PSEUDO-ENTROPY UNDER LOCAL OPERATOR QUENCHES IN 2D CFTS

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2206.11818 2301.04891

I. Background & Motivation

Background: In 2D rational conformal field theory (RCFT), when a system in vacuum is quenched by a local operator $\mathcal{O}(x)$ at $t=0$, the entanglement entropy (EE) of subsystem will produce a jump at some point[1,2], as shown in Fig. 1(a).

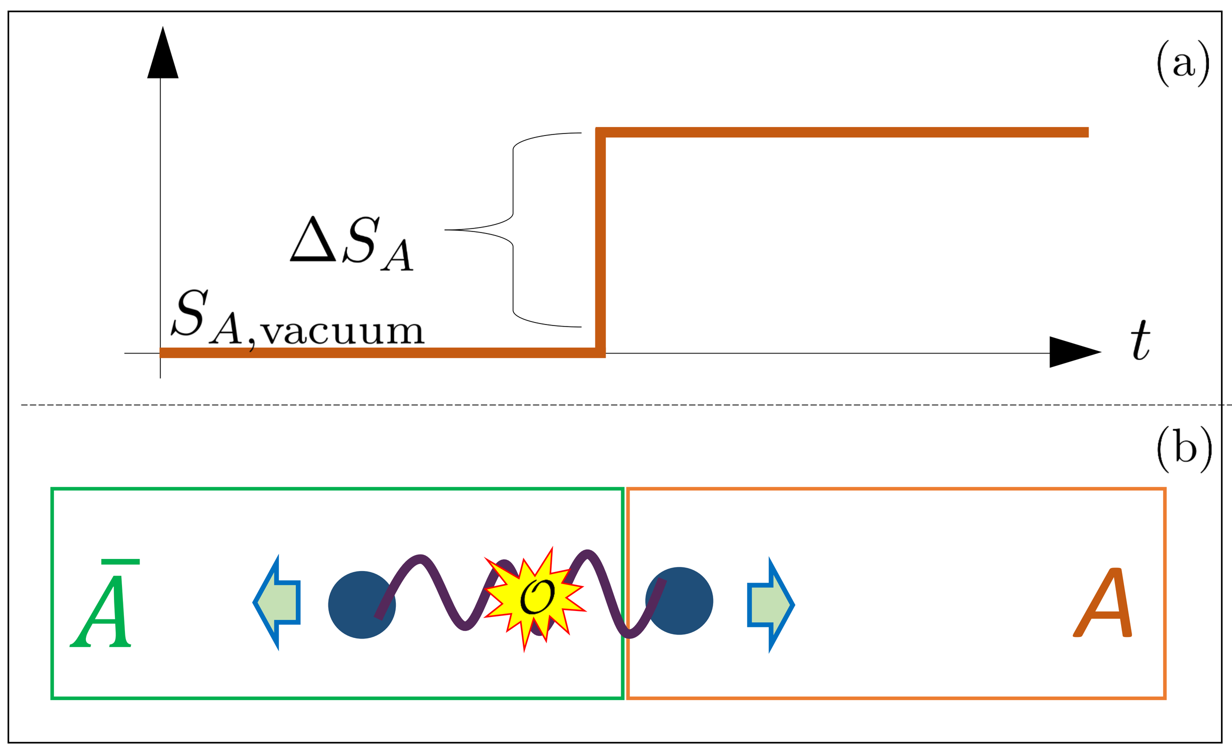

Figure 1: EE under local operator quench.

Such a result has a good explanation for quasiparticle propagation[1,3]: $t=0$, a pair of entangled quasiparticles is generated at x, each running towards infinity in opposite directions at the speed of light (see Fig. 1(b)).

Our motivation: How does the pseudo-entropy (PE) evolve under the local operator quenches, and does the picture of quasiparticle propagation still hold?

II. Setup

The PE of subsystem is defined as the $n\to 1$ limit of the so-called *pseudo-Rényi entropy* (PRE)[4].

$$S_A = -\,\mathrm{tr}[\mathcal{T}_A^{\psi|\varphi}\log \mathcal{T}_A^{\psi|\varphi}],$$
$$S_A^{(n)} = \frac{1}{1-n}\log \mathrm{tr}[(\mathcal{T}_A^{\psi|\varphi})^n]. \qquad (1)$$

$\mathcal{T}_A^{\psi|\varphi}$ called *reduced transition matrix* is the partial trace of the *transition matrix*, $\mathcal{T}_A^{\psi|\varphi}=\mathrm{tr}_{\bar{A}}[\mathcal{T}^{\psi|\varphi}]$.

$$\mathcal{T}^{\psi|\varphi} = \frac{|\psi\rangle\langle\varphi|}{\langle\varphi|\psi\rangle}. \qquad (2)$$

We are interested in the case that both ψ and φ are locally excited states with a time evolution,

$$|\psi\rangle = e^{-iH(t-i\epsilon)}\mathcal{O}_1(x_1)|\Omega\rangle,$$
$$|\varphi\rangle = e^{-iH(t-i\epsilon)}\mathcal{O}_2(x_2)|\Omega\rangle. \qquad (3)$$

$O_{1,2}$ can be primary/descendant operators or their linear combination. Ω is the vacuum. ϵ is a infinitesimal regulator.
The subsystem A is chosen to be $A=[0,L]$ or $A=[0,\infty)$.

III. Primary & Descendant Quenches

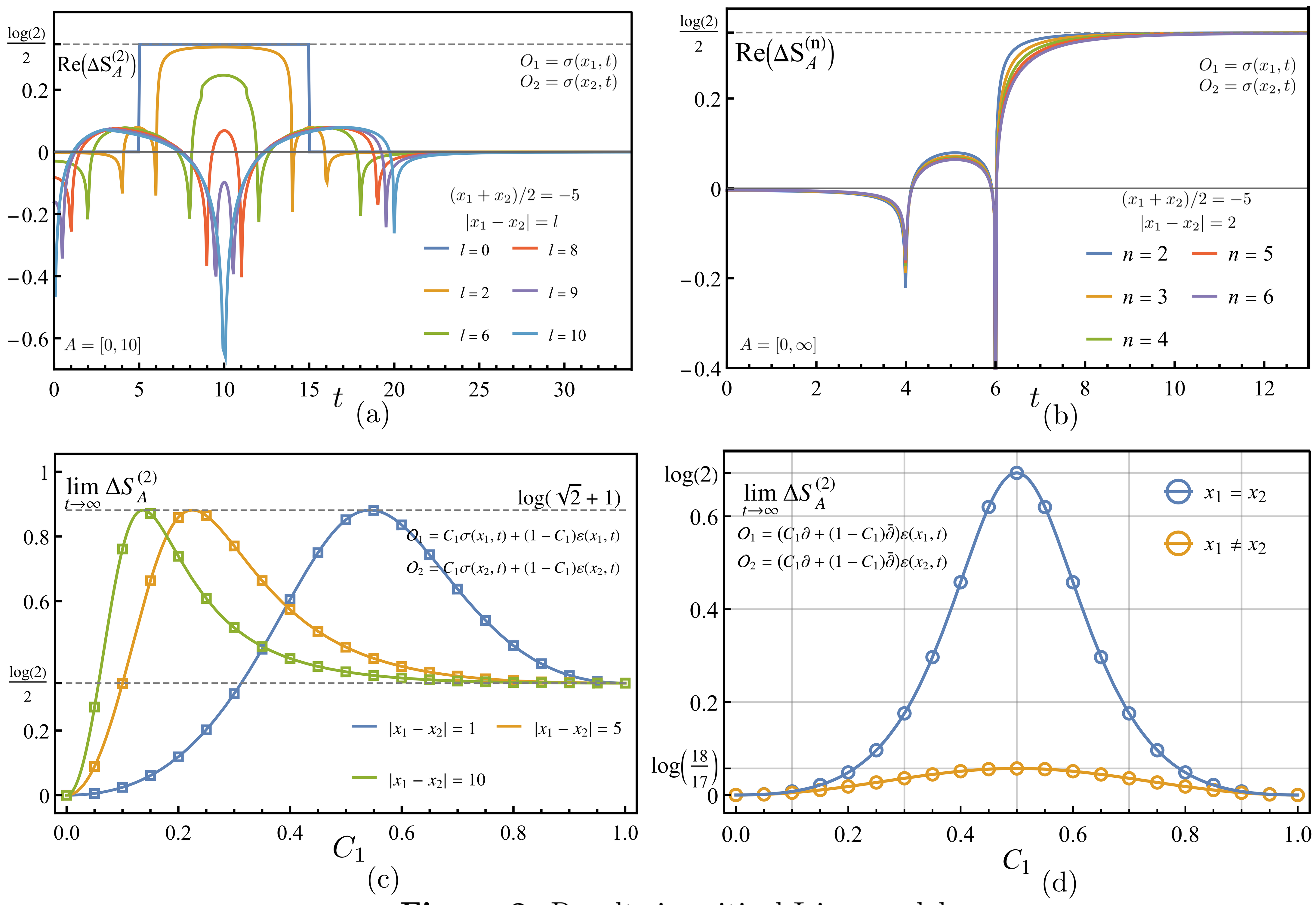

Figure 2: Results in critical Ising model.

Complete time evolution: We study the time evolution of PREs of locally primary/descendant states in specific RCFTs (e.g., critical Ising model: Fig. 2(a) and 2(b)) and find that:

- The picture of quasiparticle propagation is partially preserved.
- The PREs exhibit a universal nontrivial late-time behavior for a semi-infinite subsystem.

Late-time behavior: One can show that $\lim_{t\to\infty}\Delta S^{(n)}_{A=[0,\infty)}=\log d_{\mathcal{O}}$ holds for any pair of descendants in a conformal family: $\mathcal{O}_1=L_{-\{k\}}\bar{L}_{-\{\bar{k}\}}\mathcal{O}$, $\mathcal{O}_2=L_{-\{k'\}}\bar{L}_{-\{\bar{k}'\}}\mathcal{O}$ ($L_{-\{k\}}\equiv\prod_i L_{-k_i}$).
Linear combination: There are two types of linear combination operators:

$$\text{Type 1:}\quad \mathcal{O}_1(x_1)=\sum_q C_q\cdot\mathcal{O}_q(x_1),\quad \mathcal{O}_2(x_2)=\sum_q C'_q\cdot\mathcal{O}_q(x_2). \qquad (4)$$

$$\text{Type 2:}\quad \mathcal{O}_1(x_1)=\sum_i C_i\cdot L_{-\{k_i\}}\bar{L}_{-\{\bar{k}_i\}}\mathcal{O}(x_1),$$
$$\mathcal{O}_2(x_2)=\sum_i C'_i\cdot L_{-\{k'_i\}}\bar{L}_{-\{\bar{k}'_i\}}\mathcal{O}(x_2). \qquad (5)$$

We obtain the late-time limit of $\Delta S^{(n)}_{A=[0,\infty)}$ corresponding to each type. (Eq. (3.49) in 2206.11818 for Type 1; Eq. (49) and (51) in 2301.04891 for Type 2) Our analytical results is in agreement with numerical computations. See Fig. 2(c) for Type 1 and 2(d) for Type 2.

References

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Based on our (upcoming) papers: Song He, Jie Yang, YXZ, Zi-Xuan Zhao, arXiv: 2301.04891; Song He, YXZ, Long Zhao, Zi-Xuan Zhao, arXiv: 2309.xxxx Wu-Zhong Guo, Song He, YXZ, JHEP 09 (2022) 094, arXiv: 2206.11818;