Exploring supersymmetric wormholes in $\mathcal{N} = 2$ SYK with chords

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"Exploring supersymmetric wormholes in $\mathcal{N} = 2$ SYK with chords"
2308.16283
w/ Jan Boruch & Henry W. Lin

Motivation

why $\mathcal{N} = 2$ SUSY SYK?

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- want to understand BH in regime where Quantum Gravity is important

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- want to understand BH in regime where Quantum Gravity is important
- this is at very low temperature, i.e. ground state sector
- SUSY SYK has a lot of zero-energy ground states

Motivation

- $\mathcal{N} = 2$ SUSY SYK in double-scaling limit using chord diagrams [BerkoozBruknerNarovlanskyRaz]'20

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- Goal: go beyond super-Schwarzian
  - count ground state from bulk, understand quantum corrections (super-JT [StanfordWitten]'20)
  - length of wormhole quantum corrections (super-JT [LinMaldacenaRozenbergShan]'22)

Majorana SYK v.s. $\mathcal{N} = 2$ SUSY SYK

Majorana SYK

- N Majorana fermions ψ_i

$$\{\psi_i, \psi_j\} = 2\delta_{ij}$$

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- Hamiltonian

$$H = i^{p/2} \sum_I C_I \Psi_I$$

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- N complex fermions ψ_i

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- supercharges

$$Q = \sum_I C_I \Psi_I \qquad \bar{Q} = \sum_I C_I^* \overline{\Psi}_I$$

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$$H = \frac{1}{2}(Q\bar{Q} + \bar{Q}Q)$$

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- $U(1)$ R charge

$$\gamma = \frac{1}{2p} \sum_{i=1}^{N} (\bar{\psi}_i \psi_i - \psi_i \bar{\psi}_i)$$

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- average over couplings

$$\langle C_I \rangle = 0 \qquad \langle C_I C_{I'} \rangle = 2 \binom{N}{p}^{-1} \mathcal{J}^2 \delta_{I,I'}$$

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double scaled SYK

double-scaling limit

$$N \to \infty \qquad p \to \infty \qquad \lambda = \frac{2p^2}{N} \quad \text{fixed}$$

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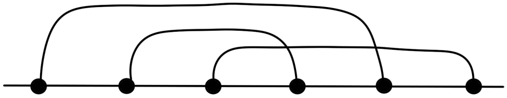
$$N \to \infty \qquad p \to \infty \qquad \lambda = \frac{2p^2}{N} \quad \text{fixed}$$

define

$$q = e^{-\lambda}$$

Majorana SYK

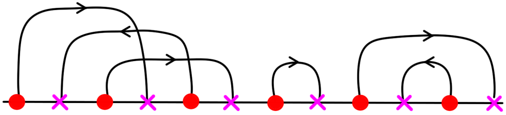
moment $m_k = \mathrm{tr}(H^k)$



$\mathrm{tr}(\Psi \quad \Psi \quad \Psi \quad \Psi \quad \Psi \quad \Psi)$

$\mathcal{N} = 2$ SUSY SYK

$m_k = 2^{-k}\left(\mathrm{tr}((Q\bar{Q})^k) + \mathrm{tr}((\bar{Q}Q)^k)\right)$



$\mathrm{tr}(\Psi \quad \bar{\Psi} \quad \Psi \quad \bar{\Psi} \quad \Psi \quad \bar{\Psi} \quad \Psi \quad \bar{\Psi} \quad \Psi \quad \bar{\Psi} \quad \Psi \quad \bar{\Psi})$

Chord diagrams

Majorana SYK

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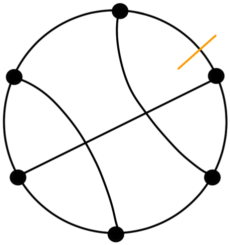

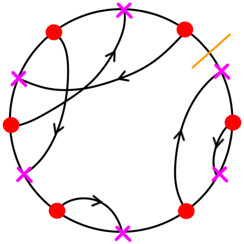

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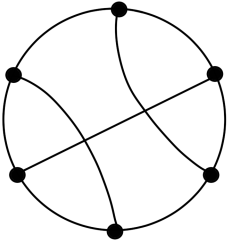

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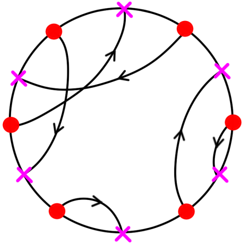
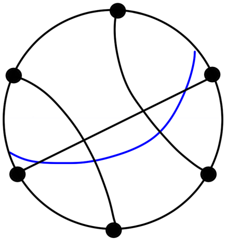


Hilbert space

Majorana SYK



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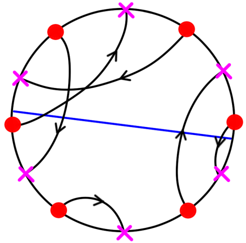

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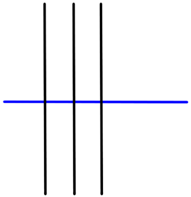
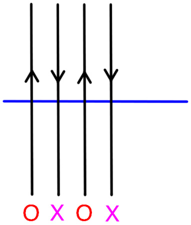


$|3\rangle$

$\mathcal{N} = 2$ SUSY SYK



$|OXOX\rangle = |2, OX\rangle$

Hilbert space

Majorana SYK

$$|n\rangle$$

$\mathcal{N} = 2$ SUSY SYK

$$|n, XO\rangle = |\overbrace{XOXO\cdots XO}^{n \text{ pairs of } XO}\rangle$$

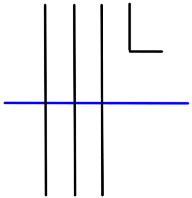
$$|n, OX\rangle = |\overbrace{OXOX\cdots OX}^{n \text{ pairs of } OX}\rangle$$

$$|n, XX\rangle = |\overbrace{XOXO\cdots XO}^{n \text{ pairs of } XO}X\rangle$$

$$|n, OO\rangle = |\overbrace{OXOX\cdots OX}^{n \text{ pairs of } OX}O\rangle$$

Majorana SYK

$\mathcal{N} = 2$ SUSY SYK



$H_R |3\rangle \supset |4\rangle$

Hilbert space

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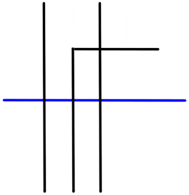
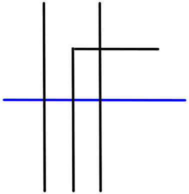


$H_R \, |3\rangle \supset |2\rangle$

Hilbert space

Majorana SYK



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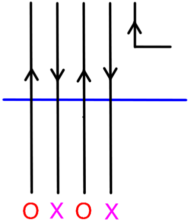

$Q_R\,|OXOX\rangle \supset |OXOXO\rangle$

Hilbert space

Majorana SYK

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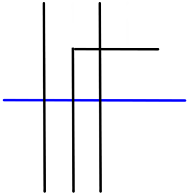
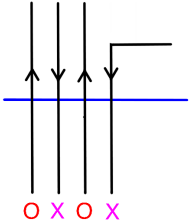


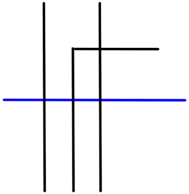
$H_R \left|3\right\rangle \supset \left|2\right\rangle$



$Q_R \left|OXOX\right\rangle \supset \left|OXO\right\rangle$

Hilbert space

Majorana SYK



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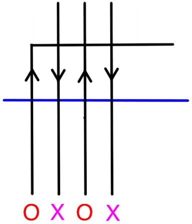

$\bar{Q}_R \left|OXOX\right\rangle \supset \left|XOX\right\rangle$

Hilbert space

$$\{|n, XO\rangle, |n, OX\rangle, |n, XX\rangle, |n, OO\rangle\}$$

Hilbert space

emergence of fermion in the bulk

$$\{\underbrace{|n, XO\rangle, |n, OX\rangle}_{\text{bosonic}}, \underbrace{|n, XX\rangle, |n, OO\rangle}_{\text{fermionic}}\}$$

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There are 4 supercharges

$$Q_L, \bar{Q}_L, Q_R, \bar{Q}_R$$

Finding the ground state

- ground state: $|\Psi, j\rangle$ such that

$$Q_R |\Psi, j\rangle = \bar{Q}_R |\Psi, j\rangle = 0$$

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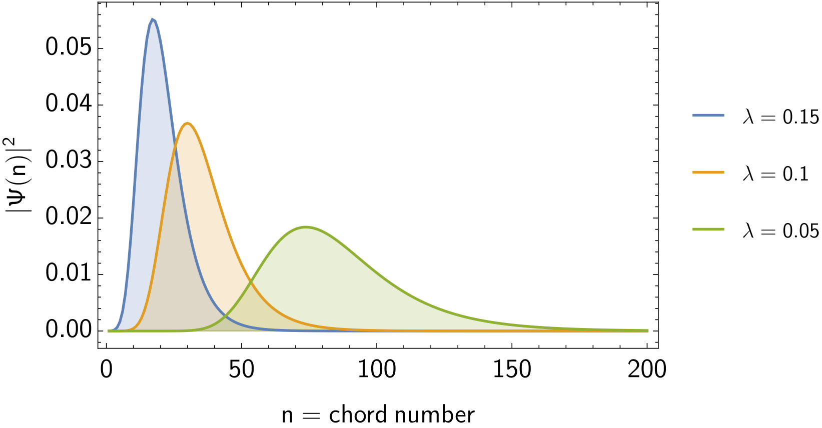
- expand

$$|\Psi, j\rangle = \sum_{n=0}^{\infty} q^{-n/4} \left(\alpha_n |n, XO\rangle + \beta_n |n, OX\rangle\right)$$

wavefunction



fraction of ground states in sector j

- want to calculate

$$D(j) = \frac{\#\,\text{ground states of charge } j}{\#\,\text{all states of charge } j} = \frac{Z(j,\infty)}{Z(j,0)}$$

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$$Z(j,\beta) = \mathrm{tr}_j\left(e^{-\beta H}\right) = \langle \Omega, j|\, e^{-\beta H}\, |\Omega, j\rangle$$

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- $\beta \to \infty$

$$Z(j,\infty) = \frac{\langle \Omega, j|\Psi, j\rangle \langle \Psi, j|\Omega, j\rangle}{\langle \Psi, j|\Psi, j\rangle}$$

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- double-scaling limit

$$D(j) = 2\cos(\pi j)\, e^{-\pi^2/(4\lambda)} + 2\cos(3\pi j) e^{-9\pi^2/(4\lambda)} + 2\cos(5\pi j) e^{-25\pi^2/(4\lambda)} + \cdots$$

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- double-scaling limit (same as microscopic calculation)

$$D(j) = \underbrace{2\cos(\pi j)\, e^{-\pi^2/(4\lambda)}}_{\text{Schwarzian}} + \underbrace{2\cos(3\pi j) e^{-9\pi^2/(4\lambda)} + 2\cos(5\pi j) e^{-25\pi^2/(4\lambda)} + \cdots}_{\text{non-perturbative corrections}}$$

physical interpretation of $Z(j, \infty)$

$$Z(j,\infty) = \frac{\langle \Omega, j | \Psi, j \rangle \langle \Psi, j | \Omega, j \rangle}{\langle \Psi, j | \Psi, j \rangle} = \text{probability of wormhole having zero chords}$$

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This is a bulk quantity

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