p)^{-2k_{\mathrm{mean}}L}$

$\gamma^{2L} = (1-p)^{-2L}$

GSE: Optimization in $\mathrm{SPAN}\{\rho^m\}$
N. Yoshioka, et al, (2022).

PEC: Quasiprobabilistic $\mathcal{E}^{-1}$
K. Temme, et al, (2017).

Rescaling: $\langle X\rangle_{\mathrm{est}} = (1-p)^{-k_{\mathrm{mean}}L}\langle X\rangle_{\mathrm{est}}$

Cost-optimal

Summary of today's talk

- What is the theoretical limit of the sampling cost of unbiased QEM?

- Exponential growth with circuit depth
  - For general circuit

$$N \geq O(\gamma^{2L})$$

L: circuit depth
$\gamma > 1$:Noise-dependent constants

- Exponential growth with qubit count
  - For scrambling circuit

$$\mathbb{E}_U[N] \geq O(\nu^{nL})$$

n: qubit count
$\nu > 1$:Noise-dependent constants

- What is the cost-optimal QEM method achieving the limit?

Rescaling the measurement result is cost-optimal

$$\langle X \rangle_{\mathrm{est}} = (1-p)^{-k_{\mathrm{mean}}L} \langle X \rangle_{\mathrm{noisy}}$$

Future direction

- What is the tradeoff between bias and sampling costs in QEM ?
  - Our work : cost bound for unbiased QEM.
  - c.f.) Cost bound for biased QEM R. Takagi et al., 2022. E. Quek et al., 2022
- What is the cost-optimal QEM method for structured circuit ?
  - Is the rescaling method still effective for structured circuit ?
  - If not, what other method will be useful ?
- Can we unify QEC and QEM ?
  - What is the optimal countermeasure in this regime?

Summary of today's talk

- What is the theoretical limit of the sampling cost of unbiased QEM?

- Exponential growth with circuit depth
  - For general circuit

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- What is the cost-optimal QEM method achieving the limit?

Rescaling the measurement result is cost-optimal

$$\langle X \rangle_{\mathrm{est}} = (1-p)^{-k_{\mathrm{mean}}L} \langle X \rangle_{\mathrm{noisy}}$$