Is action complexity better for de Sitter in Jackiw-Teitelboim gravity?

Based on arXiv:2303.05025 [hep-th], in collaboration with T. Anegawa, N. Iizuka, and S. K. Sake

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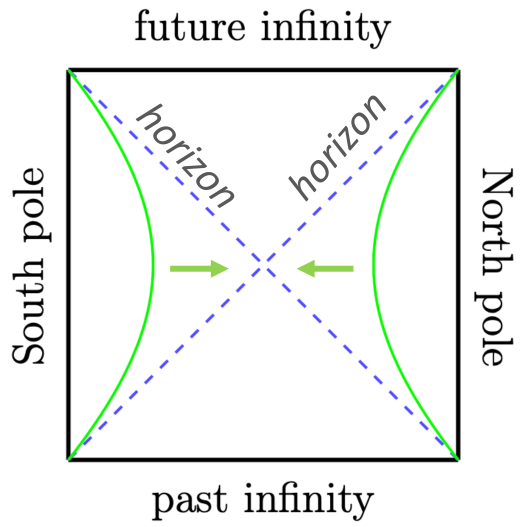
A probe of cosmological horizons?

In **de Sitter** (dS) spacetime, a **cosmological horizon** arises due to the inflationary expansion



Recently, **gravity in dS** has been conjectured to be **dual** to a **quantum mechanical** system living on the stretched horizon

Holographic quantity able to **probe** the dS **expansion**?

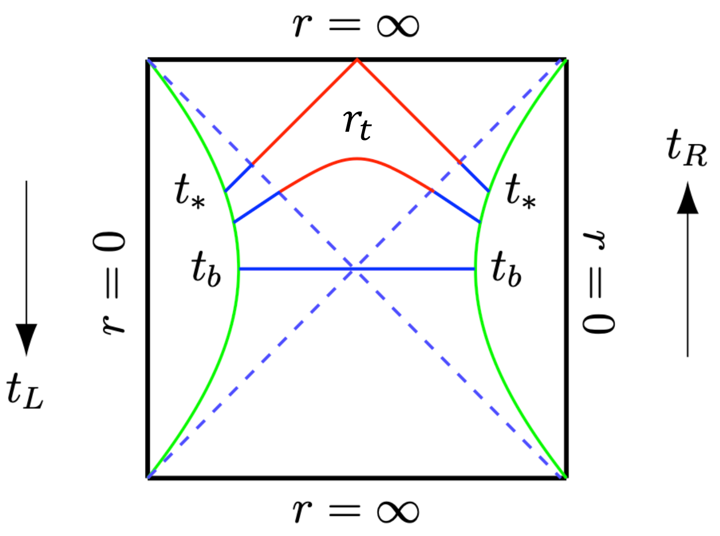
Volume complexity in dS

In analogy with AdS/CFT, a candidate is volume complexity:

$$\mathcal{C}_V(B) = \max_{\partial\Sigma = B} \frac{V(\Sigma)}{G L_{dS}}$$

CV conjecture in dS

Pure dS$_{d+1}$ spacetime ($d > 1$): $ds^2 = -f(r)\, dt^2 + \frac{dr^2}{f(r)} + r^2\, d\Omega_{d-1}^2$, $\quad f(r) = 1 - \frac{r^2}{L_{dS}^2}$



Boundary time: $t_b = t_R = -t_L$

Maximal d-dimensional slices can be anchored at $r_{st} = \rho\, L_{dS}$ ($\rho \to 1$)

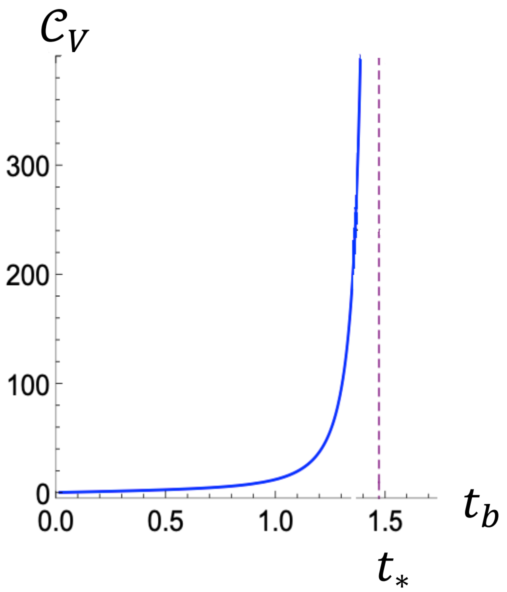
Volume and **boundary time** as functions of a conserved quantity P are [E. Jørstad et al., 2022]

$$V(P) = 2\Omega_{d-1} \int_{r_{st}}^{r_t} \frac{r^{2(d-1)}}{\sqrt{r^{2(d-1)}\, f(r) + P^2}}\, dr$$

$$t_b(P) = -\int_{r_{st}}^{r_t} \frac{P}{f(r)\sqrt{r^{2(d-1)}\, f(r) + P^2}}\, dr$$

A probe of expansion

Connected maximal surfaces exist **up to** the critical time $t_* = L_{dS} \tanh^{-1} \rho$



$$\frac{d\mathcal{C}_V}{dt_b} \approx \frac{L_{dS}^{d-1}}{(t_* - t)^d}$$

Hyperfast growth at a critical time!

$$\mathcal{C}_V \approx \frac{L_{dS}^{d-1}}{(d-1)(t_* - t)^{d-1}}$$

Complexity diverges at a critical time!

Volume complexity probes the **expansion** of dS!

Is this also true in the dS_2 case?

The two-dimensional puzzle

dS_2 is a solution of **Jackiw-Teitelboim** (JT) **gravity**:

$$I_{JT} = \frac{1}{8G} \int d^2x \sqrt{-g}\, \phi \left(R - \frac{2}{L_{dS}^2} \right) + boundary\ terms$$

Dilaton

The solutions for the **metric** and **dilaton** are:

$$ds^2 = -f(r)\, dt^2 + \frac{dr^2}{f(r)} \quad , \qquad \phi(r) = \frac{r}{L_{dS}}$$

At the critical time, defined **as in the higher-dimensional case** [E. Jørstad et al., 2022, S. Chapman et al., 2021]:

$$\lim_{t_b \to t_*} \frac{d\mathcal{C}_V}{dt_b} = \frac{const}{\sqrt{t_* - t}}$$

Hyperfast growth at a critical time!

$$\lim_{t_b \to t_*} \mathcal{C}_V = \mathcal{O}(1)$$

Complexity remains finite!

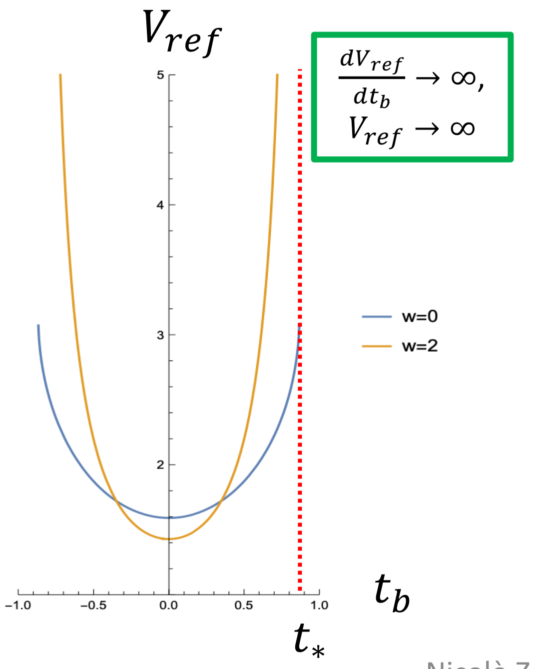
Since there is **no transverse direction**, volume remains **finite** up to the time t_*!

Don’t forget the dilaton!

Volume contains information **just** on the metric, but **dilaton is important**!

Consider a Weyl-like **field redefinition** of the metric: $g_{\mu\nu} \rightarrow \Omega(\phi)\, g_{\mu\nu}$

$$ds^2 = -\Omega(\tilde{r}) f(\tilde{r})\, dt^2 + \frac{d\tilde{r}^2}{\Omega(\tilde{r}) f(\tilde{r})} \qquad , \qquad \tilde{r}(r) = \int \Omega(\hat{r}) d\hat{r}$$



Volume and boundary time **change** as:

$$V_{ref}(P) = 2 \int_{r_{st}}^{r_t} \frac{\Omega(\phi)}{\sqrt{\Omega(\phi)\, f(r) + P^2}} dr$$

$$t_b(P) = -\int_{r_{st}}^{r_t} \frac{P}{f(r)\sqrt{\Omega(\phi)\, f(r) + P^2}} dr$$

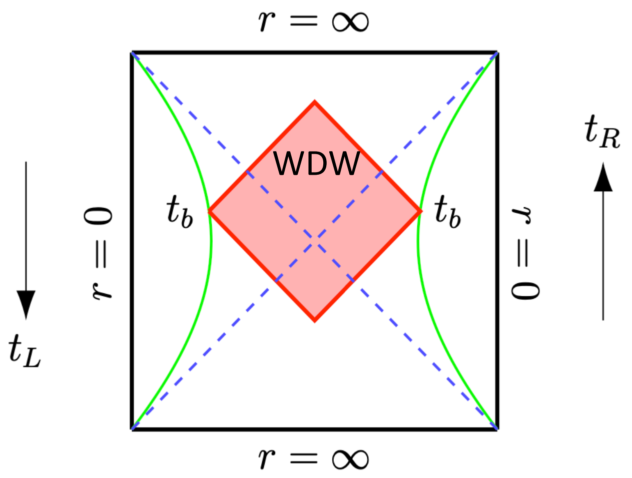
With the **choice** $\Omega(\phi) = \phi^w$,
it is possible to reproduce the **dS_3 result**!

Beyond volume complexity

The CV conjecture in dS_2 has some drawbacks:

- Volume is **not invariant** under Weyl-like field redefinition of the metric
- After the critical time, **no geodesics exist** which connect the stretched horizons

We can look at **other putative** bulk **duals** of complexity



$$\mathcal{C}_A = \frac{I(WDW)}{\pi \, \hbar}$$

Complexity = Action (CA) conjecture

[L. Susskind et al., 2015]

Dimensional reduction

dS_2 can be built by (half) **dimensional reduction** starting from dS_3 [A. Svesko et al., 2022]:

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\,d\theta^2, \quad f(r) = 1 - \frac{r^2}{L_{dS}^2}$$

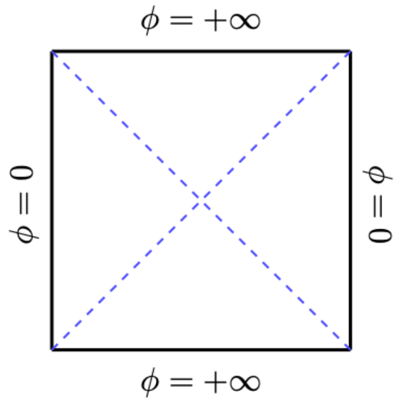
dS_3 spacetime

Half dimensional reduction

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + L_{dS}^2\,\phi(r)^2\,d\theta^2$$

dS_2 spacetime

Dilaton: replaces the higher-dimensional transverse sphere!



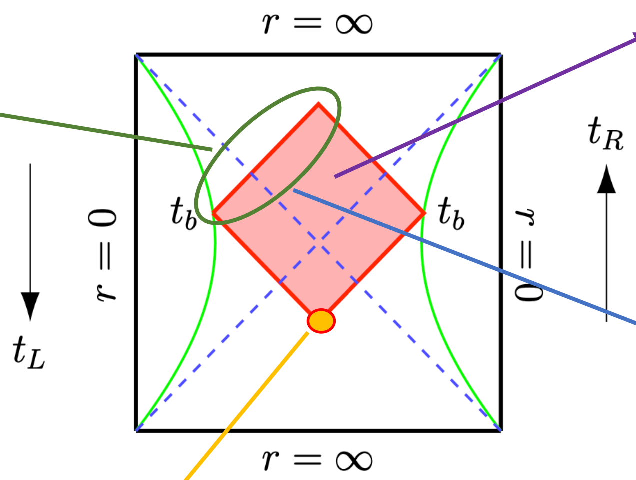
$$\phi(r) = \frac{r}{L_{dS}} \geq 0$$

Action manual in JT gravity

The JT gravity action can be directly **obtained** by the **Einstein-Hilbert action**

$$I_{3d} = \frac{1}{16\pi G_{(3)}} \int d^3X \sqrt{-g_{(3)}} \left(R_{(3)} - \frac{2}{L_{dS}^2} \right) + boundary\ terms$$

$$I_{bry} = -\frac{1}{4G_{(2)}} \int d\lambda\, \partial_\lambda \phi$$



$$I_{bulk} = \frac{1}{8G_{(2)}} \int d^2x \sqrt{-g_{(2)}}\, \phi \left(R_{(2)} - \frac{2}{L_{dS}^2} \right) = 0$$

$$I_{ct} = \frac{1}{4G_{(2)}} \int d\lambda\, \partial_\lambda \phi \log\left| \tilde{L}\, \partial_\lambda \log \phi \right|$$

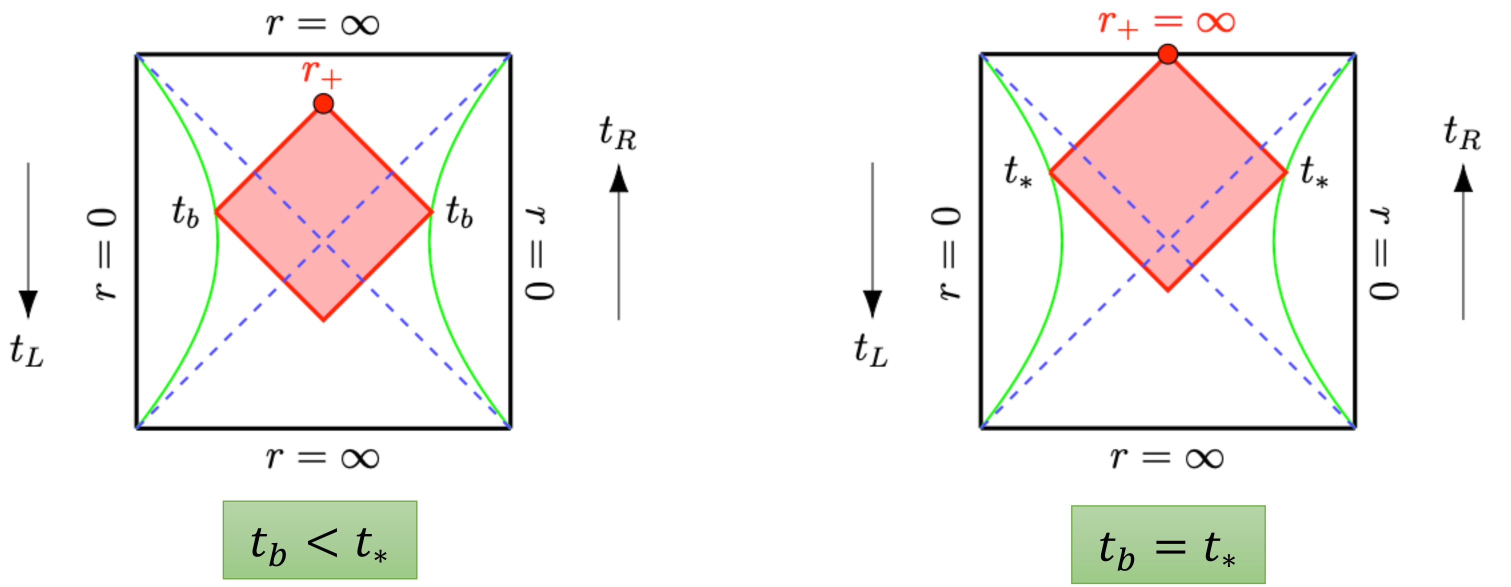
$$I_{joint} = \frac{sign(joint)}{4G_{(2)}} \phi(r_{joint}) \log\left| \frac{k_1 \cdot k_2}{2} \right|$$

${k^\mu}_i = \frac{dx^\mu}{d\lambda}$ is the null normal to the i-th null boundary

The role of the dilaton

The **upper tip** of the WDW patch meets the future infinity at the critical time

$$t_* = L_{dS} \tanh^{-1} \rho$$



At the critical time $t_b \to t_*$, action complexity behaves as:

$$\mathcal{C}_A = \frac{I_{bry} + I_{joint} + I_{ct}}{\pi} = \frac{1}{\pi G_{(2)}} \frac{L_{dS}}{(t_* - t_b)} + \cdots \approx \frac{\phi(r_+)}{G_{(2)}} + \cdots$$

Divergent!

Conclusions and remarks

	$t_b \to t_*$	$t_b > t_*$	Weyl-field redefinition
$\mathcal{C}_V$	$\mathcal{O}(1)$	No connected geodesics	Which Weyl frame?
$\mathcal{C}_A$	Divergent	Well-defined WDW patch	Invariant

Refined volume **diverges** at a critical time, as in **higher dimensions**

Action complexity is **better** than volume for dS_2 in JT gravity!

Thank you
for your
attention!