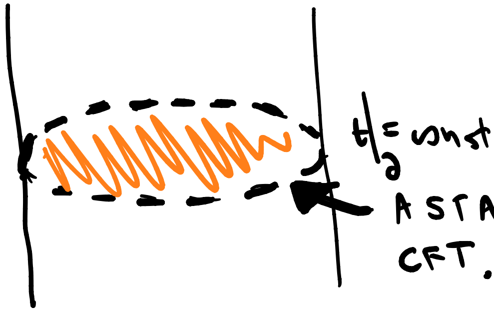
MICHAL.P.HELLER@UGENT.BE

HOLOGRAPHIC COMPLEXITY



A STATE IN A HOLOGRAPHIC CFT. FOR EMPTY AdS IT IS $|0\rangle$

STANDARD OBSERVABLES:

- $\langle O O \rangle|_{L \gg \Delta \gg 1} \sim e^{-\# L}$ (1-DIM BULK QTY) **1999+**
- RT/HRT HYPERSURFACES (CODIM.-2 BULK QTY) **2006+**

THESE LECTURES:

- HOLOGRAPHIC COMPLEXITY: AN ∞ FAMILY OF BULK OBSERVABLES DEFINED BY A BOUNDARY TIME SLICE. THEY INCLUDE VOLUMES OF MAXIMAL VOLUME TIMESLICES, DEPICTED ABOVE AS , AS WELL AS ∞-MANY OTHER BULK QTIES OF CODIM.-1 AND -0.

2014+

THESE LECTURES: A SUMMARY OF A CLASS OF APPROACHES TOWARDS UNDERSTANDING HOLOGRAPHIC COMPLEXITY FROM QUANTUM FIELD THEORY (2017-)

RESOURCES:

ORIGINAL ARTICLES (LISTED AS WE PROCEED)

2XXX.XXXXX REVIEW WITH CAPUTA, CHAPMAN, SWINGLE AND YOSHIDA

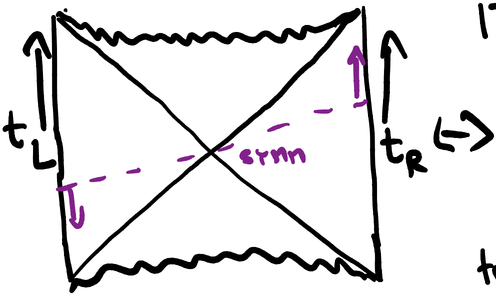
2110.14672 PEDAGOGICAL REVIEW BY BY CHAPMAN AND POLICASTRO

1810.11563 LECTURE NOTES BY SUSSKIND

WHY INTERESTING:

- NATURAL PROBES OF THE BLACK HOLE INTERIOR

ETERNAL AdS BLACK HOLE $\longleftrightarrow$ THERMOFIELD DOUBLE STATE



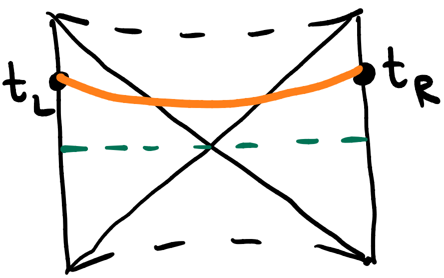
$\longleftrightarrow$

$$|TFD\rangle \sim e^{-iH_L t_L}\, e^{-iH_R t_R} \overbrace{\sum_E e^{-\frac{\beta E}{2}} |E\rangle_L |E\rangle_R}^{|TFD_0\rangle}$$

$$= \sum_E e^{-iE(t_L+t_R)} e^{-\frac{\beta E}{2}} |E\rangle_L |E_R\rangle$$

$(t_L = -t_R$ is $\boxed{\text{THE SYMMETRY}})$

$$\mathrm{tr}_{L/R} |TFD\rangle\langle TFD| \sim e^{-\beta H_{R/L}}$$



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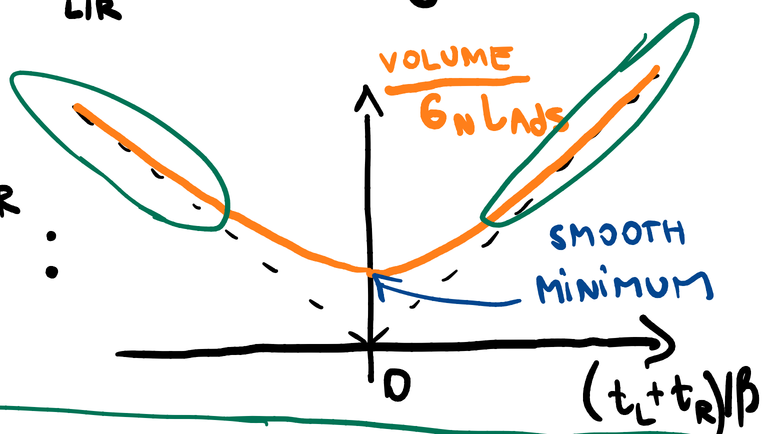
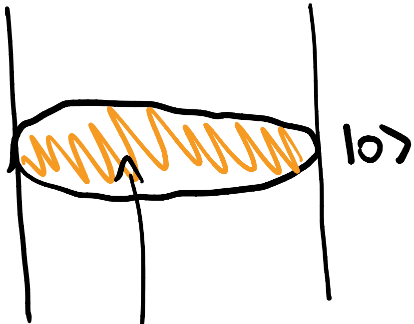


PERSISTENT LINEAR GROWTH AT A CLASSICAL LEVEL AFTER A SHORT TRANSIENT BEHAVIOUR

- CONNECTION WITH TENSOR NETWORKS

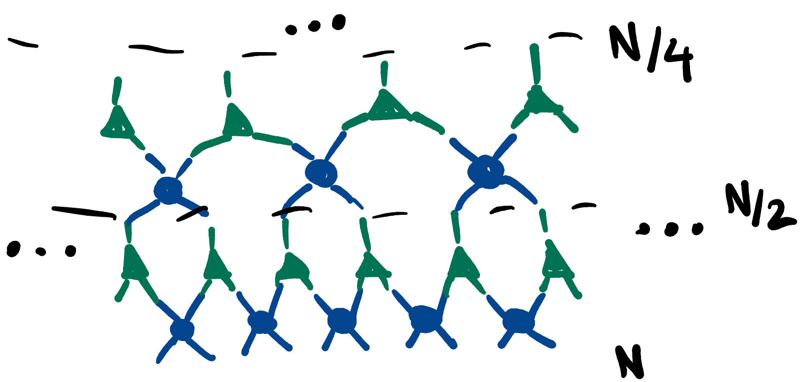


$$ds^2 = \boxed{\frac{dz^2 + dx^2}{z^2}} - \frac{dt^2}{z^2}$$

$H_2 =$

CONJECTURE: THE H_2 SLICE OF  IS A HOLOGRAPHIC DUAL TO A MULTISCALE ENTANGLEMENT RENORMALIZATION ANSATZ (MERA) 0905.1317 BY SWINGLE

MERA: QUANT-PH/0610099 BY VIDAL



EFFICIENT APPROX. OF $|0\rangle$ OF A CRITICAL SYSTEM

IF CORRECT, THEN THE VOLUME OF  CORRESPONDS TO THE TOTAL NUMBER OF TENSORS IN MERA

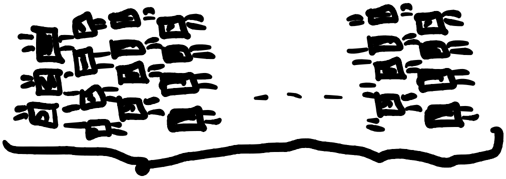
- CONNECTION WITH COMPLEXITY

HINT 1: THERE IS A SENSE IN WHICH MERA IS AN OPTIMAL TENSOR NETWORK REPRESENTING GROUND STATES OF CRITICAL SYSTEMS

1502.05385 EUCLIDEAN PATH INTEGRAL $\longrightarrow$ MERA
BY EVENBLY AND VIDAL

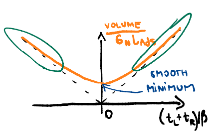
HINT 2: THE CIRCUIT MODEL OF UNITARY TIME EVOLUTION 1406.2678 BY STANFORD AND SUSSKIND

ASPECT I: $e^{-iH_L t_L}$:



TENSORS GROWS $\sim t_L$

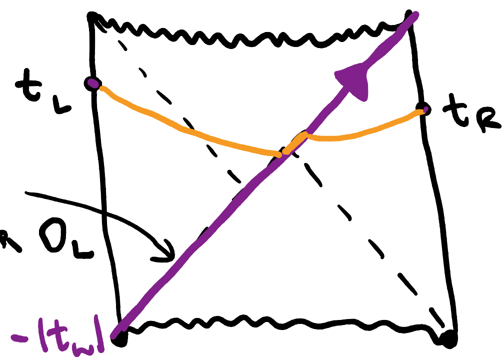
C.F. WITH



(IF CORRECT, THE PRESENCE OF THE SMOOTH MINIMUM INDICATES VOLUME $\neq$ THE TOTAL NUMBER OF TENSORS

ASPECT II: RESPONSE TO BULK SHOCKWAVES (THE SWITCHBACK EFFECT)

THE SHOCK GENERATED BY THE INSERTION OF A LOCAL OPERATOR O_L (OR A SMEARED ONE) IN THE PAST (PRECURSOR W)



$\longleftrightarrow |\psi\rangle$

$$\text{VOLUME} \sim 2|t_w| + t_L - t_R - 2t_*$$

LET'S ANALYZE IT FROM THE POINT OF VIEW OF THE BDRY TIME EVOLUTION:

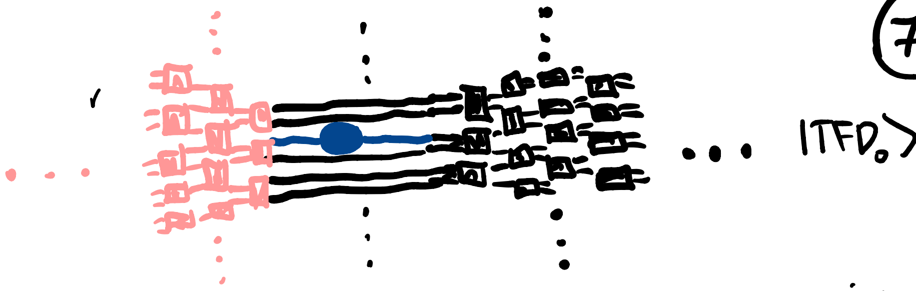
$$|\psi\rangle = e^{-iH_R t_R} e^{-iH_L t_L} e^{iH_L(-|t_w|)} O_L e^{-iH_L(-|t_w|)} |TFD_0\rangle =$$

COMMUTES

$$e^{-iH_L(|t_w|+t_L)} O_L e^{+iH_L|t_w|} e^{-iH_L t_R} \underbrace{\cancel{e^{-i(H_R-H_L)t_R}}}_{\text{SYMMETRY}} |TFD_0\rangle =$$

$$\underbrace{e^{-iH_L(|t_w|+t_L)}}_{U(|t_w|+t_L)} O_L \underbrace{e^{iH_L(|t_w|-t_R)}}_{\sim U^\dagger(|t_w|-t_R)} |TFD_0\rangle$$

LET'S LOOK NOW AT THE CIRCUIT MODEL

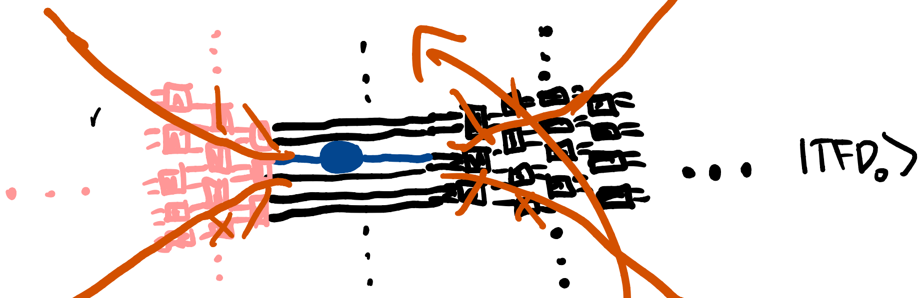


\# OF LAYERS IN U $\sim |t_w| + t_L$

O_L

\# OF LAYERS IN $U^\dagger$ $\sim |t_w| - t_R$

BECAUSE OF THE LOCAL NATURE OF TENSORS AND THE $U^\dagger O_L U$ STRUCTURE MANY CANCELLATION AMONG TENSORS



HERE AND THERE WE GET CANCELATIONS UNTIL THE PERTURBATION SPREADS THROUGH THE WHOLE SYSTEM (SCRAMBLES)

IF THIS TAKES t_s, THEN WE GET THE NUMBER OF TIME EVOLUTION GATES AS $\sim |t_w| + t_L - t_s + |t_w| - t_R - t_s$

$$\text{VOLUME} \sim 2|t_W| + t_L - t_R - 2t_*$$

WE GET AN AGREEMENT WITH $t_* = t_s$

THE PUNCHLINE: VOLUMES OF MAXIMAL VOLUME SLICES ARE CONSISTENT WITH AN INTERPRETATION OF A QUANTUM CIRCUIT LENGTH SUBJECT TO (SOME) OPTIMIZATION → COMPLEXITY = VOLUME (C_V) HOLOGRAPHIC COMPLEXITY PROPOSAL.

- MANY HOLOGRAPHIC COMPLEXITY PROPOSALS

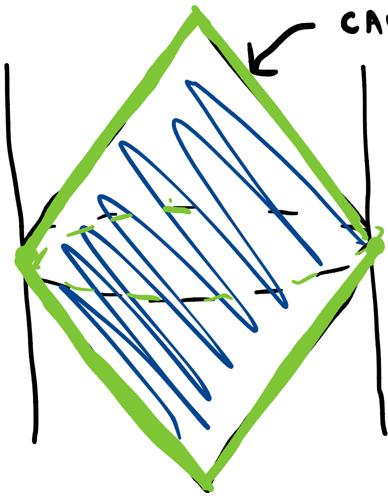
CHOSEN DEFINING FEATURES IN ANTI-DE SITTER BLACK HOLE SPACETIMES

- LATE TIME LINEAR GROWTH
- SWITCHBACK

INSTANCES:

- COMPEXITY = ACTION (CODIM.-0)

1509.07876 BY BROWN, ROBERTS, SUSSKIND, SWINGLE, ZHAO



CAUSAL DEVELOPEMENT (WHEELER-DEWITT PATCH)

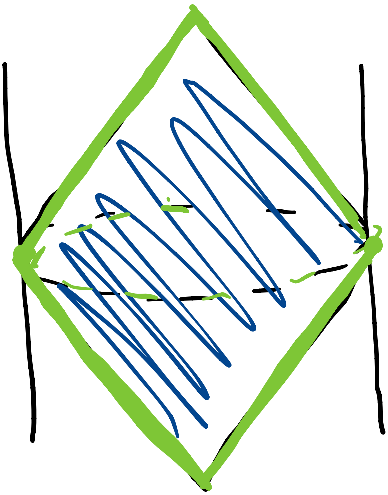
$$C_A = -\frac{1}{16\pi G_N}\Big[\int_{WdW} d^{d+1}x \left(R + \frac{d(d-1)}{L_{AdS}}\right) - 2\int d\lambda\, d\alpha^{d-1}\sqrt{\gamma}\,\Theta\log(\Theta L_{CT}) + \dots\Big]$$

ENSURES REPARAMETRIZATION INVARIANCE OF THE ACTION WITH RESPECT TO λ, α^i DIFFEOMORPHISM ON THE NULL BOUNDARY

KEY MOTIVATION: FIXING UNIQUELY THE OVERALL NORMALIZATION.

PRICE: ONE-PARAMETER FREEDOM GIVEN BY L_{CT}. IT DOES NOT AFFECT TIME DEPENDENCE, BUT APPEARS IN THE ULTRAVIOLATE DIVERGENCE.

- COMPLEXITY = VOLUME 2.0 (CODIM.-0)



$$C_{V2.0} \sim \int_{WdW} d^{d+1}x \sqrt{-g}$$

THE EASIEST TO DETERMINE: NO MINIMIZATION NEEDED, JUST TRACING LIGHTRAYS AND EVALUATING ENCLOSED SPACETIME VOLUME.

SO FAR, RATHER RIGID BULK QTIES:
ONE CAN CHANGE THEIR OVERALL NORMALIZATION AND FOR C_A WE HAD L_{CT}

- "COMPLEXITY = ANYTHING"
  2111.02429 BY BELIN, MYERS, RUAN, SÁROSI, SPERANZA
  2210.09647

KEY IDEA:

IMPOSE
- LATE TIME LINEAR GROWTH
- SWITCHBACK

IN HOLOGRAPHIC BLACK HOLE SPACETIMES TO SINGLE OUT COVARIANTLY DEFINED HOLOGRAPHIC COMPLEXITY CARRIERS:

STEP 1: EXTREMIZE A COVARIANT FUNCTIONAL DEFINING A BULK OBJECT ANCHORED AT A BOUNDARY TIME SLICE

SEEN EXAMPLE: MAX $\int d^d x \sqrt{g_{\text{IND}}}$ TO FIND A MAXIMAL VOLUME SLICE

STEP 2: EVALUATE IN GENERAL ANOTHER COVARIANT FUNCTIONAL ON THIS GEOMETRIC LOCI

SEEN EXAMPLE: $\int d^d x \sqrt{g_{\text{IND}}}$ ON THE MAXIMAL VOLUME SLICE

OUTCOME: FURTHER CONTINUUM OF HOLOGRAPHIC COMPLEXITY PROPOSALS

GR POINT OF VIEW: INTERESTING GEOMETRIC OBJECTS

COMMENT: C_V, C_A, $C_{V2.0}$ AND STUDIED C_{ANY} PROPOSALS ARE DIVERGENT BECAUSE OF THE NEAR BDRY CONTRIBUTIONS ASSOCIATED WITH THEIR BDRY TIME SLICE ANCHORED NATURE.

$$\Downarrow$$

TWO POSSIBLE AND NOT MUTUALLY EXCLUSIVE APPROCHES:

- USE THE DIVERGENCE AS A GUIDANCE FOR INTERPRETING HOLOGRAPHIC COMPLEXITY AKIN TO ENTANGLEMENT ENTROPY UV DIVERGENCES

- CONSTRUCT UV-FINITE COMBINATIONS LIKE

C(EXCITED GEOMETRY) − C(EMPTY AdS)

C(ETERNAL BH) − 2 C(EMPTY AdS)

C(FIXED BULK, BDRY TIME 1) − C(FIXED BULK, BDRY TIME 2)

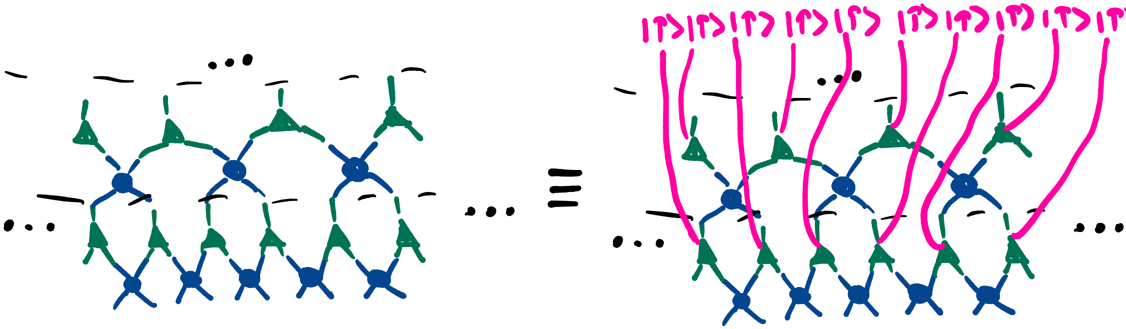
...

COMPLEXITY OF FORMATION

THE GOAL FOR THE REST OF THE LECTURES: DISCUSS IDEAS TOWARDS UNDERSTANDING HOLOGRAPHIC COMPLEXITY FROM QUANTUM FIELD THEORY AND TENSOR NETWORKS.

ATTEMPT 1: INSPIRED BY CONTINUOUS MERA (cMERA)

1707.08582 WITH CHAPMAN, MARROCHIO, PASTAWSKI



ONE CAN REINTERPRET MERA AS A UNITARY CIRCUIT ACTING ON A SPATIALLY DISENTANGLED STATE $\ldots|\uparrow\rangle|\uparrow\rangle|\uparrow\rangle|\uparrow\rangle|\uparrow\rangle|\uparrow\rangle|\uparrow\rangle|\uparrow\rangle|\uparrow\rangle|\uparrow\rangle\ldots$ ISOMETRIES OF MERA GET PROMOTED TO UNITARIES WHOSE ROLE IS TO BRING IN NEW QUBITS TO PASS TO A NEW SCALE WHERES DISENTANGLERS ARE VIEWED AS ENTANGLING OPERATIONS, THE STRUCTURE IS

$$|0\rangle_{\text{critical system}} \approx |\text{MERA}\rangle = \ldots \delta U_D\, \delta U_I\, \delta U_D\, \delta U_I\, |\text{ALL}\uparrow\rangle$$

$$|\text{TARGET}\rangle = \text{UNITARY CIRCUIT}\ |\text{REFERENCE}\rangle$$

THIS PERSPECTIVE HAS A CONTINUUM GENERALIZATION REALIZED FOR FREE QUANTUM FIELDS (AS WELL AS LEADING ORDER PERTURBATIVE BOSONIC QUANTUM FIELD THEORY)

HERE FREE BOSONS.

$|ALL \uparrow\rangle$: VACUUM $|R\rangle$ OF $H_R = \frac{1}{2}\int d^d x \left(\pi^2 + M^2\phi^2\right)$

TARGET STATE $|T\rangle$: VACUUM OF $H_{QFT} = \frac{1}{2}\int d^d x \left(\pi^2 + (\partial_x \phi)^2\right)$

BOTH STATES ARE GAUSSIAN AND ONE CAN (APPROXIMATELY IN CONTINUUM) TRANSFORM BETWEEN THEM BY APPLYING QUADRATIC OPERATORS IN ϕ AND π

THE INDEPENDENT CORRELATIONS ARE GIVEN BY

$$\langle R|\,\phi(\vec{k})\,\phi(\vec{k}')\,|R\rangle = \frac{1}{2M}\,\delta^{d-1}(\vec{k}+\vec{k}')$$

$$\langle T|\,\phi(\vec{k})\,\phi(\vec{k}')\,|T\rangle = \frac{1}{2|\vec{k}|}\,\delta^{d-1}(\vec{k}+\vec{k}')$$

DISPERSION RELATIONS

ONE UNITARY THAT APPROX. DOES THE JOB IS

TRIVIAL DECOMPOSITION INTO A CONTINUOUS CIRCUIT

$$|0\rangle \approx |T\rangle = \mathcal{P}e^{-i\int_0^1 ds \int_{|\vec{k}|\leq\Lambda} d^{d-1}k\,\left(\phi(\vec{k})\pi(-\vec{k}) + \pi(\vec{k})\phi(-\vec{k})\right)\frac{1}{4}\log\frac{M}{|\vec{k}|}}|R\rangle$$

CIRCUIT COST : $\sim \int_0^1 ds \int_{|\vec{k}|\leq\Lambda} d^{d-1} \left|\log\frac{M}{|\vec{k}|}\right| \sim \Lambda^{d-1}\left|\log\frac{M}{\Lambda}\right|$

FOR $M\sim\Lambda$ THIS IS THE LEADING UV DIVERGENCE OF C_V AND $C_{V2.0}$ AND FOR $M\ll\Lambda$ THIS IS THE C_A'S LEADING UV DIVERGENCE UPON IDENTIFYING $\frac{L_{CT}}{L_{AdS}} \sim \frac{\Lambda}{M}$

GENERALIZING FROM IT TO DEFINE COMPLEXITY

$$|T\rangle = U\,|R\rangle$$

CIRCUIT COMPLEXITY OF U:

DECOMPOSITION OF A UNITARY INTO A CIRCUIT

$$U = \mathcal{P}e^{-i\int_0^1 \sum_I K_I\, Y^I(s)\, ds}$$

HERMITEAN GENERATORS

HOW MUCH EACH GENERATOR IS USED AT EACH INFINITESIMAL LAYER

INTRODUCE A COST FOR EACH INFINITESIMAL LAYER WITH A TOTAL COST OF A GIVEN CIRCUIT BEING AN INTEGRAL OVER ALL ITS LAYERS

$$\mathrm{COST}_{L_1} = \int_0^1 \sum_I p_I |Y^I(s)|\, ds \quad , \; p_I > 0$$

$$\mathrm{COST}_{L_2} = \int_0^1 \sqrt{\sum_{IJ} p_{IJ} Y^I(s) Y^J(s)}\, ds \quad , \; p_{IJ} \text{ POSITIVE DEFINITE}$$

• • •

IMPORTANT: COSTS DO NOT DEPEND ON $\dot{Y}^I$ ETC!

$$\text{NIELSEN COMPLEXITY}[U] = \text{MIN}_{\substack{\text{OVER ALL CIRCUITS} \\ \text{GIVING } U}} \begin{pmatrix} \text{A CHOSEN} \\ \text{COST} \end{pmatrix}$$

QUANT-PH/0502070 BY NIELSEN

1707.08570 BY JEFFERSON AND MYERS

ONE CAN USE IT TO DEFINE COMPLEXITY OF STATES

L_2 NORM A

$$\text{STATE COMPLEXITY}[|T\rangle] = \text{MIN}_{U:\, |T\rangle = U|R\rangle} \; \text{NIELSEN COMPLEXITY}[U]$$

COMMENT: FOR K_I FORMING A LIE ALGEBRA, L_2 NORM ASSIGNS A RIEMANNIAN GEOMETRY TO THE LIE GROUP AND FINDING OPTIMAL CIRCUITS REDUCES TO FINDING GEODESICS.

ONE CAN DEFINE COMPLEXITY OF STATES ON ITS OWN USING THE HILBERT SPACE DISTANCE (THE FUBINI-STUDY DISTANCE):

$$|\Psi(s')\rangle = e^{-i\int_0^{s'} Q(s)\,ds}\,|R\rangle, \quad Q(s) = \sum_I K_I Y^I(s), \quad |\Psi(1)\rangle = |T\rangle$$

$$\text{FUBINI-STUDY COMPLEXITY} =$$

$$= \text{MIN} \int_0^1 ds \sqrt{\langle\Psi(s)|Q(s)^2|\Psi(s)\rangle - \langle\Psi(s)|Q(s)|\Psi(s)\rangle^2}$$

1707.08582 WITH CHAPMAN, MARROCHIO, PASTAWSKI

IN THE CONTEXT OF QFT, THE ADVANTAGE OF BOTH APPROACHES IS THAT ONE DOES NOT INTRODUCE DISCRETE GATES, BUT CAN OPERATE WITH GENERATORS THAT CAN BE (PRODUCTS OF) LOCAL OPERATORS, SO SOMETHING NATURAL ON BOTH SIDES OF AdS/CFT

COMMENT: AT THIS LEVEL s IS AN AUXILIARY PARAMETER

ONE CAN SHOW THAT

$$|0\rangle \approx |T\rangle = P e^{-i\int_0^1 ds \int_{|\vec{k}|\leq\Lambda} d^{d-1}k \, (\phi(\vec{k})\pi(-\vec{k}) + \pi(\vec{k})\phi(-\vec{k}))\frac{1}{4}\log\frac{M}{\delta|\vec{k}|}} |R\rangle$$

IS AN OPTIMAL GAUSSIAN CIRCUIT FOR A PARTICULAR COST. THIS CORROBORATES THE MATCH OF DIVERGENCES WITH $C_{V,A,V2.0}$ REPORTED EARLIER

1707.08582 WITH CHAPMAN, MARROCHIO, PASTAWSKI

1707.08570 BY JEFFERSON AND MYERS

IN FREE QFTs $|TFD(t_L, t_R\rangle$ STATES ARE GAUSSIAN. ONE CAN AGAIN REPEAT ANALYSIS USING PARTICULAR COST WITH THE RESULT FOR $d \geq 3$

$$\text{COMPLEXITY}\left[|TFD(t_L=0, t_R=0)\rangle\right] - 2\,\text{COMPLEXITY}\left[|0\rangle\right]$$

$$\sim C_{V|A|V2.0}\left[\text{BLACK HOLE}(t_L=0, t_R=0)\right] - C_{V|A|V2.0}\left[\begin{matrix}\text{EMPTY}\\ \text{AdS}\end{matrix}\right]$$

$\sim$ THERMAL ENTROPY

HOWEVER, TIME DEPENDENCE IN FREE QFTs DOES NOT EXHIBIT PERSISTENT LINEAR GROWTH. THIS SHOULD COME AS NO SURPRISE SINCE TIME DEPENDENCE AT WEAK AND STRONG COUPING IN GENERAL IS VERY DIFFERENT:

$$\frac{R}{S}\Big|_{\lambda\to 0} \sim \frac{1}{\lambda^2\log(1/\lambda)} \quad \text{vs.} \quad \frac{R}{S}\Big|_{\lambda\to\infty} = \frac{1}{4\pi}$$

1810.05151 WITH CHAPMAN, EISERT, HACKL, JEFFERSON, MARROCHIO, MYERS

ATTEMPT 2 (IN FACT 1) 1703.00456 BY CAPUTA, KUNDU, MIYAJI, TAKAYANAGI, WATANABE

IN CFT_2:

ω VANISHES HERE

MATRIX ELEMS. OF ρ_β ~ EUCLIDEAN PATH INTEGRAL ON $d\tau^2+dx^2$ ~ EUCLIDEAN PATH INTEGRAL ON $e^{2\omega}(d\tau^2+dx^2)$ WITH $\omega(\tau,x)\neq 0$

β β ω VANISHES HERE

IDEA: $P e^{-\int_0^\beta d\tau \int_{-\infty}^{\infty} T_{\tau\tau}\, dx}$

NON-UNITARY CIRCUIT

OTHER CIRCUIT DEPENDENT ON ω

POSTULATED CIRCUIT COST:

$$\text{COST} \sim \int d\tau\, dx \left[\frac{1}{L^2} e^{2\omega} + (\partial_\tau \omega)^2 + (\partial_x \omega)^2 \right]$$

THIS COST IS ~ log OF THE CHANGE OF NORMALIZATION OF g_{ab} UNDER $(d\tau^2 + dx^2) \to e^{2\omega}(d\tau^2 + dx^2)$

THE LIOUVILLE ACTION CAN BE VIEWED AS A COVARIANT FUNCTIONAL OF THE SOURCE (THE METRIC IN WHICH A GIVEN CFT LIVES):

$$\text{COST} \sim \int d^2x \sqrt{g} \left(\frac{1}{L^2} + \frac{1}{4} \partial_\mu \chi \partial^\mu \chi \right) \quad \text{WITH} \quad \chi = \int d^2y \sqrt{g(y)}\, \Box^{-1}(x,y) R(y)$$

METRIC + (EUCLIDEAN) TIME FOLIATION $\to$ CIRCUIT

ONE CAN SHOW THAT FOR $\frac{1}{L^2} \gg (\partial_\tau \omega)^2, (\partial_x \omega)^2$ THE LIOUVILLE ACTION IS A NIELSEN COST FUNCTION FOR $\{K^I\} \equiv \{T^{\mu\nu}\}$

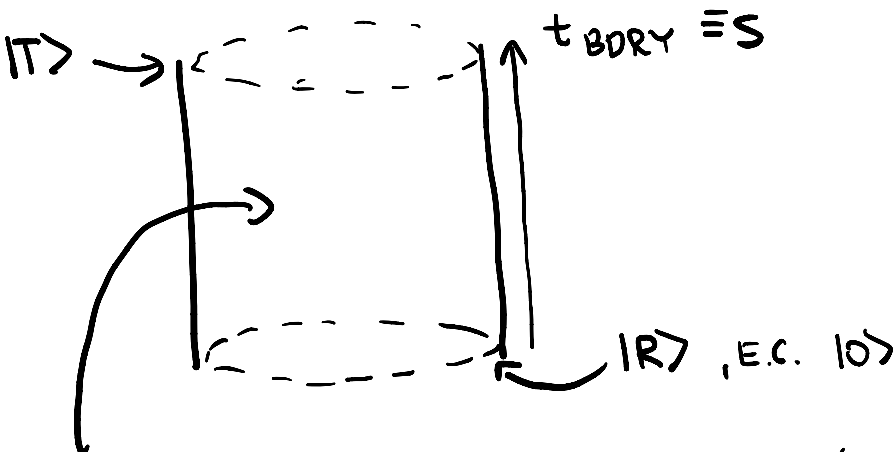
1904.02713 WITH CAMARGO, JEFFERSON, KNAUTE

KEY TAKE HOMES:

- CIRCUITS ARE DEFINED IN TERMS OF A QFT SOURCE (AND TIME FOLIATION)
- CIRCUIT PARAMETER IS THE EUCLIDEAN TIME
- OPTIMIZATION LEADS TO $\partial_\tau \omega, \partial_x \omega = O\left(\frac{1}{L}\right)$, WHICH IS NOT A SELF CONSISTENT QFT RESULT

ATTEMPT 3 : USE THE PHYSICAL TIME AS A CIRCUIT PARAMETER s.

THIS SIMPLE PERSPECTIVE ALLOWS TO EMBED THE CIRCUIT COMPLEXITY PROBLEM IN HOLOGRAPHY AS A QUENCH PROBLEM:
2112.12158 WITH ERDMENGER, FLORY, GERBERSHAGEN, WEIGEL



NONTRIVIAL EVOLUTION USING HAMILONIAN WHICH IS TIME DEPENDENT DUE TO NONTRIVIAL SOURCES.
ONE CIRCUIT ACTING ON $|R\rangle$ = ONE GEOMETRY

THE DIFFICULTY LIES OF COURSE IN ENSURING ONE GETS A DESIRED $|T\rangle$ FROM $|R\rangle$.

THE KEY INSIGHT OF 2212.00043 WITH ERDMENGER, GERBERSHAGEN, WEIGEL: IN A GRAVITY DUAL TO A CIRCUIT, THE FUBINI-STUDY COST IS ALWAYS HOLOGRAPHIC.

LOCAL OPERATORS

$$H(t) = \sum_I O_I J^I$$

THEIR SOURCES

SCHEMATIC

$$\rightarrow \; d\,\mathrm{cost}^2_{FS} = \langle\Psi(s)|H(s)^2|\Psi(s)\rangle - \langle\Psi(s)|H(s)|\Psi(s)\rangle^2$$

THIS EXPRESSION IS HOLOGRAPHIC SINCE IT REDUCES TO A SUM OF 1- AND 2-POINT FUNCTIONS OF LOCAL OPERATORS, WHICH ALWAYS HAVE A HOLOGRAPHIC REALIZATION.

(ALBEIT IN GENERAL REQUIRE BACKREACTION SIMILAR TO RENYI'S)

COMMENT: 2103.06920 BY CHAGNET, CHAPMAN, DE BOER, ZUKOWSKI CONSTRUCTS A BULK DUAL TO FUBINI-STUDY COMPLEXITY WITHIN SO(2,d) ACTING ON $|E\rangle$ IN CFT_d

ATTEMPT 4

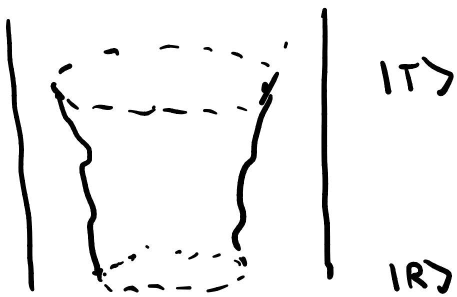
2101.01185 WITH CHANDRA, DE BOER, FLORY,
2203.08842 HÖRTNER, ROLPH

(SIMILAR IDEAS ALSO IN 2012.05247
BY CAPUTA, KRUTHOFF, PARRIKAR, 2011.08188
BY BORUCH, CAPUTA, TAKAYANAGI, 2104.00010 + GE)

A POSSIBILITY: VIEWING GEOMETRIC HOLOGRAPHIC CARRIERS AS ARISING FROM WITHIN A PARTICULAR SPACETIME DUE TO OPTIMIZATION OF A GEOMETRIC COST FUNCTIONAL (+ POSSIBLY ADDITIONAL CONDITIONS)

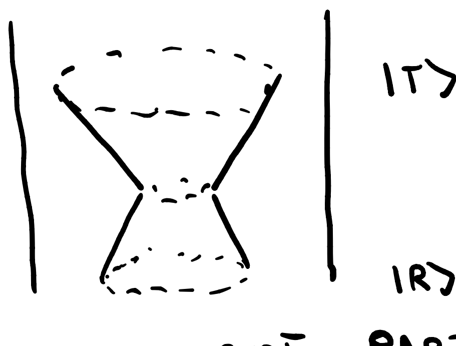
VERY SIMILAR TO C_{ANY} WITH THE EXCEPTION THAT THE TWO INVOLVED FUNCTIONALS ARE THE SAME.

EXAMPLE (LORENTZIAN)



BULK COST $= C_{BULK} \int d^{d+1}x \sqrt{-g} + C_{BDRY} \int d^{d}x \sqrt{-g_{IND}}$

MINIMIZATION WHILE REQUIRING THE BDRY IS TIMELIKE GIVES US A PART OF WdW AND $C_{V2.0}$



+ : WE CAN GET PARTS OF HOLOGRAPHIC PROPOSALS

− : WE DO NOT NECESSARILY KNOW WHAT THE COST COUNTS

A FEATURE: CIRCUITS NONLOCAL IN DUAL HOLOGRAPHIC CFT. PERHAPS BEST ADDRESSED WITHIN $T\bar{T}$ HOLOGRAPHY.

ATTEMPT 5: LESSONS FROM LOWER-DIMENSIONAL EXAMPLES

QUANTUM JT GRAVITY: SATURATION OF A GENERALIZATION OF THE VOLUME
2107.06287 BY ILIESIU, MEZEI, SÁROSI

JT GRAVITY DISK LEVEL: EXACT MATCH OF THE KRYLOV COMPLEXITY WITH C_V
2305.04355 BY RABINOVICI, SÁNCHEZ-GARRIDO, SHIR, SONNER

OUTLOOK

- IN THE COURSE OF THE PAST DECADE WE GOT MUCH CLOSER TO UNDERSTANDING HOLOGRAPHIC COMPLEXITY
- WE HAVE FIRST MATCHES BETWEEN THE BOUNDARY AND THE **BULK**
- WE NEED TO KEEP PUSHING TO DEVELOP A COMPREHENSIVE PICTURE

SOME OPEN PROBLEMS (RANDOM ORDER):

- OTHER THAN 2103.06920 BULK MANIFESTATIONS OF THE FUBINI-STUDY COMPLEXITY?

- GRAVITY DUAL TO MERA? NOVEL, HOLOGRAPHY INSPIRED TENSOR NETWORKS FOR QFTs?

- BOUNDARY INTERPRETATION OF BULK COSTS LEADING TO C_{ANY} PROPOSALS?

- IS REGULARIZED VOLUME IN AdS_{3+} ALSO DUAL TO KRYLOV COMPLEXITY (GOOD STARTING POINT: INTERPRETING C_V FOR LOCAL CONFORMAL TRASFORMATIONS AS IN 1806.08376 BY FLORY, MIEKLEY AND IN 2112.12158. THE LATTER REF. FOUND DISCREPANCY BETWEEN ΔC_V AND C_{FS} AT FOURTH ORDER IN δf: $\sigma \to f(\sigma) = \sigma + \delta f(\sigma)$)?

- GENERALIZATIONS OF KRYLOV COMPLEXITY AND HOLOGRAPHY ($\sum_n n|n\rangle\langle n| \to \sum_n p_n |n\rangle\langle n|$)?

- CIRCUIT COMPLEXITY IN DSSYK AND C_{ANY} IN JT GRAVITY?

- DOES COUNTING EUCLIDEAN (HERMITEAN) AND LORENTZIAN (UNITARY) GATES IN THE SAME WAY MAKES THE MOST SENSE?

- HOLOGRAPHIC COMPLEXITY BEYOND AdS?

- KRYLOV COMPLEXITY AND SWITCHBACK?