Quantifying EPR: the resource theory of nonclassicality of common-cause assemblages

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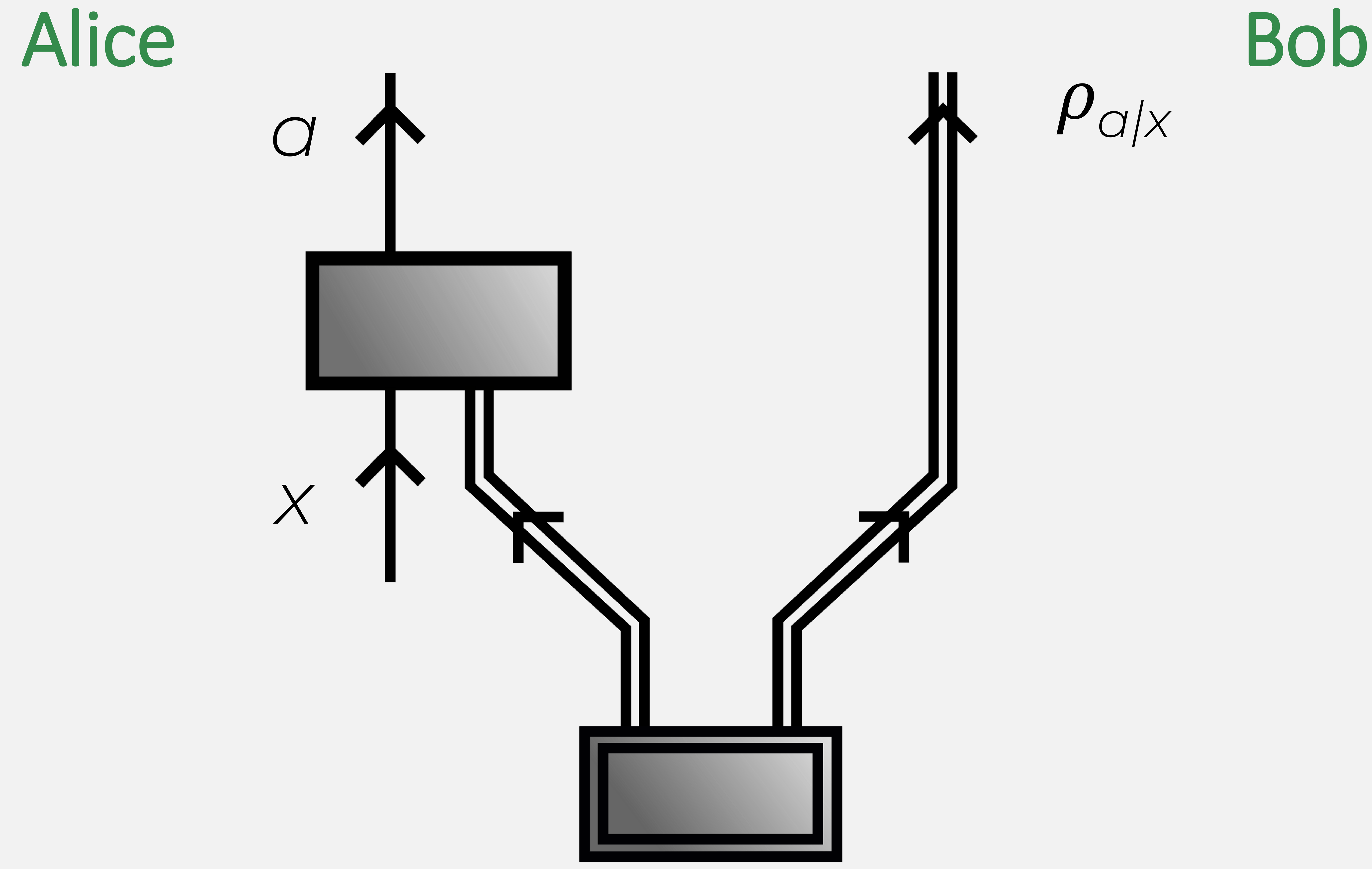
Standard EPR scenario

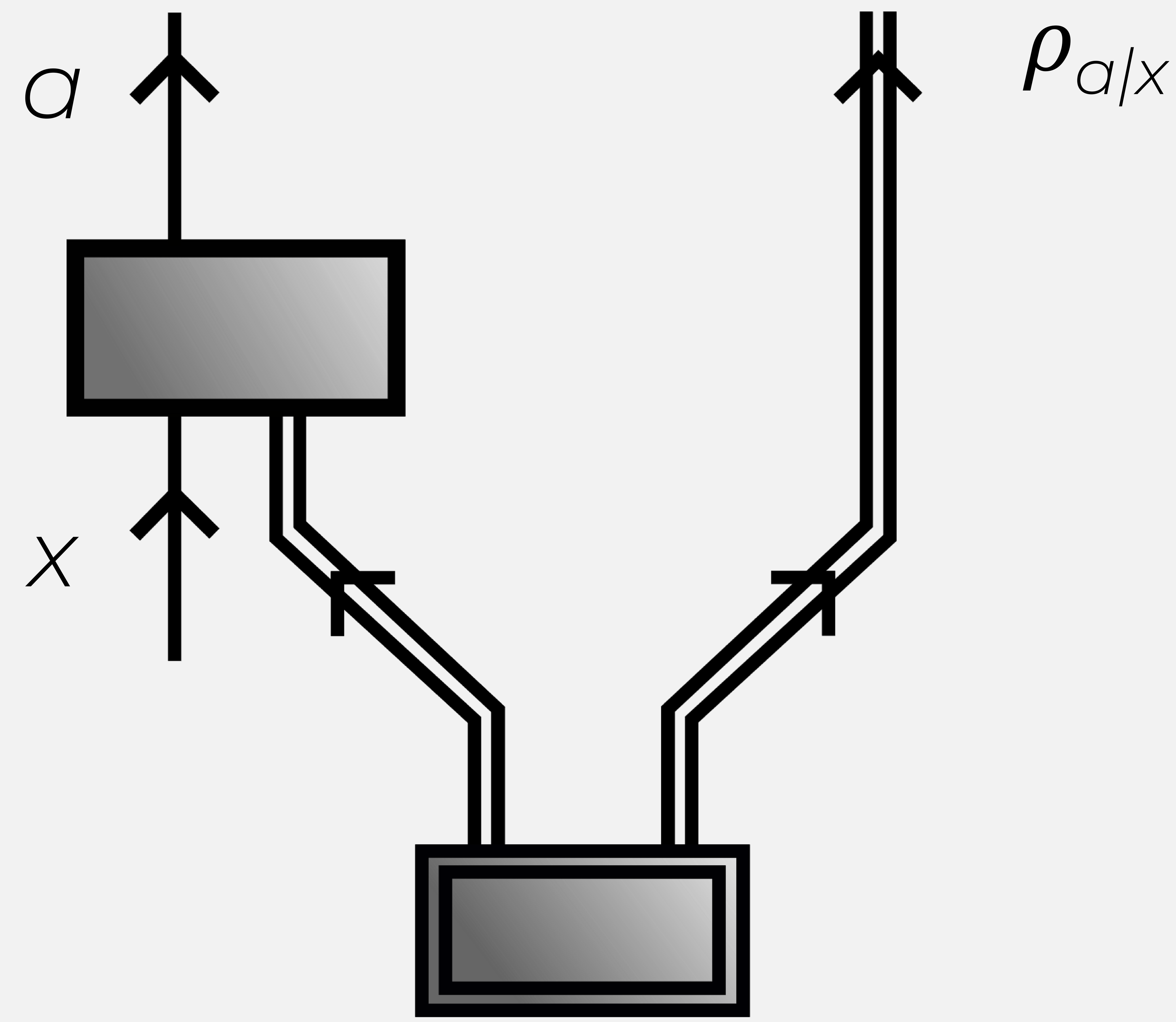
A form of **nonclassical correlations** that arise when one considers measurements performed on half of a bipartite system prepared on an entangled state.



Quantum assemblage

$\boldsymbol{\Sigma}_{A|X} = \{\boldsymbol{\sigma}_{a|x}\}_{\{a,x\}}$

$\boldsymbol{\sigma}_{a|x} = p(a|x)\, \boldsymbol{\rho}_{a|x}$



Multiple applications in quantum information protocols

The resource theory of assemblages

Resource theories allow one to formally quantify physical resources. The structure of a resource theory is completely determined by the set of **free operations**.

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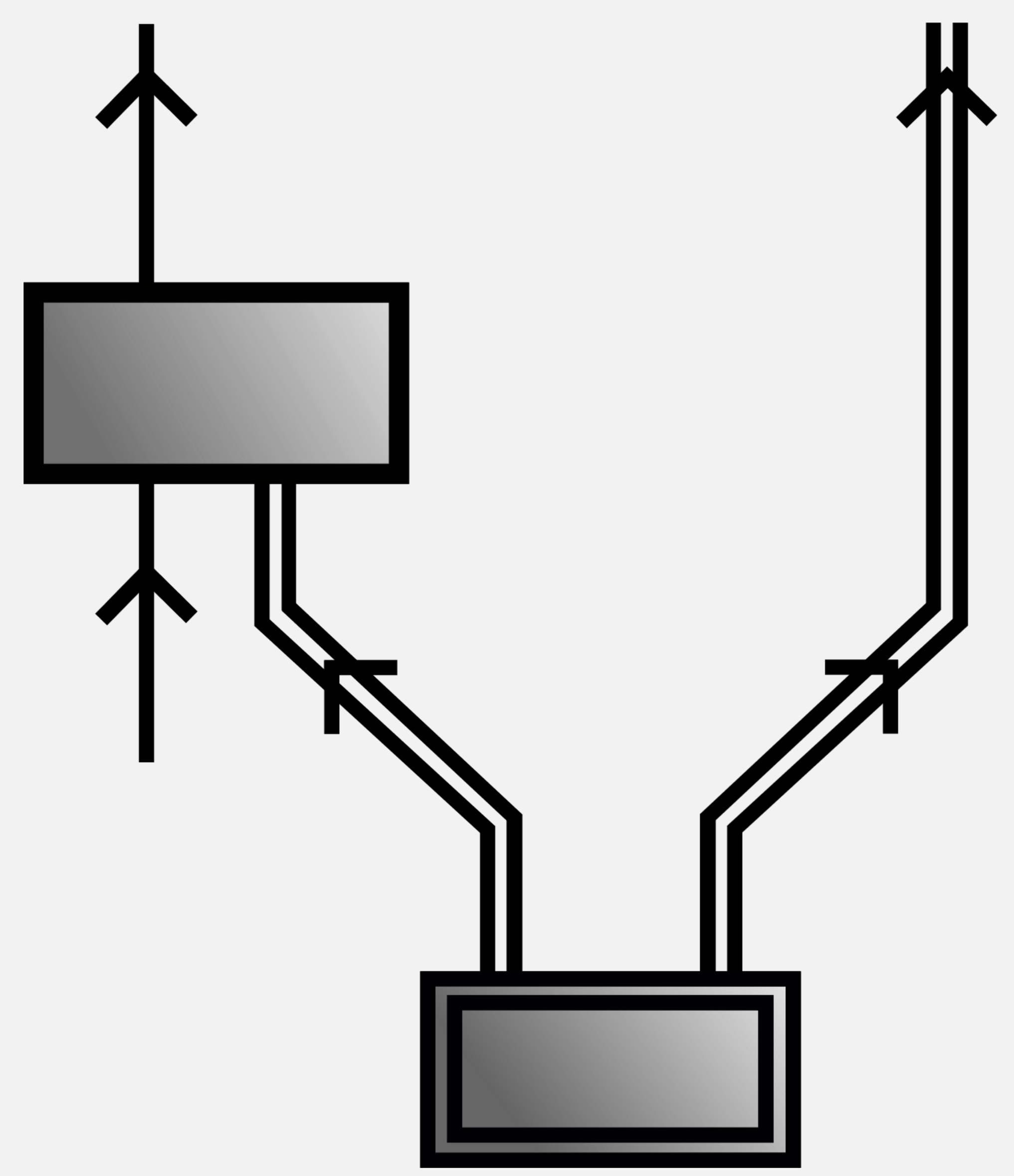
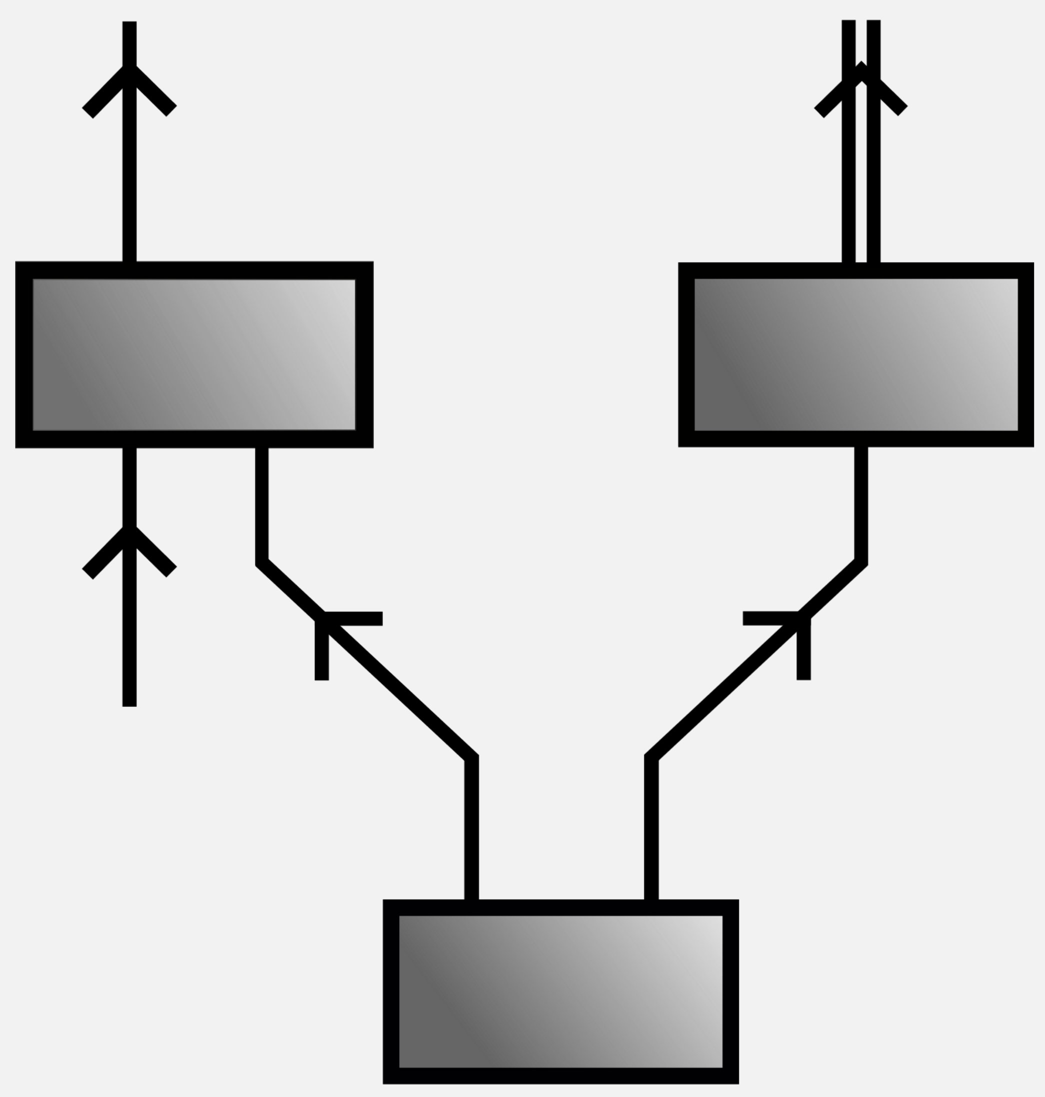
Free operations: **local operations and shared randomness**

General assemblage:



Free assemblage:



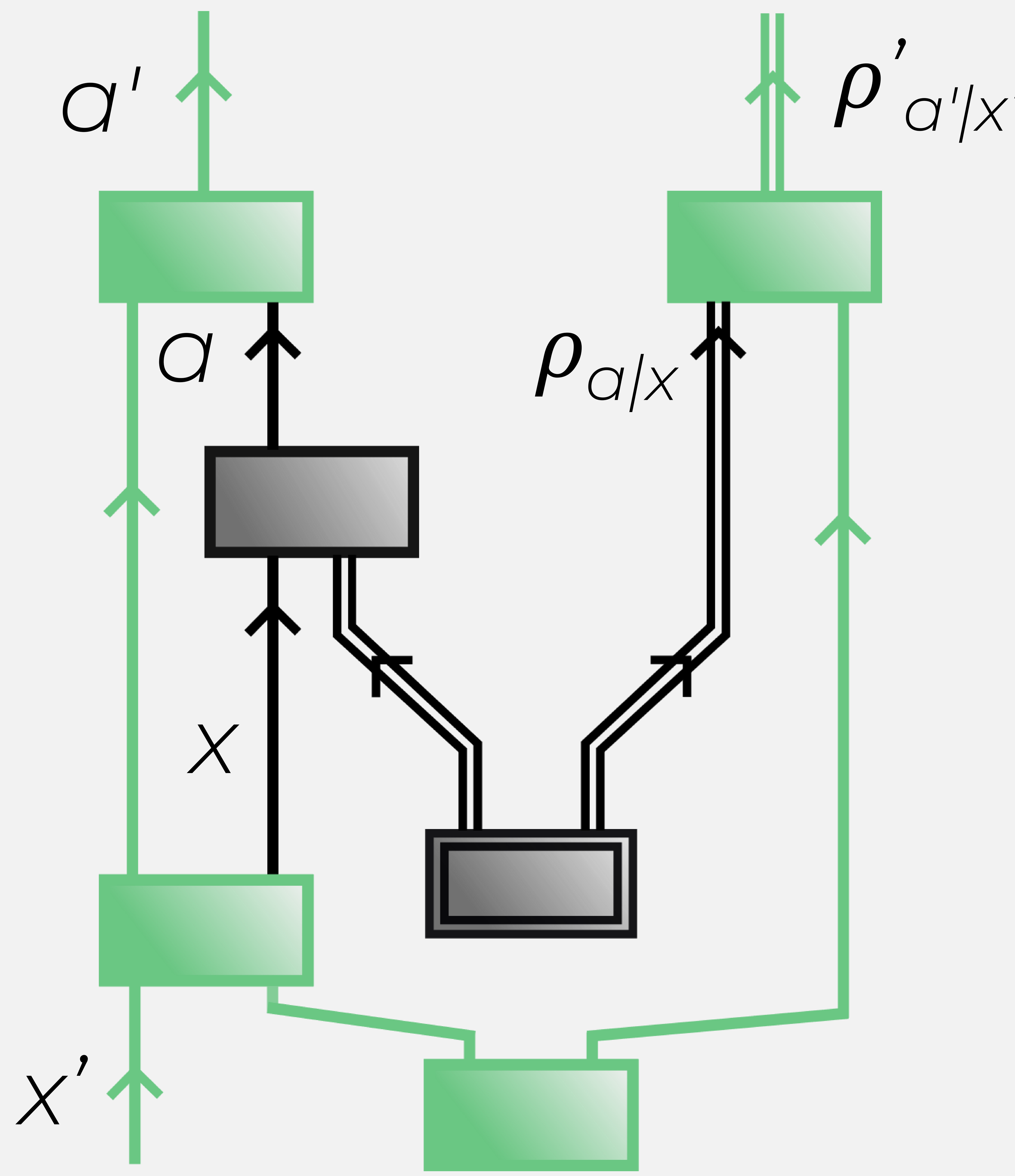
$$\boldsymbol{\sigma}_{a|x} = \Sigma_\lambda \, p(a\,|x\,\lambda)\, \boldsymbol{\sigma}_\lambda$$

(unsteerable assemblage)

LOSR operations

$\boldsymbol{\Sigma}_{A|X} = \{\boldsymbol{\sigma}_{a|x}\}_{\{a,x\}}$

$\boldsymbol{\sigma}_{a|x} = p(a|x)\,\boldsymbol{\rho}_{a|x}$



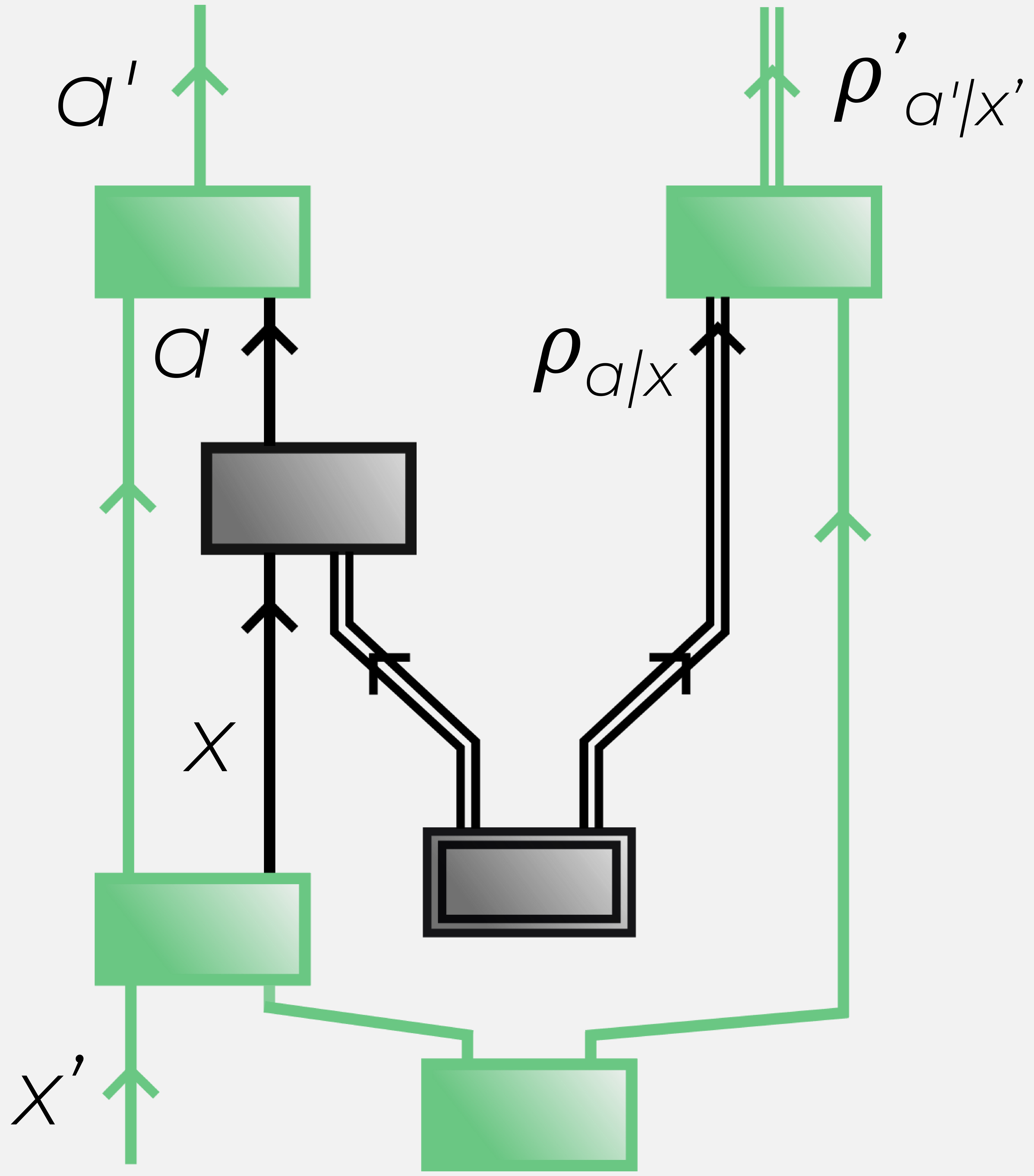
$\boldsymbol{\Sigma}_{A'|X'} = \{\boldsymbol{\sigma}_{a'|x'}\}_{\{a',x'\}}$

$\boldsymbol{\sigma}'_{a'|x'} = p(a'|x')\,\boldsymbol{\rho}'_{a'|x'}$

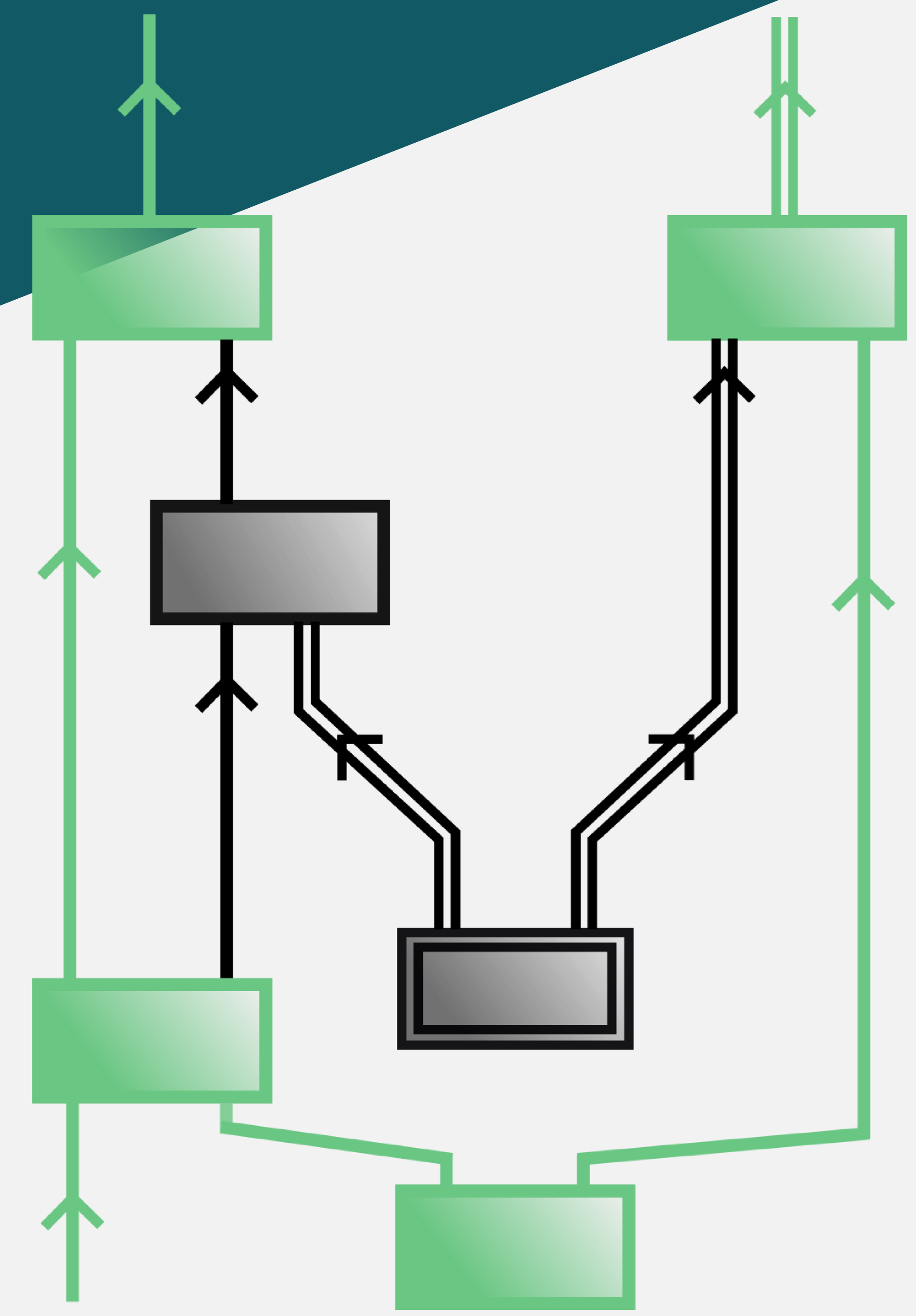
Properties of the pre-order

Assemblage conversion under LOSR

One assemblage is said to be ***more nonclassical*** than another if it can be freely converted to the latter.



Assemblage conversion under LOSR can be tested using a single instance of a semidefinite program



$$
\begin{aligned}
&\textit{given} \quad \{\sigma_{a|x}\}_{a,x}\,,\ \{\sigma_{a'|x'}\}_{a',x'}\,,\ \{D(a'|a,x',\lambda)\}_{\lambda,a',a,x'}\,,\ \{D(x|x',\lambda)\}_{\lambda,x,x'}\\
&\textit{find} \quad \{W_\lambda\}_\lambda\\
&\textit{s.t.} \quad \begin{cases}
W_\lambda \geq 0\,,\\
\mathrm{tr}_{B'}\{W_\lambda\} \propto \frac{1}{d}\,\mathbb{I}_B \quad \forall\lambda\,,\\
\sum_\lambda \mathrm{tr}_{B'}\{W_\lambda\} = \frac{1}{d}\,\mathbb{I}_B\,,\\
\sigma_{a'|x'} = \sum_\lambda \sum_{a,x} D(a'|a,x',\lambda)\, D(x|x',\lambda)\, d\,\mathrm{tr}_B\left\{W_\lambda\, \mathbb{I}_{B'} \otimes \sigma^T_{a|x}\right\}\,.
\end{cases}
\end{aligned}
$$

Structure of the pre-order

We find an **infinite family of incomparable resources** (none of them can be converted into any other).

$$\sigma^{\theta}_{a|x} = \mathrm{tr}_{\mathrm{A}} \left\{ \widetilde{M}_{a|x} \otimes \mathbb{I} \left|\theta\right\rangle \left\langle\theta\right| \right\} ,$$

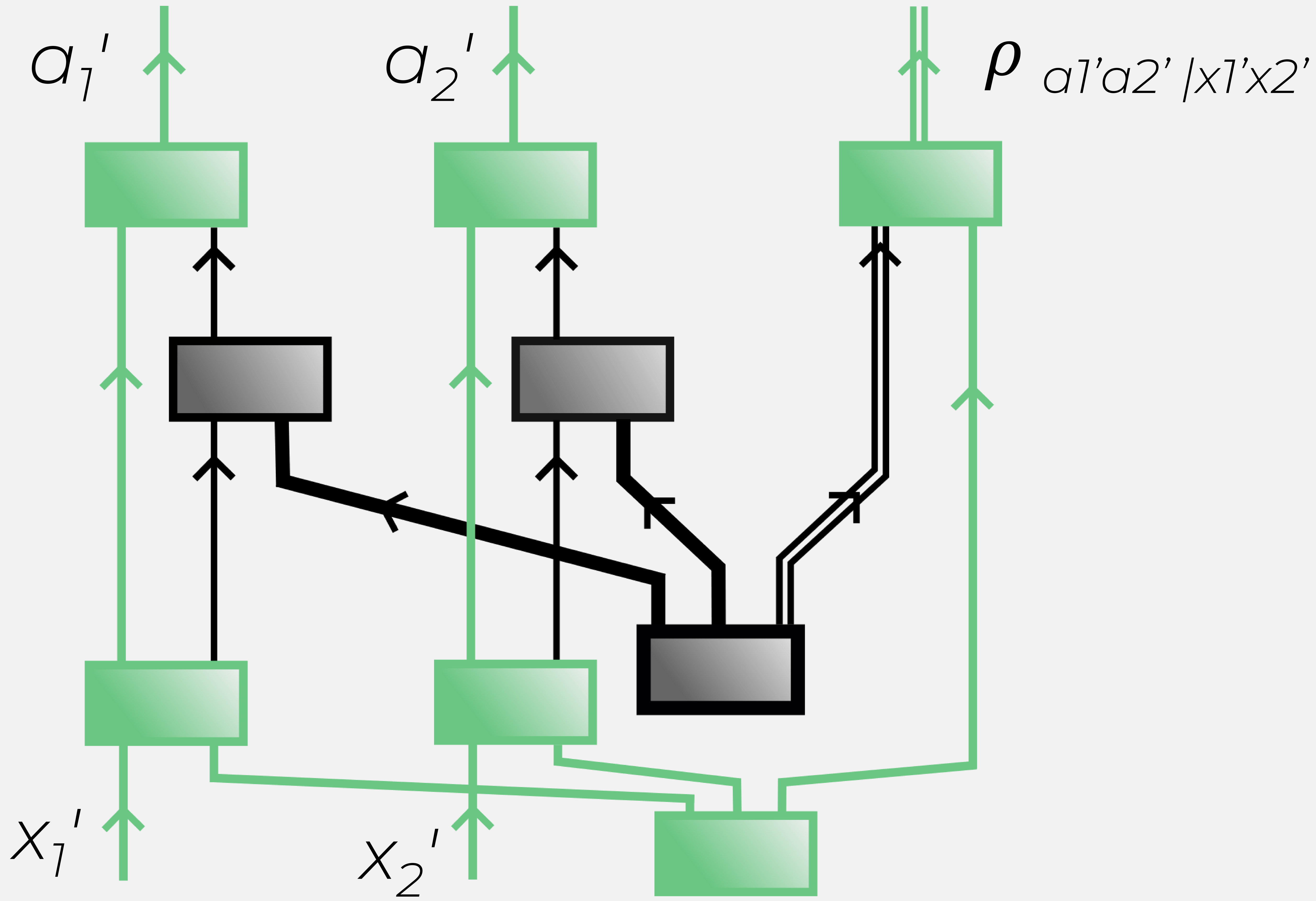
$$\left|\theta\right\rangle = \cos\theta \left|00\right\rangle + \sin\theta \left|11\right\rangle ,$$

$$\widetilde{M}_{a|0} = \frac{\mathbb{I} + (-1)^a \sigma_z}{2}, \quad \widetilde{M}_{a|1} = \frac{\mathbb{I} + (-1)^a \sigma_x}{2} .$$

We confirm this result with our SDP and analytically using **EPR monotones** that we develop.

Multipartite scenario

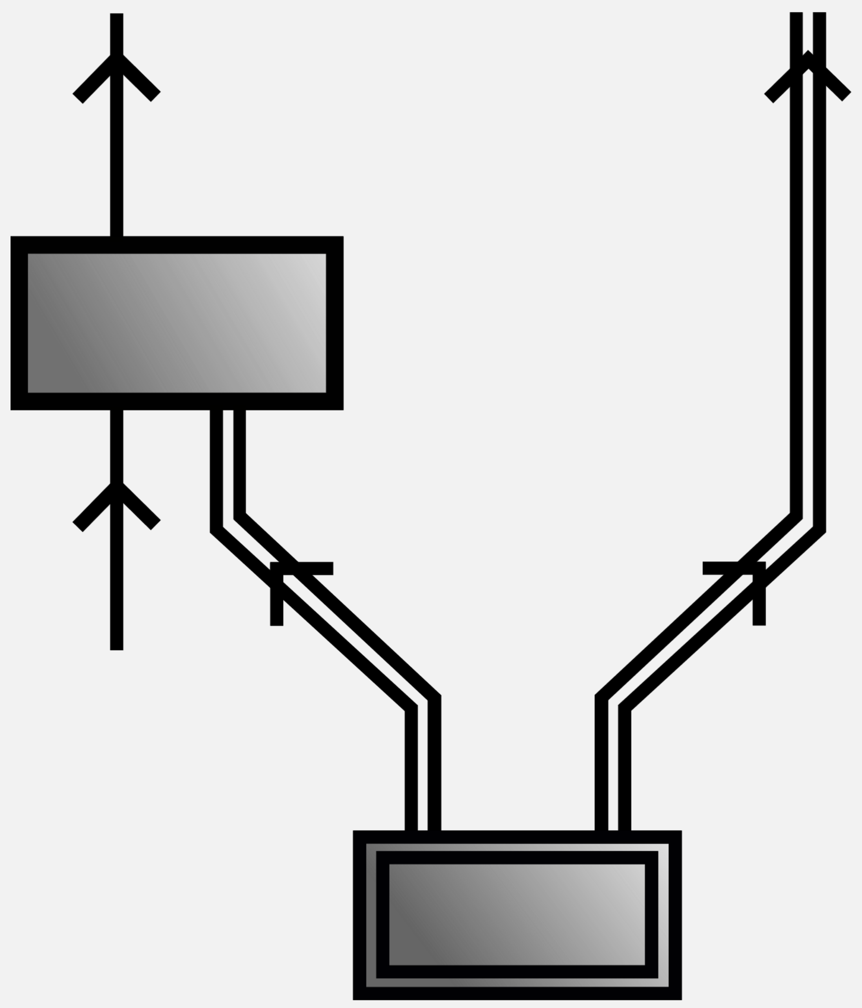
Multipartite EPR scenarios



- Assemblage conversion under LOSR can be tested using a single instance of a semidefinite program
- Family of incomparable resources

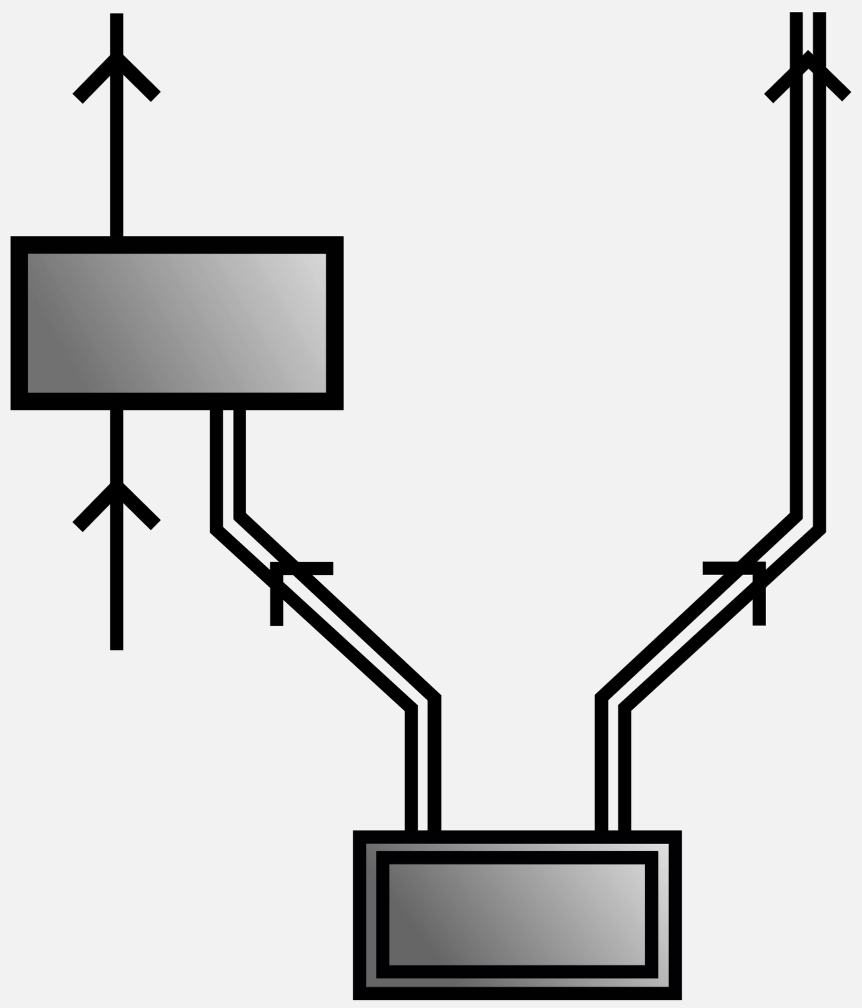
Bipartite generalizations

Bipartite generalizations

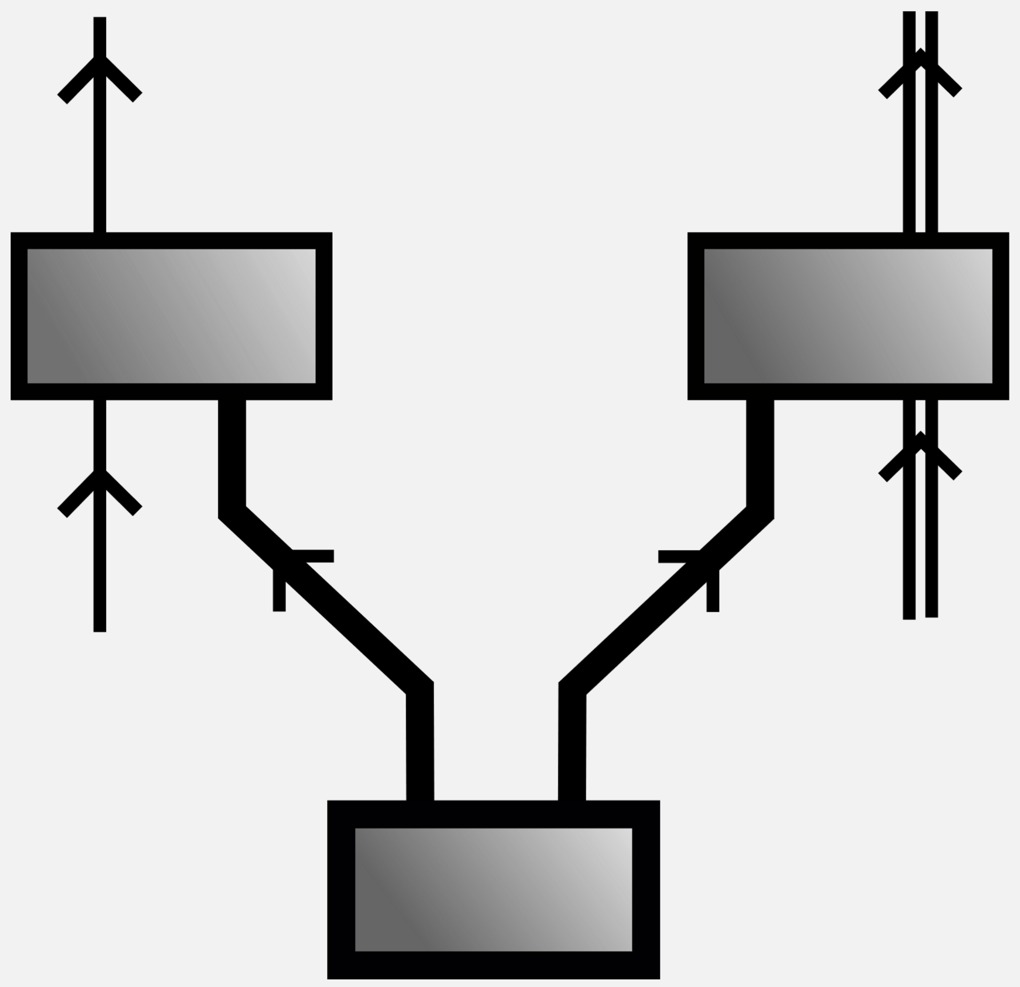


EPR

Bipartite generalizations

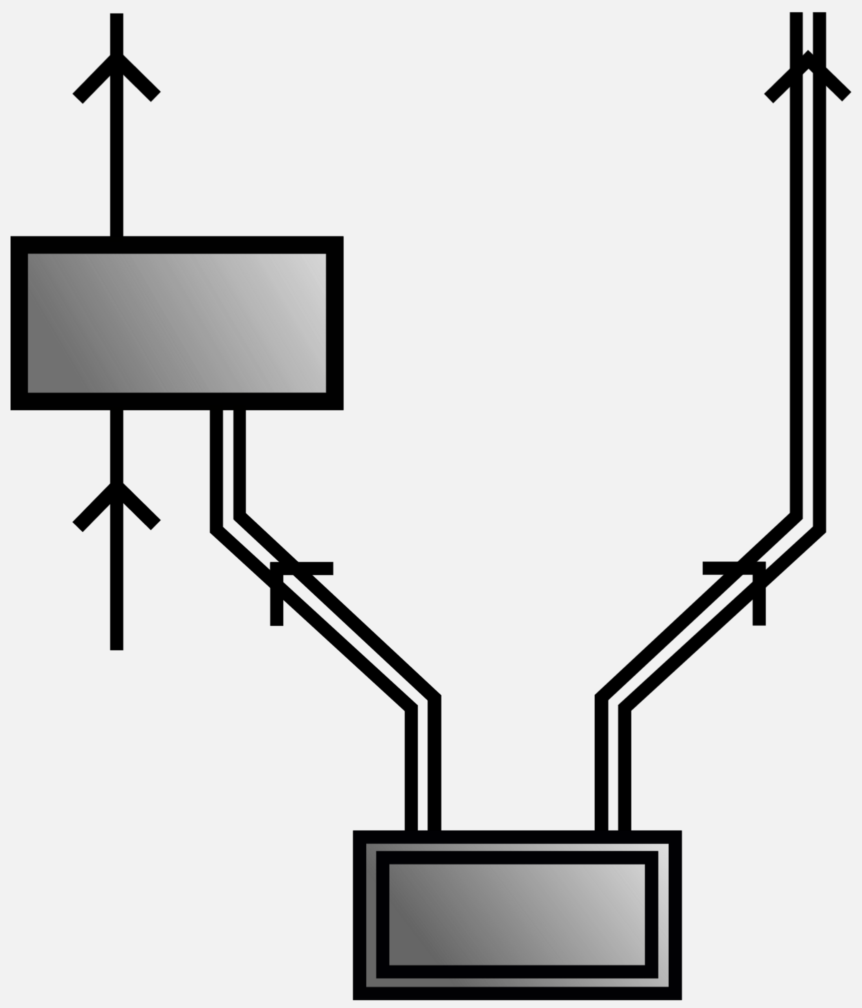


EPR

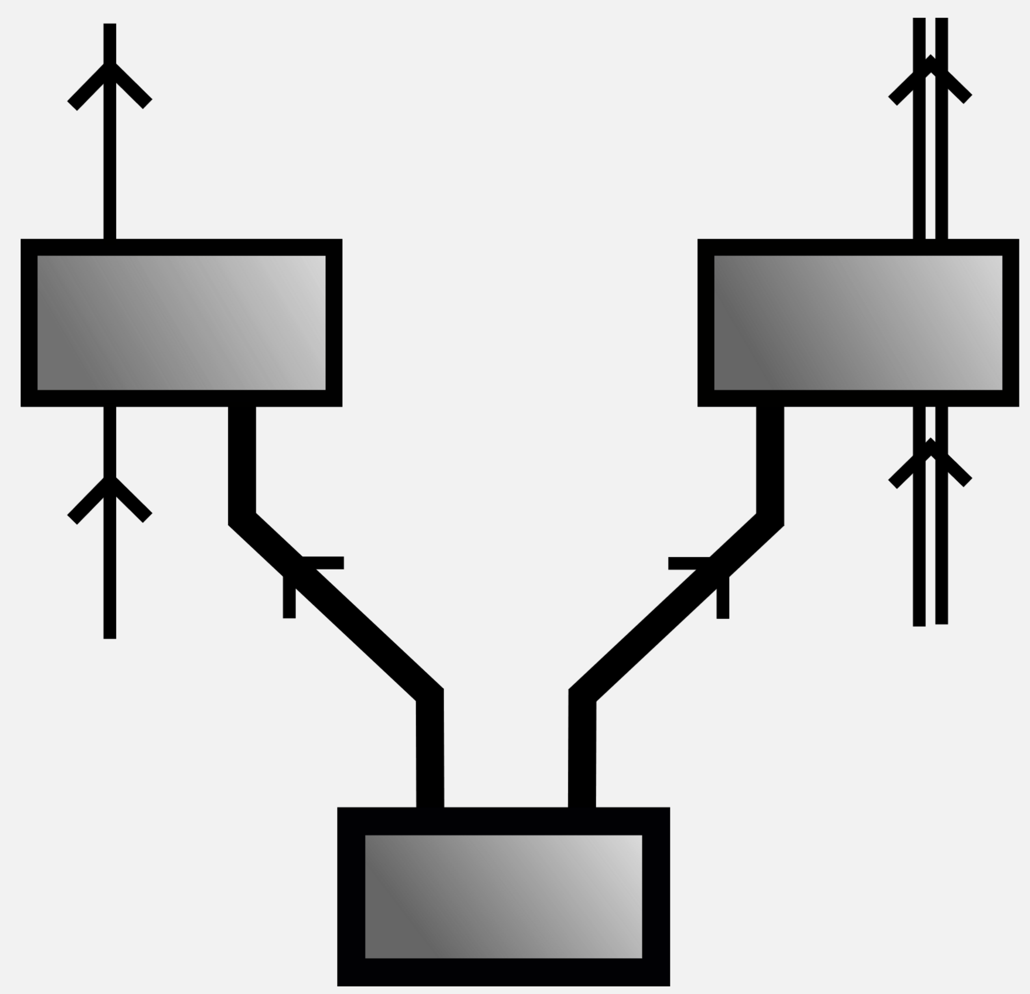


Channel

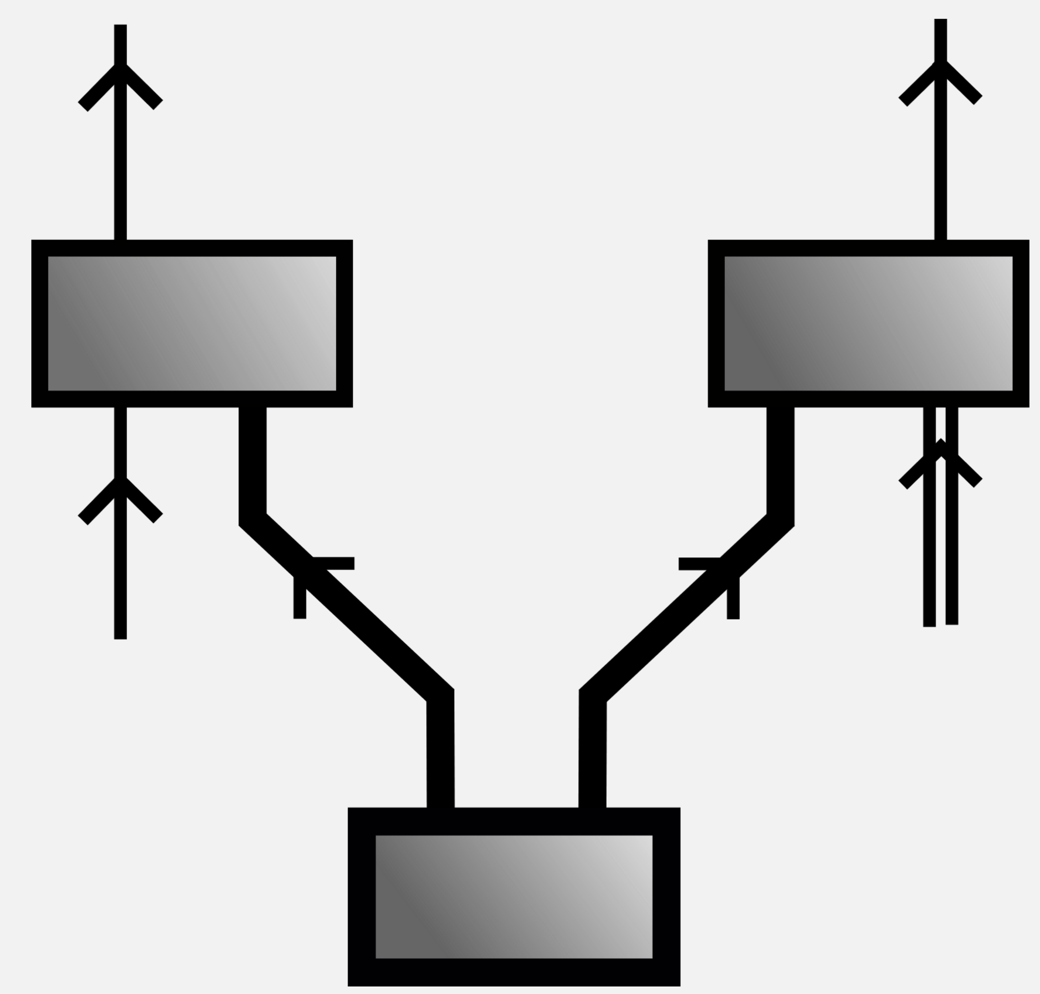
Bipartite generalizations



EPR

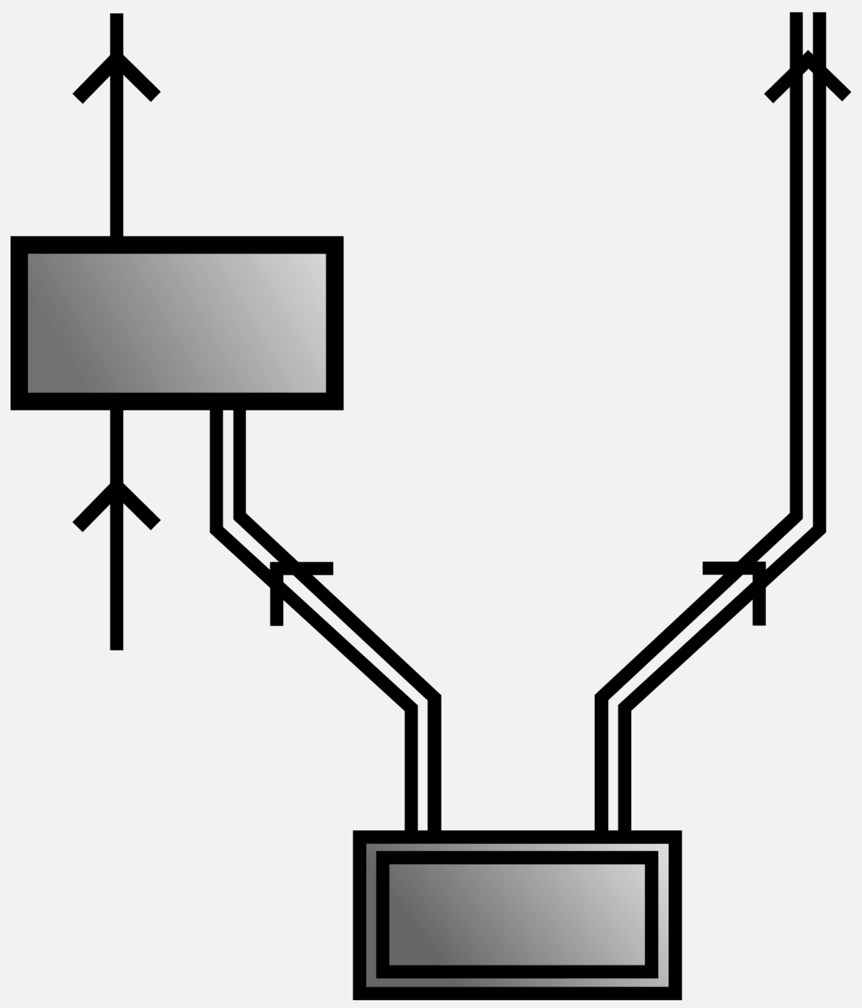


Channel

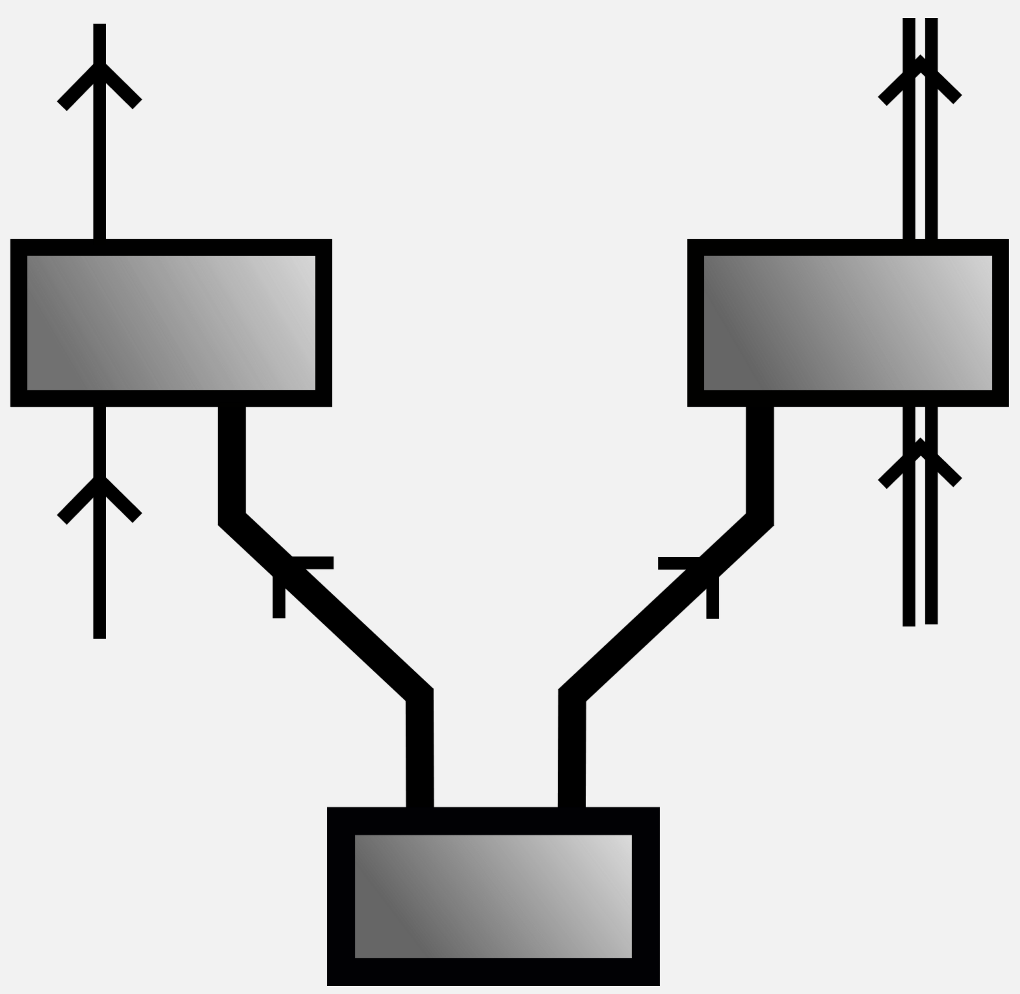


MDI

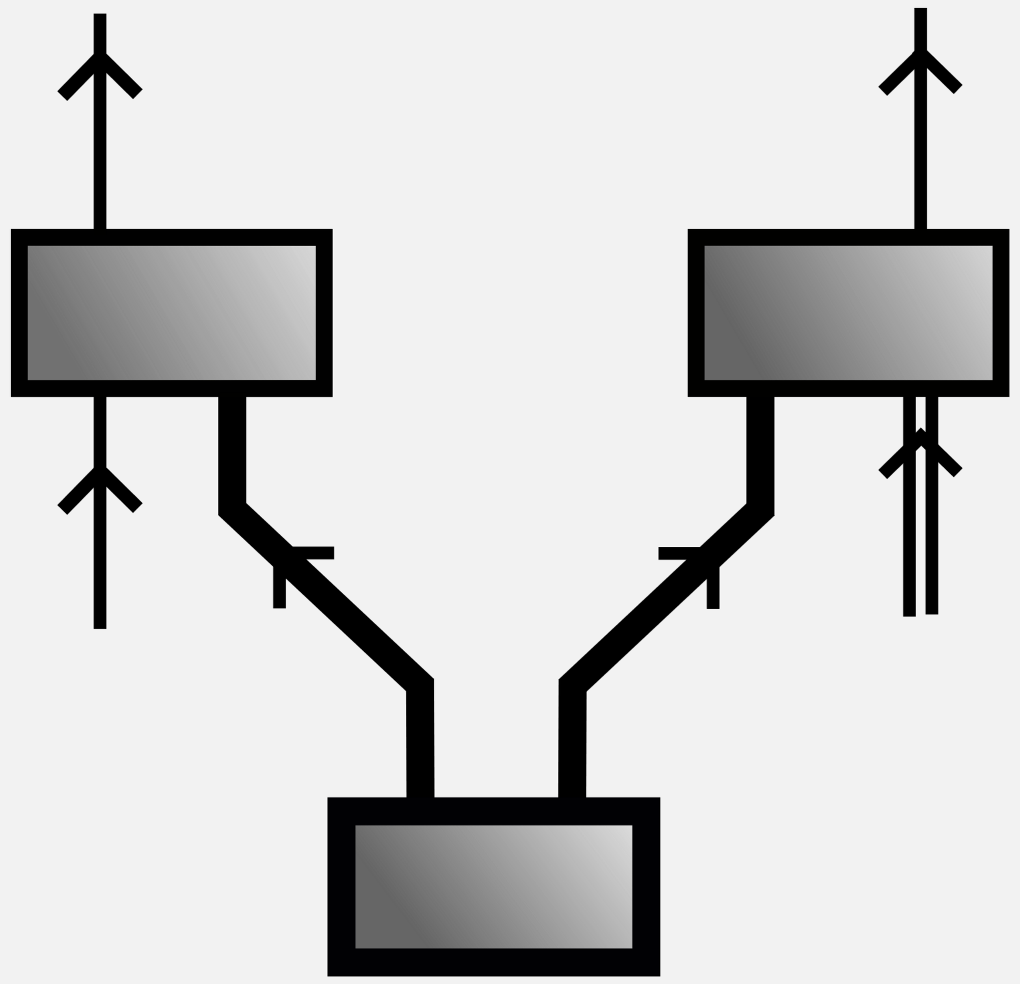
Bipartite generalizations



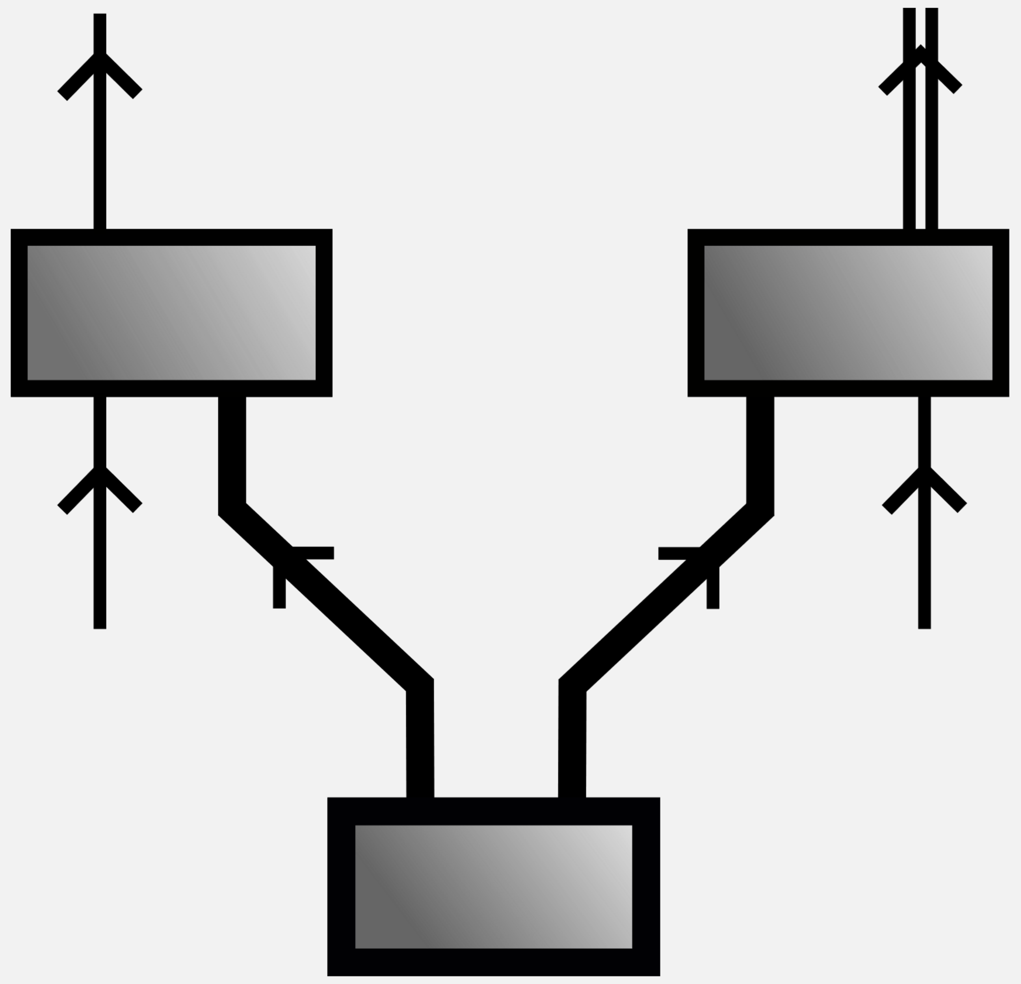
EPR



Channel

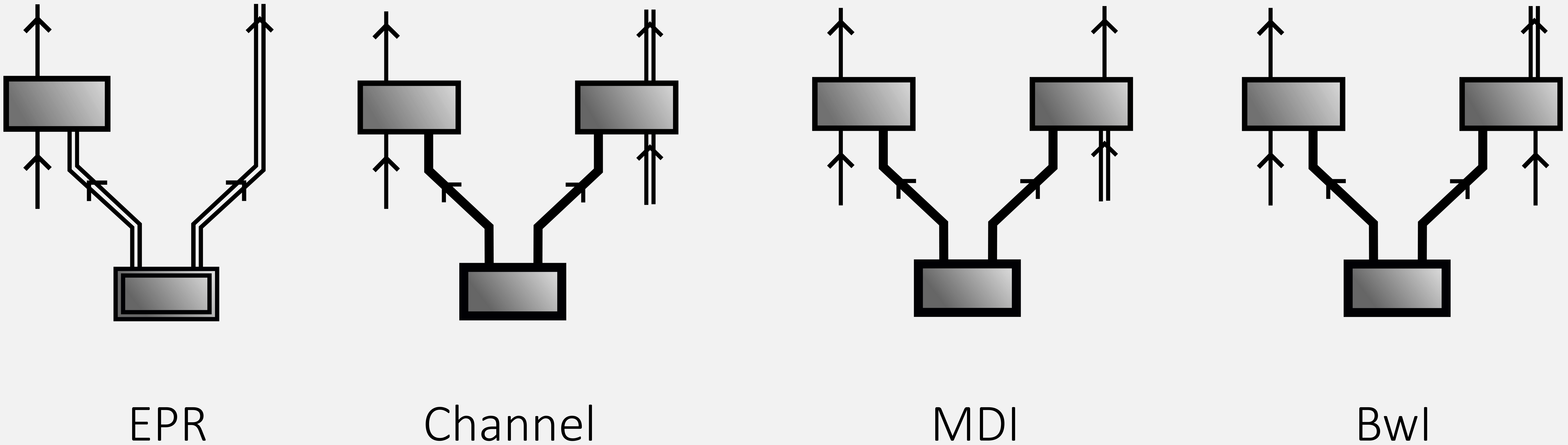


MDI



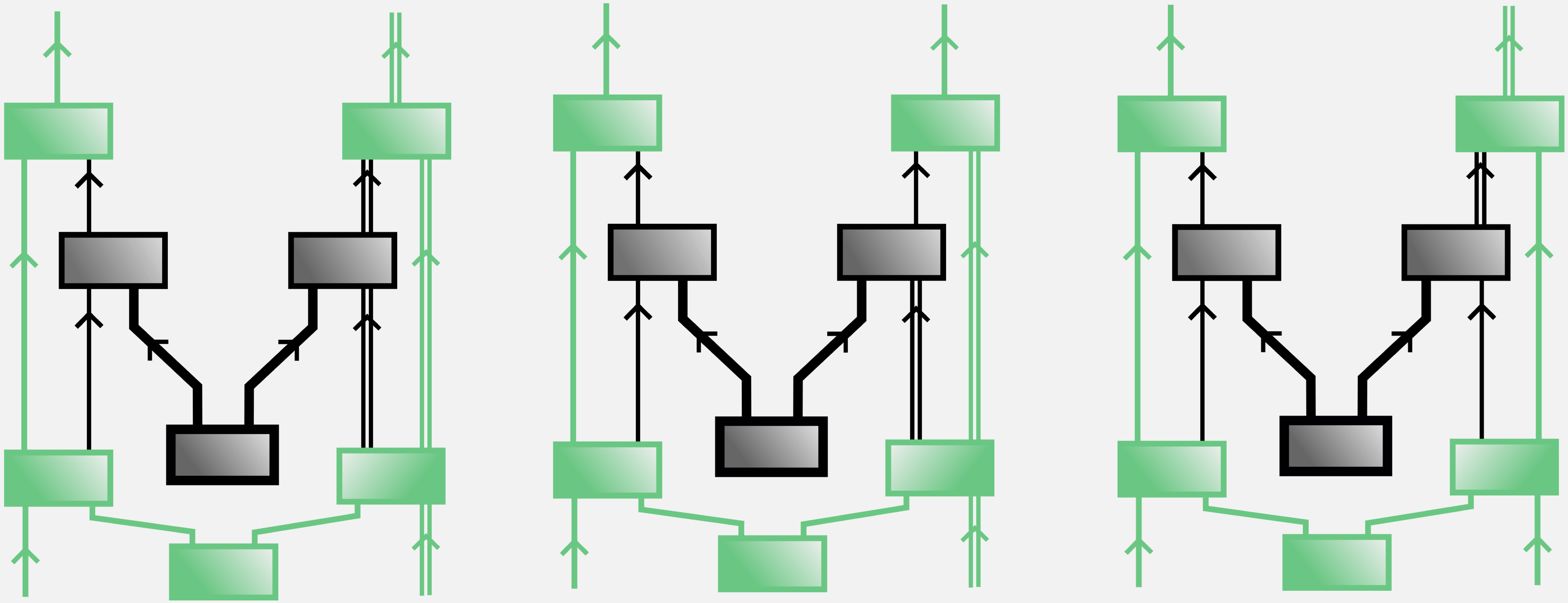
BwI

Bipartite generalizations



Free resources = classical common cause

LOSR transformations

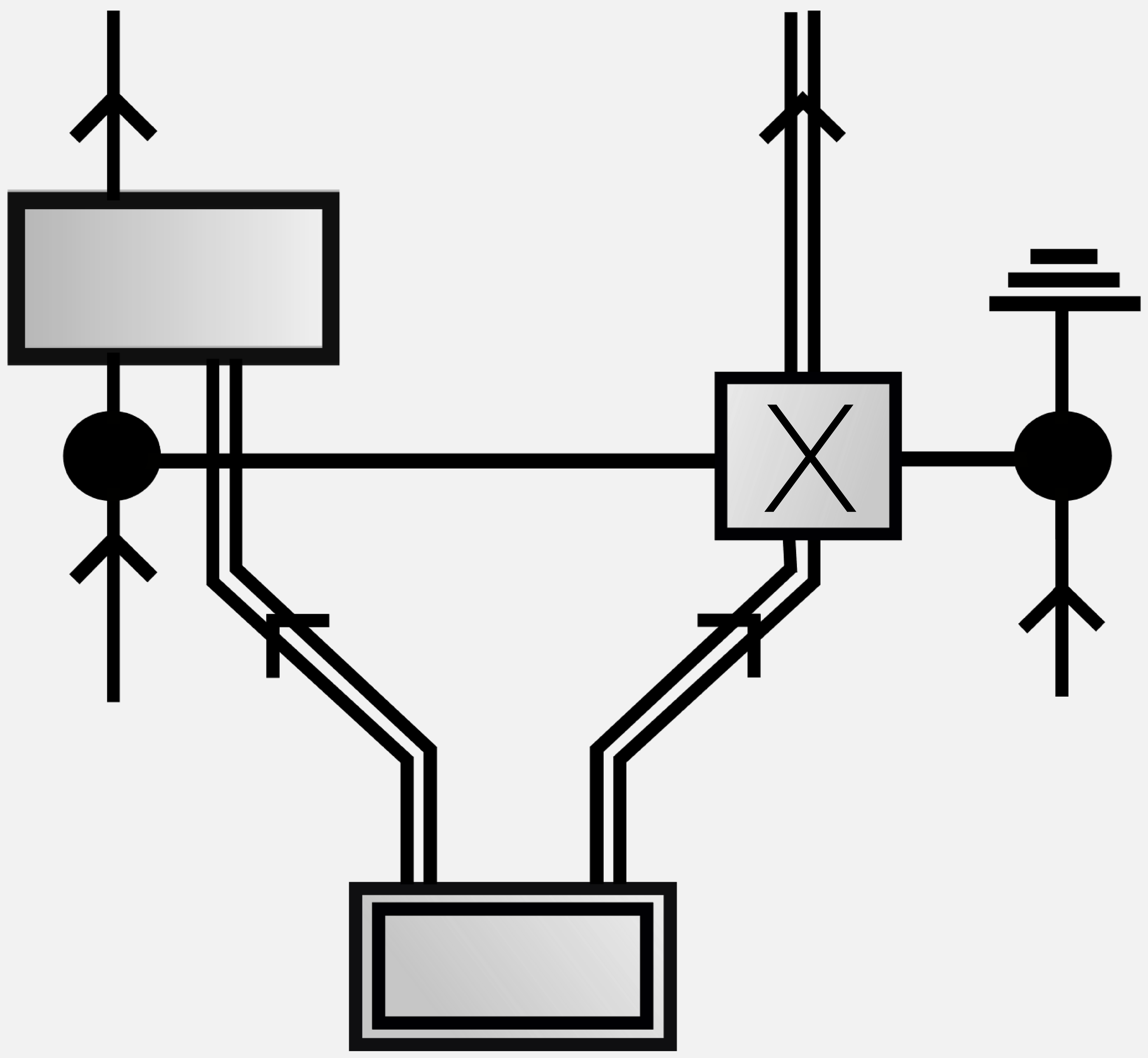


Channel MDI BwI

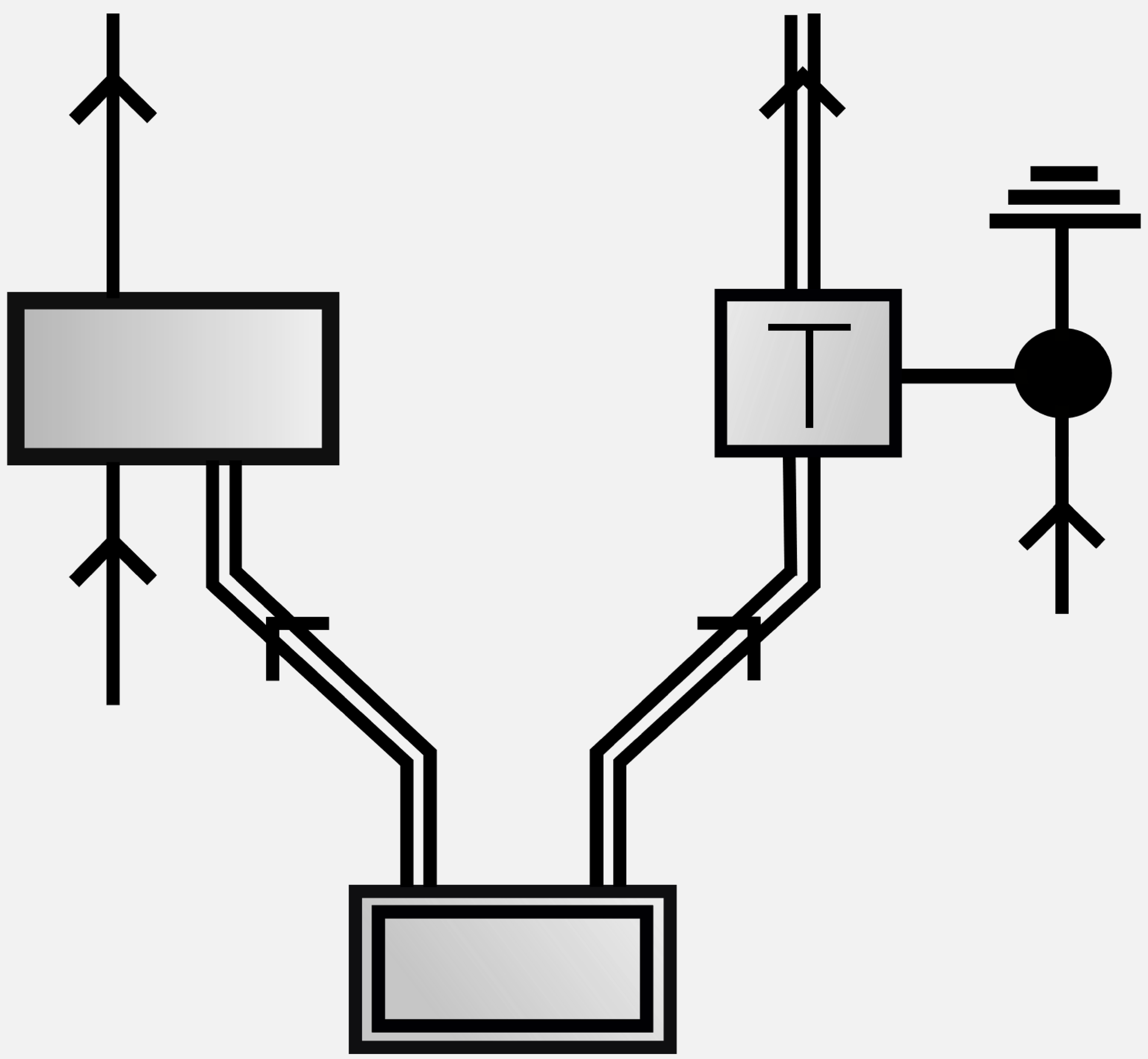
Assemblage conversion under LOSR can be tested using a single instance of a semidefinite program

Postquantum Bob-with-input assemblages

PR-box assemblage



PTP assemblage

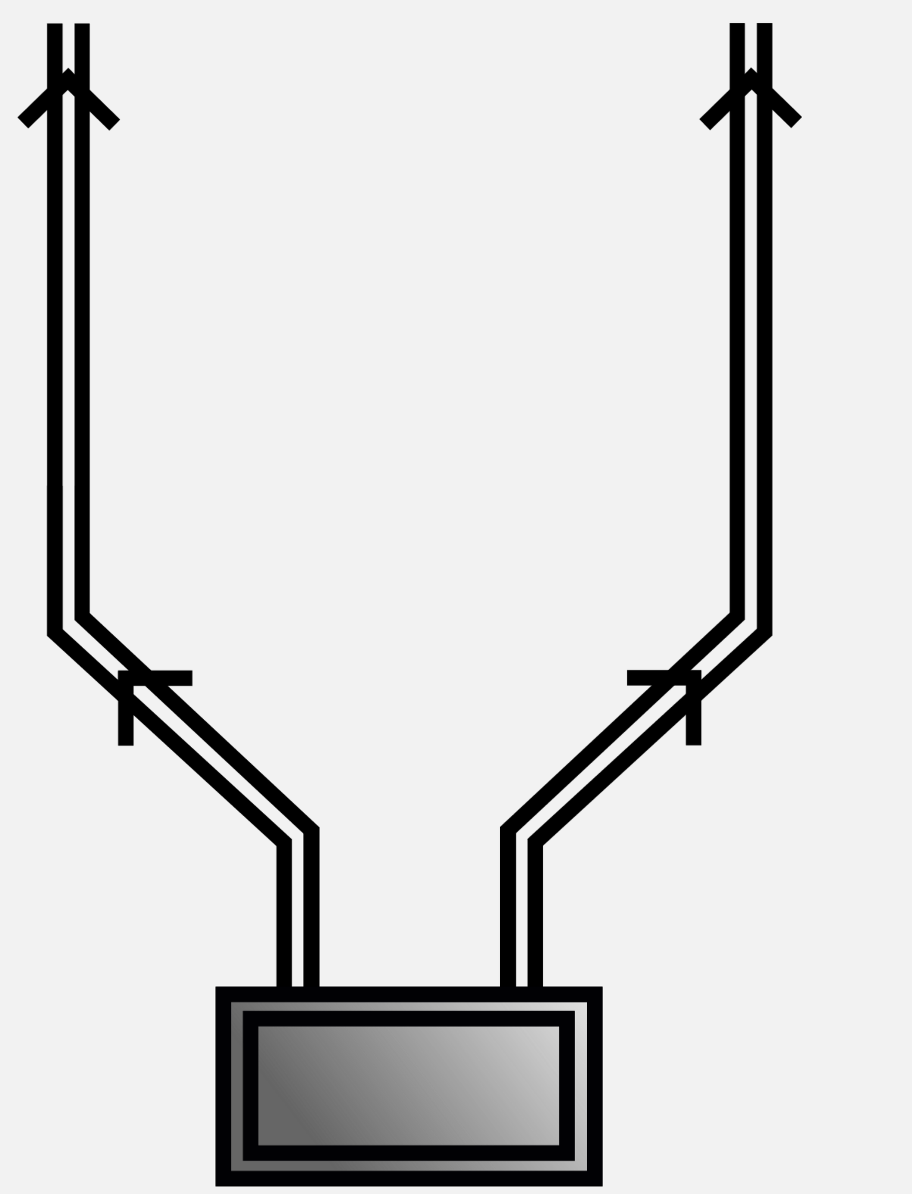


- Incomparable under LOSR
  - PR -> PTP under LOSE

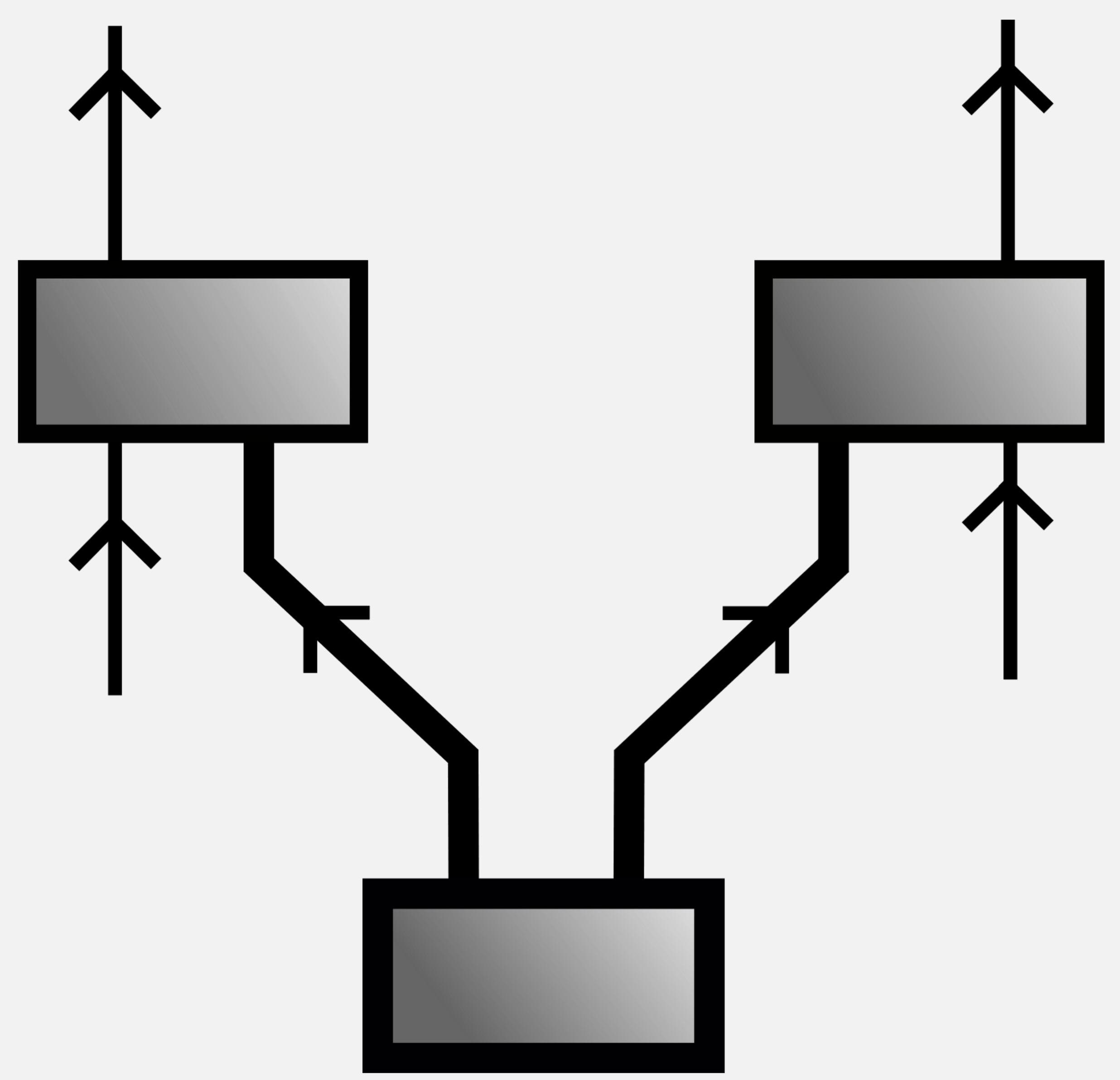
Measurement-device-independent scenario

A hierarchy of SDPs to test membership to the quantum set

Common-cause resources



Entanglement
arXiv:2004.09194



Bell scenarios
arXiv:1903.06311

Other common-cause processes:
arXiv:1909.04065

Final remarks

Resource theory of common-cause assemblages

(standard bipartite & multipartite, channel,
Bob-with-input, measurement-device-independent)

arXiv:2111.10244

Resource theory of common-cause assemblages

(standard bipartite & multipartite, channel, Bob-with-input, measurement-device-independent)

Unified notion of common-cause resources

(EPR, Bell,entanglement)

arXiv:2111.10244

Resource theory of common-cause assemblages

(standard bipartite & multipartite, channel,
Bob-with-input, measurement-device-independent)

Unified notion of common-cause resources

(EPR, Bell,entanglement)

Testing conversions is numerically tractable

(interesting properties of the pre-order)

Thank you!

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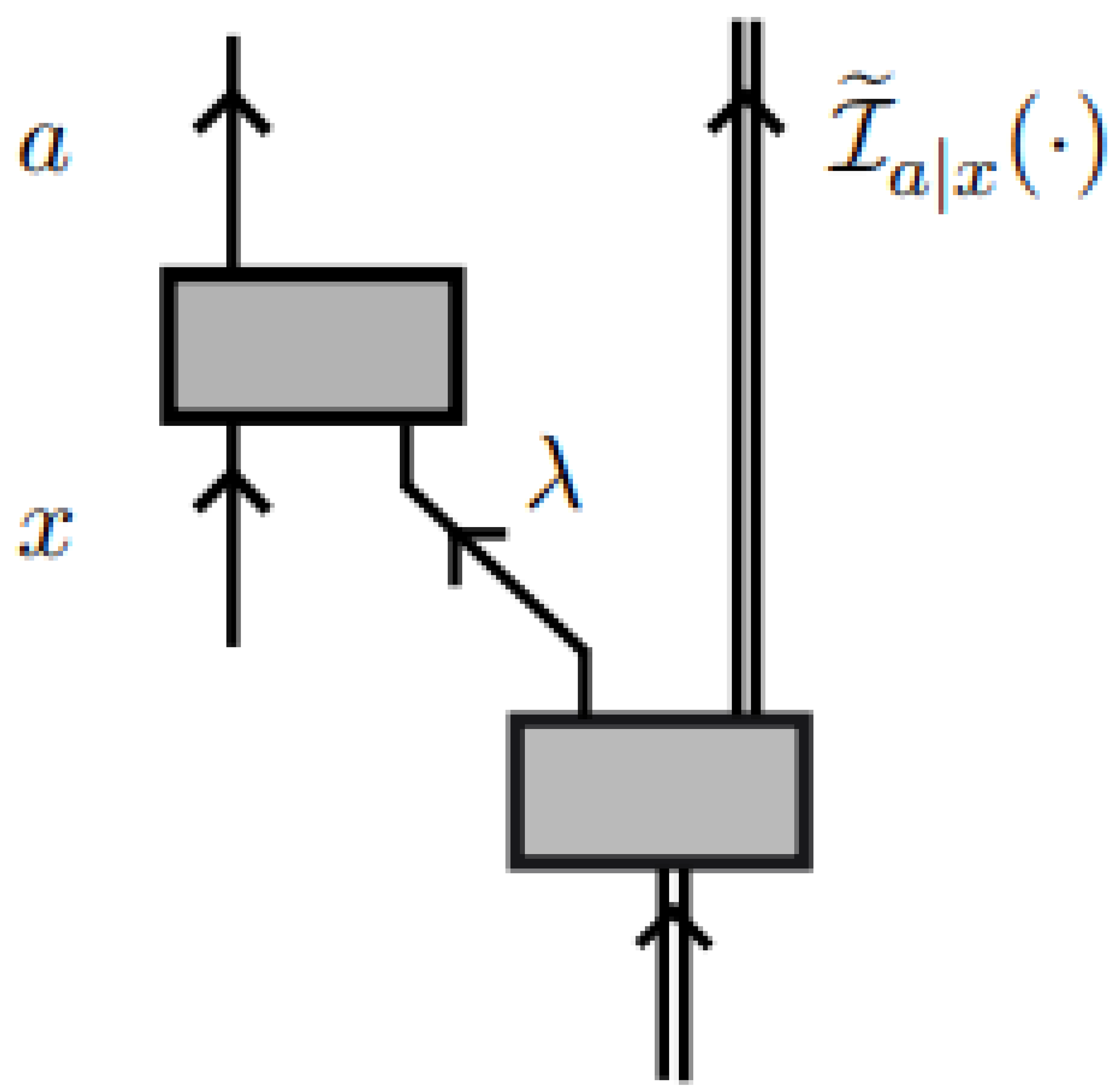
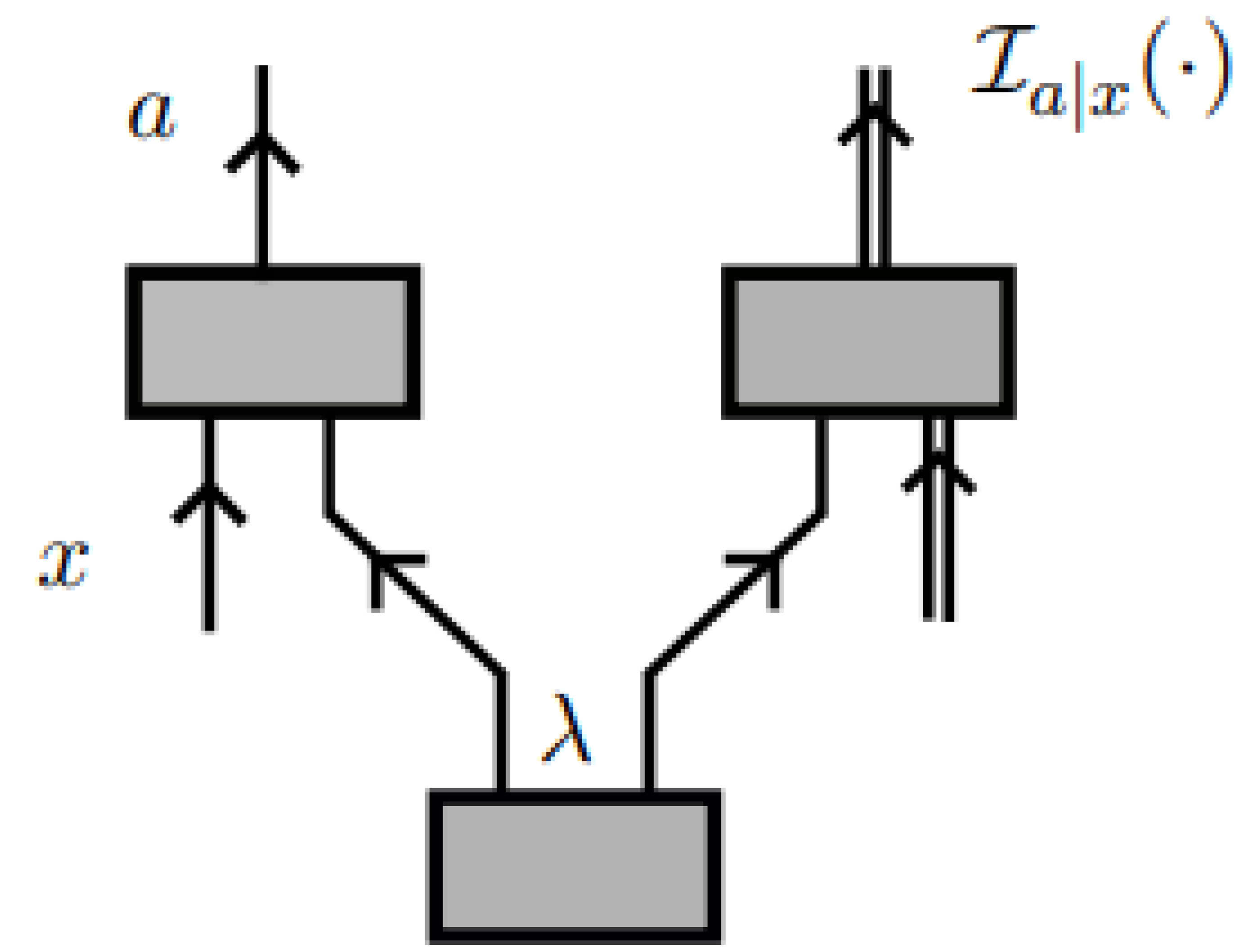
arXiv: 2111.10244, 2209.10177

One-way signalling



LOCC-free channel assemblage
that allows signalling from Bob to Alice

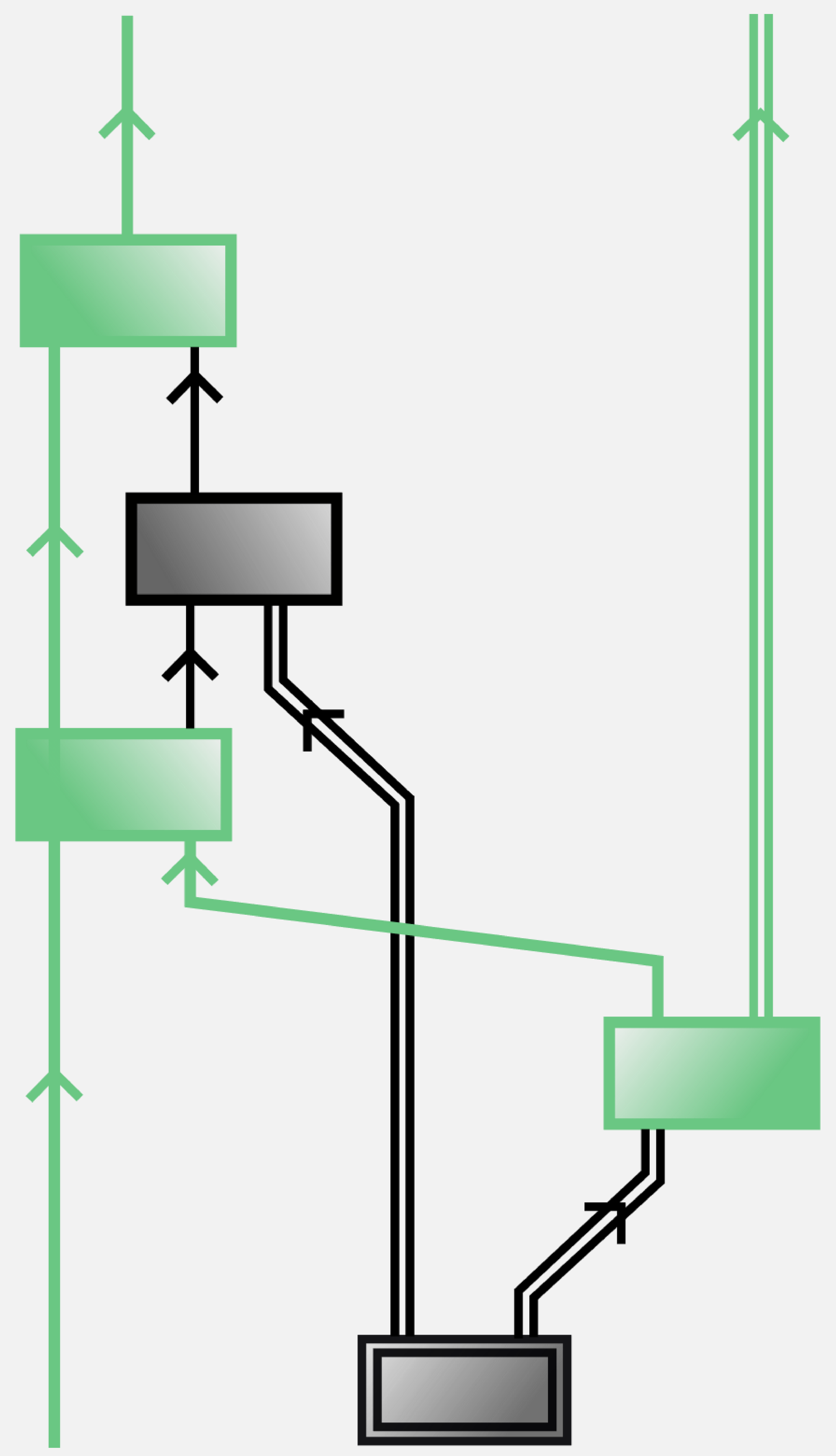


LOSR-free channel assemblage

Comparison to prior work

Rodrigo Gallego and Leandro Aolita. ***Resource theory of steering***. Physical Review X, 5 (4):041008, 2015
https://doi.org/10.1103/PhysRevX.5.041008

Free operations: Stochastic local operations and one-way classical communication from Bob to Alice (**S-1W-LOCC**)



Differences:

- Different pre-orders

Conceptual advantages:

- Clear physical motivation
- Unification of every type of nonclassical correlation in Bell-like scenarios

Technical advantages:

- simpler to characterize and study
- direct generalizations: multipartite EPR scenarios, Bob-with-input EPR scenarios and channel EPR scenarios

Structure of the pre-order

Family of assemblages indexed by two parameters:

$$\mathsf{S} = \left\{ \mathbf{\Sigma}^{\theta,p}_{\mathbb{A}|\mathbb{X}} \,\middle|\, \theta \in (0, \pi/4] \,, p \in [0,1] \right\} ,$$

$$\text{where} \quad \mathbf{\Sigma}^{\theta,p}_{\mathbb{A}|\mathbb{X}} = \left\{ p\, \sigma^{\theta}_{a|x} + (1-p)\, \frac{\mathbb{I}}{4} \right\}_{a\in\mathbb{A},\, x\in\mathbb{X}} ,$$

$$\text{with} \quad \sigma^{\theta}_{a|x} = \mathrm{tr}_{\mathrm{A}} \left\{ \widetilde{M}_{a|x} \otimes \mathbb{I}\, |\theta\rangle \langle\theta| \right\} ,$$

$$|\theta\rangle = \cos\theta\, |00\rangle + \sin\theta\, |11\rangle \,,$$

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